PHYSICAL REVIEW VOLUME 188, NUMBER 1 5 DECEMBER 1969

Amplitude Effects for the Oscillating Disk in Liquid Helium II†

Robert P. Roger and R. G. Hussey
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
(Received 12 May 1969)

The amplitude variation of the logarithmic decrement D of a disk (radius ≈ 2.5 cm) performing torsional oscillations in liquid He II has been found to depend on the history of the helium bath. Two types of damping curves are observed: (i) The first (called "steep") occurs when the height h of the bath above the disk is greater than some critical height h', $(15.8 \leq h' \leq 16.8$ cm) and the time interval Δt between reaching the λ temperature T_λ and the initial deflection of the disk is short ($\Delta t < 30$ min); (ii) the second curve (called "flat") occurs when either $h < h'$ or, if $h > h'$, then Δt must be long ($\Delta t > 60$ min). There is a critical velocity v_c associated with each type of curve. For the steep curves, v_{c1} increases with temperature (from 1.33 to 2.15 °K) and varies with period as $\tau^{-1/2}$ for periods between 3.9 and 19.6 sec. For the flat curves, v_{c2} is independent of temperature (from 1.4 to 2.156 °K), is larger than v_{c1}, and has a period dependence given by $v_{c2}\tau^{1/2} = 1.4$ (cgs units) for periods between 3.9 and 45.0 sec. The difference between steep and flat curves is attributed to the presence of a long-lived motion of the superfluid (possibly an ordered array of quantized vortex lines) in the subcritical flow for the steep case and the absence of this motion for the flat case. The low-amplitude logarithmic decrement D_0 is found to be the same for the steep and flat curves. The relative change in D_0 from the calculated value $[(D_0 - D_{\rm calc})/D_0]$ is found to increase as h/λ decreases (λ is the penetration depth). The excess damping slightly above v_c cannot be explained by the effect of mutual friction. The relative change in moment of inertia $(I_a - I_{a0})/I_{a0}$ at high amplitudes (up to 2.5 rad) indicates that, for the steep curves, about 30% of the superfluid is entrained for a 4-sec period and 5% for a 12-sec period. No entrainment is indicated for the flat curves.

I. INTRODUCTION

When a disk oscillates in liquid helium II at very low amplitudes, the logarithmic decrement D_n is independent of amplitude and can be used to determine $\eta_n \rho_n$, the product of normal-fluid viscosity and density. Hollis Hallett[1] was the first to report that above a certain critical amplitude φ_c, the logarithmic decrement increases with amplitude; the critical velocity is then taken to be the velocity of the edge of the disk

$$v_c = 2\pi\varphi_c R/\tau \quad , \tag{1}$$

where R is the disk radius, and τ is the period.

Experiments with oscillating disks, cylinders, and spheres have shown agreement on the following results: (i) the critical velocity varies with temperature as ρ_s^{-n} $(0.25 \leq n \leq 0.5)$,[1–4] where ρ_s is the superfluid density; (ii) roughness of the oscillating boundary surface lowers the value of v_c[2,4]; and (iii) the excess decrement $(D - D_n)$ above φ_c varies approximately as ρ_s.[1,2] A major point of disagreement is the period dependence of the critical velocity: some investigators have found $v_c \propto \tau^{-1/2}$,[1,3] and others have found v_c to be independent of period.[2,4] Other results of oscillating boundary experiments are the following: (a) v_c depends on the size of the helium bath[4]; (b) the effective density at high amplitudes (as revealed by an oscillating pile of disks) exceeds ρ_n and may approach the total density $\rho(=\rho_s + \rho_n)$[1]; and (c) the effective kinematic viscosity at high amplitude is less than η_n/ρ_n.[5]

Donnelly and Hollis Hallett[6] reviewed experiments of this type and succeeded in associating a similarity parameter with v_c. Hall[7] has connected this similarity parameter with superfluid turbulence by assuming that v_c occurs when the average spacing between quantized vortex lines becomes of the order of the penetration depth λ [defined as $(\eta_n\tau/\pi\rho_n)^{1/2}$]. However, both of these results depend upon the critical velocity varying as $\tau^{-1/2}$.

We have measured the amplitude dependence of the logarithmic decrement of an oscillating disk over a wider range of periods (3.9 to 45.0 sec) than before. Our major results are: (i) both v_c and the shape of the damping curves are strongly affected by the previous history of the helium bath; (ii) when the helium bath is undisturbed for a sufficient time (about 1 h) after lowering the temperature below the λ point, v_c is independent of temperature and varies as $\tau^{-1/2}$.

II. EXPERIMENT

The torsion pendulum cryostat used in this investigation is shown in Fig. 1. Both ends of the

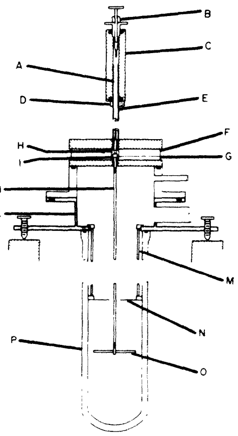

FIG. 1. Experimental cryostat. The torsional oscillations were excited by deflecting the magnet H with a set of external Helmholtz coils.

