Comments on Bronzan's Two-Meson Solution of the Charged Scalar Static Model

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A discrepancy is found between the two-meson solution of charged scalar theory given by Bronzan and the isobar spectrum known from strong-coupling theory. The discrepancy is traced to a failure of production amplitudes to satisfy crossing symmetry.

In recent years some attention has been given to the importance of three-body intermediate states in the computation of two-particle scattering amplitudes.[1] A serious drawback is the lack of exactly soluble field theories which include crossing symmetry. Such examples would obviously answer questions about approximations to more realistic field theories and provide some answers to the vexing question about the relative importance of crossing versus three-body unitarity.

A few static models including three-body intermediate states have been solved, most notably that of Amado,[2] but these unfortunately do not preserve crossing.

In a recent series of papers by Bronzan[3,4] a solution of the charged scalar static model which includes the sector of two-meson intermediate states has been presented. The purpose of this paper is to point out some curious features of Bronzan's solution which tend to make it unreliable at least for large couplings.

If the Bronzan solution is analytically continued to large renormalized coupling, a neutral bound state is formed.[4] The appearance of a neutral isobar is in conflict with the old strong-coupling (SC) theories[5] of the charged scalar model. SC theory predicts for the spectrum of isobars

$$M_Q \propto Q(Q-1)/g^2, \tag{1}$$

where Q is the isobar charge and g is the renormalized Yukawa coupling constant. On the other hand, Goebel[6] has shown that the one-meson solution gives isobars consistent with the old SC theory. One is now faced with a dilemma: If the SC theory is correct, either the Bronzan solution is inconsistent or the n-meson approximation does not converge smoothly to the SC solution.

The resolution of the paradox lies in the study of the original integral equation for a five-point function solved by Bronzan [Eq. (21) of Ref. 3]:

$$Q_-(\omega_1,\omega_2)=\frac{(2\omega_1\Omega)^{1/2}}{u(\omega_1)}\sum_s\left[\frac{\langle n|j^+(0)|s\rangle\langle s|j(0)|\pi_{k_1}{}^-p,\text{ out}\rangle}{E_s-m-\omega_2-i\epsilon}+\frac{\langle n|j(0)|s\rangle\langle s|j^+(0)|\pi_{k_1}{}^-p,\text{ out}\rangle}{E_s-m+\omega_2-\omega_1+i\epsilon}\right], \tag{2}$$

where all the symbols are defined in Ref. 3.

Equation (2) may be represented schematically by Fig. 1. If the sum over intermediate states[5] is truncated with at most one intermediate meson, one obtains the terms in Fig. 2. Bronzan solves the equation corresponding to Fig. 2 and subsequently arrives at a scattering amplitude which preserves three-particle unitarity.

The fourth term in Fig. 2 has serious problems which eventually lead to the spurious neutral isobar of Bronzan's solution. Note that as $g\to\infty$, term 4 develops a pole corresponding to the $(-)$ isobar. There is no other pole around to cancel it, so that one gets a large production amplitude. But it is known[7] from SC theory that pole terms with the correct residues cancel pairwise as $g\to\infty$, and furthermore that the multiple-meson production amplitudes themselves tend to zero.

By retaining only the fly-by terms for three-meson intermediate states, we obtain the two additional terms shown in Fig. 3. Now notice that the first term of Fig. 3 develops a pole corresponding to the $(++)$ isobar and this term will cancel the pole mentioned

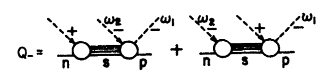

FIG. 1. Dispersion graphs for Eq. (2).

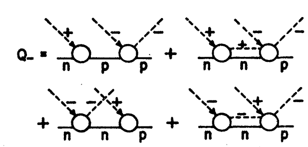

FIG. 2. Dispersion graphs remaining after retaining only one- and two-particle intermediate states.

[1] See, e.g., R. Aaron, R. D. Amado, and J. E. Young, Phys. Rev. **174**, 2022 (1968).
[2] R. D. Amado, Phys. Rev. **122**, 696 (1961); J. B. Bronzan, *ibid.* **139**, B751 (1965).
[3] J. B. Bronzan, J. Math. Phys. **7**, 1351 (1966).
[4] J. B. Bronzan, Phys. Rev. **154**, 1545 (1967).
[5] G. Wentzel, Helv. Phys. Acta **13**, 169 (1940); R. Serber and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).
[6] C. J. Goebel, Phys. Rev. **109**, 1846 (1958).

