at 8 BeV/c are given in Fig. 3. The density matrix elements obtained indicate that the typical qualitative behavior of OPEA calculations are reproduced in all the reactions mentioned above, but no experimental data are available for comparison.

It has been observed that the OPE-δ model gives qualitatively reasonable spin-density matrix elements for quasi-two-body reactions in which the OPE mechanism is expected to dominate. The agreement between the OPE-δ model and the OPEA model predictions for spin-density matrix elements is in general quite good. However, a significant quantitative difference was observed in the OPEA and OPE-δ predictions for diagonal density matrix elements $\rho_{0,0}$ and $\rho_{1,1}$ for the reaction $\pi+N\rightarrow f^0+N$ at 6 BeV/c. The difference between the two models in this can can be traced to a large $J=\frac{3}{2}$ Kronecker-δ term in the OPE amplitude $t_{1,-\frac{1}{2};-\frac{1}{2}}{}^{J=\frac{3}{2}}$ which is not completely absorbed in the OPEA model but is completely deleted in the OPE-δ model. That this term is large is a property of the $\bar{N}N\pi$ vertex. This mechnism should be expected to give rise to even larger discrepancies with the OPEA model in reactions where higher-spin bosons are produced with the final-state nucleon. This is because the large Kronecker-δ terms would appear in higher partial waves which are even less completely absorbed in the OPEA model. Unfortunately, no experimental data are available for a definitive discrimination between the density matrix elements predicted by the OPE-δ and the OPEA models.

Finally, it must be emphasized that the OPE-δ model derives its usefulness in terms of being able to generate OPEA-model-type predictions for spin-density matrix elements without partial-wave expansion. Hence, it will be useful as a tool for a quick generation of OPEA-model-type results as a part of a larger scheme of calculations.

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Vector-Meson Dominance and Pion Production*

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On the basis of the current-field-identity relation, a unified treatment and quantitatively successful calculation of pion production by ρ mesons and photons is presented. The model, which is independent of free parameters, explains the empirically established facts about strong mass dependences in the ρ density matrix elements (in particular, also in the combination $\rho_{11}+\rho_{1-1}$) considered as a function of the mass k^2 of the ρ meson. These facts have been the basis for a recent criticism of the vector-meson-dominance hypothesis in pion production. But our results indicate that the current-field-identity relation is not in conflict with experiment. We emphasize in our discussion that no model-independent formulation of vector-meson dominance is possible in pion production, according to our present knowledge.

I. INTRODUCTION

A BASIC ingredient to any concept of ρ dominance in electroproduction of pions is the relation

$$\langle\pi N'|J_\mu{}^V(0)|N\rangle=-(m_\rho{}^2/f_\rho)\langle\pi N'|\rho_\mu(0)|N\rangle$$
$$=[m_\rho{}^2/(m_\rho{}^2-k^2)](1/f_\rho)\langle\pi N'|J_\mu{}^\rho(0)|N\rangle. \quad (1)$$

Here, $k^2=(N-N'-\pi)^2$ and the momenta and all other quantum numbers of the pion and nucleons are denoted by π, N, and N'. Equation (1) is a direct consequence of the more general current-field-identity relation[1]

$$J_\mu{}^V(x)=-(m_\rho{}^2/f_\rho)\rho_\mu(x). \quad (2)$$

This equation relates the hadronic isovector part of the electromagnetic current $J_\mu{}^V(x)$ to the phenomenological field $\rho_\mu(x)$ of the ρ meson $[J_\mu{}^\rho(x)=-(\Box+m_\rho{}^2)\rho_\mu(x)]$. Through charge independence and time-reversal invariance of the nuclear forces, Eq. (1) connects specifically the isovector part of the amplitudes for the pion-production processes by real or virtual photons

$$\gamma p\rightarrow\pi^+n \quad\text{and}\quad \gamma n\rightarrow\pi^-p \quad (3)$$

to the ρ^0-production processes by pions

$$\pi^-p\rightarrow\rho^0 n. \quad (4)$$

Strictly speaking, comparison of the reactions (3) and (4) requires an analytic continuation in k^2 of the matrix elements of Eq. (1). To separate the ρ pole in the neighborhood of the ρ resonance, we write (1) in the form

$$\langle\pi N'|J_\mu{}^V(0)|N\rangle\approx\frac{m_\rho{}^2}{m_\rho{}^2-k^2-im_\rho\Gamma}\frac{1}{f_\rho}\langle\pi N'|\hat{J}_\mu{}^\rho|N\rangle. \quad (5)$$

Here $\hat{J}_\mu{}^\rho$ is defined as the modified current operator in

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[1] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

the sense of Ref. 1; Γ is the width of the ρ meson. The matrix elements of $\hat{J}_\mu{}^\rho$ do not vanish at $k^2=m_\rho{}^2$ contrary to $J_\mu{}^\rho$. They are related by time-reversal invariance to the measured ρ-production amplitudes of the process (4).

Postulating Eq. (1) is not enough to draw directly practical advantage in situations where $k^2\neq m_\rho{}^2$, i.e., where the matrix element on the right-hand side of (1) is not measurable. Equation (1) has then to be supplemented by a postulate which specifies the conditions under which the dependence of $\langle\pi N'|J_\mu{}^\rho|N\rangle$ on the mass k^2 can be neglected. So far, the physicists who applied (1) assumed the existence of such situations; some attempts to specify them have been made by Białas and Zalewski.[2] These authors understood ρ dominance in photoproduction to imply Eq. (1) together with a postulate on the mass independence of suitably chosen matrix elements of $\langle\pi N'|J_\mu{}^\rho(0)|N\rangle$.