quartz torsion fiber A were glued into half-inch pieces of No. 23-gauge needle with Araldite Epoxy. The needle on the upper end of the fiber was then cemented into a piece of No. 8-gauge needle which was fastened to the head piece B. The head, which was used to adjust the zero, was held vacuum tight by an O-ring connection to the vertically adjustable sleeve C, which was fixed in a given position on the stainless-steel tube E by the clamp D. The lower end of the quartz fiber was similarly connected to the aluminum-mirror mount I. The front surface mirror G was placed so that its reflecting side lay on the axis of rotation. Just above the mirror, in the center of the mount, was placed a cylindrical piece of magnetized steel H, which was used in connection with a set of external Helmholtz coils to deflect the system to a desired amplitude. After the initial deflection the Helmholtz coils were removed. To the bottom of the mirror-mount was cemented a long glass tube J, of length 80 cm and o.d. 0.31 cm. The lower end of the tube was fastened to the disk O. Three highly polished disks were used (see Table I). The disk could be pulled up to the shield N by means of the sleeve C. This was done to prevent the surface of the disk from being contaminated with frost or dust particles during the precooling and transferring processes. The shield is a piece of aluminum foil which extends to the walls of the glass Dewar P and has a 13-mm-diam hole in its center. At its outer edge, the shield is attached to a brass ring which is hung by three thin-walled stainless-steel tubes M (o.d. 1.5 mm) from the pumping collar L. When the disk was lowered, it was 4.6 cm from the shield and 10.5 cm from the bottom of the helium flask. The i.d. of the helium flask was 7.0 cm.

The amplitude of oscillation was measured by reading the leading edge of the focused image of a straight-filament light source on a circular scale of radius $\frac{1}{3}$ m. The viewing chamber F was turned down from a Plexiglas tube to a thickness of 0.0794 cm and radius of 4.85 cm. The width of the light image on the scale was 0.75 mm. A similar light, focused onto a slit placed in front of a photomultiplier tube, was used to measure the period of oscillation. The width of the light at this slit was 1.5 mm and the slit was 0.5 mm wide. The distance from the mirror to the photomultiplier was 1.5 m. The pulse from the photomultiplier was fed into an amplifier pulse shaper and a scale-of-two circuit (to eliminate the mid-period pulse). The resulting square wave pulses were differentiated and used to activate a 1-MHz time base counter. Period accuracy at low amplitudes varied from 0.1 msec at the short periods to 1 msec at the longer periods.

The temperature was regulated with an elastic membrane regulator of the Walker type,[8] and was measured with an oil manometer. The oil used had a density of 0.981 g/cm^3.

Measurements were made of the amplitude dependence of the logarithmic decrement and period of each torsion fiber in vacuum ($\sim 2\times10^{-6}$ mm Hg). The vacuum decrement for all fibers used was essentially independent of amplitude and ranged from 0.00096 for the thin fibers to 0.00007 for the thicker ones. The vacuum period varied with amplitude (at 1.5 rad the change was 100 msec for a 12-sec period and less than 1 msec for a 4-sec period), but the change was reproducible, so calibration curves were taken before and after each run and the liquid-helium data were corrected for the vacuum variation. Measurements of the period in the liquid were made by measuring single periods; at the low amplitudes several periods were averaged and a smooth curve drawn through these points was used in analyzing the re-

TABLE I. Experimental disks (dimensions at liquid-helium temperatures).

Disk	Radius (cm)	Thickness (cm)	Moment of inertia (g cm^2)	Material
A	2.5103	0.1566	82.576	Brass
B	2.5318	0.2976	161.164	Brass
C	2.5307	0.1584	28.428	Aluminum

sults. The damping was small enough so that the logarithmic decrement could be determined from the expression

$$D=[\ln(\varphi_i/\varphi_j)]/(j-i)\,, \tag{2}$$

where φ_i is the ith amplitude. This expression is a factor of 2π larger than the one used by Hollis Hallett.[1] For runs with the smallest damping, the number of amplitudes $(j-i)$ used in Eq. (2) ranged typically from 7 at the high amplitudes to 30 at the low amplitudes; for runs with larger damping, the range was typically 4–15. Values of the critical velocity determined from semilog plots of φ_i versus i were in very good agreement with those obtained from curves of D versus φ. The logarithmic decrements measured in He II ranged from 0.0022 for disk B to 0.045 for disk C.

III. RESULTS

A. Damping Curves

Figure 2 shows the amplitude dependence of the logarithmic decrement D of disk C taken on two separate days at approximately the same temperature, and Fig. 3 shows curves of D versus φ for disk C taken at the same temperature for two consecutive runs on the same day. For convenience, we describe the upper curve as "steep" and the lower curve as "flat." As long as the temperature remained below the λ temperature T_λ, the damping curves were all of the same type, i.e., either all steep or all flat. We have found that the difference between steep and flat curves is not due to (i) roughness of the disk surface, (ii) the manner of deflecting the disk (whether by many small impulses or by a few large impulses), or (iii) the position of the disk as the temperature passed through T_λ (whether lowered or held against the shield N). The difference is due to

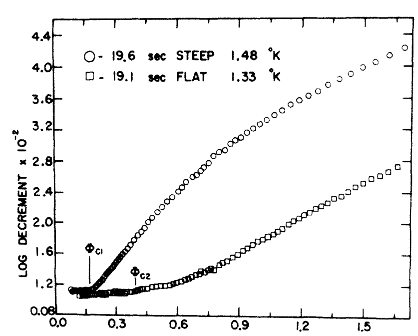

FIG. 2. Logarithmic decrement versus amplitude for two 19-sec runs with disk C, taken on two separate days.

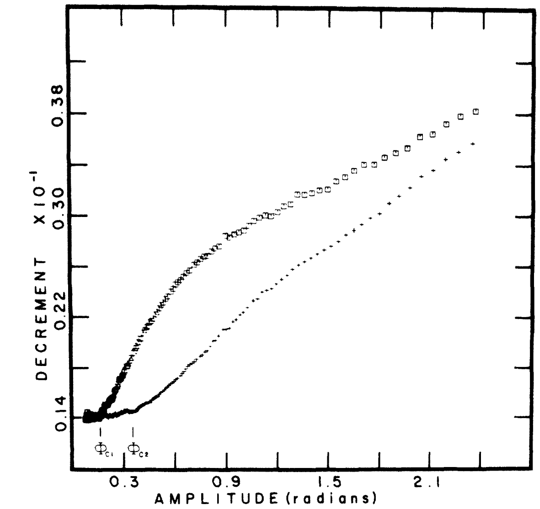

FIG. 3. Two curves of logarithmic decrement versus amplitude for disk C taken on the same day with a period of 12.4 sec at 1.83 °K. The steep curve was taken first, the bath temperature was raised above T_λ the then the flat curve was taken.

