MAGNETOSTRICTION WITH SMALL MAGNETIZING FIELDS.

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SYNOPSIS.

Piezo-electric Method of Measuring Extremely Small Magnetostriction Effects.—In this method, suggested by Pupin, the wire to be studied is surrounded by a solenoid through which an alternating current of known strength and frequency is sent, and the vibrations set up in the wire are transmitted to a piezo-electric crystal and the resulting electric charge is amplified and measured. (1) For elongations down to 2×10^{-9} the wire was hung directly from a *quartz crystal.* To measure the charge produced by a given magnetizing current, an electromotive force of the same frequency and adjusted by means of a Pupin wave balance to the proper phase, was fed to the amplifier through a vacuum tube in parallel with the one connected to the crystal, and the potential was varied until the note produced by the crystal was balanced out. The force corresponding to a given charge was determined with the aid of a condenser, one plate of which was suspended from the crystal. (2) By using the torsion of a *Rochelle salt* crystal, the sensitiveness was extended to 2×10^{-11}, and (3) by tuning both the crystal and the wire to resonance with the frequency used, a sensitiveness of 3×10^{-12} was reached. The last two arrangements were calibrated in absolute units by comparison of the results with those obtained with quartz. The smallest change previously measured was about 10^{-8}.

Magnetostriction with Small Magnetizing Fields.—An *iron* wire containing 0.1 per cent. C, and a wire of *electrolytic nickel* were studied. In each case the curve for magnetostriction as a function of the field is nearly linear, but slightly concave downward, and apparently passes through the origin. For fields of 1, 0.05 and 0.002 gauss, the ratio of magnetostriction to field strength is respectively 3, 2.4 and 4×10^{-9} for iron, and 2, 1 and 3×10^{-9} for nickel. The probable error is from 7 to 10 per cent. *The effect of increasing the tension* is to decrease the elongation of both metals. A rod of *bismuth* 8 cm. long was tested in a field of 12 gauss using the resonance method, but gave a negative result, indicating that the effect is at least 10,000 times less than for iron.

Pupin Wave Balance for varying the Phase of an Alternating Electromotive Force is described.

HISTORICAL.

OBSERVATIONS on magnetostriction, that is, on the change in the length of a piece of iron when subjected to a magnetic field, were made by Joule[1] in 1847. In 1873 Mayer[2] made a series of careful measurements of the effect and obtained coefficients of elongation and contraction which ranged from 4.86×10^{-7} to 4.78×10^{-6}.

A very thorough study of magnetostriction was made by Bidwell[3]

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[1] J. P. Joule, "On the Effects of Magnetostriction upon the Dimensions of Iron and Steel Bars," Phil. Mag., XXX., 1847, pp. 76, 225.

[2] A. M. Mayer, "On the Magnetic Elongation of Rods of Iron and Steel," Phil. Mag., XLVI., 1873, p. 177.

[3] Shelford Bidwell, "On the Changes produced in the Length of Rods of Iron, Steel, and Nickel," Proc. Roy. Soc., Vol. 38, 1885, p. 265; Vol. 40, 1886, pp. 109, 257; "On the Changes

in investigations extending from 1885 to 1890. An optical lever was used to measure the change in length, and he was able to detect a change of 10^{-7} times the length of his specimen with accuracy. The iron was found to increase in length with increasing magnetization up to a certain critical value, at which the maximum elongation was reached. Beyond the critical value, the elongation diminished, and a large magnetizing field produced contraction. Nickel contracted at first, and continued to contract, even with magnetizing forces far greater than those which produced the maximum elongation in iron. Tension decreased the elongation in iron, and increased the contraction.

Nagaoka[1] in 1894 examined the hysteresis effects attending magnetostriction. He used an optical lever and a micrometer microscope, and was able to detect changes in length of about 10^{-7} centimeters.

In 1898 Stevens[2] measured the change in length by observing the shift produced in interference bands. The smallest change that he was able to detect was 4×10^{-8} per unit length.

General Method.

The attention of these investigators was devoted to a study of magnetostriction with large fields. Their methods did not permit its study with small fields. The method of the present investigation permits of its study with very small fields, and was invented by Professor Pupin, of Columbia University.

