Longitudinal Electrical Conductivity of a Relativistic Gas in an Intense Magnetic Field

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The electrical conductivity for an electron gas is computed when a strong quantizing magnetic field is present. This strong field ($\simeq 10^{13}$ G) is thought to exist in the interior of a neutron star. The knowledge of the conductivity is of primary importance for the evaluation of the decay time of the magnetic field due to Ohmic dissipation. This is a critical quantity in most of the recent developments of pulsars as magnetized neutron stars. The chief scattering process is Coulomb scattering by ions. For magnetic field of the order of 10^{13} G, the conductivity is higher by a factor 10–10^2 with respect to the case without a magnetic field. The ion-ion correlation is included through the use of the static pair-correlation function. Its effect is that of increasing the conductivity by a factor of 2.

1. INTRODUCTION

THE relativistic transport properties of an electron gas imbedded in a system of ions has been studied in a previous paper,[1] taking into account the ion-ion interaction. In the present paper the electrical conductivity is studied when the electrons are also imbedded in a strong homogeneous and constant magnetic field. Only the problem of the longitudinal conductivity is studied in this paper. As we see later, the electron moves freely in the direction parallel to the magnetic field; this circumstance allows the application of the Boltzmann equation. When the electric field E is parallel to H, there is no need to diagonalize the Hamiltonian with both fields included as one ought to do when the transverse conductivity is studied. This last problem is treated in a separate paper.[2] We proceed to compute the electrical conductivity and then, through the use of the Wiedemann-Franz law, we can deduce the thermal conductivity. The conductive opacity is then easily computed. The Weidemann-Franz law has been shown to hold in the presence of a quantizing magnetic field by Zyrianov.[3] All of the transport properties depend on a dimensionless function $f(\phi,\mu,\theta)$, where μ is the chemical potential (plus the rest mass of the electron in units of mc^2), $\theta=H/H_c$ is the magnetic field in units of $H_c=m^2c^3/e\hbar=4.414\times10^{13}$ G, $\phi=kT/mc^2$. The results are arranged in tables for the function $f(\phi,\mu,\theta)$ for degenerate and nondegenerate cases as function of the density ρ (or, equivalently, the chemical potential) for two different values of θ: 0.1 and 1.0. Figures 1–3 show the behavior of the function f when μ varies. The discontinuities present in the degenerate case are caused by the density of final states which is a discontinuous function of the energy.[4] The behavior of $f(\phi,\mu,\theta)$ is considerably smoother at high temperatures, i.e., $T\simeq10^{10}$ °K. As seen from Figs. 1 and 2, the number of discontinuities considerably increases with decreasing magnetic field. For that reason it was not possible to include values lower than $\theta=0.1$. The problems involved in the computation of the function $f(\phi,\mu,\theta)$ were quite complex, and in the Appendix a sketch of the procedure we used is outlined.

The knowledge of the electrical conductivity allows us to compute the decay time of H due to Ohmic dissipation. This quantity has become of primary importance in problems related to pulsars which are now accepted to be magnetized neutron stars. Using Maxwell's equations and Ohm's law, one can easily prove that the diffusion equation for a magnetic field H is[5]

$$\partial H/\partial t=(c^2/4\pi\sigma_H)\nabla^2 H\,,$$

which indicates that an initial configuration of H will decay in a diffusion time τ given by

$$\tau=4\pi\sigma_H L^2c^{-2}\,,$$

where L is a length which characterizes the spatial variation of H. The main contribution to σ_H is thought to be the Coulomb scattering of electrons by ions. The elastic scattering of an electron by a Coulomb potential, taking place in a magnetic field, is computed in analogy with the Mott scattering by using the exact wave-function solution of the Dirac equation.

2. DIRAC EQUATION WITH MAGNETIC FIELD

As discussed in a previous paper,[4] an electron of mass m, charge e, in an external electromagnetic field A_μ, is described by

$$[\gamma_\mu\partial_\mu+mc/\hbar+(ie/\hbar c)\gamma_\mu A_\mu]\psi_D=0\,. \qquad (1)$$

In the special case of a pure magnetic field H, constant in time and homogeneous in space, we have

$$\mathbf{A}=\tfrac{1}{2}\mathbf{H}\times\mathbf{r}\,,\quad A_4=0\,. \qquad (2)$$

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[1] V. Canuto, Astrophys. J. (to be published).

[2] V. Canuto and C. Chiuderi (unpublished).

[3] P. S. Zyrianov, Fiz. Metal. i Metalloved **18**, 161 (1964).

[4] V. Canuto and H.-Y. Chiu, Phys. Rev. **173**, 1210 (1968); **173**, 1220 (1968); **173**, 1229 (1968).

[5] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 1962), p. 313.

If H is directed along the z axis, i.e., $H_x=H_y=0$, then

$$A_x=-\tfrac{1}{2}yH\,,\quad A_y=\tfrac{1}{2}xH\,,\quad A_z=0. \tag{3}$$

Because of the nature of the problem we treat later, it is more convenient to work with a different ψ, gauge transformed in the following way:

$$\psi=\psi_D\, e^{if(xy)}\,, \tag{4}$$

with $f(xy)$ chosen in such a way that

$$A_x=-yH\,,\quad A_y=A_z=0. \tag{5}$$

Clearly,

$$f(xy)=-\tfrac{1}{2}(e/\hbar c)xyH. \tag{6}$$

The solution to the Dirac equation can be written as[5]

$$\begin{gathered}\psi=\psi(\mathbf{r})e^{-imc^2\eta\epsilon t/\hbar}\,,\\ \psi(\mathbf{r})=u_n(\xi)e^{-\frac{1}{2}\xi^2}e^{i(k_1x+k_3z)}\,,\\ u_n(\xi)=\begin{bmatrix}c_1\bar{H}_n(\xi)\\ c_2\bar{H}_{n-1}(\xi)\\ c_3\bar{H}_n(\xi)\\ c_4\bar{H}_{n-1}(\xi)\end{bmatrix};\end{gathered} \tag{7}$$

