The surface-plasmon absorption is readily apparent in graphs of  $\epsilon_2$  computed from ellipsometry data, and the resonant wavelength is observed to relax to progressively longer wavelengths as the age of the foil increases. This provides a sensitive measure of the formation of a surface corrosion layer as a function of time and environmental conditions. On the basis of computed values of  $\epsilon_1$ , it is concluded that the Ag-vacuum resonant wavelength is approximately 3400 Å; in contrast, the resonant wavelength is always greater than or equal to 3450 Å after exposure of the foil to the atmosphere.

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# Lattice Thermal Conductivity of Superconducting Niobium Carbide\*

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The unusually low lattice thermal conductivity of the transition-metal carbides at low temperatures has been attributed to the scattering of phonons by conduction electrons. An experiment which confirms this interpretation is reported. Thermal conductivity measurements on single-crystal NbC0.96 through its superconducting critical temperature (9.8°K) show that the lattice component of the thermal conductivity, K1s, increases greatly below Te because of decreased phonon-electron scattering. The maximum increase in lattice thermal conductivity from this effect occurs at 3°K with  $K_{ls}/K_{ln}$  equal to 160. The theories of Bardeen, Rickayzen, and Tewordt and of Klemens and Tewordt for the influence of electrons and point defects on lattice conductivity provide a quantitative interpretation of the effect. The best fit to the NbC<sub>0.96</sub> data is for  $2\epsilon(o)/kT_c = 4.0$ . The behavior of NbC<sub>0.96</sub> is in contrast to that of NbC<sub>0.76</sub>, which remains in the normal state throughout the temperature interval studied and shows no increase in thermal conductivity.

# I. INTRODUCTION

**P**REVIOUS measurements<sup>1,2</sup> of the thermal conduc-tivity of Crewer W tivity of Groups IV and V transition-metal carbides have shown that strong phonon-electron scattering must be invoked to explain the low-temperature behavior of the lattice thermal conductivity. It was also noted, however, that the ordinary theory of phononelectron scattering does not apply to carbide samples which are highly carbon deficient. Hence, some experimental demonstration of the above interpretation was sought.

In this paper, we report measurements on the thermal conductivity of superconducting niobium carbide which do demonstrate experimentally the existence of strong phonon-electron scattering in the transition-metal carbides. Use is made of the fact that some of the carbides have superconducting critical temperatures in the range

of interest. Since electrons in the superconducting state do not scatter phonons, an increase in lattice conductivity below  $T_c$  is predicted and observed. A comparison of the data with the theories of Bardeen, Rickayzen, and Tewordt<sup>3</sup> (BRT) and Klemens and Tewordt<sup>4</sup> (KT) shows quantitative agreement for the effect of phononelectron scattering and point-defect scattering on the lattice thermal conductivity.

#### **II. EXPERIMENTAL**

Two specimens of single-crystal niobium carbide,  $\rm NbC_{0.76}\ ^{5}$  and  $\rm NbC_{0.96},^{6}$  were studied. The specimens, which crystallize in the rocksalt structure, are carbon

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<sup>&</sup>lt;sup>8</sup> J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959). <sup>4</sup> P. G. Klemens and L. Tewordt, Rev. Mod. Phys. 36, 118 (1964).

<sup>&</sup>lt;sup>6</sup> This specimen has been characterized in (1).

<sup>&</sup>lt;sup>6</sup> This specimer was obtained from Ventron Electronics Corporation, Alfa Crystals Division. The carbon-to-metal ratio of 0.96 was determined from measurements of the superconducting transition temperature  $T_e$  and comparison with data of Toth *et al.* See L. E. Toth, M. Ishikawa, and Y. A. Chang, Acta Met. 16, 1183 (1968).



