ON THE PRESSURE IN THE CORONA DISCHARGE.

By Jakob Kunz.

Synopsis.

Theory of Pressure Effects Due to the Corona Discharge, in Terms of Ionic Mobility. —The characteristic pressure effect is an excess pressure of from 1 to 3 cm. of water inside a tube concentric with the discharge wire. Assuming that the momentum given the ions by the electric field is transferred to the gas, the author shows that the excess pressure is $i/4\pi u$, where i is the current per unit length of wire and u is the mobility. Using this expression, the *positive mobilities in* H_2, N_2, CO_2, O_2 *and* NH_3 *are computed* from the observed pressure differences, and come out respectively 1.51, 0.50, 0.16, 0.23 and 0.31 times 10^{-2} cm./sec. per volt/cm. The first three values are about half those measured by Hess in the wind of ions from radioactive substances, while the value for O_2 is only one eighth, the low value being perhaps due to ozone. The results so far refer to the mean mobility. Assuming that there are two types of ions, the *percentage of heavy ions* comes out about 13 for O_2. The time for the pressure to rise to its limiting value is computed and comes out in rough agreement with the measurements of Fazel; and an explanation is suggested for the form of the pressure curves obtained by Fazel with A.C. discharges.

It has been shown by E. H. Warner[1] that the initial pressure increase in the corona discharge is proportional to the current, that it varies in the different gases from a small value in hydrogen to higher values in nitrogen, ammonia, oxygen and reaches the highest value in carbon dioxide for a given current. A. M. Tyndell[2] has tried to explain the phenomenon in question as a pure heat effect. He has made an experiment moreover in which the central wire was heated and thereupon the potential difference applied. Instead of increasing, the pressure decreased. This phenomenon is apparently connected with the effect observed by J. W. Davis, that in the rectification of alternating currents through the corona discharge in hydrogen, the wire which was red hot, became dark as soon as the corona appeared. In the previous investigation by means of alternating currents and an oscillograph, C. S. Fazel showed that even in this experiment of Davis and Tyndall the pressure at first in a very short time interval increases and then rapidly decreases. So far the following phenomena have been observed:

1. A very small pressure difference, hardly measurable, between the central wire and the outer cylinder.

[1] Physical Review, Vol. 10, p. 483, 1917. See also the corona discharge by E. H. Warner and J. Kunz, University of Illinois Engineering Experiment Station Bulletin No. 114, 1919.

[2] Phil. Mag., Vol. 35, p. 261, 1918.

2. A pressure increase proportional to the current in the D.C. corona, which appears in about 1.5 seconds after the beginning of the corona. This pressure increase may easily amount to 3 cm. of water.
3. With alternating current a varying pressure increase occurs of about .3 cm. of water with a frequency $2n$ when the frequency of the A.C. is equal to n. These pressure fluctuations are superimposed on the effect mentioned before.
4. A pressure decrease which occurs, when direct current or alternating current corona arises from a hot central wire, after an initial pressure increase.
5. A pressure increase due to Joule's heat which can readily be separated from the effect (2).

The principal phenomenon is the pressure increase mentioned under (2) which we shall try to explain by means of the small mobilities of ions which have been measured by Victor F. Hess [1] in the wind of ions emanating from radioactive substances.

The central wire of radius R_1 in the coaxial corona cylinder of radius R_2 has a positive charge and emits in the corona discharge n positive ions per unit length and per unit time. During the time interval dt every positive ion travels through the distance dr with the velocity $v = dr/dt$. Let the electric force at the point r be E, the charge of the ion e, then the increase of the electric impulse during dt is equal to $eEdt$. We assume that this momentum be transmitted to the molecules of the gas so that a pressure difference dp occurs between r and $r + dr$. Since n ions per unit time cross the surface of the cylinder $2\pi r$, then we get: $neEdt = dp2\pi r$. But $ne = i$ which is the corona current per unit length, hence

$$iE\frac{dr}{v} = dp2\pi r.$$

According to the definition of mobility u of the ions, we have

$$v = uE;$$

hence

$$\frac{i}{u}dr = dp2\pi r \tag{2}$$

or

$$\frac{i}{2\pi u}\frac{dr}{r} = dp \tag{3}$$

or

$$\frac{i}{2\pi u}\log\frac{R_2}{R_1} = p_2 - p_1. \tag{4}$$

[1] Sitzungsberichte der Akademie der Wissenschaften von Wien. Math. Abt., 128, Bd. Heft 6, 1919; 129, Bd. Heft 6, 1920.