$$\begin{gathered}\epsilon(x)=(1+x^2+2n\theta)^{1/2}\,,\\ n=0,1,\ldots,\infty\,,\quad x=p_z/mc\,,\quad \theta=H/H_c\,,\\ H_c=m^2c^3/e\hbar=4.414\times10^{13}\ \mathrm{G}\,,\\ \xi=y\gamma^{1/2}+k_1\gamma^{-1/2}\,,\quad \gamma=\theta\lambda\!\!\!^{-}{}_c^{-2};\end{gathered} \tag{8}$$

$$\begin{gathered}\lambda\!\!\!^{-}{}_c=\hbar/mc\,,\quad c_1=aA\,,\quad c_2=saB\,,\\ c_3=\eta sbA\,,\quad c_4=\eta bB;\end{gathered} \tag{9}$$

$$\begin{gathered}a^2=\tfrac{1}{2}(1+\eta\epsilon^{-1})\,,\quad A^2=\tfrac{1}{2}[1+sx(x^2+2n\theta)^{-1/2}]\,,\\ b^2=\tfrac{1}{2}(1-\eta\epsilon^{-1})\,,\quad B^2=\tfrac{1}{2}[1-sx(x^2+2n\theta)^{-1/2}].\end{gathered} \tag{10}$$

The two indices $\eta=\pm1$, $s=\pm1$ stand for positive and negative energies and the sign of the projection of the momentum component along the spin. $\bar{H}_n(x)$ are the Hermite polynomials normalized to 1 in the interval $-\infty$ to ∞:

$$\bar{H}_n=\gamma^{1/4}\pi^{-1/4}2^{-n/2}(n!)^{-1/2}H_n. \tag{11}$$

The wave functions ψ_D were used to construct the equations of state for an electron gas in a magnetic field by calculating the energy-momentum tensor $T_{\mu\nu}$.[4]

A simplified derivation of the same equation of state by only using the expression for the quantized energy eigenvalues was given in a recent paper.[6]

3. MOTT SCATTERING IN MAGNETIC FIELD

In analogy with the case $H=0$, we start with the interaction-Hamiltonian density

$$\mathcal{H}_{\mathrm{int}}=-ie\bar{\psi}(\mathbf{r})\gamma_\mu\psi(\mathbf{r})A_\mu(\mathbf{R}_\alpha-\mathbf{r})\,, \tag{12}$$

where $\mathbf{r}$ and $\mathbf{R}_\alpha$ represent the position vectors of the electron and of the αth ion, respectively, ψ is given by Eq. (7), and $A_\mu(\mathbf{R}_\alpha-\mathbf{r})$ is given by

$$A=0\,,\qquad A_4=\frac{Ze^2}{|\mathbf{R}_\alpha-\mathbf{r}|}=\frac{4\pi Ze^2}{L_xL_yL_z}\sum_q\frac{1}{q^2}e^{-i\mathbf{q}\cdot(\mathbf{R}_\alpha-\mathbf{r})}. \tag{13}$$

The interaction energy is now ($\bar{\psi}=\psi^\dagger\gamma_4$)

$$H_{\mathrm{int}}\equiv\int\sum_{\alpha=1}^{N_i}\mathcal{H}_{\mathrm{int}}d^3r=\frac{4\pi e^2Z}{L_xL_yL_z}\sum_{\alpha=1}^{N_i}q^{-2}I_{if}e^{-i\mathbf{q}\cdot\mathbf{R}_\alpha}\,, \tag{14}$$

$$I_{if}\equiv\int d^3r\,\psi_f^\dagger(r)e^{i\mathbf{q}\cdot\mathbf{r}}\psi_i(r)\,, \tag{15}$$

where i and f mean the set of all quantum numbers, namely, s, η, n, x [see Eqs. (7)–(10)], describing the initial and final states of the electrons. In our case $\eta_i=\eta_f=+1$, and from now on we drop these indices. Substituting Eq. (7) into Eq. (15), we obtain

$$\begin{aligned}I_{if}&=\delta_{k_1-k_1'+q_1}\delta_{k_3-k_3'+q_3}(\omega_1\langle n|n'\rangle+\omega_2\langle n-1|n'-1\rangle)\\ &\equiv\delta_{k_1-k_1'+q_1}\delta_{k_3-k_3+q_3}A_{n,n'}\,,\end{aligned} \tag{16}$$

with

$$\omega_1=c_1c_1'+c_3c_3'\,,\quad \omega_2=c_2c_2'+c_4c_4'\,,$$

and, in general,

$$\begin{aligned}\langle n|n'\rangle&=\gamma^{1/2}\pi^{-1/2}(n!n'!)^{-1/2}(2^{-n-n'})^{1/2}\\ &\quad\times\int_{-\infty}^{+\infty}dy\,H_n(y_1)H_n(y_2)\exp(iq_2y-\tfrac{1}{2}y_1^2-\tfrac{1}{2}y_2^2)\,,\\ y_1&=y\gamma^{1/2}+k_1\gamma^{-1/2}\,,\quad y_2=y\gamma^{1/2}+k_1'\gamma^{-1/2}.\end{aligned} \tag{17}$$

The integration can be performed only after specifying the sign of $(n-n')$. When $n\geqslant n'$, we denote $\langle n|n'\rangle$ by $I(n'|n)$. The result is

$$\begin{aligned}I(n'|n)&=[(n-n')!]^{-1}(n!n'!)^{-1/2}e^{-t/2}t^{(n-n')/2}\\ &\quad\times{}_1F_1(-n';n-n'+1;t)e^{-i(n-n')\phi}e^{-i\Lambda}\,,\end{aligned} \tag{18}$$

with

$$t=(2\gamma)^{-1}(q_1^2+q_2^2)\,,\quad \phi=\operatorname{arccot}(q_2/q_1)\,,\quad \Lambda=q_2(q_1+k_1)(2\gamma)^{-1};$$

${}_1F_1$ is the hypergeometric function. The result (18) can be brought into a more symmetric form by changing ${}_1F_1$ to ${}_2F_0$ (which is symmetrical in its parameters) using the relation

$${}_1F_1(-n';n-n'+1;x)=[(n-n')!/n!]\times(-)^{n'}x^{n'}\,{}_2F_0(-n',-n;-1/x)\,,$$

which gives

$$\begin{aligned}I(n'|n)&=(n!n'!)^{-1/2}e^{-t/2}t^{(n+n')/2}\\ &\quad\times{}_2F_0(-n',-n;-1/t)(-)^{n'}e^{-i(n-n')\phi}e^{-i\Lambda}\,,\end{aligned} \tag{19}$$

or

$$I(n'|n)=\Psi(n|n')(-)^{n'}e^{-i\Lambda}e^{-i(n-n')\phi}\,, \tag{20}$$

with

$$\Psi(n|n')=\Psi(n'|n).$$

[6] V. Canuto, H.-Y. Chiu, and L. Fassio-Canuto, Astrophys. Space Sci. 3, 258 (1969).