The question of whether or not there exist in electroproduction situations in which one can neglect the mentioned mass dependences without introducing too large errors has so far not been settled either theoretically or empirically. But one may have serious doubts about the existence of these happy situations on the basis of some recent experimental work.[3–5] We therefore propose to study this question theoretically by investigating a model with no free parameters. This model is even quantitatively successful enough to relate photoproduction of pions and ρ production of pions using Eq. (1).

We ignore mass dependences in $\langle\pi N'|J_\mu{}^\rho|N\rangle$ which are connected with the unsettled ambiguities in defining the field operator ρ_μ away from the mass shell. Also, we do not consider mass dependences related to nonresonant background contributions in the ρ channel of the π-π system. Instead, we are mainly concerned with mass dependences which arise, as we shall later see, from such dynamical concepts as crossing symmetry and analyticity, or which stem from a violation of the universality concept for at least some of the effective coupling constants. To a lesser extent, we are concerned with kinematic mass dependences enforced, e.g., by current conservation ($k_\mu\langle f|J^\mu|i\rangle=0$), although they may be important in practice.

The model will be introduced in Sec. II, and the results will be presented in Sec. III. These results are further discussed in Sec. IV, where general conclusions relevant to the definition of vector-meson dominance are presented. Pion photoproduction in the framework of vector-meson dominance was first treated by Beder.[6] This treatment is in general terms and does not deal in particular with the question of mass dependences in detail. This question of mass dependences has been attacked in a model-independent way by Fraas and Schildknecht[7] as well as by Meiere[8] and by LeBellac and Plaut.[9] However, our conclusions are different, since our model contradicts some of their assumptions (see the discussion in Sec. IV).

II. MODEL FOR AMPLITUDES $\langle\pi N'|J_\mu{}^V|N\rangle$ AT HIGH ENERGIES AND $k^2\neq0$

To construct a model for the scalar isovector amplitudes $A_i{}^V(s,t,k^2)$ $(i=1,\cdots,6)$ appearing in the covariant decomposition[10] of $\langle\pi N'|J_\mu(0)|N\rangle$, we make use of our theoretical understanding of the dynamics in forward $\pi^\pm$ photoproduction above $E_\gamma=1$ GeV. At $k^2=0$, this kinematical region—including the by now famous forward peak[11]—is dominated by the real part of the isovector amplitudes $A_i{}^V$. As we have discussed some years ago,[12] these amplitudes are reasonably well predicted by fixed-t dispersion relations if one assumes that the dispersion integrals are saturated by the low-energy contributions of the integrand. It is therefore tempting to assume the same approximation for $k^2\neq0$. Thus we obtain the $A_i{}^V$ from the following relations:

$$\mathrm{Im}A_i{}^V(s,t,k^2)\approx0\quad(s>s_c)\,,$$

$$\mathrm{Re}A_i{}^V(s,t,k^2)=A_i{}^V(s,t,k^2)_{\text{pole term}}+\frac{1}{\pi}\int_{(M+\mu)^2}^{s_c}ds'\,\mathrm{Im}A_i{}^V(s',t,k^2)\times\left(\frac{1}{s'-s}\pm\frac{1}{s'-u}\right),\qquad(6)$$

where $s>s_c$, with s_c being the cutoff energy. In order to be able to use the partial-wave expansions for $\mathrm{Im}A_i{}^V$ in the integrand of (6), we have to restrict $-t$ to values not much larger than $15m_\pi{}^2/c^2$ because of the convergence of this expansion in the unphysical kinematical region. To avoid difficulties with anomalous thresholds, we

[2] A. Białas and K. Zalewski, Phys. Letters **28B**, 436 (1969).

[3] Geweniger, P. Heide, U. Kötz, R. A. Lewis, P. Schmüser, H. J. Skronn, H. Wahl, and K. Wegener, Phys. Letters **28B**, 155 (1968).

[4] L. J. Gutay, F. T. Meiere, J. H. Scharenguivel, D. H. Miller, R. J. Miller, S. Lichtman, and R. B. Willman, Phys. Rev. Letters **22**, 424 (1969).

[5] R. Diebold and J. A. Poirier, Phys. Rev. Letters **22**, 906 (1969).

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[7] H. Fraas and D. Schildknecht, Nucl. Phys. **B6**, 395 (1968).

[8] F. T. Meiere, Purdue University Report, 1969 (unpublished).

[9] M. LeBellac and G. Plaut, Nice Report, 1969 (unpublished).

[10] P. Dennery, Phys. Rev. **124**, 2000 (1961). For a very efficient and compact kinematical formalism, which we use, see G. von Gehlen, Heidelberg Report, 1968 (unpublished).

[11] In contrast to some popular statements [e.g., F. Gilman, SLAC Report No. SLAC-Pub-589, 1969 (unpublished)], this peak was known prior to experiments. The only surprising feature of this forward peak was the fact that it is so remarkably well predicted by fixed-t dispersion relations in the low-energy approximation. We pointed this out, for the first time, in a report to the Dubna Conference on Electromagnetic Interactions, February 1967 [see External Report No. 3/67-1 of the Kernforschungszentrum Karlsruhe (unpublished)].