FIG. 1. Thermal conductivity versus temperature: open circles,  $NbC_{0.96}$ ; crosses,  $NbC_{0.76}$ . The dashed line is an extrapolated curve which describes the normal-state behavior.

deficient with vacancy concentrations of 24 and 4%, respectively, in the fcc carbon sublattice. Thermal conductivity was measured using the longitudinal heat flow technique with Au-Fe thermocouples as temperature indicators. Details of the measurement have been reported.<sup>1.2</sup>

## III. RESULTS

Figure 1 exhibits the thermal conductivity of NbC<sub>0.76</sub> and NbC<sub>0.96</sub> in the temperature range 1.65–78°K. The data for NbC<sub>0.76</sub> were reported in an earlier paper,<sup>1</sup> but to facilitate comparison with NbC<sub>0.96</sub> both sets of data are included.

The most striking feature of Fig. 1 is the large increase in the total thermal conductivity K below the superconducting transition temperature  $T_c$  for NbC<sub>0.96</sub>. At 3°K, the ratio of K in the superconducting state to that in the normal state,  $K_s/K_n$ , is nearly 70. The normal-state behavior for NbC<sub>0.96</sub> below  $T_c$  was obtained by extrapolating the data in the range 10–15°K to lower temperatures. We believe that this extrapolation procedure is valid because the slopes of the K(T) curves for the carbides change only slightly below 15°K.<sup>1</sup> The superconducting transition temperature for NbC<sub>0.76</sub> is less than 2°K and could not be observed with our apparatus.

A second feature of Fig. 1 is the dependence of the thermal conductivity on vacancy concentration. At temperatures below 20°K, the thermal conductivity is lower for the crystal with the lower vacancy concentration. This result is just the opposite of the behavior expected for scattering by point defects in lattice *or* electronic thermal conduction. (At higher temperatures, the presence of point-defect scattering is more ap-

parent.<sup>2</sup>) Such behavior was observed in measurements of K for TiC<sub>x</sub> as a function of  $x^{1,2}$  and led us to invoke strong phonon-electron scattering to explain the lowtemperature data. For NbC<sub>x</sub>, as x is increased, the electronic density of states at the Fermi surface also increases, according to the specific-heat measurements of Toth *et al.*<sup>6</sup> Hence the scattering probability is increased, and the lattice conductivity is decreased. The electronic component  $K_e$  is still limited by vacancy scattering as is evidenced by measurements of the residual electrical resistivity.

## IV. DISCUSSION

To discuss the behavior of  $K_e$  and  $K_l$  in detail and to compare the present results with theory, we have calculated the superconducting and normal components of the electronic thermal conductivity  $K_{es}$  and  $K_{en}$ . The electrical resistivity of NbC<sub>0.96</sub> changes only slightly, from  $28.8 \,\mu\Omega$  cm at  $77^{\circ}$ K to  $26.7 \,\mu\Omega$  cm at  $10^{\circ}$ K. Most of the resistance is therefore attributable to elastic electron-vacancy scattering, so that  $K_{en}$  can be calculated using the Wiedemann-Franz Law. Values of  $K_{es}$  were obtained from the BRT theory for the case of strong electron-defect scattering. The resulting smoothed curves are shown in Fig. 2. As is evident from Figs. 1 and 2, the normal state values of  $K_e$  contribute significantly to K for  $NbC_{0.96}$  at low temperatures. This was not the case for  $NbC_{0.76}$  where the residual resistivity  $172 \,\mu\Omega$  cm is much higher.

From the experimental data along with the calculated values of  $K_{es}$  and  $K_{en}$ , the superconducting and normal components of the lattice thermal conductivity,  $K_{ls}$  and  $K_{ln}$ , were determined. The smoothed curves are shown in Fig. 3 along with  $K_{ln}$  for NbC<sub>0.76</sub>. Curve A is the superconducting component of the lattice thermal conductivity of NbC<sub>0.96</sub>, and curves B and C are the



FIG. 2. Electronic thermal conductivity versus temperature of NbC<sub>0.96</sub> in the normal and superconducting states.



