Extrapolation of Elastic Differential πp Cross Section to Very High Energies and the Pion Form Factor

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A least-squares fit is made of elastic πp differential cross sections and then extrapolated to infinite energies. The result is used, in the droplet model, to evaluate the pion form factor. The pion root-mean-square radius is found to be $(0.63\pm0.02)\times10^{-13}$ cm, which is slightly smaller than that of the proton.

1. INTRODUCTION

SOME time ago we proposed[1,2] a model of elastic high-energy scattering which gave a relationship between differential cross sections at infinite energy and the form factors of the hadrons involved in the collision. For the process $AB\rightarrow AB$, we obtained

$$F_A(k^2)F_B(k^2)=(\text{const})[a_{AB}(\mathbf{k})+\tfrac{1}{2}a_{AB}(\mathbf{k})\otimes a_{AB}(\mathbf{k})+\cdots], \quad (1)$$

where $F_A(k^2)$ and $F_B(k^2)$ are the form factors for A and B, respectively, and a_{AB} is the elastic amplitude at infinite energy. The symbol $\otimes$ denotes a convolution integral. The vector $\mathbf{k}$ in a_{AB} is the two-dimensional momentum-transfer vector in a plane perpendicular to the incoming beam. In the usual notation, $t=-k^2$.

Using this model, we have computed[1,2] from pp scattering data the proton form factor and found it to agree very well with the proton charge form factor measured in ep scattering experiments. We also obtained[1] the pion form factor by using a simple but crude fit of the then existing experimental πp scattering amplitude. In this paper, we report a more accurate calculation of the pion form factor using again (1) but based on a limiting $\pi^- p$ elastic differential cross section at very high energies obtained through a systematic extrapolation of presently available experimental data. Procedures of error estimation are outlined and results will be discussed.

TABLE I. Limiting elastic $\pi^- p$ differential cross sections at infinite energies.

$-t_i$ (BeV/c)^2	$(d\sigma/dt)_\infty$[mb (BeV/c)$^{-2}$]
0	26.10 ± 0.924
0.301	2.302 ± 0.129
0.785	$(7.77\pm1.75)\times10^{-2}$
1.0	$(4.12\pm0.347)\times10^{-2}$
1.2	$(9.04\pm1.40)\times10^{-3}$
1.5	$(4.49\pm0.771)\times10^{-3}$
2.0	$(3.53\pm1.23)\times10^{-4}$
2.4	$(2.45\pm2.53)\times10^{-5}$

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[1] T. T. Chou and C. N. Yang, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967), pp. 348–359.

[2] T. T. Chou and C. N. Yang, Phys. Rev. **170**, 1591 (1968).

2. LIMITING πp SCATTERING AMPLITUDE

We take $\pi^- p$ differential cross sections at various energies for eight t values from six different experiments.[3] (In some cases cross sections cannot be read off directly; thus interpolations are made.) For each t (say, $t=t_i$) the measured differential cross sections at various incoming energies are fitted by

$$\frac{d\sigma(t_i)}{dt}=\left[\frac{d\sigma(t_i)}{dt}\right]_\infty+\frac{a(t_i)}{(p_{\text{lab}})^{b(t_i)}}, \quad (2)$$

through a least-squares fitting process. Thereby we obtain the constants a, b, and also $[d\sigma(t_i)/dt]_\infty$, which is the infinite-energy limiting differential cross section at the specified t value, $t=t_i$. The results are listed in Table I, together with errors obtained through the fitting procedure.

The limiting scattering amplitude $a_{\pi p}$ is computed by assuming it to be purely real (i.e., imaginary in the usual notation) at high energies. Thus

$$a_{\pi p}(t_i)=\left[\frac{1}{\pi}\left(\frac{d\sigma(t_i)}{dt}\right)_\infty\right]^{1/2}. \quad (3)$$

For calculational convenience we fit a curve through the seven extrapolated limiting amplitudes, with the result

$$a_{\pi p}(t)=3.13e^{5.38t}+1.49e^{2.29t}\ (\text{BeV}/c)^{-2}. \quad (4)$$

Substituting (4) into (1), we obtain the normalized form-factor product $F_\pi(k^2)F_p(k^2)$, which is plotted in Fig. 1.

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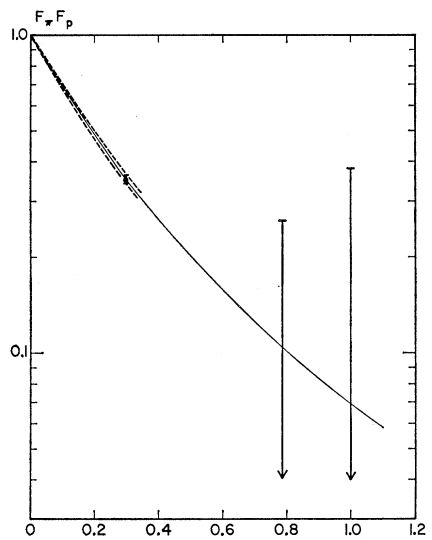

FIG. 1. Form-factor product $F_\pi F_p$ calculated from Eq. (1). The vertical lines represent the bounds of the value of $F_\pi F_p$ as calculated in this paper at the experimental values of k^2. The solid line represents $F_\pi F_p$ as calculated from Eq. (1). The dashed lines represent upper and lower bounds of $F_\pi F_p$ extrapolated into the small-k^2 region. For the method used in the error estimation, see the text. [The large errors for $t \gtrsim 0.6$ (BeV/c)^2 are due to the fact that a small fractional error in the πp scattering amplitude at small values of t can cause, through Eq. (1), large errors in $F_\pi F_p$ at large values of t.]