[12] J. Engels, W. Schmidt, and G. Schwiderski, Phys. Rev. **166**, 1343 (1968). These results can be translated, of course, into a t-channel approach at high energies using finite-energy sum rules [see A. Bietti *et al.*, Phys. Letters **26B**, 457 (1968); P. di Vecchia *et al.*, *ibid.* **27B**, 296 (1968); J. D. Jackson *et al.*, *ibid.* **29B**, 236 (1969)].

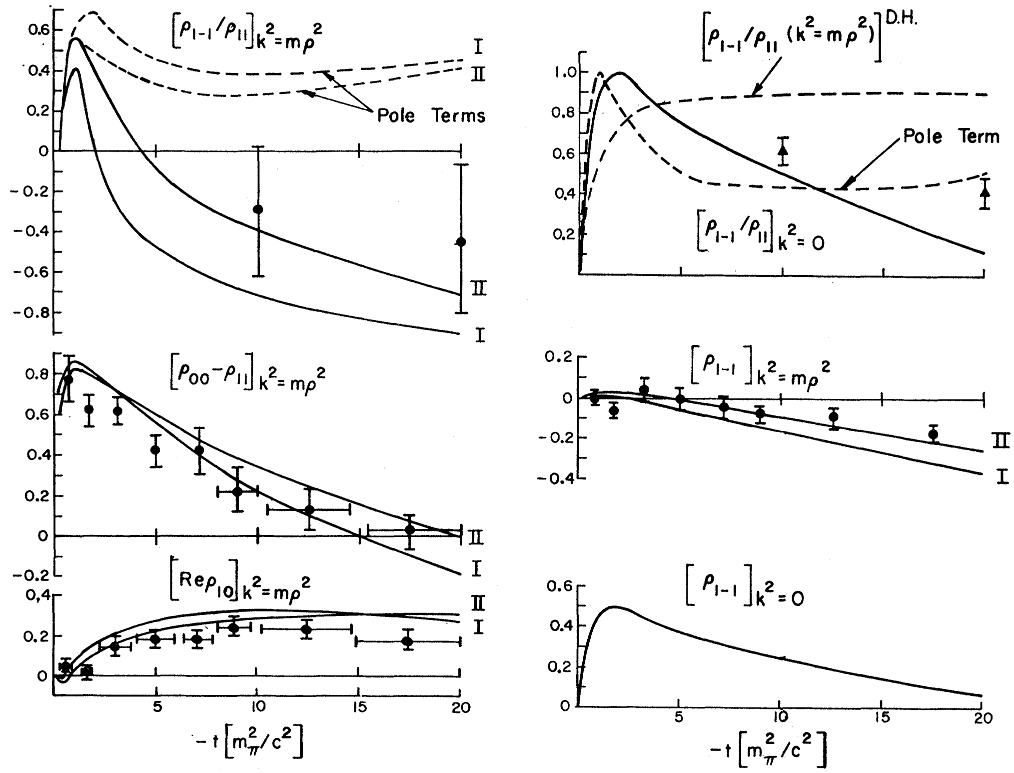

FIG. 1. Density matrix element ρ_{ik} in the helicity frame at a pion lab momentum of 4 GeV/c. The letters I and II refer to sets of coupling constants I and II in Sec. II. Results for $k^2=m_\rho^2$ and $k^2=0$ in the helicity frame are presented; $\rho_{1-1}/\rho_{11}(k^2=m_\rho^2)^{\rm DH}$ is the ratio taken with respect to the Donohue-Hoegaasen axes. Data are taken from Refs. 3 and 4.

restrict, for the time being, k^2 to values $<4m_\pi^2$. We will lift this restriction later. We note that the pole terms in (6) have as a common factor either the isovector Pauli form factors $F_{1,2}^V(k^2)$ or the pion form factor $F_\pi(k^2)$. These form factors develop the ρ pole around $k^2\approx m_\rho^2$ as postulated by (5).

For the cutoff energy s_c, we choose roughly an energy above the D_{13} pion nucleon resonance. Then by far the largest contribution to the dispersion integrals for $k^2\approx 0$ comes from the magnetic dipole excitation $M_{1+}^{3/2}(W,k^2)$.[13] This is the only contribution we shall retain in the dispersion integral even for $|k^2|\gg 0$; we postpone to a more detailed investigation refinements of this model by the inclusion of further partial amplitudes and the discussion of their contribution for $|k^2|\gg 0$.[14] For the dipole excitation $M_{1+}^{3/2}(W,k^2)$, we use the ansatz

$$M_{1+}^{3/2}(W,k^2)=P(W,k^2)M_{1+}^{3/2,\mathrm{CGLN}}(W,k^2)\,, \quad (7a)$$

where

$$M_{1+}^{3/2,\mathrm{CGLN}}(W,k^2)=\frac{1}{2f}G_M{}^V(k^2)\frac{e}{2M}\frac{k}{q}\frac{\sin\delta_{33}}{q}e^{i\delta_{33}} \quad (7b)$$

is the old result of the static theory.[9] In Eqs. (7), k and q are the initial and final momenta in the c.m. system; δ_{33} is the real part of the pion-nucleon scattering phase shift in the $I=J=\frac{3}{2}$, $P=+1$ state; $e^2=1/137.04$, $f^2=0.08$, and $G_M{}^V(k^2)$ is the magnetic Sachs form factor. Motivated by the recent work[15] on the electroproduction partial amplitudes, we shall assume that $P(W,k^2)\approx 1$ in this work.