If $n'\geqslant n$, we have from Eqs. (19) and (20),

$$I(n|n')=I(n'|n)(-)^{n-n'}e^{-2i\phi(n'-n)}. \quad (21)$$

In general, for any n and n', we have

$$\langle n|n'\rangle=\Psi(n|n')[e^{-i(n-n')\phi}(-)^{n'}\theta(n-n') + e^{-i(n'-n)\phi}(-)^{n}\theta(n'-n)-(-)^{n}\delta_{n,n'}], \quad (22)$$

with $\theta(x)=1$ for $x\geqslant 0$ and $\theta(x)=0$ for $x<0$. For completeness we give the definition of ${}_2F_0$,

$$ {}_2F_0(a,b;x)=1+\sum_{k=1}^{\infty}\frac{(a)_k(b)_kx^k}{k!}, \quad (23)$$

where $(a)_k$ is the Pockhammer symbol defined as

$$(a)_k=\prod_{s=1}^{k}(a+s-1). \quad (24)$$

The next step is to calculate the transition rate per unit time, $W(i,f)$, given by the equation

$$W(i,f)=(2\pi/\hbar)(mc^2)^{-1}|H_{\text{int}}|^2\delta(\epsilon_i-\epsilon_f).$$

Using Eqs. (14) and (15), we get

$$W(i,f)=\frac{2\pi}{\hbar}(mc^2)^{-1}\left(\frac{4\pi Ze^2}{L_xL_yL_z}\right)^2\sum_{\alpha,\beta=1}^{N_i}q^{-2}(q')^{-2} \times I_{if}(q)I_{if}^*(q')e^{-i(\mathbf{q}\cdot\mathbf{R}_\alpha-\mathbf{q}'\cdot\mathbf{R}_\beta)}. \quad (25)$$

Because the ions are unaffected by the presence of the magnetic field (because of their mass), the ion system is homogeneous, and we can therefore write

$$\sum_{\alpha=1}^{N_i}\sum_{\beta=1}^{N_i}e^{-i(\mathbf{q}\cdot\mathbf{R}_\alpha-\mathbf{q}'\cdot\mathbf{R}_\beta)}=\delta_{q-q'}N_i\left(1+\frac{N_i}{\Omega}g(q)\right) \equiv\delta_{q-q'}N_i\Phi(q), \quad (26)$$

with

$$g(q)=\int g(r)e^{i\mathbf{q}\cdot\mathbf{r}}d^3r,$$

where $g(r)$ is the two-body correlation function defined in Ref. 1:

$$g(r_1r_2)=\Omega^2N(N-1)N^{-2}\left(\int dr_1\cdots dr_Ne^{-U/kT}\right)^{-1} \times\left(\int dr_3\cdots dr_Ne^{-U/kT}\right).$$

As shown in Ref. 1, we have

$$\Phi(q)=1+3\int_0^\infty(x\xi)^{-1}\sin(x\xi)dx, \quad (27)$$

with

$$\xi=qa=(3/4\pi)^{1/3}(\Omega/N_i)^{1/3}|\mathbf{q}|.$$

If we make use of the δ function of Eq. (25), the quantity ξ becomes

$$\xi=(3/4\pi)^{1/3}(\Omega/N_i)^{1/3} \times[(k_1-k_1')^2+(k_3-k_3')^2+q_2^2]^{1/2}. \quad (28)$$

We have, finally,

$$W(i,f)=W_0\sum_{q_2}\frac{|A_{n,n'}|^2\delta(\epsilon_i-\epsilon_f)}{[(k_1-k_1')^2+(k_3-k_3')^2+q_2^2]^2} \times\Phi(k_1-k_1',k_3-k_3',q_2), \quad (29)$$

$$W_0=(2\pi/\hbar)(mc^2)^{-1}(4\pi Ze^2)^2(N_i/\Omega)(L_xL_yL_z)^{-1}.$$

From Eq. (29) we can obtain the relaxation time τ defined as ($g=2s+1=2$)

$$\tau^{-1}=\sum_f(1-k_3'/k_3)W(i,f),\quad \sum_f\equiv g\sum_{n'}\sum_{k_1'}\sum_{k_3'}$$

$$\tau^{-1}=gW_0\frac{L_xL_yL_z}{(2\pi)^3} \times\sum_{n'}\int_{-\infty}^{+\infty}dq_2\int_{-\infty}^{+\infty}dk_1'\int_{-\infty}^{+\infty}dk_3'(1-k_3'/k_3) \times\frac{|A_{n,n'}|^2\Phi\delta(\epsilon_i-\epsilon_f)}{[(k_1-k_1')^2+(k_3-k_3')^2+q_2^2]^2}, \quad (30)$$

where the summations have been converted into integration through the usual relation

$$\sum_q\to\frac{L}{2\pi}\int dq.$$

Before proceeding to the integration, we have to study the functional dependence of $A_{n,n'}$. From the definition (16) and the general formula (22) we have, after some algebra,

$$|A_{n,n'}|^2=[\omega_1\Psi(n|n')-\omega_2\Psi(n-1|n'-1)]^2 \times[\theta(n-n')+\theta(n'-n)-\delta_{n,n'}] \quad (31)$$

for any n and n'. The last square bracket is identically equal to 1 for any combination of n and n'. It is now clear that the ϕ,Λ dependence cancels and $|A_{n,n'}|^2$ depends only on the variable $t=[(k_1'-k_1)^2+q_2^2]/2\gamma$ and k_3, k_3' through ω_1 and ω_2. Introducing the new variables

