A NEW TONE GENERATOR.

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SYNOPSIS.

New Pure Tone Generator and Receiver of Sounds.—(1) *Construction and operation.* The instrument consists of a thin, non-magnetic, metallic diaphragm between two flat coils through which a constant direct current I_0 flows in such a way as to produce a radial magnetic field in the diaphragm; then when a simple harmonic alternating current I of the frequency $\omega/2\pi$ is superposed upon the direct current, circular currents are induced in the diaphragm, which thereupon is acted upon by a simple harmonic electrodynamic force and vibrates with the frequency of the alternating current. For low frequencies the electrodynamic force is approximately proportional to $\omega I_0 I \sin(\omega t + \theta)$ and the amplitude of vibration is approximately proportional to $I_0 I/\omega$. The absence of overtones is due to the absence of ferromagnetic material, and to the fact that the radial magnetic field is constant. The aperiodicity of the diaphragm renders the calculation of the performance of the instrument practicable, and eliminates distorsion, due to resonance, in the wave form of the emitted sound when the instrument is excited by a complex alternating current. When used as a generator of pure tones, the coils were connected in the circuit of a thermionic oscillator whose frequency could be varied from 500 to 25,000 vibrations per second. When used as a receiver of sound, the current generated in the coils by the motion of the diaphragm is fed into a thermionic amplifier. (2) *Quantitative study of the performance.* The distribution of the magnetic field between the coils was determined experimentally; the diaphragm current equations were deduced and solved for a particular case; the forces on various parts of the diaphragm were calculated, and thence the amplitude of vibration and the sound energy output. With an aluminum diaphragm 0.0025 cm. thick and 10 cm. in diameter, a direct current of 1 ampere, an alternating current of 0.085 ampere, and a frequency of $10^5/2\pi$, these were respectively 7×10^{-7} cm., and 9 ergs per second. By increasing both direct and alternating currents five-fold, the output could be increased over six hundred-fold. Measurements of the amplitude for various frqeuencies agreed well with the calculated values. (3) *Applications of the instrument.* Since it gives a pure tone of constant and measurable pitch and intensity over a wide range, it would serve as a *precision source of sound,* useful both for research and lecture purposes. When used as a *telephone receiver and transmitter,* actual tests have shown that the reproduction of sound is remarkably faithful.

INTRODUCTION.

In a previous publication[1] the author gave a brief description of a new instrument for generating simple harmonic sound waves. The following is a quantitative description and study of the instrument and its performance, together with a discussion of some of its applications.

[1] PHYS. REV., 17, p. 257, 1921.

I. Specifications for the Construction of the Tone Generator, and Description of a Method of Operation.

The instrument described below was used successfully at frequencies from 500 to 25,000 vib./sec. For lower frequencies the instrument should be wound with a larger number of turns, while for higher frequencies a smaller number of turns of coarser wire should be used.

Figure 1 is a section and elevation. The frame is made of hard fiber,

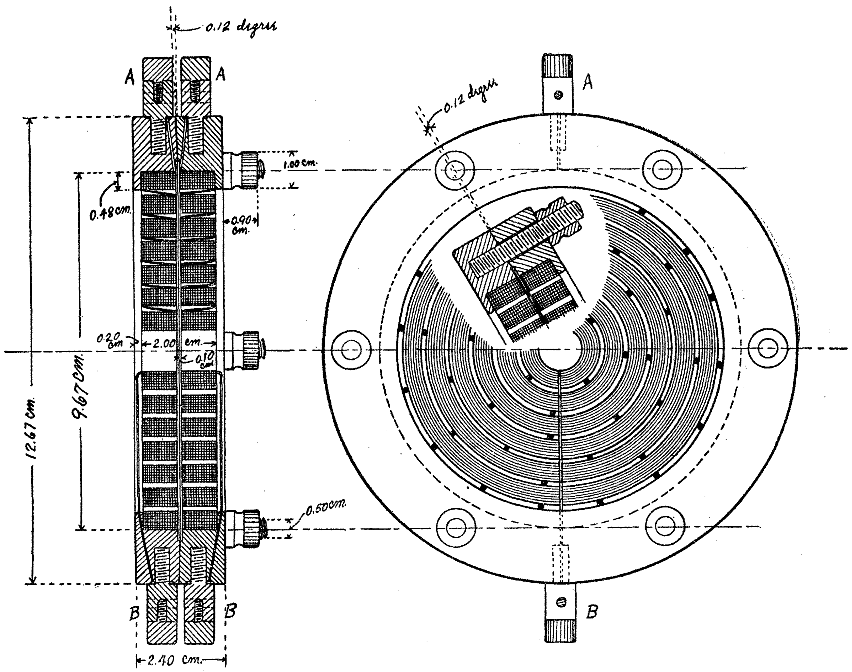

Fig. 1.