(a) the height of the bath at the time of the initial disk deflection, (b) the time interval Δt between reaching T_λ and the initial disk deflection, and, possibly, (c) the presence of the shield N. Using the cryostat in Fig. 1 (including shield N) we have found that in order to obtain a steep curve it is necessary that the bath height h above the disk at the initial disk deflection be greater than some critical value h', and that Δt be sufficiently short (probably less than 30 min). To obtain a flat curve it is necessary to have $h<h'$, or, if $h>h'$ then Δt must be sufficiently long (about 1 h). The limits on h' have been found to be $15.8 \leq h' \leq 16.8$ cm. A few runs taken without the shield indicate either that h' is lower in the absence of the shield (less than 10 cm) or that perhaps the presence of the shield is necessary to get a flat curve. It should be pointed out, however, that once a steep or flat curve was established the lowering of the liquid level below the shield did not change the type of curve.

Occasionally, curves that were neither steep nor flat were observed. These curves were either in between the steep and flat curves (8 out of 125 curves), exhibiting a critical velocity in between those for steep and flat, or lower than the flat curves ("ultraflat," 7 out of 125) – four of which exhibited critical velocities in agreement with the flat curves, and three of which had critical velocities higher than the flat ones. However, there was no correlation between the occurrence of these odd curves and the time interval Δt; in particular, one ultraflat curve was obtained with a Δt of only 10 min. Of the 125 curves, 88% were either steep or flat. On three separate occasions, both steep and flat curves were obtained on the same day by warming above T_λ. It should be emphasized that the low-amplitude damping was the same for both steep and flat curves.

Flat curves at different temperatures for a 11.54-sec period are shown in Fig. 4. Above the critical amplitude the logarithmic decrement increases and eventually becomes linear in the amplitude up to the highest amplitudes measured (2.5 rad). The slope m ($\equiv dD/d\varphi$) at the high amplitudes was found to vary as $\tau^{1/2}/I$ as shown in Fig. 5, where I is the moment of inertia of the disk, and τ is the period.

Steep curves at different temperatures for a 11.54-sec period are shown in Fig. 6. At the high amplitudes these curves also are linear in the amplitude; the slope varies as $\tau/I^{1/2}d$ as shown in Fig. 7, where d is the thickness of the disk.

Rosenblat[9] has shown that the centrifugal forces on fluid near an oscillating disk cause a radial-axial secondary flow which makes the damping of the disk change with amplitude; such changes have recently been observed in classical fluids.[10] The dashed curves in Figs. 4 and 6 indicate the changes expected for secondary flow of the normal fluid only, as given by the empirical results of Folse.[10]

On several occasions, we studied the effect of surface roughness by placing iron filings on the disk. For the steep curves, we found that surface roughness lowers the critical velocity and makes the high-amplitude part of the damping curve steeper. There was some indication that surface roughness had no effect on the flat curves, but this result is, as yet, inconclusive.

B. Critical Velocities

The critical velocities v_{c2} associated with the flat curves are shown in Fig. 8. For periods be-

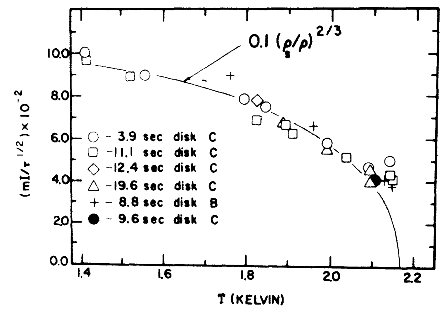

FIG. 5. Temperature dependence of the slope m of the flat damping curves at high amplitudes. I is the moment of inertia and τ is the period of oscillation.

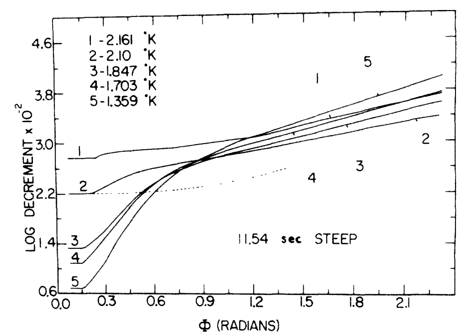

FIG. 6. Logarithmic decrement versus amplitude for curves of the steep type at five different temperatures. The broken curve represents what is expected from secondary flow of the normal fluid only.

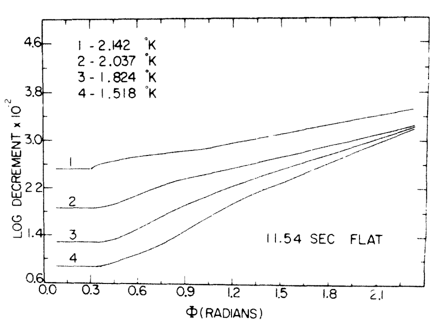

FIG. 4. Logarithmic decrement versus amplitude for curves of the flat type at four different temperatures. The broken-line curves represent what is expected from secondary flow of the normal fluid only.