3. ERROR ESTIMATION

At each point of measurement, say, $t=t_i$, $a_{\pi p}$ has an uncertainty $\delta a_{\pi p}(i)$ due to the measurement error of $d\sigma_{\pi p}/dt$ at that t value. Its magnitude is obtained from the errors tabulated in Table I. $\delta a_{\pi p}(i)$ may be regarded, for the sake of simplicity, as a step function within the interval $|t-t_i|<\frac{1}{2}\Delta t_i$, where Δt_i is a typical spacing in t between two adjacent measurements in a given experiment. We shall take $\Delta t_i=0.1$ (BeV/c)^2 for all i in later calculations. Since $\delta a_{\pi p}(i)$ for different i are independent of each other, the uncertainty of the fitted amplitude (4) can be expressed as

$$\delta a_{\pi p}=\sum_{i=1}^{7} \epsilon_i \delta a_{\pi p}(i)\,, \qquad (5)$$

where ϵ_i are random variables, each with rms expectation value equal to one.

To first order in ϵ_i, the change of $F_\pi F_p$ due to $\delta a_{\pi p}$ is

$$\delta(F_\pi F_p)=(\text{const}) \times[\delta a_{\pi p}+\delta a_{\pi p}\otimes(a_{\pi p}+a_{\pi p}\otimes a_{\pi p}+\cdots)]. \qquad (6)$$

The rms deviations, hence the upper or lower bounds, of $F_\pi F_p$ at each t_i can be determined, and are also shown in Fig. 1.

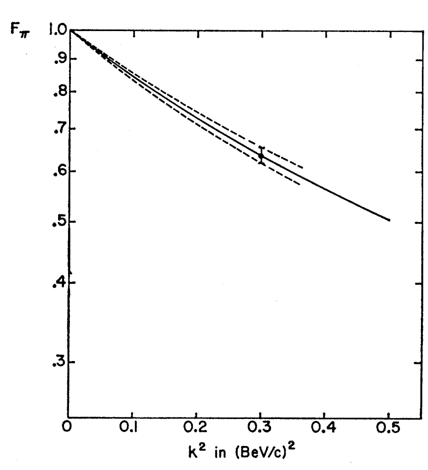

FIG. 2. Calculated pion form factor F_π shown as the solid line. This graph is obtained from the low-k^2 portion of Fig. 1 by multiplying the ordinates by F_p^{-1}. The dashed lines represent estimated upper and lower bounds of F_π.

TABLE II. List of the proton form factor F_p, the pion form factor F_π, and the form factor product $F_\pi F_p$.

k^2 (BeV/c)^2	$F_\pi F_p$	F_p	F_π	Remark
0	1.000	1.000	1.000	
0.1	0.685	0.810	0.846	
0.2	0.484	0.665	0.728	
0.3	0.352	0.553	0.636	$\Delta F_\pi \simeq \pm 0.017$
0.4	0.263	0.466	0.564	
0.5	0.201	0.399	0.505	
0.6	0.158	0.347	0.455	
0.7	0.126	0.305	0.412	
0.8	0.102	0.272	0.375	
0.9	0.084	0.245	0.341	
1.0	0.069	0.223	0.311	

4. RESULTS AND DISCUSSIONS

The proton form factor has been obtained from pp scattering data in a previous calculation.[2] After dividing it out from the form-factor product $F_\pi F_p$, we get the pion form factor which is plotted in Fig. 2. Numerical values for various form factors are listed in Table II. It is observed that the pion form factor falls off less fast than that of the proton for large t, indicating that the pion is a slightly smaller object than the proton. The rms radius of the pion[4] is found to be 0.63 ± 0.02 F compared with 0.71 F for the charge radius of the proton. These conclusions are different from our previous numerical analysis[1] because the value of the limiting $d\sigma/dt$ in πp scattering at $t \gtrsim 1$ (BeV/c)^2 as extrapolated in the present calculation and tabulated in Table I is larger than the corresponding value taken in Ref. 1 [which was obtained through a straight-line extrapolation of the curve $\lim(d\sigma/dt)_{\pi p}$ versus t from small t to $t \gtrsim 1$ (BeV/c)^2].

[4] For some experimental results of the charge radius of the pion, see C. W. Akerlof, W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Rev. **163**, 1482 (1967); C. Mistretta, D. Imrie, J. A. Appel, R. Budnitz, L. Carroll, M. Goitein, K. Hanson, and R. Wilson, Phys. Rev. Letters **20**, 1523 (1968).