We have formulated the ansatz for the amplitudes $A_i{}^V(s,t,k^2)$ for $k^2<4m_\pi^2$. But their analytical continuation to $k^2>4m_\pi^2$ is straightforward. We therefore also make the assumption that for $k^2\approx m_\rho^2$ this ansatz for $A_i{}^V(s,t,k^2)$ accounts for the main physical effects. In this approximation, $A_i{}^V(s,t,k^2)$ becomes complex for $k^2\gg 4m_\pi^2$ because of the presence of the form factors $F_{1,2}^V(k^2)$ and $F_\pi(k^2)$. We assume that these are represented for $k^2\approx m_\rho^2$ by a pole representation

$$F^\pi(k^2)\approx F^{\rho\pi\pi}m_\rho^2/(m_\rho^2-k^2-im_\rho\Gamma)\,, \quad (8a)$$

$$F_{1,2}^V(k^2)\approx F_{1,2}^{\rho NN}m_\rho^2/(m_\rho^2-k^2-im_\rho\Gamma) \quad (8b)$$

[13] G. F. Chew, M. G. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[14] To avoid spurious kinematic singularities in A_5, some authors [von Gehlen (Ref. 10), and Zagury, Phys. Rev. **165**, 1934 (1967)] introduce a t-dependent subtraction function $f(t)$ following a suggestion of S. L. Adler, Ann. Phys. (N. Y.) **50**, 189 (1968). We postpone the inclusion of this term to a more refined version of the present model. See R. Mannweiler and W. Schmidt (to be published).

[15] G. von Gehlen, Nucl. Phys. **B9**, 17 (1969).

[normalization $F^\pi(0)=F_1(0)=1, 2MF_2(0)=3.70$]. Note that in our ansatz, all parts of A_i contain the factor $(m_\rho^2-k^2-im_\rho\Gamma)^{-1}$, so that it is an over-all factor in $\langle f|J_\mu^V|i\rangle$ and drops out in (5). It remains only to specify the coupling constants $F^{\rho\pi\pi}$ and $F_{1,2}^{\rho NN}$. The Novosibirsk and Orsay experiments on $e^+e^-\to\pi^+\pi^-$ yield values of $F^{\rho\pi\pi}$ ($F^{\rho\pi\pi}=1.06\pm0.05$, Ref. 16) which deviate not more than 10% from the universality value $F^{\rho\pi\pi}=1$.[17] From a recent N/D analysis,[18] we derive at $k^2=m_\rho^2$ the values $F_1^{\rho NN}/F^{\rho\pi\pi}=\mathrm{Im}F_1/\mathrm{Im}F^\pi=0.82$ and $2MF_2^{\rho NN}/F^{\rho\pi\pi}=2M\ \mathrm{Im}F_2/\mathrm{Im}F^\pi=4.82$. The same analysis also reveals that the pole representation (8b) for the nucleon form factors is actually only a crude approximation even for $k^2\approx m_\rho^2$. Therefore, the definition of the constants $F_{1,2}^{\rho NN}$ is to some extent ambiguous, and they represent in any case only the effective $F_{1,2}^{\rho NN}$. Our choice (8b) can only be motivated by simplicity. Finally, taking from Ref. 16 the value $f_\rho^2/(4\pi)=1.90\pm0.25$, all parameters are determined, and we can make an absolute prediction of $\langle\pi N'|J_\mu^V|N\rangle$ at $k^2=0$ and $k^2=m_\rho^2$. These are the two cases in which we are interested in this work. We distinguish two sets of constants in order to identify in the following the origin of the mass dependence of the matrix elements:

$$\text{Set I: } F_\rho^{\rho\pi\pi}=1.06,\quad F_1^{\rho NN}=0.82,\quad 2MF_2^{\rho NN}=1.38\times3.70;$$
$$\text{Set II: } F^{\rho\pi\pi}=F_1^{\rho NN}=1,\quad 2MF_2^{\rho NN}=3.70.$$

The first set is the realistic set and is derived from Refs. 16 and 18 as explained previously. The second set is essentially the universality set.[17] Finally, we would like to mention that with decreasing values of t, the pole terms have to be canceled by large dispersion contributions. The model for the amplitudes is then obtained as a difference of two comparably large numbers which is, therefore, very sensitive to the approximations. All numerical results, in particular the cross section, must therefore be taken *cum grano salis* at the smallest values of t ($t\ll-10m_\pi^2$).

III. RESULTS

Using Eq. (1), the model for the electromagnetic current matrix elements and time-reversal invariance, we can predict the reaction $\pi^+n\to p\rho^0$ or, equivalently, by charge symmetry $\pi^-p\to n\rho^0$. There, the most interesting question is the prediction of the ratio ρ_{1-1}/ρ_{11} of the ρ-density matrix elements in the helicity frame. According to experiment[3,4] this ratio should show a large variation when going from $k^2=m_\rho^2$ to $k^2=0$, in particular for $-t>m_\pi^2/c^2$. In fact, our model reproduces this behavior (Fig. 1). One should note that the effect of including the dispersion contribution to the amplitudes is to obtain the right sign of ρ_{1-1} at $k^2\approx m_\rho^2$.