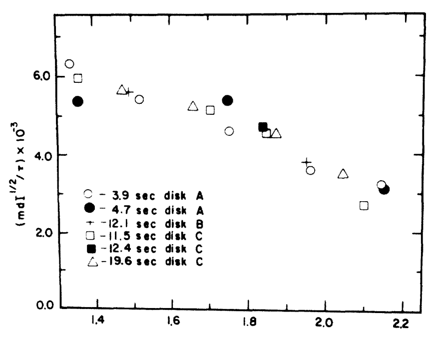

FIG. 7. Temperature dependence of the slope m of the steep damping curves at high amplitude. I is the moment of inertia, τ is the period, and d is the disk thickness.

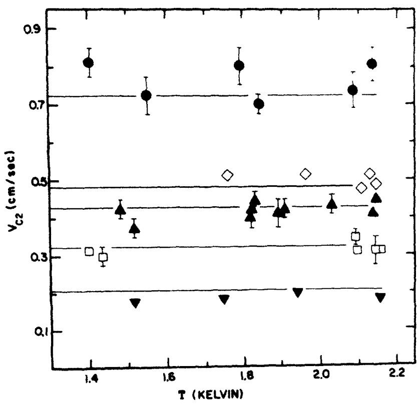

FIG. 8. Critical velocities v_{c2} associated with the flat damping curves. Symbols representing various periods of oscillation are: ● 3.9 sec, ◊ 8.84 sec, ▲ 12.0 sec, □ 19.6 sec, ▼ 45.0 sec. The solid horizontal lines correspond to the relation $v_{c2} = 1.4\,\tau^{-1/2}$

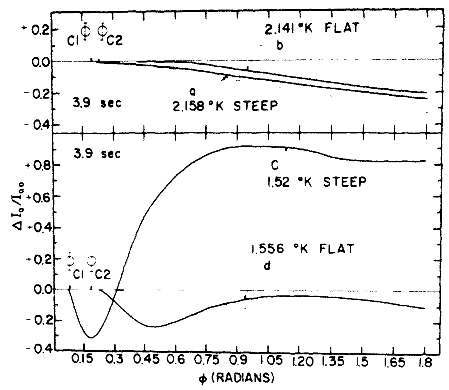

FIG. 10. Amplitude dependence of the relative change in added moment of inertia I_a for both steep and flat damping curves, at two different temperatures for a 3.9-sec period. The broken-line curves represent what is expected from secondary flow of the normal fluid only.

tween 3.9 and 45.0 sec, v_{c2} is independent of temperature. The period dependence is given by $v_{c2} = 1.4\tau^{-1/2}$ cm/sec, which is represented by the solid lines in Fig. 8. The critical velocities v_{c1} for the steep curves are shown in Fig. 9. For periods between 3.9 and 19.6 sec, we find that the period dependence at the lower temperatures is given by $v_{c1} = 0.7\tau^{-1/2}$ cm/sec. The results of Gamtsemlidze[2] ($\tau = 8.95$ sec) and Hollis Hallett[1] ($\tau = 11.0$ sec) are indicated by the broken line and the crosses, respectively.

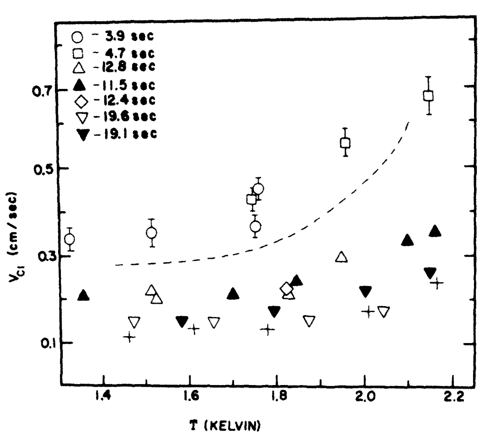

FIG. 9. Critical velocities v_{c1} associated with the steep damping curves. The broken-line curve represents the results of Gamtsemlidze for 8.95 sec, and the crosses represent the results of Hollis Hallett for 11.0 sec.

C. Period Measurements

The period of oscillation provides a measure of the added moment of inertia $I_a = \alpha(\tau^2 - \tau_v^2)$, where τ is the period of the disk oscillating in the liquid, τ_v is the period in vacuum, and α is the torsion constant. The relative change in I_a with amplitude is then given by

$$(I_a - I_{a0})/I_{a0} = (\tau^2 - \tau_0^2)/(\tau_0^2 - \tau_v^2)\,, \qquad (3)$$

where I_{a0} and τ_0 are low-amplitude values. Our results for a 3.9-sec period at a high and low temperature for both steep and flat curves are shown in Fig. 10. The low-temperature results can be compared with those in Fig. 11 for a 11.54-sec period. In both figures, the dashed curves rep-

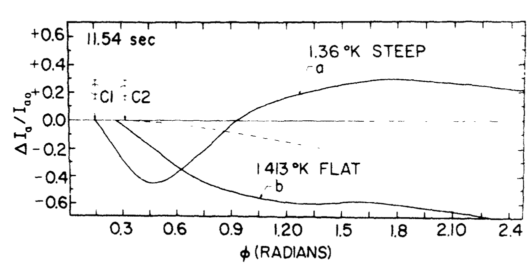

FIG. 11. Amplitude dependence of the relative change in added moment of inertia I_a for both a steep and a flat damping curve for a 11.54-sec period. The broken-line curve represents what is expected from secondary flow of the normal fluid only.

resent what is expected from secondary flow of the normal fluid only.[10]

D. Low-Amplitude Damping

We have compared our observed logarithmic decrement D_0 at low amplitude with a calculated value D_{calc} based upon η_n from the rotating cylinder viscometer[11] and ρ_n from the second sound value D_{calc} based upon η_n from the second sound velocity.[12] The expression for D_{calc} is the same as that used by Dash and Taylor,[13] but with a corner parameter of 0.30.[14] In general, D_{calc} and D_0 agree within experimental error, but at low fluid depth and large penetration depth, D_0 is significantly higher. On several occasions with the helium at constant temperature, D_0 was observed to increase as the fluid level fell to within 100 to 200 penetration depths of the disk surface. Figure 12 shows that $(D_0 - D_{\text{calc}})/D_{\text{calc}}$ depends only on the ratio h/λ, where h is the fluid level above the disk and λ is the penetration depth. The results in Fig. 12 are independent of whether the damping curve is steep or flat, or whether the shield was present, or which disk was used.

