Decays of High-Spin Objects Produced by Pion Exchange*

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The OPE-δ prescription for approximating the absorptive one-pion-exchange (OPE) predictions for spin-density matrix elements has been applied to quasi-two-body reactions with final-state particles of arbitrary spins. Results indicate that the model is able to give the qualitative features of usual absorptive-one-pion-exchange (OPEA) models. Agreement between OPE-δ and OPEA predictions is good in general. Predictions of the two models for diagonal density matrix elements for the reaction $\pi+N\rightarrow f^0+N$ tend to disagree at larger angles.

I. INTRODUCTION

IN an earlier paper,[1] one of the authors introduced a method of approximating the results of absorption-model (OPEA) calculations for spin-density matrix elements of unstable particles. The method (referred to hereafter as the OPE-δ method) consists in deleting the exceptional terms in the one-pion-exchange (OPE) partial-wave amplitudes. It was noticed in Ref. 1 that without doing a partial-wave expansion of the OPE amplitudes, one could obtain approximately the correct ratios of helicity amplitudes of the OPEA model by a simple replacement of variables in appropriate parts of the OPE amplitudes. First of all, one writes the two-body-reaction helicity amplitudes as[2]

$$\langle\lambda_c\lambda_d|B(s,t)|\lambda_a\lambda_b\rangle=\left(\frac{1-X}{2}\right)^{|\lambda-\nu|/2}\times\left(\frac{1+X}{2}\right)^{|\lambda+\nu|/2}\frac{P(\lambda,\nu;X)}{Z-X},\quad(1)$$

where X is the cosine of the center-of-mass scattering angle; $P(\lambda,\nu;X)$ is a polynomial in X; $\lambda=\lambda_c-\lambda_d$, $\nu=\lambda_a-\lambda_b$, $Z=[1+(\mu^2-t)/2qq']_{X=1}$; q and q' are the initial and final center-of-mass momenta, respectively; and μ is the exchanged mass. The OPE-δ amplitudes are given by

$$t_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}{}^{\text{OPE}-\delta}\propto\left(\frac{1-X}{2}\right)^{|\lambda-\nu|/2}\left(\frac{1+X}{2}\right)^{|\lambda+\nu|/2}\frac{P(\lambda,\nu;Z)}{Z-X},\quad(2)$$

where $P(\lambda,\nu;X)$ in Eq. (1) has been evaluated at the pion pole $X=Z$. The proportionality is assumed independent of helicity.[1] Since we will confine predictions to density matrix elements, we need not specify the OPE-δ amplitudes further.

The OPE-δ model is similar to the OPEA prescription of Durand and Chiu,[2] who, however, applied the usual absorptive corrections to the OPE-δ helicity amplitudes. The advantage of the OPE-δ model lies mainly in the simplicity and facility of the computational work involved. This is because the OPE-δ prescription does not involve the partial-wave expansion that is generally required in OPEA model calculations.[3] The OPE-δ model has been applied in the prediction of density matrix elements of ρ in the reaction $\pi+N\rightarrow\rho+N$ at 3 BeV/c.[1] It was seen that the OPE-δ predictions for the density matrix elements of ρ were in good agreement with the corresponding OPEA predictions and with the data.

In this paper, a further comparison of the OPE-δ and OPEA predictions for spin-density matrix elements with data (where available) will be made through the reactions $\pi+N\rightarrow\rho+N^*(1238)$ at 8 BeV/c and $\pi+N\rightarrow f^0+N$ at 6 BeV/c. In addition, results of OPE-δ calculations for a number of other reactions involving production of particles of higher spins by OPE's will be given. For this purpose, an appropriate expression for OPE helicity amplitudes for the production of particles of arbitrary spins shall be exhibited [Eq. (4)]. The OPE-δ prescription can then be applied to these amplitudes to generate spin-density matrix elements of the unstable product particles.