[16] J. E. Augustin *et al.*, Phys. Letters **28B**, 508 (1969).
[17] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).
[18] G. Höhler, R. Strauss, and G. H. Wunder, University of Karlsruhe Report, 1968 (unpublished).

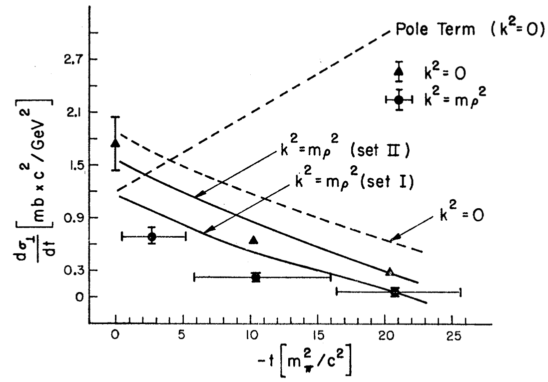

FIG. 2. Perpendicular cross section $d\sigma_\perp/dt$ [Eq. (9)] for a pion lab momentum of 4 GeV/c at $k^2=0$ (dashed lines) and $k^2=m_\rho^2$ (solid lines). Set I and II refers to coupling constants defined in Sec. II. Experimental points taken from Ref. 5.

This follows from the comparison with the pure pole-term approximation, also cited in Fig. 1. From a comparison of the other results for $(\rho_{00}-\rho_{11})$ and $\mathrm{Re}\rho_{10}$ as shown in Fig. 1, one concludes that the model reproduces the typical behavior of the density matrix elements at $k^2=m_\rho^2$ as well as at $k^2=0$.

The model is more sensitively checked by comparing differential cross sections instead of normalized density matrix elements. We are, in particular, interested in the cross section $d\sigma_\perp^\rho/dt$ for the production of pions by ρ mesons which are polarized perpendicular to the reaction plane. This is given in terms of the previous density matrix elements and the total cross section $d\sigma^\rho/dt$ for $\rho^0+p\to n+\pi^+$ by the relation

$$d\sigma_\perp^\rho/dt=(\rho_{11}+\rho_{1-1})d\sigma^\rho/dt. \qquad (9)$$

In spite of experimental uncertainties, the data at $k^2=m_\rho^2$ and $k^2=0$ reveal the systematic feature that $d\sigma_\perp^\rho/dt$ at $k^2=0$ is larger than at $k^2=m_\rho^2$. Also, this fact is reproduced by the model (Fig. 2). The comparison of the two curves for Sets I and II of the coupling constants $F^{\rho\pi\pi}$ and $F_{1,2}^{\rho NN}$ in Fig. 2 reveals that the origin of the difference is only partly due to the different couplings at $k^2=0$ and m_ρ^2. The ratio $d\sigma_\perp^\rho(k^2=0)/d\sigma_\perp^\rho(k^2=m_\rho^2)$ varies from 1.6 to 2.3 (Set I) between $-t=1m_\pi^2/c^2$ and $-t=10m_\pi^2/c^2$, respectively. In previous applications of vector-meson dominance, this mass dependence of $d\sigma_\perp/dt$ has been neglected in the application of (1). This neglect leads to an effective $f_\rho^2/4\pi$ (=1.2, Ref. 5), which is much smaller than the value of $f_\rho^2/4\pi$ (=1.90, Ref. 16) measured at $k^2=m_\rho^2$.

Let us also consider the cross section for pion production by transverse ρ mesons, defined by

$$\frac{d\sigma_{\mathrm{tr}}}{dt}=\frac{d\sigma}{dt}(\rho_{\mathrm{tr}}n\to\pi^-p)=\rho_{11}\frac{d\sigma}{dt}(\pi^-p\to\rho n), \qquad (10)$$

so that we can discuss some recent predictions by Avni, Harari, and Horovitz.[19] For $k^2=0$ the cross section[10] is proportional to the unpolarized photoproduction cross section and has, therefore, the characteristic forward peak with a width of m_π^2/c^2. It is easy to derive from our results that this characteristic forward structure survives if one goes to $k^2=m_\rho^2$ and *if* one takes ρ_{11} in the helicity frame. According to our model, ρ_{11} in the helicity frame has the following behavior: Starting at $t\approx 0$ it decreases rapidly to a minimum near $t=-m_\pi^2/c^2$ and then increases slowly. Since, according to experiment,[20] $(d\sigma/dt)(\pi^- p \to \rho n)$ is rapidly decreasing with increasing values of $-t$, the transverse cross section $\rho_{11}d\sigma/dt$ has a forward peak *in the helicity frame* similar to the photoproduction case. Obviously, this structure arises from the peripheral pion exchange contribution. It contains the scalar product $\pi\cdot\epsilon$, where ϵ^μ is the polarization vector of the ρ meson and π_μ is the four-momentum of the pion. Since $k\cdot\epsilon=0$, it follows that in the rest frame of the ρ meson $\epsilon_0=0$ and $-\boldsymbol{\pi}\cdot\boldsymbol{\varepsilon}=\pi\cdot\epsilon$. If, in contrast to the helicity axes definition, we choose $\boldsymbol{\pi}$ as the longitudinal axis (Gottfried-Jackson convention), then the pion exchange does not contribute to the transverse cross section (10), which then has no forward peaking. Avni and Harari[19] have tried to argue that this behavior of $d\sigma_{\rm tr}/dt$ is likely to be independent of the particular choice of axes, but our results disprove this conjecture.[21]

In general, one has that only $\epsilon_\perp\cdot\pi$ vanishes, where $\epsilon_\perp$ is the polarization vector perpendicular to the reaction plane, i.e., $\epsilon_\mu=\epsilon_{\mu\nu\rho\sigma}\pi^\nu N^\rho N'^\sigma$. Thus, the rapidly varying part of the pion contribution does not contribute in any frame to $d\sigma_\perp/dt=(\rho_{11}+\rho_{1-1})d\sigma/dt$. Therefore, $d\sigma_\perp/dt$ is smooth in the forward direction in any frame as, for example, in Fig. 2. The same is true for $d\sigma_{\rm tr}/dt$ [Eq. (10)] *only* if ρ_{11} is taken with respect to the Gottfried-Jackson axes.