It appears that in superfluid helium the free surface can become a serious source of energy loss in oscillating disk experiments when the fluid level is within a few hundred penetration depths. This result may explain the high values of η_n obtained with a single disk[13] (at temperatures below 1.5 °K) as compared with other methods.[11,15] Comparison with the results of Folse[16] shows that this depth effect does not occur in classical liquids.

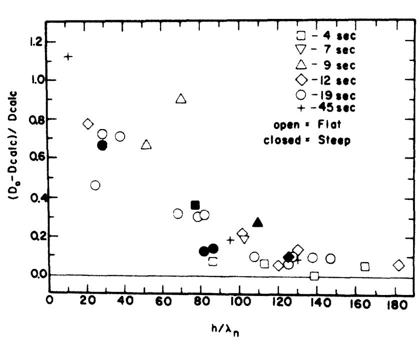

FIG. 12. Relative change of the measured low-amplitude logarithmic decrement D_0 from the calculated value D_{calc} as a function of the ratio of bath height h above the disk to the penetration depth λ_n, for both steep and flat curves.

IV. DISCUSSION

A. Types of Damping Curves

The excess energy dissipation at high amplitudes in experiments of this type is usually attributed to turbulence in the superfluid. Hall[7] and Vinen[17] have shown how the concept of superfluid turbulence can be introduced in the framework of a quantized vortex-line model.[18,19]

In the present investigation, we observe that the type of damping curve depends on the time interval Δt from reaching the λ point to the initial deflection of the disk. One explanation of this behavior is that the superfluid is created originally in a state of disorganized motion and if the system is undisturbed (the disk is not deflected) this random primordial flow eventually decays, leaving the superfluid relatively motionless, so that subsequent measurements produce flat curves; but if soon after reaching T_λ the disk is deflected to high amplitudes, then the initial random motion of the superfluid is organized into a long-lived state, and a steep curve results. However, since the low-amplitude damping is the same for both steep and flat curves, this long-lived motion of the superfluid must not interact with the disk at low amplitude. In the framework of the vortex-line picture, we could interpret the long-lived superfluid motion in the steep case as some sort of organized array of quantized vortex lines. It should be noted, however, that whatever the subcritical vortex configuration is for the steep curves, it remains present for long periods of time (at least 8 h for a series of curves and 70 min in between curves) and may persist indefinitely. We never observed a transition from one type of curve to another, as long as the temperature remained below T_λ.

Results which depend on the previous history of the bath have been observed in a number of other liquid-helium experiments, especially in those dealing with thermal conterflow in wide channels[20-23] (diam > 10^{-3} cm). All of the history effects in the thermal counterflow experiments were attributed to the presence of lingering vorticity in the superfluid. In the rotating-beaker experiment of Reppy and Lane,[24] two different results were obtained: Usually the superfluid would absorb angular momentum from the coasting beaker, occasionally it would not; these results also were attributed to the presence or absence of primordial vortex lines.

Judging from the magnitude and temperature dependence of the critical velocities and the qualitative shape of the logarithmic decrement curves for a single disk observed by both Hollis Hallett[1] and Gamtsemlidze,[2] we can classify their data in the steep category. However, some of the damping curves shown for a pile of disks in the pioneering work of Hollis Hallett are qualitatively different, and may be of the flat type.

B. Critical Velocities

The temperature independence of v_{c2} (between 1.4 and 2.15 °K) reported here is in sharp disagreement with the $\rho_s^{-1/2}$ dependence found in most previous experiments of this type. However, there have been many other types of experiment which have yielded temperature-independent critical velocities. For example, Ricci and Vicentini-Missoni[25] have used a beam of ions to study heat flow in wide channels (0.8 cm) and have found critical velocities which were constant from 0.89 to 1.92 °K; Vermeer *et al.* and Van Alphen, *et al.*[26] have performed measurements of gravitational superfluid flow in wide channels with the normal fluid held motionless by means of superleaks and have found v_c to be independent of temperature for a wide range of channel size (from 0.44 to 0.0015 cm); Keller and Hammel[27] have studied isothermal gravitational flow through two narrow slits (2.29×10^{-4} cm and 3.1×10^{-5} cm) and have found v_c to be independent of temperature except for a small region in the vicinity of T_λ, where v_c falls rapidly to zero. Similar behavior is also found in film flow of He II,[28,29] although the results are complicated by uncertainties in the film thickness and the appropriate value of ρ_s.

Keller and Hammel have observed that the narrow region $T_\lambda - T$, in which v_c drops from its constant low-temperature value to zero at T_λ, decreases with increasing channel width; Clow and Reppy[30] have seen the same result with a superfluid gyroscope. In view of this it is understandable that we observe no decrease in v_{c2} up to the highest temperature (2.156 °K), since our smallest characteristic dimension was $\lambda \approx 0.014$ cm. Langer and Fisher[31] have developed a theory to explain this decrease of v_c near T_λ based on the idea that near T_λ there should be an intrinsic critical velocity for the superfluid proportional to the superfluid density ρ_s.