II. OPE-δ AMPLITUDES

The OPE helicity amplitudes for the reaction

$$\pi+N\rightarrow c(S_c)+d(S_d),\quad(3)$$

where c is a boson resonance with spin S_c and d is a baryon resonance with spin S_d, are easily deduced from crossing symmetry and parity conservation. To within a helicity-independent proportionality constant K, the Born-approximation helicity amplitudes are given by

$$t_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}{}^{\text{OPE}}(X)=Kd_{0,\lambda_c}{}^{S_c}(\psi_c)[d_{\frac{1}{2},\lambda_b}{}^{\frac{1}{2}}(\psi_b)d_{\frac{1}{2},\lambda_d}{}^{S_d}(\psi_d)+(-1)^{S_d+\frac{1}{2}}\eta_d d_{-\frac{1}{2},\lambda_b}{}^{\frac{1}{2}}(\psi_b)d_{-\frac{1}{2},\lambda_d}{}^{S_d}(\psi_d)],\quad(4)$$

where η_d is the parity of d, λ_i is the helicity of particle i, and ψ_i is the crossing angle defined by Trueman and Wick[4] for particle i. Equation (4) can also be understood as being the amplitude which gives the correct results for density matrix elements for particles b, c, and d. Evaluated in their respective rest frames with respect

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[1] P. K. Williams, Phys. Rev. **181**, 1963 (1969).
[2] L. Durand and Y. Chiu, Phys. Rev. **137**, B1530 (1965).
[3] G. Kane, Phys. Rev. **163**, 1544 (1967).
[4] T. Trueman and G. Wick, Ann. Phys. (N. Y.) **26**, 322 (1964).