and the clamps and binding posts of brass. Each section of the pancake coils contains 98 turns of No. 22 wire, wound in 7 layers. The inside diameter of the innermost section is 1.11 cm., while that of the others is obtained by adding 1.27 cm. successively. These sections were wound on forms, shellaced, baked, and then bound at intervals with silk threads (not shown in figure). The sections mounted concentrically, are held in position by hard wood plugs. The sections of each coil are connected so that the direction of winding is continuous. The frame was turned after completing the coils, and the latter were glued in place with shellac. The taper on the clamping surfaces stretches the diaphragm enough to prevent appreciable sagging.

Any kind of metallic diaphragm will work; the author uses sheet aluminum 0.0025 cm. in thickness. Of the non-magnetic metals, aluminum gives the best results. An iron diaphragm gives more intense sounds at low frequencies for a given electrical input, although it will not emit a pure tone even when the instrument is excited by a simple harmonic alternating current. The diaphragm should be thin, as increasing the thickness decreases the intensity of sound emitted for a given electrical input, particularly for high frequencies. The reasons for the above may be drawn from the quantitative discussion of the performance of the instrument.

Figure 2 shows a method of using the instrument. The inductance

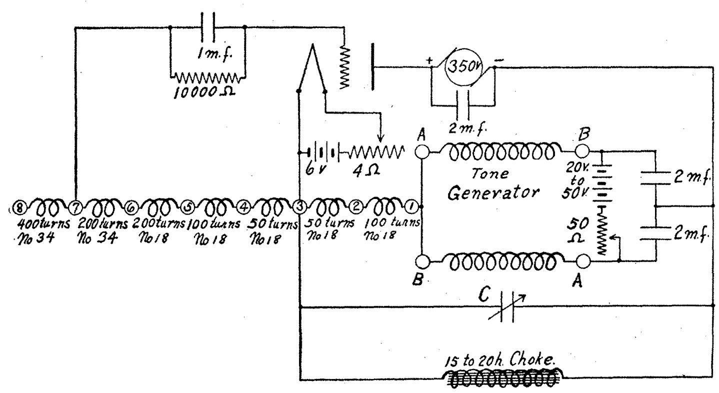

Fig. 2.

with the binding posts 1 to 8, is wound on a wooden core 20 cm. in diameter, and 6.5 cm. long. Coil 1 to 2 is innermost, and the others follow in numerical order. The wire is double cotton covered, and the layers are separated by friction tape. The best conditions for oscillation at any frequency can be obtained by adjustment of grid and filament connections to this inductance. In the actual circuit, *C*, and the choke coil are connected directly to the filament, and a single wire leads from this point to the inductance. The choke coil is home made. *C* consists of two 1 m.f. divided mica condensers, and five oil-filled rotary condensers whose combined maximum capacity is a little greater than the smallest step obtainable with the mica condensers. The other condensers are telephone condensers. The generator may be replaced by a battery, in which case no by-pass condenser is needed. The voltage shown is suitable for the Western Electric 205 B tube. The intensity of the sound from the instrument is controlled by the rheostat in series with the battery furnishing the polarizing current. The frequency of oscillation was measured with an improvised wave meter using a vacuum thermocouple for indicating resonance. If desired the instrument may be operated from a separate generator.

II. Quantitative Investigation of the Performance of the Instrument.

1. *Determination of the Distribution of the Magnetic Field in the Space occupied by the Diaphragm between the Pancake Coils.*—To simplify matters, the field of only one of the coils was investigated. Obviously the field is symmetrical about the coil axis. A flat spiral, of pitch 0.084 cm., consisting of 54 turns of No. 36 wire was wound in a shallow groove on the face of a fiber disc 10 cm. in diameter.