Vinen[32] distinguishes between two types of critical velocity, an "ideal" type in which the subcritical superfluid flow is frictionless and free of vortex lines, and a "nonideal" type in which vortex lines are present in the subcritical flow. One would expect that the nonideal critical velocity would ordinarily be lower than the ideal type because the vortex lines in the subcritical region would act as nuclei for the growth of more line. However, it is difficult to determine what types of experiment give ideal critical velocities; persistent current and film flow experiments are suggested by Vinen. Since these experiments lead to critical velocities which are probably temperature-independent (except near T_λ), it is possible that temperature independence may be an identifying feature of ideal critical velocities. If these ideas are correct, it is reasonable to identify the critical velocities obtained from our flat damping curves as ideal and those obtained from the steep damping curves as nonideal.

Wilks[32a] has correlated critical velocities in wide channels ($>10^{-3}$ cm) by means of the expression

$$v_c = (4\hbar/Md)\,[\ln(d/3a_0)] \quad , \tag{4}$$

where d is a characteristic length, $a_0 = 1.2\times10^{-8}$ cm, and M is the mass of a helium atom. Since a_0 is very small, the logarithmic factor is relatively insensitive to the value of d, so Eq. (4) implies $v_c d \approx$ const. Therefore, our result $v_{c2}\tau^{1/2} = 1.4$ suggests that d is a penetration depth based on an effective kinematic viscosity ν which is independent of temperature. The value of ν obtained in this manner is $\nu = 1.2\times10^{-4}$ cm²/sec, which is smaller than the normal-fluid kinematic viscosity ν_n. We associate with v_{c2} a critical value of the Reynolds number (Re) defined by

$$\mathrm{Re} = v\lambda/\nu = v(\tau/\pi\nu)^{1/2} \quad . \tag{5}$$

Re has a critical value of 72.

Van Alphen *et al.*[33] have proposed that in experiments where the normal-fluid velocity v_n is not negligible the critical velocities are related to classical turbulence in the normal fluid. They propose a Reynolds number $\mathrm{Re}_{v_n} = d\rho v_n/\eta_n$ which is based on the normal-fluid viscosity η_n and the total density ρ. Using $d = (\eta_n\tau/\pi\rho)^{1/2}$ and $v_n = v_{c2}$, we find that the critical value of their Reynolds number ranges from 64 to 84 between 2.15 and 1.3 °K. Our value of 72 is in good agreement, as would be expected since our effective ν is approximately equal to η_n/ρ. In spite of this agreement, we do not believe that a transition to classical turbulence takes place at this value of Re. We have made measurements in helium gas at 4.2 °K and in liquid helium I at 2.4 °K up to Re = 430 and have observed only gradual increases in damping consistent with the small increases expected from secondary flow.[10] The measurements of Folse extended up to Re = 210 in water; no sharp changes in damping were seen. A transition to turbulence would presumably be accompanied by sharp increases in damping above the critical amplitude, such as those observed by Benson and Hollis Hallett[3] for an oscillating sphere in helium gas (Re = 230) and in helium I (Re = 210). Therefore, we conclude that although Re_{v_n} may be an important similarity parameter for liquid-helium critical velocities, the critical value of 72 does not correspond to classical turbulence in the normal fluid.

C. Other Results

1. *Effect of Mutual Friction*

Zwanikken[34] derived an expression for the excess energy dissipation due to the mutual friction[35] between the normal and superfluids in the case of the oscillating disk. Hollis Hallett[1] applied Zwanikken's analysis to his results and found the predicted energy dissipation to be smaller than observed and to have a different temperature variation. However, there appears to be a sign error in Hollis Hallett's expression. Furthermore, to apply Zwanikken's result to our disk requires more terms than orginally included because of the large radius of our disk. Therefore, we have extended Zwanikken's analysis and have found that the logarithmic decrement due to mutual friction can be written

$$D = A\rho_s\rho_n\lambda(\pi^2R^4/8I) \times \epsilon[1+\Phi_1\epsilon+\Phi_2\epsilon^2+\Phi_3\epsilon^3+\Phi_4\epsilon^4], \quad (6)$$

where A is the Gorter-Mellink mutual friction parameter, λ is the normal-fluid penetration depth, ϵ is defined to be $\omega R^2\varphi^2$(φ is the amplitude), and the Φ_i terms depend only on temperature. Tabulated values of the major temperature-dependent terms are given in Table II. The values of A were read from a drawing in a paper by Vinen[23] (the value at 2.15 °K was obtained by extrapolation). Using the table and Eq. (6) to calculate the damping at an amplitude just above the critical amplitude, and comparing the result with our data, we find that the calculated value is several orders of magnitude too large and highly temperature-dependent. For example, at $\varphi = 0.6$ rad, $\tau = 12$ sec, and $T = 1.9$ °K, the calculated excess decrement is 1.76, and the observed value is 0.002. However, if we try to obtain better agreement with the flat-curve results by assuming that, instead of the amplitude φ, a relative amplitude $\varphi - \varphi_0$ is the relevant quantity, we obtain the results shown in Fig. 13 where the upper curve is for a 12-sec period and the lower curve is for a 4-sec period. The points correspond to the observed damping at an amplitude of 0.6 rad for the 12-sec period and 0.3 rad for the 4-sec period. The curves were obtained by matching the data at 2.1 °K, which required $\epsilon = 0.042$ for both periods. The result, however, is still in disagreement with observation, since the experimental points show that the excess damping is approximately independent of temperature, and the theory predicts a strong temperature dependence. In summary, we find that the mutual friction force would give an excess decrement which is orders of magnitude larger than observed, and that, even when the theory is adjusted to fit the data at 2.1 °K, the temperature dependence is incorrect.