[7] The argument that all pole terms of a multiple-meson production amplitude cancel in the SC limit is originally due to Goebel (private communication); cf. Ref. 6 and A. N. Kaufman, Phys. Rev. **92**, 1468 (1953).

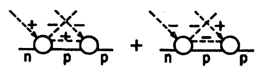

FIG. 3. Additional terms obtained by retaining the fly-by terms in three-meson intermediate states.

earlier. In the Bronzan solution the $(++)$ isobar cannot appear, so a spurious pole is generated from term one of Fig. 2 to cancel the lone pole from term 4. This spurious pole is of course the neutral isobar.

It is important to notice that a pole must appear as g increases to cancel the $(-)$ pole with residue $\propto g^2$ $(\to \infty)$, since Q_- satisfies unitarity and is therefore bounded in the physical region.

One further point is that the two terms of Fig. 3 are the terms which must be included to maintain crossing symmetry. This last is not obvious from the single-dispersion relation, but may be seen by contracting out the remaining meson to obtain a double-dispersion relation for Q_-.

Unfortunately, retaining the terms corresponding to Fig. 3 has the effect of rendering the integral equation intractible. The conclusion of these observations is that crossing symmetry in the production amplitudes is extremely important at least when the coupling is very large.

We can make no statement about the consistency of the solution for intermediate values of the coupling, but the failure at large coupling tends to make the solution suspect at intermediate values as well. We can also make no statement about the effect of production amplitudes in nonstatic models. However, in the crossing-symmetric static models, the results from SC theory suggest that bound states appear only in channels which have attractive crossed-pole terms in the scattering amplitude.

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Relation between Nonlinear and Linear Realizations of Chiral $SU(2)\times SU(2)$*

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It is shown that any nonlinear realization of $SU(2)\times SU(2)$ can be converted into a linear one of the type (j,j) by redefining the pion field. Explicit forms of the redefinition are derived for all values of j.

IN nonlinear realizations of chiral symmetry[1–3] the isospin operators act upon pion fields in the standard linear way, but chiral operators yield nonlinear functions of the pion field itself. Thus it would seem that these nonlinear realizations of $SU(2)\times SU(2)$ bear no relationship to the usual linear ones. Weinberg[1] has shown, however, that it is always possible to construct linear realizations out of nonlinear ones. Given an $N\times N$ matrix $\Lambda(\pi)$ which is a function of the pion field, and which behaves in a particular way under chiral transformations, he is able to identify the elements of one of its columns with a linear realization of $SU(2)\times SU(2)$. He proved that such a matrix must always exist, but he did not determine its specific form.

Coleman, Wess, and Zumino[2] have subsequently shown that if linear realizations are characterized by the eigenvalues (j^+,j^-) of the two commuting $SU(2)$ subgroups, then the ones that can be constructed from a nonlinearly transforming pion field are limited to the class $j^+=j^-=j$. They obtained these linear realizations from matrix representations of group elements of $SU(2)\times SU(2)$ by treating the pion fields as the parameters associated with pure chiral transformations. Their result is actually more general than that of Weinberg because it applies to any group which has a semisimple subgroup.

Here we wish to present an alternative approach to the problem of constructing linear realizations out of nonlinear ones. We show that the process of construction is equivalent to a redefinition of the pion field, and we derive the explicit form of this redefinition. Our approach does not require a detailed knowledge of the matrix representations of the group $SU(2)\times SU(2)$, but it can be regarded, from Weinberg's point of view, as a determination of the matrix $\Lambda(\pi)$.

We base our approach upon two properties of linear

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[1] S. Weinberg, Phys. Rev. **166**, 1568 (1968).

[2] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969).

[3] See, for example, J. Schwinger, Phys. Letters **24B**, 473 (1967); P. Chang and F. Gürsey, Phys. Rev. **164**, 1752 (1967); W. A. Bardeen and B. W. Lee, *ibid.* **177**, 2389 (1969); C. J. Isham, Nuovo Cimento **61A**, 729 (1969).