The spiral was connected to a ballistic galvanometer and a known current was established in the coil. The galvanometer deflection was determined for reversal of this current, (*a*) with the spiral and coil coaxially in contact, (*b*) when 1 mm. apart. The outside turns of the spiral were removed two at a time, making the above determinations after each removal, until only two turns remained. These determinations together with a calibration of the galvanometer yielded the necessary data for calculating the normal and radial components of the magnetic induction near the coil face. To simplify the calculation each spiral turn was assumed equivalent to a circular turn whose diameter is the mean diameter of the spiral turn.

Curve I., Fig. 3, shows the normal component of the magnetic induc-

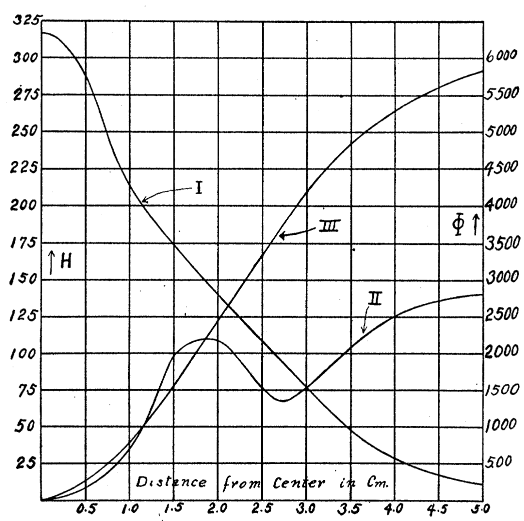

Fig. 3.

tion in C.G.S. units, as a function of the distance from the center, in the space between the two coils, when the latter are separated by a distance of 1 mm., and each carries a current of 1 ampere, their polarity being so adjusted that the unlike faces of the coils are adjacent. In this case there is no radial component of the magnetic induction in the space between the two coils. Curve III. shows the total flux through a coaxial circle between the two coils as a function of the radius of the circle, for the same conditions of relative positions of the coils, current, and polarity as specified above. Curve II. shows the distribution of the radial component of the magnetic induction when the coils are separated by a distance of 1 mm., and each carries a current of 1 ampere, their polarity being so adjusted that the like faces of the coils are adjacent. In this case there is no normal component in the space between the two coils.

2. *Calculation of the Current Induced in, and Electromagnetic Forces Acting on the Diaphragm.*—The current in the diaphragm will flow in circles coaxial with the coils. The path of each current element will have a different resistance and self inductance, a different mutual inductance with the other paths, and will be acted upon by a different induced e.m.f. The current density and phase will therefore vary from one path to the next. The diaphragm was regarded as 8 annular rings each having a width of 0.645 cm., their mean diameters ranging from 0.97 cm. to 10 cm. Each annulus was considered equivalent to a circular wire of circular cross section of the same mass, material and diameter.

Suppose the current in the coils is given by $I_0 \sin \omega t$. The maximum values of the magnetic flux through each annulus was determined from curve III., and from these were calculated the induced e.m.f.'s in the annuli owing to the current in the coils. The total rise of potential around each annulus was then equated to 0, and the resulting eight equations put in the following form:

$$\left.\begin{array}{l}
\left(\dfrac{r_1}{\omega^2} + a_{11}\right) i_1 + a_{12} i_2 + a_{13} i_3 + \cdots + a_{18} i_8 = b_1 \sin \omega t + \dfrac{\Phi_1}{\omega} \cos \omega t \\
a_{21} i_1 + \left(\dfrac{r_2}{\omega^2} + a_{22}\right) i_2 + a_{23} i_3 + \cdots + a_{28} i_8 = b_2 \sin \omega t + \dfrac{\Phi_2}{\omega} \cos \omega t \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{81} i_1 + a_{82} i_2 + a_{83} i_3 + \cdots + \left(\dfrac{r_8}{\omega^2} + a_{88}\right) i_8 = b_8 \sin \omega t + \dfrac{\Phi_8}{\omega} \cos \omega t
\end{array}\right\},$$

where

$$a_{nm} = \frac{M_{n1} M_{1m}}{r_1} + \frac{M_{n2} M_{2m}}{r_2} + \frac{M_{n3} M_{3m}}{r_3} + \cdots + \frac{M_{n8} M_{8m}}{r_8}$$

and

$$b_n = \frac{M_{n1} \Phi_1}{r_1} + \frac{M_{n2} \Phi_2}{r_2} + \frac{M_{n3} \Phi_3}{r_3} + \cdots + \frac{M_{n8} \Phi_8}{r_8}$$

i_n, r_n, and Φ_n are respectively the current in, the resistance of, and the maximum magnetic flux through, the nth annulus. The M's are the coefficients of self and mutual induction of the annuli.