Finally, we mention in passing some results for the ratio ρ_{1-1}/ρ_{11} evaluated for $k^2=m_\rho^2$ in the Donohue-Hoegaasen (DH) frame[22]: $(\rho_{1-1}/\rho_{11})^{\rm DH}$. In this frame, ${\rm Re}\rho_{10}$ vanishes for any k^2, whereas in a general frame, only for $k^2=0$. The use of this frame for a vector-meson-dominance prediction of ρ_{1-1}/ρ_{11} at $k^2=0$ has been suggested,[2] since in this frame ρ_{1-1}/ρ_{11} becomes maximal. As we see from Fig. 1, $(\rho_{1-1}/\rho_{11})^{\rm DH}$ has only the positive sign in the t range considered as a common feature with $(\rho_{1-1}/\rho_{11})^{k^2=0}$. Consequently, not much is gained by choosing the DH frame.

[19] Y. Avni and H. Harari, Phys. Rev. Letters **23**, 262 (1969); H. Harari and B. Horovitz, Phys. Letters **29B**, 314 (1969).

[20] P. B. Johnson *et al.*, Phys. Rev. **176**, 1651 (1968); W. Selove *et al.*, Phys. Rev. Letters **21**, 952 (1968).

[21] Independent from the argument presented here, one would expect that $d\sigma_{\rm tr}/dt$ is a smooth function in the mass k^2 of the vector particle. Since the arguments of Avni and Harari (Ref. 19) are obviously independent of the actual value of the mass k^2, they are also true for small values of k^2. In such a situation, their conjecture is quite obviously wrong independent from any model.

[22] J. T. Donohue and H. Hoegaasen, Phys. Letters **25B**, 554 (1967).

IV. GENERAL DISCUSSION OF RESULTS AND CONCLUSIONS

A. Meaning of Vector-Meson Dominance

The success of the model presented encourages us to draw some conclusions about ρ dominance in $\pi^\pm$ photoproduction and vector-meson dominance (VMD) in general. The results presented may, in particular, challenge a too naive concept of VMD so that the question may arise: What does VMD actually mean? A very simple-minded but safe interpretation of VMD could be based on the old concept of the isobar model in dispersion relations,[23] where we disperse this time in the mass k^2 of the π-π system. VMD then circumscribes mainly the empirical evidence that in the $I=J=1$ π-π channel, by far the strongest contribution comes from the ρ meson.[24] The word "strong" in this context is used in the sense that in the neighborhood of the ρ resonance, possible high-energy contributions summarized in subtraction constants can be neglected. Experiment has still to tell us how far this neighborhood actually reaches. The results of the model presented indicate that without introducing excessively large errors, the extrapolation is justified even for the photon mass $k^2=0$. Thus, so far VMD is just the same old idea as, for example, the s-channel isobar model of the π-N system. But there is a particular advantage this time, since the $I=J=1$ π-π system seems to be a system with much simpler structure, having one very strong resonance: the ρ peak. This peak can attract all the glamor alone, since it does not have to share it with other competing brothers. A search for other resonances in the ρ channel has been unsuccessful so far[25]; one may therefore speculate that they remain of minor importance, in general.

However, there seems to prevail to some extent the tendency to interpret VMD as a more fundamental, more powerful dynamical concept. A statement on VMD in this direction, which does not contradict the previous statement, is due to Joos,[26] who says that "the essence of vector-meson dominance is the current-field identity relation" which, in our case, is (2). Whereas our previous characterization of VMD (the isobar analog) follows also from the current-field-identity relation, further consequences can be derived from this relation. Thus one can exploit the fact that the algebra of fields is now imposed on the components of the electromagnetic current. Some asymptotic consequences for high-energy inelastic electron scattering in the limit $k^2\to\infty$ have been derived from this fact recently.[27] Those consequences should correspond, in the language of dispersion relations, to superconvergence relations and have then to be specified additionally.

[23] D. Amati and S. Fubini, Ann. Rev. Nucl. Sci. **12**, 359 (1962).

[24] See, e.g., the motivation for effective universality by M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[25] G. McClellan *et al.*, Phys. Rev. Letters **23**, 718 (1969).

[26] H. Joos, Acta Phys. Austriaca, Suppl. **IV** (1967).

[27] C. G. Callan, Jr., and D. J. Gross, Phys. Rev. Letters **22**, 156 (1969).