An alternating current of approximately five hundred cycles was applied to a helix surrounding the specimen, a stretched iron wire. Since the elongation did not depend upon the direction of the magnetizing field, the wire vibrated with a frequency twice that of the current. A piezo-electric crystal, either quartz or Rochelle salt, was attached to one end of the vibrating wire, and an alternating electromotive force was developed on the tin foil coatings of the crystal by varying the tension of the wire. This electromotive force was increased by a three-stage amplifier to such an extent that it produced a note in a telephone receiver, and thus indicated the presence of vibrations in the wire. The amplitude of these vibrations was determined by balancing out the note with a known electromotive force of opposite phase to that developed on

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produced by Magnetization in the Dimensions of Rings and Rods of Iron and of some other Metals," Proc. Roy. Soc., Vol. 43, 1888, p. 406; Phil. Trans., Vol. 179, 1888, p. 205; "On the Effect of Tension upon Magnetic Changes of Length in Wires of Iron, Nickel, and Cobalt," Proc. Roy. Soc., Vol. 47, 1890, p. 469.

[1] H. Nagaoka, "Hysteresis attending the Change of Length by Magnetostriction in Nickel and Iron," Phil. Mag., XXXVII., 1894, p. 131.

[2] J. S. Stevens, "An Application of Interference Methods to a Study of the Changes Produced in Metals by Magnetization," Phys. Rev., Vol. 7, 1898, p. 19.

the crystal. As the crystal had been calibrated by determining the potentials required to balance tensions of various amplitudes, the amplitude of the vibrations in the wire could be calculated.

With the larger magnetizing fields a quartz crystal was used. The specimen hung from the crystal, as shown in Fig. 1, and the helix through which the magnetizing current passed was wound directly on the wire. Approximately half the length of the wire was wound in this manner.

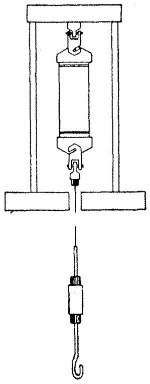

Fig. 1.

Method of suspending quartz crystal and wire.

If it is assumed that the rigidity of the crystal and its supports was large compared to that of the wire, and that the mass suspended from the wire was large compared to the mass of the wire, we can write, owing to the large frequency and large suspended mass used,

$$K = -Fl/l'sY, \tag{1}$$

where K is the amplitude of elongation per unit length due to a given magnetizing field, F the amplitude of the variation in tension due to the elongation, l the total length of the wire, l' the length of the magnetized portion, s the cross section, and Y Young's modulus for the wire.

That which was measured directly was not F but the virtual electromotive force, E, developed on the crystal. A determination of the relation between E and F was necessary.

To effect this, one plate of a parallel plate condenser hung from the quartz crystal, and the other plate was placed just below it on a board which could be raised or lowered by levelling screws. Suppose now that a known electromotive force of the same frequency as that applied to the helix is applied to the condenser. The force of attraction of the lower plate upon the upper one is easily calculated and the electromotive force developed on the crystal found in terms of the mechanical force acting on it.

The attractive force is given by the equation,

$$F_0 = AV^2(10^{-4})/72\pi d^2, \tag{2}$$

where F_0 is the virtual attractive force between the condenser plates, A the area of the plates, V the virtual electromotive force applied to the condenser, and d the distance between the plates of the condenser.

If we write, for the relation between the tension produced on the crystal and the electromotive force developed by it, $F_0 = Ez$, we have, since the attractive force F_0 is related to the amplitude F of the tension producing a virtual electromotive force E on the crystal by the equation,

$$F = \sqrt{2}\, F_0,$$

$$F = \sqrt{2}\, Ez, \quad \text{or} \quad K = -\sqrt{2}\, Ezl/l'sY = \sqrt{2}\, EzD. \tag{3}$$

The balancing electromotive force, of the same frequency as that developed by the crystal, was obtained by applying the magnetizing current to the grid of a vacuum tube. The arrangement is shown in Fig. 2. The magnetizing current, obtained from an alternating current machine giving a pure sine wave of 535 cycles, was applied to the grid of the tube T_1. A resistance between the grid of this tube and ground maintained the grid at a negative potential with respect to the filament, and the resulting distortion of the plate current gave rise to a series of overtones. The fundamental and all overtones except the first were short-circuited by a circuit TC 1, tuned to resonance at 1,070 cycles. The current of double frequency thus obtained was increased by a power tube T_2.

An electrostatic voltmeter V was used to measure the voltage, and a potentiometer P_1 to take off the amount needed for balancing.