The values of the coefficients were evaluated and the equations solved, giving the currents in the annuli, for a sine wave current whose maximum value in each coil was 0.085 amp., at a frequency given by $\omega = 10^5$. The current in an annulus may be expressed as follows $i_n = I_n \sin(\omega t + \theta_n)$. In the 4th and 5th columns of Table I. are given the values of I_n and θ_n for the successive annuli, beginning at the center. The second, third,

TABLE I.

I.	II.	III.	IV.	V.	VI.	VII.
n.	d_n Cm.	$r_n \times 10^3$ Ohms.	I_n Amperes.	θ_n Degrees.	F_n Dynes.	$A_n \times 10^7$ Cm.
1.....	0.97	5.67	1.71	56° 58′	4	0.30
2.....	2.26	12.1	2.33	43° 16′	73	2.36
3.....	3.55	20.7	2.81	41° 58′	345	7.10
4.....	4.84	28.4	3.11	40° 26′	388	5.85
5.....	6.13	35.8	3.32	38° 38′	511	6.08
6.....	7.42	43.3	2.89	43° 19′	768	7.55
7.....	8.71	50.8	2.77	43° 41′	1022	8.56
8.....	10.00	58.3	2.87	42° 52′	1263	9.22

and sixth columns of this table give respectively the diameters and resistances of the annuli, and the maximum values of the electromagnetic force acting on them. The latter are the products of the maximum current in each annulus, its circumference, and the corresponding radial component of the magnetic field for 1 amp. of direct current in each coil. The current in each annulus and the force acting on it are in phase, but the phases of the total instantaneous force and current differ slightly. These latter are given respectively by $f_t = 4423 \sin(\omega t + 41° 30')$, and $i_t = 22 \sin(\omega t + 42° 25')$.

3. *The Amplitude of Vibration of the Diaphragm, and the Amount of Sound Energy it Emits.*—From the above, the forces on the various parts of the diaphragm are nearly in phase; particularly so for those places where the force is large. Consequently, the diaphragm may be considered to vibrate as a whole. It is practically aperiodic for any frequency concerned, so that its elasticity may be neglected. The equation of motion is then

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} = F \sin \omega t,$$

where m is the mass of the diaphragm, a is the dissipative factor, and F is the maximum periodic force. The solution of the equation gives for the amplitude of vibration,

$$A = \frac{F}{\omega\sqrt{a^2 + (m\omega^2)}}\,.$$

The dissipative factor a is made up of two parts; one due to internal friction in the diaphragm, and the other to the emission of sound energy. The former is readily seen to be negligible compared to $m\omega$, while the latter is equal to the product of the velocity of sound, the density of air, and the area of the diaphragm. For $\omega = 10^5$, a^2 is only 0.0025 of $(m\omega)^2$, and consequently it may be neglected. The amplitude then becomes $A = F/m\omega^2$. Substituting $F = 4423$ dynes, $m = 0.60$ gm. and $\omega = 10^5$, the amplitude comes out 7.37×10^{-7} cm. The sound energy emitted from both sides of the diaphragm per second is 9.1 ergs. By increasing both the direct current and the alternating current in the coils five-fold, which is feasible, the amplitude of vibration would be 25, and the energy output 625 times as great; *i.e.*, an output of 5.7×10^3 ergs/sec.

A better idea of the actual performance of the diaphragm is obtained by calculating the amplitudes of the separate annuli. Making the same assumptions as before these amplitudes were found and are given in the seventh column of Table I. An appendix to this paper gives some measurements of the amplitude at various places on the diaphragm.

From the current equations and the preceding discussion it is seen why a thin diaphragm should be used. For high frequencies r_n/ω^2 is small compared to a_{nn}, so that the currents in the diaphragm, and the forces acting on it are independent of its resistance. Increasing the thickness, therefore, does not appreciably increase the force, so that the increased mass results in a proportionally decreased amplitude of vibration. At low frequencies where r_n/ω^2 is of more importance, a thin diaphragm is still preferable, although the advantage is not so marked.

An iron diaphragm will not execute a simple harmonic motion with a sine wave current in the coils, for the magnetic flux through such a diaphragm will not be proportional to this current.