$$(2\gamma)^{1/2}u_1=k_1-k_1',\quad (2\gamma)^{1/2}u_2=q_2,$$
$$u-u'=k_3-k_3',\quad t=u_1^2+u_2^2,$$

and writing explicitly the variables that $|A_{n,n'}|^2$ depends on, we have

$$\tau^{-1}=(4gZ^2e^4)(\hbar mc^2)^{-1}(N_i/\Omega)(2\gamma)^{-1}\sum_{n'}\int du_1\int du_2 \times\int du'(1-u'/u)\frac{|A_{n,n'}|^2\Phi\delta(g(u,u'))}{[u_1^2+u_2^2+(u-u')^2]}, \quad (32)$$

$$g(u,u')=(1+2u^2\theta+2n\theta)^{1/2}-(1+2u'^2\theta+2n'\theta)^{1/2}.$$

The functional form of Φ is still given by Eq. (27), with the variable ξ defined as

$$\xi=2^{1/2}(3/4\pi)^{1/3}(\Omega/N_i\lambda_c^3)^{1/3}\theta^{1/2}\times[u_1^2+u_2^2+(u-u')^2]^{1/2}. \quad (33)$$

Because the τ integrand depends on u_1 and u_2 only via the combination $u_1^2+u_2^2$, we can use the relation

$$\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\, f(x^2+y^2)=\pi\int_0^{\infty}f(s)ds$$

to obtain

$$\tau^{-1}=\tau_1^{-1}\sum_{n'}\int_{-\infty}^{+\infty}du'\left(1-\frac{u'}{u}\right)R(u,u')\delta[g(u,u')], \quad (34)$$

with

$$\tau_1^{-1}=(4\pi/g2^{1/2})\alpha^2Z^2(N_i/\Omega)c\lambda_c^2\theta^{-1/2}, \quad (35)$$

$$R(u,u')=\int_0^{\infty}dt\frac{|A_{n,n'}(t,u,u')|^2}{[t+(u-u')^2]^2}\Phi(\xi). \quad (36)$$

Splitting Eq. (34) into two integrations, from $-\infty$ to 0 and from 0 to ∞, changing from u to $-u$ in the first, and then shifting again from the variable u to E defined as

$$E^2=1+2n\theta+2u^2\theta=a_n^2+2u^2\theta, \quad (37)$$

and, finally, using in each integral the δ function on the energy, we can perform the integration obtaining as a final result

$$\frac{\tau_0}{\tau_n}=\sum_{n'=0}^{\infty}E(E^2-a_{n'}^2)^{-1/2}\left[1+\left(\frac{E^2-a_{n'}^2}{E^2-a_n^2}\right)^{1/2}\right]R_+ +\sum_{n'=0}^{\infty}E(E^2-a_{n'}^2)^{-1/2}\left[1-\left(\frac{E^2-a_{n'}^2}{E^2-a_n^2}\right)^{1/2}\right]R_-, \quad (38)$$

with

$$R_{\pm}=R(u,\mp u'), \quad u'=(2\theta)^{-1/2}(E^2-a_n^2)^{1/2}, \quad (39)$$

$$\tau_0^{-1}=2\pi g\alpha^2Z^2(N_i/\Omega)c\lambda_c^2\theta^{-1}. \quad (40)$$

As we see later [Eq. (55)], $A(u,u')$ is an even function in the variables u and u'. Although n' could, in principle, take any value $0\leq n'\leq\infty$, the requirement that $E^2-a_{n'}^2>0$ (for a given energy E) imposes an upper limit chosen to satisfy this condition.

4. LONGITUDINAL ELECTRICAL CONDUCTIVITY

We now calculate the current induced by an electric field F in the z direction. As explained in Ref. 7, the Boltzmann equation can be used in this case essentially because the part of the electron wave function in the direction of the field is still a plane wave [Eq. (7)] like the case of zero magnetic field. Without entering in details which are fully explained in the previous reference, we give only the results. If f_n is the probability of occupation of state $|n\rangle$, the solution of the Boltzmann equation is

$$f_n=f_n^0+e\hbar^{-1}F\tau_n\partial f_n^0/\partial k_3, \quad (41)$$

where F is the electric field, f_n^0 is the Fermi distribution, $f_n^0=(1+e^{(\epsilon_n-\mu)/\phi})^{-1}$, $\phi=kT/mc^2$. By definition, the

[7] A. H. Kahn and H. P. R. Frederikse, Solid State Phys. 9, 257 (1959).

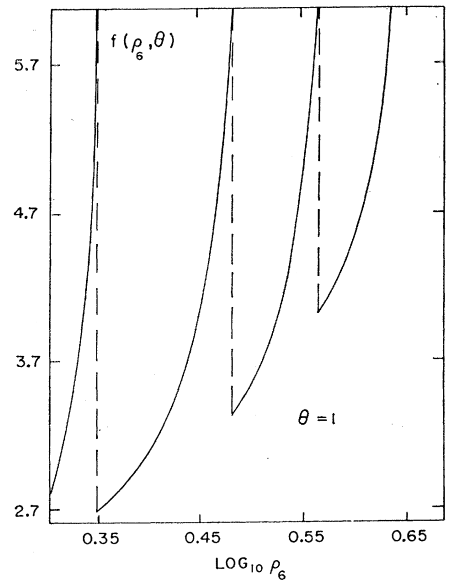

FIG. 1. The function $f(\mu,\theta)$, Eq. (4.9), as a function of the matter density $\rho_6=10^{-6}\rho$ for $\theta=H/H_c=1$. As explained in the text, $\rho_6^{2/3}=\mu^2-1$. The undulating behavior is related to the density of the final states.