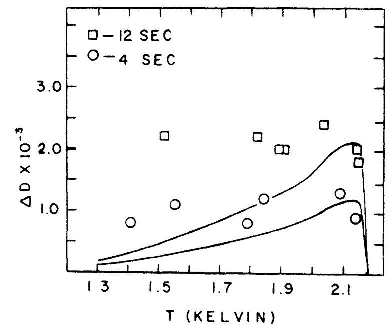

FIG. 13. Comparison of the observed excess damping (ΔD) for a 4-sec period at 0.3 rad and a 12-sec period at 0.6 rad with values calculated from the mutual friction theory of Zwanikken (solid curves). The curves are made to agree with the observed values at 2.1 °K by means of reduced amplitude ($\varphi - \varphi_0$).

2. *Period Measurements*

The relative change in added moment of inertia I_a shown in Figs. 10 and 11 gives a measure of the amount of fluid being dragged by the disk. If the superfluid is "entrained" at high amplitudes, as suggested by Donnelly and Hollis Hallett,[6] then

TABLE II. Parameters used in the calculation of energy dissipation due to mutual friction.

T (°K)	A (cm sec/g)	ρ_s (g/cm^3)	Φ_1 (sec/cm^2)	Φ_2 (sec/cm^2)^2	Φ_3 (sec/cm^2)^3	Φ_4 (sec/cm^2)^4
2.15	250	0.01644	1.233	449.6	369.2	77 900
2.1	190	0.03617	2.062	175.4	239.4	11 340
2.0	130	0.0632	2.465	44.22	66.39	561.8
1.9	105	0.0838	2.640	14.72	22.24	49.25
1.8	88	0.0999	2.637	6.340	7.477	5.697
1.6	60	0.1218	2.192	2.372	1.359	0.4973
1.4	38	0.1344	1.532	1.229	0.535	0.1113
1.3	28	0.1383	1.162	0.746	0.264	0.0439

we would expect I_a to be significantly increased, particularly at the lower temperatures. Such behavior is indeed observed for the steep curves. If the effective viscosity is assumed to be approximately η_n, then the fraction of superfluid entrained can be calculated: The result for the data of Figs. 10 and 11 is 30% for the 3.9-sec period at 1.52 °K but only 5% for the 11.54-sec period at 1.36 °K. The flat curves, on the other hand, show no evidence of entrainment and are in qualitative agreement with the results for classical liquids. At temperatures close to T_λ the results for both steep and flat curves are consistent with the changes observed in ordinary liquids (dashed lines). Due to the fiber correction and the difficulty of determining I_{a0} precisely, the results shown in Figs. 10 and 11 are subject to large error, so only qualitative conclusions can be drawn. However, the minimum in $\Delta I_a/I_{a0}$ seen at the lower temperatures was observed repeatedly, so it is believed to be a real effect, although its origin is obscure.

V. SUMMARY AND CONCLUSIONS

Our major conclusions are: (i) that the damping curves and critical velocities observed in liquid helium II with an oscillating disk are strongly dependent on the history of the helium bath, and (ii) that when the bath is not disturbed for about 1 h after reaching T_λ, the resulting critical velocities are independent of temperature and vary with period as $\tau^{-1/2}$.

We have conjectured that when the disk is set in motion shortly after reaching T_λ, the initial random motion of the superfluid is organized into a long-lived state of motion. This long-lived state has the following features: (i) it does not interact with the low-amplitude motion of the disk. (ii) it causes the critical velocities to depend upon temperature and disk roughness, (iii) it causes the superfluid to be entrained at high amplitudes, and (iv) it persists as long as the temperature remains below T_λ. These results suggest that the long-lived state consists of a stable array of vortex lines and that the resulting critical velocities are of the nonideal type.

When the disk is not deflected for 1 h after reaching T_λ, the initial unorganized motion of the superfluid dies out and consequent measurements reveal the following features: (i) the critical velocities are higher than previously observed, are independent of temperature, and (possibly) are independent of disk roughness, and (ii) the superfluid is not entrained at high amplitudes. Since the subcritical flow should be free of vorticity in this case, the critical velocities are presumed to be of the ideal type. From these critical velocities we obtain a temperature-independent kinematic viscosity, $\nu = 1.2\times10^{-4}$ cm^2/sec, which is then used to calculate a critical Reynolds number, Re = 72. This value is in good agreement with the value calculated from the Reynolds number proposed by Van Alphen *et al.*[33] However, we reject the idea that these critical velocities are due to turbulence in the normal fluid, because (i) the excess damping is much larger than observed in ordinary liquids (see Fig. 4), and (ii) no such critical value of the Reynolds number has been observed with oscillating disks in ordinary liquids.

The excess decrement at an amplitude slightly above critical was found to be temperature-independent for the flat curves. A calculation of the excess decrement expected from mutual friction (based on a theory of Zwanikken) gave values which changed strongly with temperature and were much larger than observed.

The value of the low-amplitude damping for both steep and flat curves was observed to increase when the bath height fell to within about 100 penetration depths of the disk, suggesting a connection between the observed damping and the energy dissipated at the free surface of the helium bath. This effect does not occur in classical liquids.

---

†Work supported by the National Science Foundation.

[1]A. C. Hollis Hallett, Proc. Roy. Soc. (London) A210, 404 (1952).

[2]G. A. Gamtsemlidze, Zh. Eksperim. i Teor. Fiz. 34, 1434 (1958) [English transl.: Soviet Phys. – JETP 37, 678 (1960)].

[3]C. B. Benson and A. C. Hollis Hallett, Can. J. Phys. 34, 668 (1956).

[4]R. Dean Taylor and J. G. Dash, in Proceedings of the Fifth International Conference on Low Temperature Physics and Chemistry, edited by J. R. Dillinger (The University of Wisconsin Press, Madison, Wisc., 1958). p. 164.

[5]E. L. Andronikashvili, G. A. Gamtsemlidze, and Sh. A. Dzhaparidze, Zh. Eksperim. i Teor. Fiz. 50, 856 (1966) [English transl.: Soviet Phys. – JETP 23, 568 (1966)].