FIG. 3. Lattice thermal conductivity versus temperature: A, superconducting NbC<sub>0.96</sub>; B, normal NbC<sub>0.76</sub>; C, normal NbC<sub>0.96</sub>.

normal lattice components of NbC<sub>0.76</sub> and NbC<sub>0.96</sub>, respectively. In the superconducting state, the calculated value of the phonon mean free path at 1.65°K is close to the smallest dimension of the sample. Hence, boundary scattering of phonons lowers  $K_{ls}$  below the conductivity maximum. In the temperature region 3–9.8°K, the increase in  $K_{ls}$  of NbC<sub>0.96</sub> is due to decreased phonon-electron scattering with  $K_{ls}(3^{\circ}K)/K_{ls}(9.8^{\circ}K)$ = 17 and  $K_{ls}(3^{\circ}K)/K_{ln}(3^{\circ}K)$  = 160. The latter ratio demonstrates that phonon-electron scattering is indeed a strong scattering mechanism in the carbides.

The large ratio of  $K_{ls}/K_{ln}$  below  $T_c$  suggested a comparison with the theory of the thermal conductivity of superconductors by Bardeen, Rickayzen, and Tewordt.<sup>3</sup> BRT have calculated the effect of electrons on the lattice conductivity below  $T_c$ . The BRT theory, which is based on the BCS theory of superconductivity,<sup>7</sup> predicts that in the weak-coupling limit, the ratio  $K_{ls}/K_{ln}$ should be a universal function of  $T/T_c$ . For NbC<sub>0.96</sub>,  $T_c/\theta_D$  equals 0.0162, and the weak-coupling approximation should be valid. This result is supported by lowtemperature measurements of the electronic heat capacity of NbC<sub>0.98</sub>,<sup>8</sup> which closely follow the values calculated for the BCS weak-coupling case.

Curve C of Fig. 4 shows the BRT result for  $K_{ls}/K_{ln}$ as a function of  $T/T_c$  for  $2\epsilon(0)/kT_c$  equal to 4. The quantity  $2\epsilon(0)$  is the gap energy at T=0. If the ratio  $2\epsilon(0)/kT_c$  is reduced to the BCS value of 3.5,  $K_{ls}/K_{ln}$  will be lowered. Since the gap energy for NbC<sub>0.96</sub> has not yet been measured, the value of  $2\epsilon(0)/kT_c$  is treated as a parameter. Finally, curve C is not extended below  $T/T_c=0.4$  because boundary scattering begins to dominate at low temperatures, and the theory is inapplicable.

In addition to boundary scattering, point-defect scattering can also affect  $K_{ls}/K_{ln}$ . Klemens and Tewordt<sup>4</sup> have shown that point defects can significantly reduce  $K_{ls}/K_{ln}$ , which was calculated on the basis of only phonon-electron scattering. The effect is more pronounced at lower reduced temperatures where the phonon-electron scattering in the superconducting state is greatly decreased. For the carbides, it has been shown<sup>2</sup> that the main contribution to the scattering coefficient comes from force-constant differences rather than mass differences. In the KT notation,  $\alpha$  for NbC<sub>0.96</sub> equals  $1.8 \times 10^{-5}$  for mass-defect scattering and  $2.8 \times 10^{-3}$  for scattering due to force-constant differences. The coefficient  $\alpha$  measures the scattering strength and concentration of point defects. Curve B of Fig. 4 is the calculated value of  $K_{ls}/K_{ln}$  for  $\alpha = 2.8 \times 10^{-3}$  and  $2\epsilon(0)/kT_c$ =4.0. The latter ratio gives the best visual fit to the smoothed data, curve A. Since 4.0 is the upper limit of



FIG. 4. Ratio of the lattice thermal conductivity in the superconducting and normal states versus reduced temperature: A, derived data of NbC<sub>0.96</sub>; B, BRT calculated curve including pointdefect scattering for NbC<sub>0.96</sub>; C, BRT calculated curve. Curves B and C are shown for  $2\epsilon(0)/kT_c=4.0$ .

<sup>&</sup>lt;sup>7</sup> J. Bardeen, L. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

<sup>&</sup>lt;sup>8</sup> T. H. Geballe, B. T. Mathias, J. P. Remeika, A. M. Clogston, V. B. Compton, J. P. Maita, and H. J. Williams, Physics 2, 293 (1966).