The phase of the balancing electromotive force was regulated so as to be opposite to that produced by the crystal by a wave balance, *WB*, developed by Professor Pupin. It consists of a number of coils in series, with a condenser between each pair of coils and ground. These coils, which constitute the primary, are wound on a single tube in such a way that a secondary coil fitting over the tube can be moved along it. When an alternating electromotive force is applied to the first coil it is transmitted along the coils with a varying phase and a continuously diminishing amplitude. The attenuation has such a value that no stationary wave effects due to reflection at the end of the wave balance are appreciable. An electromotive force is induced in the secondary, the phase and amplitude of which depend upon its position on the primary coil.

In the particular wave balance used, the size and number of coils and condensers were such that there was a change of phase of about two hundred degrees from one end of the primary coil to the other. Thus, with the aid of a reversing switch S_2 in the secondary circuit, an electromotive force of any desired phase could be obtained for balancing.

To balance the electromotive force from the crystal and from the wave balance two repeating tubes, T_3 and T_4, Fig. 2, were connected in parallel as shown. The amplification ratio, A, of the two tubes was determined by sending to the grid of T_4, through a potentiometer P_2–P_3, and a two-way switch S_3, the same current that was supplied to thew avebalance and its tube.

A test of the constancy of this ratio for various settings of potentiometer 3 was made, and the average variation from the mean found to be only about one per cent.

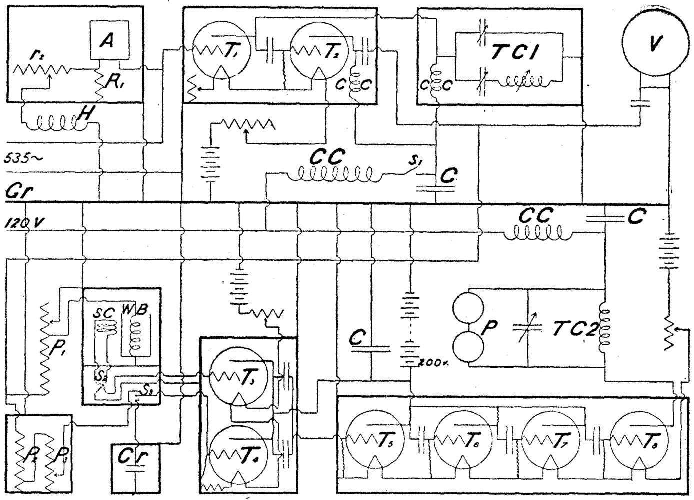

Fig. 2.
Diagram of electrical apparatus.

Equation (3) can now be written in the form

$$K = \sqrt{2}\, DzAP_1WV, \tag{4}$$

where P_1 is the ratio of potentiometer 1, W the ratio of the wave balance, and V the voltage applied to potentiometer 1.

The combined current from the two repeating tubes passed through a three-stage amplifier, made up of tubes T_5, T_6, T_7, the plates of which were fed from a two hundred volt dry battery. T_8 was a power tube used to supply the current to actuate the telephones P. A circuit TC 2 was adjusted to resonance for the frequency of the note which was to be heard in the telephones.

Complete electrical shielding was found necessary to prevent the amplifier from picking up stray oscillations which interfered with the balancing. This shielding is indicated in the figure by heavy lines. All parts of the circuit were grounded radially to the heavy ground wire, Gr.

The magnetizing helix surrounding the iron wire is shown at H. It was fed from the alternator through an ammeter A and a variable resistance r_2, which regulated the current flowing in the helix, in shunt with R_1. The amplitude of the magnetizing force is given by

$$H = 4\pi nI\sqrt{2}/10, \tag{5}$$

where *n* is the number of turns of wire per centimeter in the helix. The factor $\sqrt{2}$ is introduced since I, as determined from the ammeter reading, is the virtual current.

The coils CC were choke coils of sufficient impedance to prevent commutation from the one hundred and twenty volt machine entering different parts of the circuit, and to prevent oscillations set up at one point from reaching a point where they were not desired. The condensers C served the same purpose by furnishing a ground for any current which might get by the choke coils.

Use of Rochelle Salt.

Below one gauss, the results with quartz were not dependable, so Rochelle salt was used for the lower fields. The absolute value of K could not be calculated with the Rochelle salt, but the shape of the curve could be determined, and matched up with the curve obtained by the quartz for the same region, thus getting the value of K indirectly.

