lishing Co., Amsterdam, 1961), Chap. XII, § 5, p. 437.

[10]For example, in the weak-coupling low-density case, it was shown in I that $\chi = \zeta$.

[11]We use the word existence in the common elementary sense, as is done also in Refs. 4 and 7. For further discussion of our condition, see B. Bonifacio, L. M. Narducci, and E. Montaldi, Nuovo Cimento 47, 890 (1967).

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Addendum to Distortion Effects in the Elastic Scattering of 100- to 400-eV Electrons from Helium

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The differential cross section for the elastic scattering of 500-eV electrons from helium has recently been measured. The extended-polarization potential method used in our previous work is here applied at 500 eV and yields cross sections in good agreement with experiment. In addition, the explicit formulas used in this and our previous work for approximating the higher angular momentum phase shifts are displayed.

The differential cross section for the elastic scattering of 500-eV electrons from helium over a range of angles from 2° to 60° has recently been measured by Bromberg.[1] The relative success in the title work[2] (hereafter referred to as I) of the extended-polarization potential (EP) method in predicting high-energy differential cross sections for electrons off helium prompted us to extend this analysis to 500 eV. The results of this calculation are shown by the EP curve in Fig. 1 in comparison with Bromberg's data. Also, for comparison, the Born-approximation calculations of Khare and Moiseiwitsch,[3] as quoted by Bromberg,[1] are shown in the figure. It is again evident that the long-range distortion interaction is dominant in small-angle scattering and that the Born approximation is incapable of describing the scattering even at energies as high as 500 eV.

The calculations leading to the EP curve in Fig. 1 were essentially the same as those described in I, except that the phase shifts for angular momentum $l = 0$ to 100 were used here to predict the cross section at small but nonzero scattering angles (only the first 50 phase shifts were used in I). In addition, for zero scattering angle, the differential cross section was obtained by explicitly summing the first 10 000 phase shifts and then adding to this a small correction obtained by approximating the sum over all remaining l values by an integral.

The approximations used in I and the present analysis to estimate the phase shifts for $l > 10$ are defined in Eqs. (I3) - (I5). Considering first the contributions due to the long-range distortion interactions, we find, after substituting (I4) into (I3), that

$$\eta_l = \frac{\pi}{2} \sum_{n=1}^{3} \frac{a_n}{(2n-1)!} \frac{\partial^{2n-1}}{\partial (d_n^{\,2})^{2n-1}} \times [I_{l+1/2}(kd_n) K_{l+1/2}(kd_n)] \quad , \tag{1}$$

where I and K are the modified Bessel functions

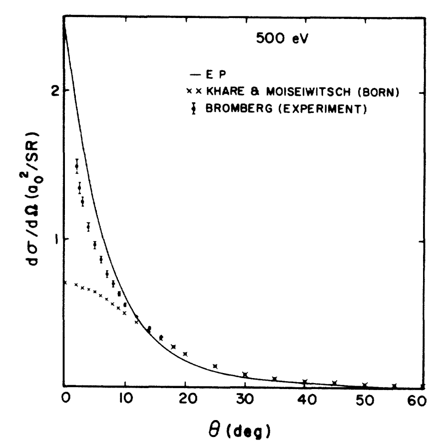

FIG. 1. Differential cross sections for the elastic scattering of 500-eV electrons from helium. Curve EP represents the extended-polarization potential calculations of this work. The crosses are from the calculations of Khare and Moiseiwitsch (Ref. 2) as quoted in Ref. 1. The experimental data are from Ref. 1.

$$I_\nu(x) = i^{-\nu} J_\nu(ix) \quad , \tag{2a}$$

$$K_\nu(x) = \tfrac{1}{2}\pi i^{\nu+1} H_\nu^{(1)}(ix) \quad , \tag{2b}$$

and the a_n and d_n are defined in I. However, rather than use algorithms for these functions, we expand their product as

$$\begin{aligned} I_{l+1/2}(z)K_{l+1/2}(z) &= (2l+1)^{-1}[1 - 2z^2/(2l+3) \\ &\times (2l-1) + 6z^4/(2l+5)(2l+3)(2l-1)(2l-3) \\ &- 20z^6/(2l+7)(2l+5)(2l+3)(2l-1) \\ &\times (2l-3)(2l-5) + \cdots] \ , \end{aligned} \tag{3}$$

take the indicated derivatives, and evaluate the result, keeping sufficient terms to give accuracy to three significant figures.

It should be noted that the first term in the sum (1) arises from the longest range $-\alpha/R^4$ dipole polarization interaction, and the corresponding result of substituting (3) into (1) and keeping the term to lowest order in k^2 is

$$\eta_l \sim \pi\alpha k^2/(2l+3)(2l+1)(2l-1) \quad , \tag{4}$$

in agreement with the modified effective-range theory analysis of O'Malley, Spruch, and Rosenberg.[4]

The contribution to the phase shifts for $l > 10$ from the short-range static potential (V_S) of the atom was obtained as follows: The potential V_S was evaluated from the Hartree-Fock wave function for helium as given by Clementi.[5] This wave function has the form

$$\Psi_{1s}(\vec{r}) = (4\pi)^{-1/2} \textstyle\sum_i C_i e^{-Z_i r} \quad , \tag{5}$$

where the parameters C_i and Z_i are tabulated by Clementi.[5] The corresponding form for V_S is then (in a.u.)

$$\begin{aligned} V_S(r) &= -\frac{4}{r} + 4\int |\Psi_{1s}(\vec{x})|^2 \frac{d\vec{x}}{|\vec{r}-\vec{x}|} \quad , \\ &= -2 \sum_{i,j} C_i C_j \left(\frac{2}{r} + Z_{ij}\right) e^{-Z_{ij} r} \quad , \end{aligned} \tag{6}$$

where $Z_{ij} = Z_i + Z_j$.

Upon substituting (6) into the semiclassical formula (I5) for phase shift, we obtain

$$\eta_l = \frac{2}{k} \sum_{i,j} C_i C_j [K_0(Z_{ij} r_0) + \tfrac{1}{2} Z_{ij} r_0 K_1(Z_{ij} r_0)] \quad , \tag{7}$$

where K is the modified Bessel function defined in (2b) and $r_0 = (l + \frac{1}{2})/k$ is the classical turning point. An algorithum for computing K_0 and K_1 is available[6] and was used in evaluating Eq. (7).

In the present work it was found that the contribution of (7) to the phase shifts was negligible compared to that from the long-range interactions (1) for $l > 30$.

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