The pressure difference between the cylinder and the wire is proportional to the current and inversely proportional to the mobility of the ions. If, however, we attach to the corona cylinder an open manometer, we measure the pressure difference between the inner side of the cylinder and the outer pressure $p_2 - p_0$ and not $p_2 - p_1$. We determine the pressure difference $p_2 - p_0$ in the following way:

$$\int_{R_1}^{r} dp = \frac{i}{2\pi u}\int_{R_1}^{r}\frac{dr}{r},$$

$$p - p_1 = \frac{i}{2\pi u}(\log r - \log R_1). \tag{5}$$

The conservation of mass gives the following equation:

$$\int_{R_1}^{R_2} p2\pi r dr = p_0\pi(R_2^2 - R_1^2). \tag{6}$$

From (5) we substitute p in (6) and carry out the integration.

$$p_1\pi(R_2^2 - R_1^2) - \frac{i}{2u}\log R_1(R_2^2 - R_1^2) + \frac{i}{u}\int_{R_1}^{R_2} r\log r dr = p_0\pi(R_2^2 - R_1^2).$$

or

$$p_1 = p_0 + \frac{i}{2\pi u}\log R_1 - \frac{i}{4\pi u}\left[\frac{R_2^2\log R_2^2 - R_1^2\log R_1^2}{R_2^2 - R_1^2} - 1\right]. \tag{7}$$

We have moreover:

$$\int_{r}^{R_2} dp = \frac{i}{2\pi u}\int_{r}^{R_2}\frac{dr}{r} = \frac{i}{2\pi u}(\log R_2 - \log r) = p_2 - p. \tag{8}$$

We substitute this last p in equation (6) and obtain:

$$p_2 = p_0 + \frac{i}{2\pi u}\log R_2 - \frac{i}{4\pi u}\left[\frac{R_2^2\log R_2^2 - R_1^2\log R_1^2}{R_2^2 - R_1^2} - 1\right]. \tag{9}$$

From (7) and (9) we obtain again equation (4). The pressure difference $p_2 - p_0$ is now given by the expression:

$$p_2 - p_0 = \frac{i}{2\pi u}\left[\log R_2 - \left(\frac{R_2^2\log R_2 - R_1^2\log R_1}{R_2^2 - R_1^2} - \frac{1}{2}\right)\right].$$

In our experiments R_2 and R_1 had the following values:

$$R_2 = 2.25 \text{ cm.}, \quad R_2^2 = 5.05, \quad \log R_2 = 0.81,$$
$$R_1 = .01 \text{ cm.}, \quad R_1^2 = 10^{-4}, \quad \log R_1 = -4.6.$$

Therefore approximately

$$\frac{R_2^2 \log R_2 - R_1^2 \log R_1}{R_2^2 - R_1^2} = \log R_2$$

and

$$p_2 - p_0 = \frac{i}{4\pi u}$$

or

$$u = \frac{i}{4\pi(p_2 - p_0)}. \tag{10}$$

The total current measured in the corona tube of length l was I, hence $i = I/l$ and

$$n = \frac{I}{4\pi l(p_2 - p_0)} = \frac{I}{4\pi l \Delta p}. \tag{11}$$

This pressure increase Δp is assumed to be due entirely to the corona wind and not partly to the heat of Joule. This latter pressure increase, according to the measurement of Warner, is only about 1/2 per cent. of the observed pressure increase in the first second. The necessary small correction is already contained in the values of Δp that are used in the calculation of the mobilities.

We apply formula (11) at first to the straight lines of Fig. 45, page 85 of the bulletin of Warner and Kunz, in order to determine the ratio of the mobilities of the ions in various gases. We choose always the same current $I = 2.10^{-4}$ amp. and obtain the following ratios:

$$\frac{U_{H_2}}{U_{N_2}} = \frac{p_{N_2}}{p_{H_2}} = \frac{1.3}{0.4} = 3.2$$

while Hess gives 3.5.

$$\frac{U_{H_2}}{U_{CO_2}} = \frac{4.2}{0.4} = 10.5;$$

Hess gives 8.3

$$\frac{U_{H_2}}{U_{NH_3}} = \frac{2.2}{.4} = 5.5$$

$$\frac{U_{H_2}}{U_{O_2}} = \frac{3}{.4} = 7.5.$$

These ratios agree in order of magnitude and in their order with those of Hess. The same values can hardly be expected, as impurities, water vapor, dust particles, for instance, of disintegrated metal have a large influence on the mobilities.