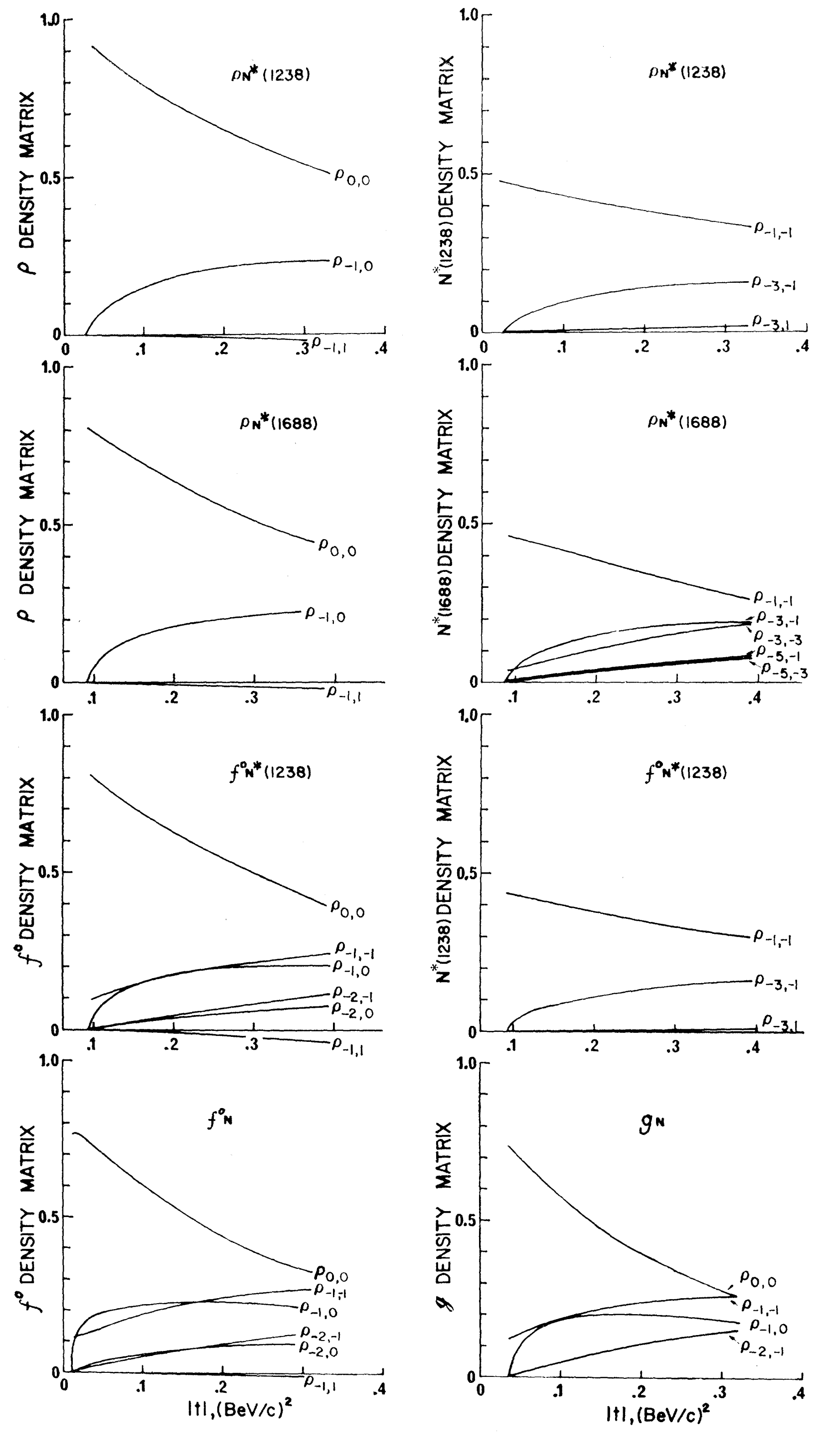

FIG. 1. OPE-δ predictions for the density matrix elements of the final-state resonance particles for the reactions $\pi+N\rightarrow\rho+N^*(1238)$, $\pi+N\rightarrow\rho+N^*(1688)$, $\pi+N\rightarrow f^0+N^*(1238)$, $\pi+N\rightarrow f^0+N$, and $\pi+N\rightarrow g+N$ at 8 BeV/c.

to "canonical"[5] axes, these density matrix elements take on especially simple constant values (e.g., for particle c in its rest frame, with respect to the direction of particle a, $\rho_{ij}=\delta_{i0}\delta_{j0}$, etc.).

The OPE amplitudes are related to functions $P(\lambda,\nu;X)$ defined in Eq. (1) by

$$P(\lambda,\nu;X)=\left(\frac{1-X}{2}\right)^{-|\lambda-\nu|/2}\left(\frac{1+X}{2}\right)^{-|\lambda+\nu|/2}\times(Z-X)t_{\lambda_c,\lambda_d;\lambda_b}{}^{\mathrm{OPE}}(X). \quad (5)$$

[5] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

One can evaluate Eq. (5) at $X=Z$ and substitute the result in Eq. (2) to obtain the OPE-δ amplitudes

$$t_{\lambda_c,\lambda_d;\lambda_b}{}^{\mathrm{OPE}-\delta}(X)\propto\left(\frac{1-X}{1-Z}\right)^{+|\lambda-\nu|/2}\left(\frac{1+X}{1+Z}\right)^{+|\lambda+\nu|/2}\frac{1}{Z-X}\times\left[(Z-X)t_{\lambda_c,\lambda_d;\lambda_b}{}^{\mathrm{OPE}}(X)\right]_{X=Z}, \quad (6)$$

with t^{OPE} given by Eq. (4). Equation (6) is directly useful in calculating density matrix elements for particles c and d in Eq. (3).

III. RESULTS AND DISCUSSION

A number of computations have been performed on the $\pi+N$ reactions with the two-body final states ρN^* (1238), $\rho N^*(1688)$, f^0N, $f^0N^*(1238)$, and gN. Only spin-density matrix elements with respect to the "canonical"[5] axes will be shown. The results for $\pi+N\rightarrow\rho N^*$ (1238) at 8 BeV/c are plotted in Fig. 1 together with two somewhat differing OPEA results[6,7] and experimental data.[7] Figure 1 shows good agreement between the OPE-δ and the OPEA calculations for diagonal matrix elements. The OPE-δ predictions for $\rho_{1,0}$ and $\rho_{3,1}$ are somewhat larger than those of both OPEA calculations, but the disagreement between the two OPEA calculations is equally pronounced. None of these three

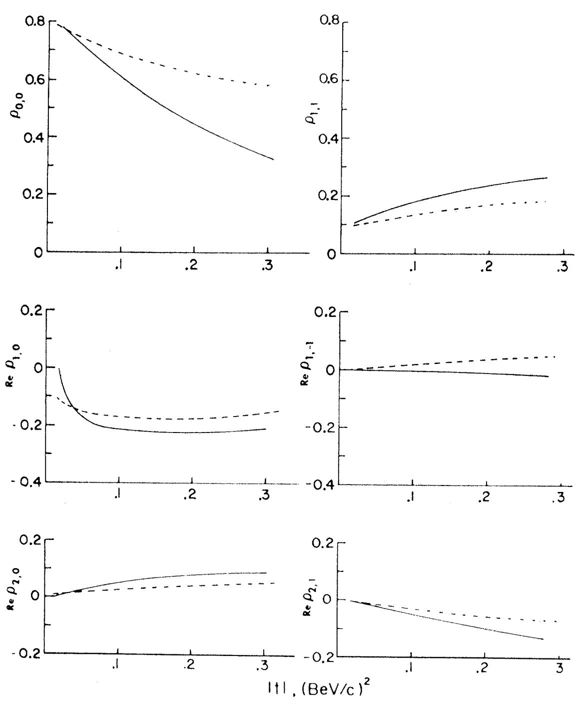

FIG. 2. Density matrix elements of the f^0 for the reaction $\pi+N\rightarrow f^0+N$ at 6 BeV/c. Solid curves are OPE-δ predictions, dashed curves are OPEA predictions from Ref. 8.