III. Some Applications of the Instrument.

1. *Precision Source of Sound.*—A practically pure tone can be obtained whose frequency and intensity can be maintained practically constant for an indefinite period, can be varied continuously over a very wide range, and can be determined with considerable accuracy. The practical absence of eddy current and magnetic hysteresis energy losses makes it possible to operate the instrument at very high frequencies. The natural period of the diaphragm is so low that it introduces no irregularities in the response in the region of operation.

Mr. C. E. Lane has recently used this instrument to determine the minimum flux of sound energy for audition at several frequencies, and it is intended that this work shall be carried further in the near future. In this connection it may be mentioned that the instrument may be constructed with one coil, so that one side of the diaphragm is free. This has the advantage that the waves are unbroken in leaving the instrument. The resulting tone is somewhat impure, because the radial component of the magnetic field, having an alternating component, is no longer constant. The impurities consist of the octave and higher partials of the fundamental, and can be eliminated by a sound filter. With a direct current large compared to the alternating current, these impurities are negligible. Above 10,000 vib./sec. the impurities are inaudible unless the fundamental is very intense.

The instrument is suitable for the demonstration of diffraction, interference, and reflection effects. At high frequencies, the sound goes out in a well-defined beam, which makes possible some very striking qualitative demonstrations.

For reproducing speech the instrument promises interesting development. It has been used successfully as a generator and as a receiver of voice currents. The coils were specially designed to suit each of these uses. As a generator, the vibration of the diaphragm induces alternating currents in the coils, and these currents are fed into a thermionic amplifier. The voice spoken into one instrument is reproduced with remarkable faithfulness in another connected in the plate circuit of the amplifier. This is due partly to the absence of eddy-current and magnetic hysteresis energy losses, and partly to the aperiodicity of the diaphragm. As a loud-speaking generator of sound, the instrument has been used to address an audience of more than one hundred persons. The distinctness of the words, and the quality of the voice of the instrument approached very closely those of the speaker who operated the generator of voice currents.

For voice frequencies the current equations are much simplified. Only the terms containing r_n/ω^2 and the cosine terms need be retained, so that the diaphragm currents are approximately proportional to the frequency, and are nearly in phase. Consequently, the force acting on the diaphragm, for a given alternating current in the coils, is proportional to the frequency, and the amplitude of vibration is inversely proportional to the frequency. This would cause distortion if the voice currents were of the same wave shape as the sound waves which gave rise to them. But if the same kind of instrument is used as a generator, the wave form of its electromotive force is such that the distortion arising in the receiver results in the production of sound waves of the same character as the original ones. As a matter of fact, actual trials have shown that the distortion, as perceived by the ear, even when using a carbon granule transmitter, is very small.

IV. Appendix. Measurements of the Amplitude of Vibration of the Diaphragm.

Mr. C. E. Lane, while a graduate student in this laboratory, conducted the experimental work described below. Using an electric micrometer for amplitude measurements, the amplitude of vibration at a definite frequency, of an ordinary telephone receiver, was first shown to be proportional to the current through the receiver, and then the ratio of amplitude to current was determined for several frequencies. The tone generator and telephone receiver were excited by alternating current of the same frequency, and a brass tube, of internal diameter 0.4 cm., leading to a tuned resonator carrying a Rayleigh disc, was placed close to the diaphragms, alternately of the tone generator and the telephone receiver. The current in the latter was adjusted till the deflection of the disc was the same for the two. The amplitude for the telephone receiver was calculated from the amplitude-current ratio, and it was assumed that this also gave the amplitude for the tone generator. In this manner the amplitude was determined at various places on the diaphragm, and its distribution was shown to agree closely with that given in Table I. The root mean square of the amplitude was calculated for the whole diaphragm and compared with the amplitude calculated as outlined in section II. Table II. shows the results of these measurements reduced to the standard condition of 1 ampere of direct current and 0.1 ampere of alternating current in the coils. The measured and calculated amplitude agree within the limits of experimental error.

Table II.

Frequency.	Measured Amplitude.	Calculated Amplitude.
1440....................	4.64×10^{-5} cm.	4.96×10^{-5} cm.
1600....................	4.16 "	4.52 "
2200....................	4.04 "	3.24 "
3700....................	1.72 "	1.88 "

The writer takes pleasure in acknowledging the assistance rendered by Mr. C. E. Lane and other students in this laboratory, in parts of this work. Credit is also due Mr. J. B. Dempster for suggestions in design and help in the construction of apparatus.

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