In pion electroproduction, no *direct*, practical, model-independent consequence could be drawn from the current-field-identity relation (2). As we discussed in the Introduction, in order to relate π production by ρ mesons and by virtual photons, Eq. (2) has to be supplemented by a postulate concerning the mass independence of certain matrix elements appearing in (1). Thus Schildknecht—emphasizing the practical point of view—formulates[28] the following for a general strong-interaction process $V+A\rightarrow B$, which involves the vector meson V and the particle groups A and B, and which is compared to $\gamma+A\rightarrow B$: "The assumption of VMD is the following one: $e_{\rm tr}{}^{\mu}T_{\mu}(V+A\rightarrow B)$, or rather the invariant amplitudes appearing therein, are varying slowly as a function of the mass of the vector meson m_V at high energies." Here e^{μ} is the polarization vector, and the index tr denotes the transverse component. Now, as has been stressed several times, only the transverse component of the polarization vector, which is perpendicular to the reaction plane (and thus proportional to $\epsilon_{\mu\nu\rho\sigma}\pi^{\nu}N^{\rho}N'^{\sigma}$ in our case), seems to have the least unambiguous meaning. But in Sec. III we showed that the corresponding cross section $d\sigma_{\perp}{}^{\rho}/dt$ varies to a large degree when going from $k^2=m_{\rho}{}^2$ to $k^2=0$. As we pointed out, this variation is due to the deviation of the coupling constants $F^{\rho\pi\pi}$ and $F_{1,2}{}^{\rho NN}$ from their universality values given by the Set II in Sec. II and from mass dependences entering the dispersion relations (6) through kinematic factors. This fact is a vital blow against any hope that mass-independent observables formed out of the matrix elements $\langle\pi N'|J_{\mu}{}^{\rho}(0)|N\rangle$ may be found at all. The perpendicular cross section is the only natural observable which is not affected by the presence of a large, although not uniquely defined, longitudinal contribution, which has to vanish in the limit $k^2\rightarrow 0$.

In passing, we note that because of this experience with the perpendicular cross section, it is evident that not much can be gained by the suggestion[2] to take the density matrix elements of the DH frame[22] instead of the usual helicity density matrix elements. The results presented in Fig. 1 for the crucial asymmetry coefficient ρ_{1-1}/ρ_{11} evaluated in the DH frame show that the asymmetry has a completely different t dependence than the asymmetry for $k^2=0$. As we already pointed out, the only common feature is the same positive sign for the present range of the t values.

B. General Argument Concerning Mass Dependences

In order to show that the results in Sec. III on the mass dependence of the observables may not be accidental, we include here a general argument on the mass dependence of the scalar amplitudes $A_i{}^{\rho}(s,t,k^2)$ appearing in the decomposition of $\langle\pi N'|J_{\mu}{}^{\rho}(0)|N\rangle$. These amplitudes, or an appropriate subset of them, are the primary quantities which, according to recent suggestions,[7-9] should be subjected to the VMD hypothesis, i.e., they should be approximately independent of k^2. We want to demonstrate that appreciable mass dependences in $A_i{}^{\rho}$ have to be expected as a consequence of such general dynamical laws as analyticity, crossing symmetry, and superconvergence relations. These mass dependences are in addition to the effects arising from the variation of the coupling constants with mass (sets I and II, Sec. II). The amplitudes $A_i{}^{\rho}$ fulfill fixed-t dispersion relations which are analogous to (6). For even symmetry under the exchange of s and u (t fixed), we can write, using $s+t+u=2M^2+k^2+m_{\pi}{}^2$,

$$\begin{aligned}\mathrm{Re}A_i{}^{\rho}(s,t,k^2)&\\ &=R_i(k^2)\frac{t-m_{\pi}{}^2-k^2}{(M^2-s)(M^2-u)}+\frac{1}{\pi}\mathrm{P}\int ds'\,\mathrm{Im}A_i{}^{\rho}(s',t,k^2)\\ &\quad\times\frac{2(s'-M^2)+t-m_{\pi}{}^2-k^2}{(s'-s)(s'-u)},\quad i=3,5,6.\quad(11)\end{aligned}$$

(R_i= residue of the pole terms.) For values of $s\gg M^2$ and $|t|\approx m_{\pi}{}^2$, the k^2 dependence in u can be completely neglected. We thus see that the pole term is in these cases very sensitive to k^2. With respect to the dispersion integrals, we again assume that the main contribution comes from the low-energy part of the integrand, so that s' is typically of the order $(M+2m_{\pi})^2$: the approximate energy of the first pion nucleon isobar. Then the change of the kinematical factor in the integrand of (11) is $\approx(4m_{\pi}M+k^2)/4m_{\pi}M=2.1$ when one goes from the mass of the ρ meson $k^2=0.585$ GeV^2 to the photon mass $k^2=0$. Thus the individual contributions to $\mathrm{Re}A_i{}^{\rho}(s,t,k^2)$ for $i=3$, 5, 6 may change with k^2 for s and t fixed, to a large extent because of the kinematical factors which are a consequence of crossing symmetry. Now, the net variation in $\mathrm{Re}A_i{}^{\rho}(s,t,k^2)$ depends on the relative sign of the pole-term and dispersion contributions and the sense of variation in the residues $R_i(k^2)$ and $\mathrm{Im}A_i{}^{\rho}(s,t,k^2)$. Thus, it may happen that in some exceptional cases, these individual k^2 dependences cancel out altogether (e.g., in certain kinematic regions of the s, t plane). But it is very unlikely to have mass independence of the $A_i{}^{\rho}(s,t,k^2)$ in general. In fact, our explicit results in Sec. III show the existence of this type of mass dependence in the kinematical region which is presently under experimental investigation.