[6] J. Donohue, Ph.D. thesis, University of Illinois, 1967 (unpublished).

[7] Aachen-Berlin-CERN Collaboration, Phys. Letters **22**, 533 (1966).

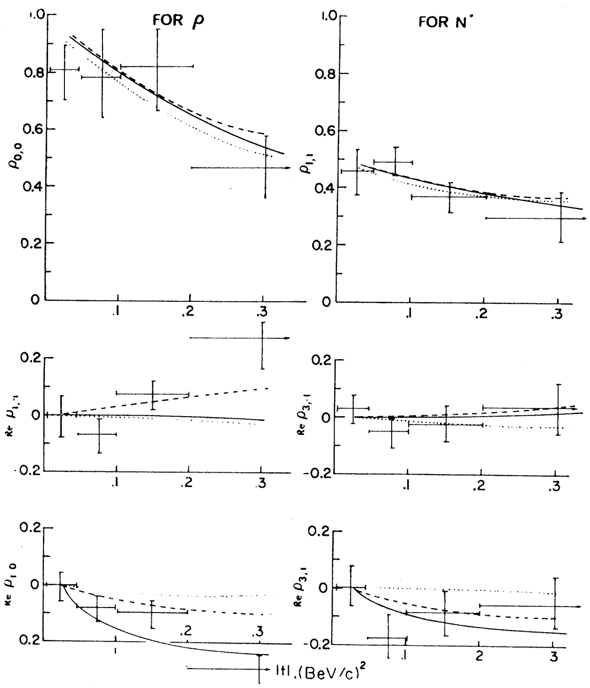

FIG. 3. Density matrix elements of the ρ and N^* for the reaction $\pi+N\rightarrow\rho+N^*(1238)$ at 8 BeV/c. Solid surces are OPE-δ predictions, dashed curves are OPEA-IPR predictions from Ref. 7, and dotted curves are OPEA-EPWS predictions from Ref. 6. Data are from Ref. 7.

models seems to be preferred in comparison with experimental data. In general, the agreement between OPE-δ and OPEA predictions for $\pi N\rightarrow\rho N^*(1238)$ is comparable to that of the reaction $\pi N\rightarrow\rho N$.[1]

Predicted density matrix elements for the reaction $\pi+N\rightarrow f^0+N$ at 6 BeV/c and corresponding OPEA results[8] are plotted in Fig. 2. It indicates that the predictions of the two models do exhibit the same qualitative behavior, but the discrepancy between these is in general more pronounced than in the ρN^* case. The difference between the predictions of the two models for the diagonal matrix elements is particularly large at larger angles. Experimental data for the diagonal density matrix elements in this reaction are available[9] but the experimental numbers do not seem reliable enough[10] to allow definitive discrimination between the two models.

Some of the more interesting OPE-δ spin-density matrix elements for the reactions

$$\begin{aligned}\pi+N&\rightarrow\rho+N^*(1688)\\&\rightarrow f^0+N\\&\rightarrow f^0+N^*(1238)\\&\rightarrow g?+N\end{aligned}$$

[8] P. C. M. Yock and D. Gordon, Phys. Rev. **157**, 1362 (1967).

[9] J. Poirer, N. Biswas, N. Cason, I. Derado, V. Kennedy, W. Shepard, and E. Synn, Phys. Rev. **163**, 1462 (1967).

[10] The experimentally determined diagonal (2,2) density matrix element for f^0 given in Ref. 8 turned out to be a significantly negative number.

at 8 BeV/c are given in Fig. 3. The density matrix elements obtained indicate that the typical qualitative behavior of OPEA calculations are reproduced in all the reactions mentioned above, but no experimental data are available for comparison.