BRT theory, the value of  $2\epsilon(0)/kT_c$  for NbC<sub>0.96</sub> is fairly closely determined by the present results. Curves for the choice  $2\epsilon(0)/kT_c=3.5$  and 3.7 were also calculated. With the inclusion of point-defect scattering, known to be present here, these values are clearly too low to fit the data.

The value of 4.0 for  $2\epsilon(0)/kT_c$  corresponds to a value of the zero-temperature energy gap of 0.0034 eV. This value is subject to some uncertainty. First, it must be noted that  $K_{ln}$  was derived from an extrapolated curve, and this procedure introduces an estimated uncertainty of  $\pm 10\%$  into  $K_n$ . Second, for the special case of the carbides, the applicability of the BRT theory must be further examined. Although NbC<sub>0.96</sub> appears to satisfy the weak-coupling limit treated by BRT, a second factor must also be considered. In the BRT theory, the electron mean free path is assumed to be longer than the dominant phonon wavelength. Studies of the carbides indicate that for high vacancy concentrations,  $\lambda_e$ is small compared to an average phonon wavelength at low temperatures. This result alters the frequency dependence of the phonon relaxation time.<sup>9</sup> Resistivity measurements indicate, however, that the electron mean free path  $\lambda_e$  in NbC<sub>0.96</sub> is much greater than  $\lambda_e$ for NbC<sub>0.76</sub>, and thus the ordinary theory of phononelectron scattering may apply to  $NbC_{0.96}$ . This result is consistent with the observed higher slope value of  $\ln K$  versus  $\ln T$  for NbC<sub>0.96</sub>. In Fig. 3, the slope of curve C for  $NbC_{0.96}$  is slightly greater than 2 in the low-temperature region. This result is in contrast to the data for the other carbides which always gave slope values between 1 and 2. A slope value greater than 2 suggests that the ordinary theory of phonon-electron scattering is applicable to NbC<sub>0.96</sub> since this theory predicts a slope of 2. When combined with the boundary scattering, the slope will lie between 2 and 3. A similar behavior was noted for TiC specimens with higher carbon contents.<sup>1,2</sup> The uncertainty in  $K_{ln}$  for NbC<sub>0.96</sub> due to extrapolation of the data below  $T_c$  should not alter any of the above qualitative remarks, however. The above discussion thus suggests that the largest uncertainty in  $2\epsilon(0)$  arises from the need to extrapolate  $K_n$  below  $T_c$ .

The value of  $2\epsilon(0)/kT_c$  for niobium metal is 3.64 from the work of Carlson and Satterthwaite.<sup>10</sup> This value was obtained by fitting the  $k_e$  data to the theory of Kadanoff and Martin.<sup>11</sup> Kadanoff and Martin modified the BRT treatment of  $k_e$  for the case of strong electron-phonon scattering. This modification is not required for the carbides because the electron scattering is defectcontrolled and, hence, elastic. Niobium metal also differs from its carbide in the fact that heat is transported primarily by electrons in the former. Hence, as the superconducting transition of niobium is reached from above, the measured thermal conductivity initially decreases, since electrons in the superconducting state cannot carry thermal energy. In niobium carbide, the measured thermal conductivity increases below  $T_c$  as electrons are removed as phonon scatterers. The values of  $T_c$  for Nb and NbC are similar, 9.3 and 9.8°K, respectively. At temperatures considerably below  $T_c$ (centered at approximately 2°K) a "phonon bump" appears in K versus T for Nb. The effect is smaller than in NbC<sub>0.96</sub>, but has the same origin. Other examples of the enhancement of lattice conductivity below the superconducting transition temperature are discussed by Mendelssohn and Rosenberg.<sup>12</sup>

To summarize our results, we have shown that measurements of the thermal conductivity of superconducting niobium carbide demonstrate experimentally the existence of strong phonon-electron scattering in materials of this type. The results are in reasonable agreement with theory for the effect of electrons and point defects on the lattice conductivity in the superconducting state.

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