The Rochelle salt crystal used was obtained through the kindness of Dr. A. M. Nicolson of the Western Electric Company. The mounting was the same as he had used in a phonograph transmitter,[1] and the method of attaching it to the wire is shown in Fig. 3. The iron wire hung from a brass block, the vibration of which was communicated to the crystal. The block was supported by a brass wire. In this way the weights suspended from the iron wire were supported by the brass wire and produced no strain on the crystal, but the elasticity of the brass wire permitted the vibrations due to the magnetostriction to be transmitted to the crystal. The vibration produced in the crystal was a torsional one, to which the crystal was most sensitive.

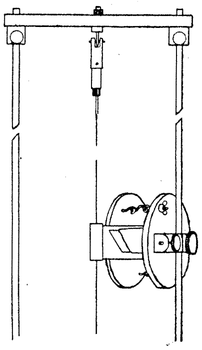

Fig. 3.

Method of supporting Rochelle salt crystal and wire.

Resonance Method.

To increase the sensitiveness of the method still more, Professor Pupin suggested using a wire and a crystal resonant to the frequency of the impressed vibrations.

A Rochelle salt crystal which, with its mounting, had a natural frequency between 1,050 and 1,100 vibrations per second was used in connection with a wire which had a length of one wave-length for longi-

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[1] A. McL. Nicolson, "The Piezo-Electric Effect in the Composite Rochelle Salt Crystal," Proc. Am. I. E. E., Vol. 38, p. 1315, Nov., 1919.

tudinal vibrations of this frequency. The resonant crystal was furnished by Dr. Nicolson.

This wire was stretched in a horizontal position and held tightly by passing over a pulley and hanging weights from the end. Connection between the wire and the crystal was made in such a manner that the crystal could be moved back and forth from the end of the wire until a point was found where the note was of maximum intensity.

The alternator previously used was replaced by an oscillator, so that the frequency could be varied over a small range. To tune the circuit, the length of the wire was changed by moving a stop at its far end until a note of maximum intensity was produced when the oscillator was adjusted to a frequency corresponding to the length of the wire. This was taken to be the resonant frequency of the crystal.

Calculation of the absolute value of K could not be made directly, so the shape of the curve was obtained, and the ordinates multiplied by the proper constant to make them fit the previous curves.

METHOD OF EXPERIMENTATION.

For any run with the quartz crystal the switch S_3, Fig. 2, was thrown so as to connect the grid of T_4 with the potentiometer 3, and the note produced by the current from potentiometers 2 and 3 was balanced out by adjusting potentiometer 1 and the wave balance. This balance was made by setting the potentiometer and wave balance alternately for minimum sound in the telephone, until no sound was heard.

After measuring the amplification ratio, the crystal was connected to the grid of T_4, and a run was made by balancing out the note produced by the crystal under varying conditions.

RESULTS WITH QUARTZ.

Runs with the quartz crystal were made with an iron wire containing about one tenth of one per cent. of carbon, and with a wire of electrolytic nickel. Both the iron and the nickel wires were stretched to remove kinks, then annealed by heating to a red heat.

The smallest weights used were found to be necessary to keep the wire stretched; the largest weights were about as much as could be used without producing a permanent stretch in the wire. The mean results of several runs under different conditions are shown in Figs. 4, 5, and 6.

The general character of the effect agrees with that which Bidwell obtained for small fields. The fact that a contraction is obtained in the case of nickel does not appear from the present method, which measures only the magnitude of the change, but does not indicate whether it is an elongation or a contraction.

The precision of the results shown in these curves is from seven to ten per cent.

The results with quartz correspond to those of Bidwell. This, together with the fact that a brass wire in place of the iron gave no sound, shows

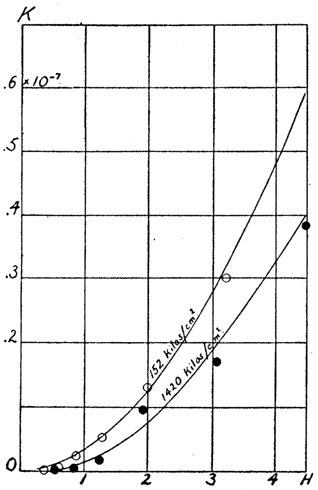

Fig. 4.

Amplitude of elongation per unit length vs. amplitude of magnetizing field (gauss). Low and high tensions. Quartz crystal. Iron wire.