It has been observed that the OPE-δ model gives qualitatively reasonable spin-density matrix elements for quasi-two-body reactions in which the OPE mechanism is expected to dominate. The agreement between the OPE-δ model and the OPEA model predictions for spin-density matrix elements is in general quite good. However, a significant quantitative difference was observed in the OPEA and OPE-δ predictions for diagonal density matrix elements $\rho_{0,0}$ and $\rho_{1,1}$ for the reaction $\pi+N\to f^0+N$ at 6 BeV/c. The difference between the two models in this can can be traced to a large $J=\frac{3}{2}$ Kronecker-δ term in the OPE amplitude $t_{1,-\frac{1}{2};-\frac{1}{2}}{}^{J=\frac{3}{2}}$ which is not completely absorbed in the OPEA model but is completely deleted in the OPE-δ model. That this term is large is a property of the $\bar{N}N\pi$ vertex. This mechnism should be expected to give rise to even larger discrepancies with the OPEA model in reactions where higher-spin bosons are produced with the final-state nucleon. This is because the large Kronecker-δ terms would appear in higher partial waves which are even less completely absorbed in the OPEA model. Unfortunately, no experimental data are available for a definitive discrimination between the density matrix elements predicted by the OPE-δ and the OPEA models.

Finally, it must be emphasized that the OPE-δ model derives its usefulness in terms of being able to generate OPEA-model-type predictions for spin-density matrix elements without partial-wave expansion. Hence, it will be useful as a tool for a quick generation of OPEA-model-type results as a part of a larger scheme of calculations.

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Vector-Meson Dominance and Pion Production*

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On the basis of the current-field-identity relation, a unified treatment and quantitatively successful calculation of pion production by ρ mesons and photons is presented. The model, which is independent of free parameters, explains the empirically established facts about strong mass dependences in the ρ density matrix elements (in particular, also in the combination $\rho_{11}+\rho_{1-1}$) considered as a function of the mass k^2 of the ρ meson. These facts have been the basis for a recent criticism of the vector-meson-dominance hypothesis in pion production. But our results indicate that the current-field-identity relation is not in conflict with experiment. We emphasize in our discussion that no model-independent formulation of vector-meson dominance is possible in pion production, according to our present knowledge.

I. INTRODUCTION

A BASIC ingredient to any concept of ρ dominance in electroproduction of pions is the relation

$$\langle\pi N'|J_\mu{}^V(0)|N\rangle=-(m_\rho{}^2/f_\rho)\langle\pi N'|\rho_\mu(0)|N\rangle =[m_\rho{}^2/(m_\rho{}^2-k^2)](1/f_\rho)\langle\pi N'|J_\mu{}^\rho(0)|N\rangle. \quad (1)$$

Here, $k^2=(N-N'-\pi)^2$ and the momenta and all other quantum numbers of the pion and nucleons are denoted by π, N, and N'. Equation (1) is a direct consequence of the more general current-field-identity relation[1]

$$J_\mu{}^V(x)=-(m_\rho{}^2/f_\rho)\rho_\mu(x). \quad (2)$$

This equation relates the hadronic isovector part of the electromagnetic current $J_\mu{}^V(x)$ to the phenomenological field $\rho_\mu(x)$ of the ρ meson $[J_\mu{}^\rho(x)=-(\Box+m_\rho{}^2)\rho_\mu(x)]$. Through charge independence and time-reversal invariance of the nuclear forces, Eq. (1) connects specifically the isovector part of the amplitudes for the pion-production processes by real or virtual photons

$$\gamma p\to\pi^+n \quad\text{and}\quad \gamma n\to\pi^-p \quad (3)$$

to the ρ^0-production processes by pions

$$\pi^-p\to\rho^0n. \quad (4)$$

Strictly speaking, comparison of the reactions (3) and (4) requires an analytic continuation in k^2 of the matrix elements of Eq. (1). To separate the ρ pole in the neighborhood of the ρ resonance, we write (1) in the form

$$\langle\pi N'|J_\mu{}^V(0)|N\rangle\approx\frac{m_\rho{}^2}{m_\rho{}^2-k^2-im_\rho\Gamma}\frac{1}{f_\rho}\langle\pi N'|\hat{J}_\mu{}^\rho|N\rangle. \quad (5)$$

Here $\hat{J}_\mu{}^\rho$ is defined as the modified current operator in

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† On leave from Institut für Experimentelle Kernphysik, Kernforschungszentrum Karlsruhe, Germany.
[1] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).