We proceed to calculate the mobilities of the ions. From the straight lines we find for hydrogen the following values:

$$I = 2.10^{-4} \text{ amp.}; \quad 1 = 24 \text{ cm.}; \quad \Delta p_{H_2} = 0.45 \text{ cm. } H_2O$$

$$U_{H_2} = \frac{2.10^{-4}\, 3.10^9}{4\pi \cdot 24 \cdots 45 \cdot 981 \cdot 300} = 1.51\ 10^{-2} \frac{\text{cm.}}{\text{sec.}} : \frac{\text{volt}}{\text{cm.}}.$$

The results are collected in the following table.

Mobilities $\frac{\text{cm.}}{\text{sec.}} : \frac{\text{volt}}{\text{cm.}}$

	Corona.	α articles, Hess.	Point Discharge, Chattock.
U_{H_2}	$1.51 \cdot 10^{-2}$	$3.73 \cdot 10^{-2}$	5.4
U_{N_2}	$.505 \cdot 10^{-2}$	$1.08 \cdot 10^{-2}$	
U_{CO_2}	$.161 \cdot 10^{-2}$	$.45 \cdot 10^{-2}$	.83
U_{O_2}	$.23 \cdot 10^{-2}$	$1.98 \cdot 10^{-2}$	1.3
U_{NH_3}	$.311 \cdot 10^{-2}$		

In most cases the mobilities are only about two times smaller than those determined by Hess; the mobility of oxygen however is 8.6 times smaller than the value obtained by Hess; this may be due to the formation of ozone which accompanies the corona discharge in oxygen. Conspicuous in the corona discharge as well as in the ionization by α particles are the small values of the mobilities compared with those of the point discharge for instance. We may expect other analogies in those forms of discharge which are distinguished by small mobilities of the ions.

We proceed to calculate the fraction of the heavy ions of Langevin. We write the formula (11) in the form:

$$\Delta p = \frac{en}{u} \frac{1}{4\pi},$$

or for one type of ions:

$$\Delta p_s = \frac{e}{4\pi} \frac{n_s}{u_s},$$

$$\Delta P = \Sigma \Delta p_s = \frac{e}{4\pi} \sum_{s=1}^{v} \frac{n_s}{u_s},$$

when there are v types of ions present. If $v = 2$, then

$$\Delta P = \frac{e}{4\pi} \left(\frac{n_1}{u_1} + \frac{n_2}{u_2} \right) \text{ and } i = e(n_1 + n_2).$$

The fraction of heavy ions is then put equal to α so that

$$n_2 = \alpha n,$$
$$n_1 = (1 - \alpha) n$$

and

$$\Delta P = \frac{en}{4\pi}\left(\frac{1-\alpha}{u_1} + \frac{\alpha}{u_2}\right) = \frac{i}{4\pi}\left[\alpha\left(\frac{u_1 - u_2}{u_1 u_2}\right) + \frac{1}{u_1}\right],$$

$$\alpha = \frac{1}{u_1 - u_2}\left[\frac{\Delta P 4\pi}{i} u_1 u_2 - u_2\right]$$

or in per cent.

$$\alpha = \frac{100 u_2}{u_1 - u_2}\left(\frac{\Delta P 4\pi u_1}{i} - 1\right).$$

For air we have according to Pollock [1]

$$u_1 = 1.3 \text{ (volt)}; \qquad u_2 = .0003 \text{ (volt)}.$$

For electrostatic units of the field we multiply these numbers by 300;

$$i = \frac{I}{l} = \frac{2.10^{-4} \cdot 3.10^9}{24},$$

$$\Delta P = 3.981 \frac{\text{dynes}}{\text{cm.}^2} \text{ for oxygen.}$$

With these numbers we obtain $\alpha = 13.4$ per cent.

We shall now approximately determine the time which the pressure increase requires for its development. Let the average velocity of the ions be $v = uE$, then the average time required by the ions to pass through the distance $R_2 - R_1$ will be equal to

$$t = \frac{R_2 - R_1}{\bar{v}} \text{ and } \bar{E} = \frac{V}{R_2 - R_1},$$

where V is the potential difference between the cylinder and the wire.

$$t = \frac{(R_2 - R_1)^2}{u.V} = \frac{R_2^2}{u.V}.$$

For $R_2 = 2.25$ cm., $u = 5.05 \cdot 10^{-3}$ (volt), $V = 15{,}000$ volts, we obtain $t = .067$ sec. If however we take into account the heavy ions, for which $u = 3 \cdot 10^{-4}$, then we find $t = 1.1$ seconds, while C. S. Fazel found about 1.5 seconds. For the high mobilities $u = 1.3$ (volt) we find for $t = 2.6 \cdot 10^{-4}$ seconds. For the explanation of the large pressure increase we have to assume the formation of heavy ions with very small mobilities. Apparently we can test this conclusion by means of alternating currents. Before doing this, we shall call attention to one or two corrections which have to be made in formula 10. If the ions which arise from the positive wire do not impart their momentum to the gas, they will impart it to the cathode and the gas pressure will not change.