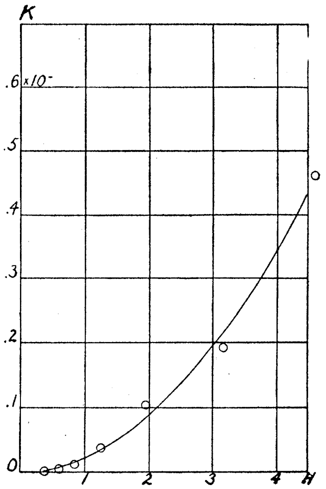

Fig. 5.

Amplitude of elongation per unit length vs. amplitude of magnetizing field (gauss). Low tension. Quartz crystal. Nickel wire.

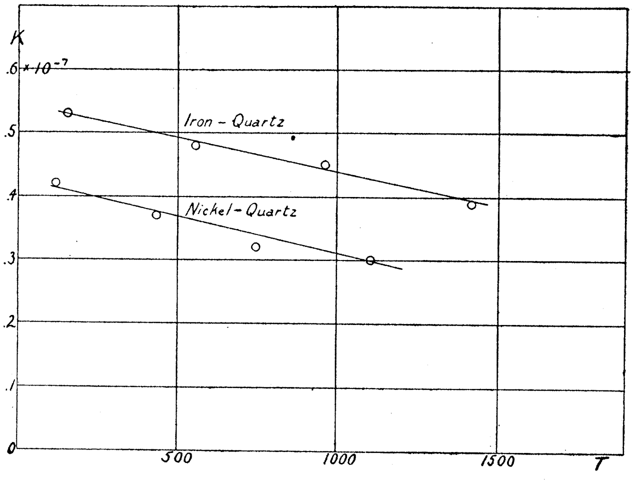

Fig. 6.

Amplitude of elongation per unit length vs. tension in wire (kgms./cm.²). Quartz crystal. Iron and Nickel Wires.

that the effect is due to magnetostriction, rather than to any direct magnetic effect of the helix.

RESULTS WITH ROCHELLE SALT.

The results with quartz were most dependable above one gauss. The Rochelle salt gave curves that checked with those of quartz if the magnitude of the field was not greater than two gauss. For this reason

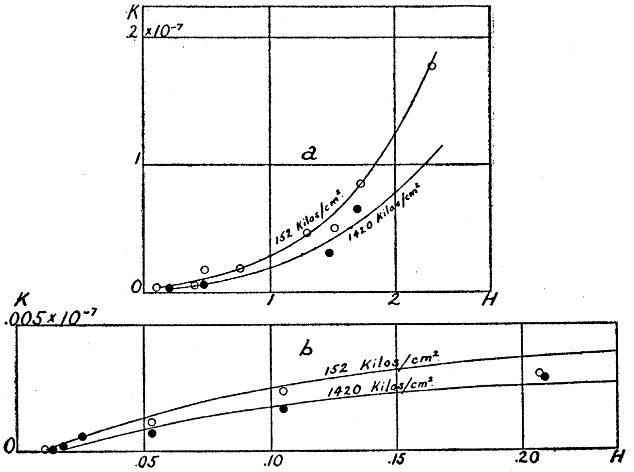

Fig. 7.

Amplitude of elongation per unit length vs. amplitude of magnetizing field (gauss). Low and high tensions. Rochelle salt crystal. Iron wire.

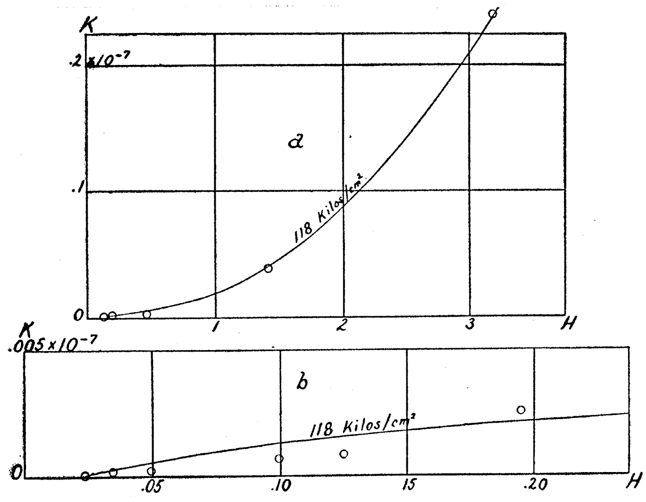

Fig. 8.