[6]R. J. Donnelly and A. C. Hollis Hallett, Ann. Phys. (N. Y.) 3, 320 (1958).

[7]H. E. Hall, Advan. Phys. (Phil. Mag. Suppl.) 9, 89 (1960).

[8]E. J. Walker, Rev. Sci. Instr. 30, 834 (1959).

[9]S. Rosenblat, J. Fluid Mech. 6, 206 (1959).

[10]R. F. Folse, R. G. Hussey, and J. S. Bosnak, Phys. Fluids, 11, 2508 (1968).

[11]A. D. B. Woods and A. C. Hollis Hallett, Can. J. Phys. 41, 596 (1963).

[12]R. G. Hussey, B. J. Good, and J. M. Reynolds, Phys. Fluids 10, 89 (1967).

[13]J. G. Dash and R. D. Taylor, Phys. Rev. 105, 7

(1957).
[14]R. F. Folse and R. G. Hussey, Phys. Fluids 11, 2278 (1968).
[15]D. F. Brewer and D. O. Edwards, Proc. Roy. Soc. (London) A251, 247 (1959).
[16]R. F. Folse (unpublished).
[17]W. F. Vinen, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1961), Vol. 3, p. 1.
[18]L. Onsager, Nuovo Cimento Suppl. 6, 249 (1949).
[19]R. P. Feynman, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1955), Vol. 1, p. 36.
[20]D. F. Brewer, D. O. Edwards, and K. Mendelssohn, Phil. Mag. 1, 1130 (1958).
[21]D. F. Brewer and D. O. Edwards, Phil. Mag. 6, 775 (1961).
[22]K. Mendelssohn and W. A. Steele, Proc. Phys. Soc. (London) A73, 144 (1959).
[23]W. F. Vinen, Proc. Roy. Soc. (London) A240, 128 (1957).
[24]J. D. Reppy and C. T. Lane, Phys. Rev. 140, 106 (1965).
[25]M. V. Ricci and M. Vicentini-Missoni, Phys. Rev. 158, 153 (1967).
[26]W. Vermeer, W. M. Van Alphen, J. F. Olijhoek, K. W. Taconis, and R. De Bruyn Ouboter, Phys. Letters 18, 265 (1965); W. M. Van Alphen, J. F. Olijhoek, R. De Bruyn Ouboter, and K. W. Taconis, Physica 32, 1901 (1966).
[27]W. E. Keller and E. F. Hammel, Physics 2, 221 (1966).
[28]K. Mendelssohn and G. K. White, Proc. Phys. Soc. (London) A63, 1328 (1950).
[29]G. R. Hebert, K. L. Chopra, and J. B. Brown, Phys. Rev. 106, 391 (1957).
[30]J. R. Clow and J. D. Reppy, Phys. Rev. Letters 19, 291 (1967).
[31]J. S. Langer and M. E. Fisher, Phys. Rev. Letters 19, 560 (1967).
[32]W. F. Vinen, in Liquid Helium, edited by G. Careri (Academic Press Inc., New York, 1963), p. 336.
[32a]J. Wilks, The Properties of Liquid and Solid Helium (Clarendon Press, Oxford, England, 1967), p. 392.
[33]W. M. Van Alphen, G. J. Haasteren, R. De Bruyn Ouboter, and K. W. Taconis, Phys. Letters 20, 474 (1966).
[34]G. C. J. Zwanikken, Physica 16, 805 (1950).
[35]C. J. Gorter and J. H. Mellink, Physica 15, 285 (1949); C. J. Gorter, P. W. Kasteleijn, and J. H. Mellink, *ibid.* 16, 113 (1950).

PHYSICAL REVIEW VOLUME 188, NUMBER 1 5 DECEMBER 1969

Submillimeter-Wave Spectra of Ammonia and Phosphine†

Paul Helminger and Walter Gordy
Department of Physics, Duke University, Durham, North Carolina 27706
(Received 28 July 1969)

Rotational transitions of different isotopic species of ammonia and phosphine have been observed in the $\frac{1}{2}$- to 1-mm region. The frequency of the $J=0\rightarrow 1$ transition of $^{14}NH_3$ is $572\,496.69\pm 0.60$ Mc/sec and that for $^{15}NH_3$ is $572\,053.18\pm 0.50$ Mc/sec. Spectral constants B_0 observed for the different isotopic species (in Mc/sec) are: 298 114.68 for $^{14}NH_3$, 297 359.32 for $^{15}NH_3$, 154 173.25 for $^{14}ND_3$, and 153 600.82 for $^{15}ND_3$. Structural dimensions for the ground vibrational state of ammonia obtained by isotopic substitution are 1.0136 Å for the bond length and 107°3′ for the bond angle. For PH_3, the rotational constants obtained (in Mc/sec) are: $B_0=133\,480.15$, $D_J=3.95$, and $D_{JK}=-5.18$; for PD_3 they are $B_0=69\,471.09$, $D_J=1.02$, and $D_{JK}=-1.31$. An upper limit of $\frac{1}{2}$ Mc has been put on the unknown inversion frequency of PH_3.

INTRODUCTION

Although there has been an extensive amount of work done in the centimeter-wave region on the inversion spectrum of ammonia and its hyperfine structure, there have been no previous measurements of the pure rotational spectrum by means of microwave techniques, except for the deuterated species.[1] As a consequence, no microwave determination of the ground-state rotational constant B_0 for NH_3 has been available. Work in the infrared region[2-5] has provided a value for B_0 through the analysis of rotation and rotation-vibration spectra, but a much more precise value can be expected from microwave measurements. The reason for the delay in this measurement for this light simple