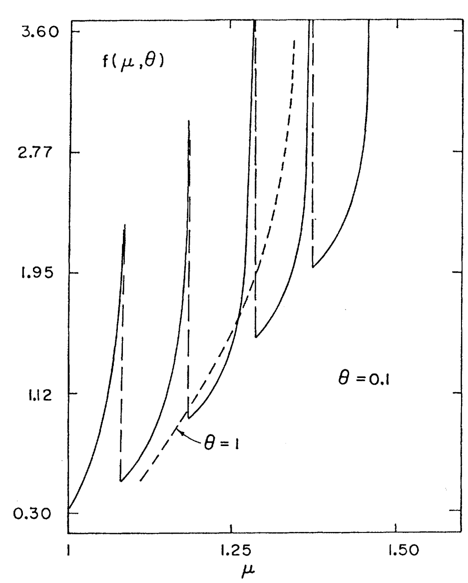

FIG. 2. The function $f(\mu,\theta)$ as a function of μ for $H/H_c=\theta=0.1$. The number of discontinuities is enormously increased compared with the case $H/H_c=\theta=1$, also shown in the figure.

TABLE I. The function $f(\rho_6,\theta)$, Eq. (4.9), as a function of the density ρ_6, for $\theta=1$. Each group defined the values of the function inside of a jump. In this table, $\Phi=1$; see Eq. (50).

$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$
0.359	2.8	1.064	15.4	1.287	25.7	1.422	33.9
0.391	3.6	1.069	15.8	1.290	26.1	1.424	34.4
0.422	4.7	1.075	16.2	1.293	26.4	1.426	34.7
0.477	3.5	1.085	16.2	1.298	26.2	1.430	34.6
0.501	4.3	1.089	16.7	1.301	26.7	1.431	35.0
0.525	5.3	1.094	17.0	1.304	27.0	1.433	35.3
0.567	4.3	1.104	17.1	1.309	26.6	1.437	35.1
0.587	5.0	1.108	17.5	1.311	27.1	1.439	35.5
0.606	5.9	1.113	17.8	1.314	27.4	1.440	35.9
0.640	5.1	1.122	17.8	1.319	27.3	1.444	35.6
0.656	5.8	1.126	18.2	1.321	27.7	1.446	36.1
0.672	6.6	1.130	18.6	1.324	28.0	1.447	36.4
0.702	5.9	1.139	18.5	1.329	28.0	1.451	36.2
0.715	6.6	1.143	19.0	1.331	28.4	1.452	36.6
0.729	7.3	1.147	19.3	1.334	28.7	1.454	37.0
0.754	6.7	1.155	19.3	1.338	28.5	1.457	36.8
0.766	7.4	1.159	19.8	1.341	28.9	1.459	37.2
0.778	8.1	1.163	20.1	1.343	29.2	1.461	37.5
0.800	7.5	1.171	20.0	1.348	29.2	1.464	37.1
0.811	8.2	1.175	20.5	1.350	29.6	1.466	37.5
0.821	8.9	1.179	20.8	1.352	29.9	1.467	37.9
0.841	8.4	1.186	20.7	1.357	29.7	1.471	37.6
0.850	9.0	1.190	21.1	1.359	30.1	1.472	38.1
0.860	9.7	1.193	21.5	1.361	30.4	1.474	38.4
0.878	9.2	1.200	21.3	1.366	30.3	1.477	38.2
0.886	9.9	1.204	21.8	1.368	30.7	1.479	38.6
0.895	10.5	1.207	22.1	1.370	31.0	1.480	39.0
0.911	10.1	1.214	21.9	1.374	30.8	1.483	38.8
0.919	10.7	1.218	22.4	1.377	31.2	1.485	39.2
0.927	11.3	1.221	22.7	1.379	31.5	1.486	39.5
0.942	11.0	1.228	22.6	1.383	31.4	1.489	39.3
0.949	11.6	1.231	23.1	1.385	31.8	1.491	39.7
0.956	12.2	1.234	23.4	1.387	32.1	1.493	40.0
0.970	11.9	1.240	23.2	1.391	31.9	1.496	39.8
0.976	12.5	1.244	23.7	1.393	32.4	1.497	40.2
0.983	13.1	1.247	24.0	1.395	32.7	1.499	40.5
0.996	12.8	1.253	23.8	1.399	32.5	1.501	40.2
1.002	13.4	1.256	24.2	1.401	32.9	1.503	40.7
1.008	13.9	1.259	24.6	1.403	33.2	1.504	41.0
1.020	13.7	1.265	24.4	1.407	32.9	1.507	40.9
1.026	14.3	1.268	24.9	1.409	33.4	1.509	41.3
1.032	14.8	1.271	25.2	1.411	33.7	1.510	41.6
1.043	14.6	1.276	25.0	1.415	33.5	1.513	41.3
1.048	15.0	1.279	25.4	1.417	33.9	1.514	41.7
1.054	15.4	1.282	25.8	1.418	34.2	1.516	42.0

current J is found by averaging the z component of the current

$$j_\mu = iec\int \bar{\psi}(r)\gamma_\mu\psi(r)d^3r \tag{42}$$

over the distribution function f, in the following way:

$$J\equiv J_3 = -\sum_n \int j_z f_n dk_z N(k_z)\,, \tag{43}$$

where $N(k_z)dk_z$ is the density of states between k_z and k_z+dk_z and is given by[4]

$$N(k_z)dk_z = g(2\pi)^{-2}\lambda_c^{-2}\theta dk_z\,, \tag{44}$$

where $g=2$. Using Eq. (43) and Ohm's law, $\mathbf{J}=\sigma_H\mathbf{F}$, we obtain

$$\sigma_H \equiv \frac{J}{F} = -egh^{-1}\sum_n \int_{-\infty}^{+\infty} j_z\tau_n \frac{\partial f_n^0}{\partial k_z} dk_z N(k_z)\,. \tag{45}$$