Amplitude of elongation per unit length vs. amplitude of magnetizing field (gauss). Low tension. Rochelle salt crystal. Nickel wire.

the multiplying factor applied to the Rochelle salt values was such as to make them fit the quartz curve most closely between one and two gauss.

The results are plotted in Figs. 7 *a* and *b*, and 8 *a* and *b*. The larger scales show the lower portions of the curves. A blank run with a brass wire gave no effect.

With the resonant wire and crystal the greatest effect was obtained when the tension in the wire was greatest, so the runs were made with the maximum tension, 1420 kilos per square centimeter, previously used. The results with the iron wire are plotted in Fig. 9 *a*, *b*, *c*, on successively larger scales. Comparison of Figs. 7*a* and 9*a* shows that the resonant curve fits the previous one for values of H less than 0.6 gauss. Above this value the resonant curve is less, doubtless due to excessive damping in the case of the vibrations of greater amplitude. The curves shown in Fig. 9 *b* and *c* are more dependable than those in Fig. 7*b* for the lower portion of the curve.

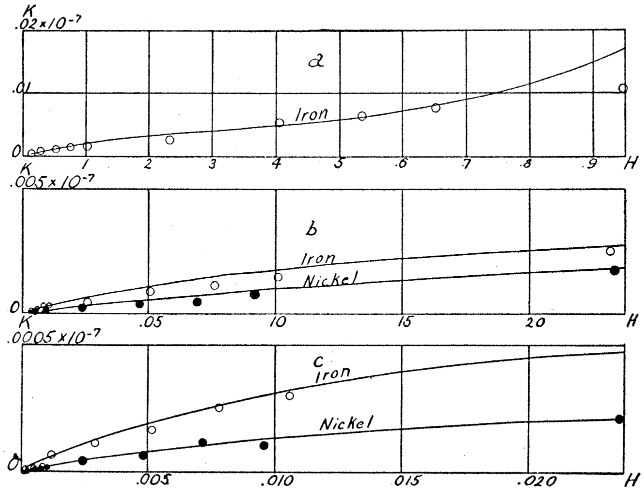

Fig. 9.

Amplitude of elongation per unit length vs. amplitude of magnetizing field (gauss). Resonant wire and crystal. Iron and Nickel Wires.

Runs were made with the nickel wire in the same manner as with the iron, except that a tension of 747 kilos per square centimeter was used. The results are shown in Fig. 9 *b* and *c*. Since no run with the quartz crystal and nickel wire was made at this tension, the curves for nickel in Fig. 9 *b* and *c* can only be regarded as a rough approximation to the absolute value. Their shape, however, can be depended on.

By means of the resonant method the curve has been extended to 0.002 of a gauss in the case of iron, and to 0.005 of a gauss in the case of nickel. At these values the effect could just be detected. A brass wire substituted for the iron or nickel produced no effect.

In each case the shape of the curve was the same; that is, nearly linear but slightly concave downward in its lower portion, apparently passing through the origin.

EXPERIMENTS WITH BISMUTH.

In the account of Bidwell's work[1] he tells of detecting magnetostriction in bismuth. The effect was small but measurable. Later Wills[2] undertook to measure the effect. Using specially prepared very pure bismuth and an apparatus which would have detected a change of 10^{-8} times the length of the specimen, one centimeter, he was unable to observe any effect with a field as high as 3,200 gauss.

If Bidwell's results were correct, a field of seven gauss should give an elongation great enough to come within the limits of the resonant method, about 3×10^{-12}. A rod of bismuth about eight centimeters long and one half a centimeter in diameter was wound with a helix to within about one centimeter of each end, and attached to a brass wire of the resonant length.

The power of the amplifier was increased, and a field of twelve gauss applied to the bismuth, but without producing any effect. It would seem, then, that magnetostriction is lacking in bismuth, at least in fields of twelve gauss.

The writer wishes to express his obligations to Professors Pupin and Wills for frequent suggestions made during the development of the method and the progress of the investigation, and to the Western Electric Company and Dr. Nicolson for their kindness in furnishing the crystals of Rochelle salt, and in giving suggestions as to their use.

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August, 1921.

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[1] Shelford Bidwell, "On the Changes Produced by Magnetization in the Dimensions of Rings and Rods of Iron and of Some other Metals," Phil. Trans., Vol. 179, 1888, p. 205.

[2] A. P. Wills, "On Magnetostriction in Bismuth," PHYS. REV., Vol. 15, 1902, p. 1.