The momentum imparted to the surface $2\pi R_2$ of the cylinder per unit time will be equal to

$$u_{R_2}E_{R_2}mn = (uE)_{R_2}\frac{m}{e}ne = (uE)_{R_2}\frac{m}{e}i = \Delta p_i 2\pi R_2.$$

In general only a fraction of the ions will transmit their momentum directly to the electrode, *i.e.*, to the outer cylinder, so that the pressure increase observed corresponds only to a certain fraction of the current; in formula (10) we have to diminish the current or to add to the pressure Δp a small positive correction Δp_i. We would have: $i/4\pi u = \Delta p + \Delta p_i$. In the manometer we observe only Δp. If the current $i = 0$, then $\Delta p = -\Delta p_i$; this seems to occur in the case of hydrogen according to Fig. 45 on page 85 of the bulletin. If on the other hand the gas molecules are reflected from the cylinder, then the gas pressure on an external surface element will be increased and we shall have a correction Δp_r in the form:

$$\frac{i}{4\pi u} = \Delta p - \Delta p_r.$$

If now $i = 0$, then $\Delta p = \Delta p_r$; a trace of this may occur in the straight lines of NH_3, O_2 and CO_2 in Fig. 45, where the straight lines meet the axis of ordinates a little above the origin. This effect of course could not be a stationary one.

From the explanation of the principal phenomenon it follows, that if we push the glass tube of the manometer from the cylinder more and more toward the wire, the pressure difference $p_2 - p_0$ can not change, provided that in that point the discharge does not change. If with this method nevertheless very small pressure differences have been observed for the two limiting positions of the gas tube, they can only be reduced to small secondary changes in the discharge. There exists indeed a pressure difference $p_1 - p_2$ which we have calculated, but it cannot be measured with the manometric method here indicated.

The straight lines which express the relation between the current and the pressure increase $p_2 - p_0$, have only been obtained for the positive corona, in which the wire is positive. If the wire is charged negatively, we observe also a pressure increase, which however is subject to strong fluctuations, so that regular observations cannot be repeated. This is apparently due to the character of the negative discharge, which does not spread out evenly over the wire, but consists of beautiful little beads, which are partly stationary, partly travel along the wire. For very short time intervals, however, immediately after the potential difference has been applied, the negative corona appears in the form of a fine uniform cylinder of light, which covers the whole wire. During this interval the negative corona pressure is probably stable.

If now we produce the corona with alternating current of frequency n, then during every positive and negative half wave the light ions will travel through the whole layer of gas from the wire to the cylinder, producing a partial pressure increase with a frequency $2n$. This phenomenon has been observed by C. S. Fazel.

If the wire is positive, large positive ions of small mobility will be formed, to which corresponds a large pressure increase dp according to the equation $i/u\, dr = dp 2\pi r$. These ions, however, will not reach the cylinder during the first positive half wave of the A.C. The corresponding pressure increase will not reach the manometer; if now the negative wave of the A.C. acts, the large ions may be neutralized or even become negative, they will stand still or travel a little further; if afterwards the wire becomes positive again, the quick positive ions will also charge positively those previously formed large ions, which will travel a little further toward the cylinder, which they will reach only after a certain number of cycles. Thus the resultant pressure increase will be equal to that of a corresponding direct current; upon this positive pressure increase are superimposed small regular fluctuations with frequency $2n$. This phenomenon also has been observed by C. S. Fazel.

It has been shown that the characteristic pressure increase in the corona discharge is due to the fact that the ions impart their momentum to the gas. The pressure increase is exactly proportional to the current. The mobilities are of the same order of magnitude as those determined by Hess. If we assume only ordinary ions and heavy Langevin ions, we find that the latter amount to about 13 per cent. of the total ionization. Of course it is possible that ions of widely different mobilities are formed as those studied by O. Blackwood[1] and P. J. Nolan.[2] The mobilities of the ions formed in the corona discharge will be measured by an independent method. The average time required for the formation of the pressure increase has been calculated. The pressure increase observed by C. S. Fazel with alternating currents has been explained.

I am glad to thank Dr. K. F. Herzfeld of the Institute for Theoretical Physics of the University of Munich for many valuable suggestions.

LABORATORY OF PHYSICS,
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October, 1921.

[1] PHYSICAL REVIEW, Vol. XVI., p. 85, 1920.
[2] PHYSICAL REVIEW, Vol. XVIII., p. 185, 1921.