Using the eigenfunctions ψ [Eq. (7)], it is easy to show that

$$j_z = ecE^{-1}(E^2-a_n^2)^{1/2}\,, \tag{46}$$

as expected from the relativistic definition of velocity. Substituting Eqs. (38), (44), and (46) into Eq. (45), we obtain the final expression for the conductivity as

$$\sigma_H/\bar{\sigma}_H = \theta^2 f(\phi,\mu,\theta)\,, \tag{47}$$

with

$$\bar{\sigma}_H = 4[(2\pi)^3\lambda_c^3 ZN_i/\Omega]^{-1}(\alpha Zh/mc^2)^{-1}\,, \tag{48}$$

$$f(\phi,\mu,\theta) = \int_1^\infty \frac{dE}{E^2}\sum_{n=0}^\infty (E^2-a_n^2)^{1/2}\frac{\partial f_0}{\partial E}\Bigg/ \left\{\sum_{n'=0}^\infty (E^2-a_{n'}^2)^{-1/2}\left[1+\left(\frac{E^2-a_{n'}^2}{E^2-a_n^2}\right)^{1/2}\right]R_- + \sum_{n'=n}^\infty (E^2-a_{n'}^2)^{-1/2}\left[1-\left(\frac{E^2-a_{n'}^2}{E^2-a_n^2}\right)^{1/2}\right]R_+\right\}, \tag{49}$$

$$R_\pm = \int_0^\infty \frac{dt|A_n^{n'}(t,E)|^2\Phi_\pm(t,E)}{\{t+(2\theta)^{-1}[(E^2-a_n^2)^{1/2}\mp(E^2-a_{n'}^2)^{1/2}]^2\}^2}, \tag{50}$$

$$\Phi_\pm = 1+3\int_0^\infty (x\xi_\pm)^{-1}\sin(x\xi_\pm)dx\,, \tag{51}$$

$$\xi_\pm = 2.69547\rho^{-1/3}Z^{1/3}\theta^{1/2}\{t+(2\theta)^{-1}[(E^2-a_n^2)^{1/2} \mp(E^2-a_{n'}^2)^{1/2}]^2\}^{1/2}, \tag{52}$$

$$|A_{n,n'}|^2 = \omega_1^2\Psi^2(n|n') + \omega_2^2\Psi^2(n-1|n'-1) - 2\omega_1\omega_2\Psi(n|n')\Psi(n-1|n'-1)\,, \tag{53}$$

$$\Psi(n|n') = (n!n'!)^{-1/2}e^{-t/2}t^{(n+n')/2} \times {}_2F_0(-n', -n; -1/t)\,. \tag{54}$$

The spin average is easily performed; we have

$$\begin{aligned}
\omega_1^2 &\to \sum_{s,s'}\omega_1^2(s,s') = \tfrac{1}{2}[1+E^{-2} \pm E^{-2}(E^2-a_n^2)^{1/2}(E^2-a_{n'}^2)^{1/2}],\\
\omega_2^2 &\to \sum_{s,s'}\omega_2^2(s,s') = \tfrac{1}{2}[1+E^{-2} \pm E^{-2}(E^2-a_n^2)^{1/2}(E^2-a_{n'}^2)^{1/2}],\\
\omega_1\omega_2 &\to \sum_{s,s'}\omega_1(ss')\omega_2(ss') = \theta E^{-2}(nn')^{1/2}.
\end{aligned} \tag{55}$$

5. CONDUCTIVE OPACITY COEFFICIENT

The thermal-conductivity coefficient λ is usually defined through the relation

$$Q = -\lambda_H dT/dx\,, \tag{56}$$

where Q has the dimensions of erg cm^{-2} sec^{-1} and λ of erg cm^{-1} sec^{-1} deg^{-1}. The Wiedemann-Franz law relates the parameter λ with the electrical conductivity σ via the relation[8]

$$\lambda_H = (\tfrac{1}{3}\pi^2)(k/e)^2\sigma_H T\,, \tag{57}$$

when $k = 1.38024\times 10^{-16}$ erg deg^{-1} is the Boltzmann constant. This equation has been shown to hold in a quantizing magnetic field by Zyrianov.[3] The conductive opacity coefficient $k_c{}^H$, defined as

$$k_c{}^H = (4a_c/3\rho)\lambda_H{}^{-1}T^3\,, \tag{58}$$

after using Eqs. (57) and (47), becomes

$$k_c{}^H = \bar{k}_c{}^H\theta^2 f(\phi,\mu,\theta)\,, \tag{59}$$

$$\bar{k}_c{}^H = \tfrac{3}{5}(2\pi)^3 m_H{}^{-1}\alpha^2\phi^2\lambda_c{}^2Z^2\mu_i{}^{-1}. \tag{60}$$

In Eq. (60), α is the fine-structure constant ($\alpha = 1/137$), $\phi = kT/mc^2$, m_H is the proton mass, and μ_i enters in because of the relation between N_i/Ω and total density ρ. For the completely ionized system and when only one type of ion is present, $\mu_i = A$. Numerically, Eq. (60) becomes

$$\bar{k}_c{}^H = 20.26T_6{}^2(Z^2/A)\times 10^{-8}\ \mathrm{cm^2\,g^{-1}}\,, \tag{61}$$

where $T_6 \equiv 10^{-6}\times T$ in °K.

TABLE II. The function $f(\rho_6,\theta)$ for $\theta = 0.1$ as a function of μ. Because of too many jumps (as explained in the text), the computation was stopped at $\mu \simeq 5$. In Fig. 2 only the first four jumps are shown. Here, too, $\phi = 1$ [Eq. (50)].