We arrive at a similar conclusion for the amplitudes which are odd under crossing symmetry if we assume additionally the validity of certain superconvergence relations. For odd crossing symmetry, we write ($i=1$, 2, 4)

$$\mathrm{Re}A_i{}^{\rho}(s,t,k^2)=-\frac{2}{s}\left[R_i(k^2)+\frac{1}{\pi}\mathrm{P}\int ds'\,\mathrm{Im}A_i{}^{\rho}(s',t,k^2)\right]$$

$$+R_i(k^2)\frac{(s-M^2)(2M^2+k^2+\mu^2-t)-M^2(k^2+\mu^2-t)}{s(M^2-s)(M^2-u)}$$

[28] D. Schildknecht, DESY Report No. 69/10, 1969 (unpublished).

$$+\frac{1}{\pi}\mathrm{P}\int ds'\,\mathrm{Im}A_i{}^{\rho}(s',t',k^2)\frac{1}{s}\times\frac{2s'^2+(s-2s')(2M^2+k^2+\mu^2-t)}{(s'-s)(s'-u)}. \quad (12)$$

Superconvergence relations, with the two terms in the first bracket canceling, have been postulated[29,30] and could be confirmed at least for $k^2=0$.[30] To obtain some feeling for possible mass dependences in this case, we calculate as in the previous case the dependence of the kinematical factors on k^2. Assuming again that the main contribution to the dispersion integral comes from the region $s'=(M+2m_\pi)^2$, we obtain for the pole term a variation by a factor 1.33, and for the dispersion contribution a factor 1.27 at $s=(4M)^2$ when going from $k^2=0.585$ GeV^2 (ρ mass) to $k^2=0$ with s and t fixed. Thus, this time the dependence of the kinematic factors on k^2 is less strong than in the previous case. However, mass dependences may show up if the term in square brackets in (12) is canceled by the superconvergence relation

$$R_i(k^2)=-\frac{1}{\pi}\int ds'\,\mathrm{Im}A_i{}^{\rho}(s',t,k^2).$$

We remark here, incidentally, that at $t=0$ and $k^2=0$, the perpendicular cross section (9) is given by

$$\frac{d\sigma_\perp}{dt}\sim\frac{d\sigma}{dt}\approx\frac{3}{16\pi}|A_1|^2, \qquad s\gg M^2 \quad (13)$$

Thus in forward direction, $d\sigma_\perp/dt$ is determined by A_1 alone. The validity of the superconvergence relation for $i=1$,[29] *and* the assumption of saturation of the remaining dispersion integrals by low-energy contributions, are at variance with experiment. Under these assumptions, the amplitude A_1 would behave like $1/s^2$ for $s\to\infty$, whereas experiment shows a $1/s$ behavior. In fact, it has been shown by Dietz and Korth[30] that high-energy contributions are vital to guarantee the validity of the superconvergence relation for A_1. Therefore, low-energy contributions can saturate only the unsubtracted dispersion contributions (6) and not the subtracted dispersion contributions (12). We mention in this context that the variation of $d\sigma_\perp/dt$ in k^2 at $t\approx0$ is mainly an effect of the variation of the coupling constants, since the dispersion contribution is small for $t\approx0$. Thus, we expect this variation even at asymptotic energies $s\to\infty$.

[29] M. B. Halpern, Phys. Rev. **160**, 1611 (1967).

[30] K. Dietz and W. Korth, University of Bonn Report No. 2-28, 1967 (unpublished); Phys. Letters **26B**, 394 (1968).

C. Conclusions with Respect to Other Situations

We have shown in the previous discussion how intimately the question of whether there exists mass independence of the scalar amplitudes $A_i{}^{\rho}(s,t,k^2)$ depends on further dynamical assumptions such as analyticity, crossing symmetry, and superconvergence relations. Further mass dependences in $A_i{}^{\rho}(s,t,k^2)$ arise from the variation of the $\rho N\bar{N}$ coupling constants $F_{1,2}{}^{\rho NN}$ considered as a function of the mass k^2 of the ρ meson. Therefore, one should not be surprised if mass dependences also show up in other reactions like ρ production, where the production mechanism is completely different (diffraction production) from pion production. It is very likely that the same dynamical laws and facts again account for mass dependences of the amplitudes $\rho+N\to\rho'+N'$ if one takes the mass of the first ρ meson to zero where one then has photoproduction of ρ mesons. Here, difficulties in applying VMD in the usual sense have recently been reported.[31]

The VMD hypothesis has stimulated much experimental progress and many theoretical efforts. One is therefore not inclined to give up this idea easily. But we have learned that a too simple-minded understanding of the meaning of VMD leads to failure. The mass dependence of the scalar amplitudes $A_i{}^V$ for a process initiated by real or virtual photons is not just simply given by the ρ propagator $(m_\rho{}^2-k^2)^{-1}$. Such a simple picture may evolve for some particular cases like the reaction $e^+e^-\to\pi^+\pi^-$, which has a very simple structure, but in general the situation is more complicated. Finally, we would like to mention that, in general, for masses much smaller than the photon mass ($k^2\ll0$), higher-mass contributions cannot be neglected. The example of the nucleon form factors serves as a clear illustration of this fact.[18]

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[31] G. McClellan *et al.*, Phys. Rev. Letters **22**, 377 (1969).