$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$
−0.820	1.1	0.095	12.2	0.370	22.4	0.532	30.8
−0.777	1.5	0.101	12.6	0.374	22.8	0.534	31.2
−0.737	2.0	0.107	12.9	0.377	23.1	0.537	31.6
−0.666	1.6	0.120	13.0	0.384	23.0	0.541	31.5
−0.634	2.1	0.126	13.4	0.387	23.4	0.543	31.9
−0.603	2.6	0.132	13.7	0.390	23.7	0.545	32.2
−0.548	2.2	0.144	13.8	0.396	23.4	0.549	32.0
−0.522	2.7	0.150	14.2	0.399	23.8	0.551	32.4
−0.497	3.2	0.155	14.5	0.402	24.2	0.554	32.7
−0.452	2.8	0.166	14.5	0.409	24.1	0.558	32.5
−0.431	3.3	0.172	14.9	0.412	24.5	0.560	33.0
−0.410	3.8	0.177	15.2	0.415	24.8	0.562	33.3
−0.372	3.5	0.188	15.2	0.420	24.8	0.566	33.1
−0.354	4.0	0.193	15.6	0.423	25.2	0.568	33.5
−0.336	4.5	0.198	16.0	0.426	25.5	0.570	33.8
−0.303	4.2	0.208	16.0	0.432	25.2	0.574	33.7
−0.288	4.7	0.213	16.4	0.435	25.7	0.576	34.1
−0.273	5.2	0.218	16.7	0.438	26.0	0.578	34.4
−0.244	4.9	0.228	16.7	0.443	25.9	0.582	34.0
−0.230	5.4	0.233	17.1	0.446	26.3	0.584	34.5
−0.216	5.9	0.237	17.4	0.449	26.6	0.586	34.8
−0.191	5.7	0.246	17.4	0.454	26.5	0.589	34.6
−0.178	6.2	0.251	17.8	0.457	26.9	0.591	35.0
−0.166	6.7	0.255	18.1	0.460	27.2	0.593	35.3
−0.143	6.4	0.264	18.0	0.465	27.1	0.597	35.2
−0.132	6.9	0.268	18.4	0.467	27.5	0.599	35.6
−0.121	7.4	0.273	18.7	0.470	27.8	0.601	35.9
−0.100	7.2	0.281	18.6	0.475	27.6	0.604	35.7
−0.090	7.7	0.285	19.0	0.478	28.0	0.606	36.1
−0.080	8.2	0.289	19.3	0.480	28.3	0.608	36.4
−0.061	8.0	0.297	19.3	0.485	28.2	0.612	36.2
−0.052	8.5	0.301	19.7	0.488	28.6	0.613	36.6
−0.043	9.0	0.305	20.0	0.490	28.9	0.615	36.9
−0.025	8.8	0.313	19.9	0.495	28.8	0.619	36.7
−0.017	9.4	0.317	20.3	0.498	29.2	0.620	37.1
−0.008	9.9	0.321	20.7	0.500	29.5	0.622	37.4
0.008	9.7	0.328	20.5	0.505	29.3	0.626	37.2
0.016	10.2	0.332	20.9	0.507	29.7	0.627	37.6
0.024	10.7	0.336	21.2	0.509	30.0	0.629	37.9
0.039	10.5	0.343	21.2	0.514	29.8	0.632	37.9
0.046	11.0	0.346	21.6	0.516	30.2	0.634	38.3
0.053	11.5	0.350	21.9	0.519	30.5	0.636	38.6
0.068	11.4	0.357	21.7	0.523	30.4	0.639	38.3
0.075	11.8	0.360	22.1	0.525	30.8	0.641	38.7
0.081	12.1	0.364	22.4	0.528	31.1	0.642	39.0

TABLE III. Co mparison between the conductivities in units of $\bar{\sigma}_H$ [see Eq. (48)] with and without the magnetic field, for $\theta = 1$ and $\Phi = 1$; $\sigma_H \gg \sigma_0$.

$\log_{10}\rho_6$	$3\sigma_0$	$\sigma_H,\ \theta = 1$	$\sigma_H/\sigma_0,\ \theta = 1$
0.4	0.155	3.53	68.32
0.6	0.286	5.79	60.73
0.8	0.573	7.44	43.51
1.0	0.895	13.71	45.95
1.2	1.531	22.85	44.77
1.4	2.570	35.60	41.55
1.6	4.266	60.48	42.48

TABLE IV. The function $f(\rho_6,\theta)$ including the ion-ion correlation Φ, for $\theta = 1$ and $Z = 20$. The parameter $\Gamma \equiv [(Ze)^2/kT] \times [\tfrac{4}{3}\pi(N_i/\Omega)]^{1/3}$ which measures the strength of the ion-ion correlation is taken equal to 100. Compared with Table I, i.e., $\Phi = 1$, the function $f(\rho_6,\theta)$ is increased up to a factor of 2.

$\log_{10}\rho_6$	$f(\rho_6,\theta)$	$\log_{10}\rho_6$	$f(\rho_6,\theta)$
0.35869	2.65077	1.06433	23.65639
0.39114	3.50828	1.06946	24.65825
0.42204	4.60602	1.07456	25.46954
0.47683	3.49817	1.08451	25.49251
0.50117	4.33834	1.08936	26.50386
0.52464	5.35837	1.09418	27.31723
0.56729	4.44158	1.10360	27.26363
0.58669	5.27978	1.10820	28.28908
0.60553	6.26141	1.11277	29.10674
0.64035	5.47310	1.12171	28.91310
0.65645	6.31962	1.12609	29.96494
0.67216	7.28090	1.13043	30.79442
0.70153	6.58780	1.13894	30.53162
0.71526	7.44959	1.14311	31.60017
0.72870	8.40082	1.14725	32.43800
0.75406	7.78230	1.15536	31.86873
0.76602	8.66409	1.15934	32.96918
0.77776	9.61211	1.16330	33.82063
0.80005	9.05432	1.17105	33.54079
0.81063	9.95980	1.17486	34.61751
0.82104	10.90955	1.17864	35.45503
0.84092	10.40237	1.18607	35.28603
0.85040	11.33491	1.18972	36.34979
0.85975	12.29074	1.19334	37.17818
0.87766	11.82560	1.20047	36.32027
0.88625	12.78871	1.20397	37.40823
0.89472	13.75377	1.20745	38.24104
0.91103	13.32390	1.21430	37.95444
0.91887	14.32039	1.21766	39.02263
0.92662	15.29583	1.22101	39.84346
0.94157	14.89740	1.22760	39.15213
0.94879	15.93048	1.23084	40.25760
0.95592	16.91925	1.23407	41.09760
0.96972	16.54680	1.24041	40.36420
0.97640	17.62074	1.24354	41.50366
0.98301	18.62857	1.24665	42.35660
0.99583	18.27303	1.25277	41.65547
1.00204	19.39587	1.25579	42.76546
1.00820	20.43629	1.25879	43.60252
1.02015	20.06915	1.26471	43.01193
1.02596	21.24089	1.26762	44.10635
1.03172	22.31667	1.27052	44.92992
1.04293	21.80699	1.27625	44.09698
1.04838	22.80507	1.27907	45.17487
1.05379	23.62039	1.28187	45.98706

[8] R. E. Marshak, Ann. Acad. Sci. (N. Y.) **41**, 49 (1941).

TABLE V. Numerical solution of Eq. (64). The table gives μ as a function of ρ_6, ϕ, for $\theta=1$. The few zeros in the corner mean that no solution was found for this range of the variables.

$\log_{10}\rho_6$ \ $\log_{10}\phi$	−1.2	−1.0	−0.8	−0.6	−0.4	−0.2	0.0	0.2
−0.523	1.0156	0.9766	0.8984	0.7031	0.2344	0.0	0.0	0.0
−0.423	1.0352	1.0156	0.9375	0.7813	0.3125	0.1563	0.0	0.0
−0.323	1.0547	1.0352	0.9766	0.8203	0.4297	0.1732	0.0	0.0
−0.223	1.0937	1.0742	1.0547	0.8984	0.5469	0.1732	0.0	0.0
−0.123	1.1328	1.1328	1.1328	0.9766	0.6250	0.1758	0.0391	0.0
−0.023	1.2109	1.2109	1.2109	1.0742	0.7422	0.2236	0.1732	0.0
+0.077	1.2891	1.3086	1.3086	1.1523	0.8398	0.2734	0.1732	0.0
0.177	1.4258	1.4453	1.4453	1.2500	0.9570	0.3516	0.1732	0.1563
0.277	1.6016	1.6211	1.6406	1.3574	1.0742	0.4688	0.1732	0.1732
0.377	1.8750	1.8750	1.8750	1.4648	1.2109	0.6055	0.2236	0.1732
0.477	2.2266	2.2266	2.2266	1.5820	1.3379	0.7813	0.2646	0.1732
0.577	2.6953	2.6953	2.6953	1.6895	1.4746	0.9473	0.3000	0.1732
0.677	3.3203	3.3203	3.3203	1.8262	1.6211	1.1230	0.3606	0.2236
0.777	4.1016	4.1016	4.1016	1.9629	1.7676	1.3086	0.4395	0.2236
0.877	5.1172	5.1172	5.1172	2.1240	1.9287	1.4990	0.5750	0.2646
0.977	6.4062	6.4062	6.4062	2.2998	2.0898	1.6992	0.7813	0.3000
1.077	8.0078	8.0078	8.0078	2.5098	2.2632	1.9043	1.0449	0.3606
1.177	0.0	0.0	0.0	2.7881	2.4561	2.1265	1.3135	0.4123
1.277	0.0	0.0	0.0	3.1641	2.6562	2.3584	1.5967	0.5000
1.377	0.0	0.0	0.0	3.6914	2.8760	2.6001	1.8896	0.6250
1.477	0.0	0.0	0.0	4.4141	3.1201	2.8564	2.1948	0.8185

6. NUMERICAL RESULTS AND CONCLUSIONS

The set of equations (49)–(55) define the electrical conductivity and the conductive opacity. The following successive steps have to be performed. Given the implicit relation between the density ρ and the chemical potential μ^4 valid for a degenerate magnetized electron gas, we have

$$10^{-6}\rho\equiv\rho_6=3\theta\left[\tfrac{1}{2}C_4(\mu)+\sum_{n=1}^{s}a_nC_4\frac{\mu}{a_n}\right],\qquad C_4=(\mu^2-1)^{1/2}. \tag{62}$$

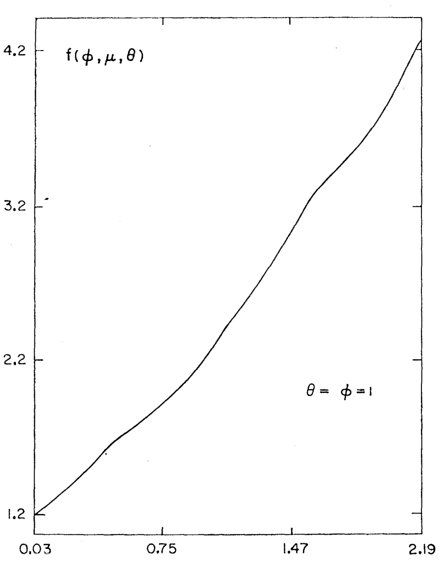

FIG. 3. For a nondegenerate electron gas, the function $f(\phi,\mu,\theta)$ has a smoother behavior than in the case of complete degeneracy (Figs. 1 and 2).

We have determined μ for values of ρ in the interval $10^6\leq\rho\leq10^9$ g/cm^3 and $\theta=1$, and $\theta=0.1$. For $\theta=1$ and $\mu\simeq2$, Eq. (62) gives almost the same value for ρ_6 as the formula for a free relativistic degenerate electron gas, i.e., $\mu^2-1=\rho_6^{2/3}$. We have therefore used this simple relation for $\theta=1$, and the function $f(\mu,\theta)$ for the degenerate case is therefore listed in Table I as $f(\rho_6,\theta)$ for values of ρ_6 in the range $0.3\leq\log_{10}\rho_6\leq1.75$. Unfortunately, because of the many jumps in the f function, any extension to higher densities would imply a time problem, although in principle there is no extra difficulty. For $\theta=0.1$, the function $f(\rho_6,\theta)$ is listed as a function of ρ_6 in Table II. Figures 1 and 2 show the function $f(\mu,\theta)$ for $\theta=1$ and 0.1. The discontinuities are related to the behavior of the density of final states.

TABLE VI. Same as Table V for $\theta=0.1$.

$\log_{10}\rho_6$ \ $\log_{10}\phi$	−1.4	−1.2	−1.0	−0.8	−0.6	−0.4	−0.2
−0.523	2.1875	2.1875	2.1875	1.0547	0.8203	0.3125	0.1367
−0.423	2.8125	2.8125	2.8125	1.0937	0.8984	0.3906	0.1514
−0.323	3.4375	3.4375	3.4375	1.1523	0.9375	0.5078	0.1660
−0.223	4.0625	4.0625	4.0625	1.2109	1.0156	0.6250	0.1807
−0.123	5.0000	5.0000	5.0000	1.2695	1.0937	0.7031	0.2051
−0.023	6.2500	6.2500	6.2500	1.3281	1.1719	0.8203	0.2344
0.077	8.1250	8.1250	8.1250	1.4062	1.2500	0.9180	0.2930
0.177	0.0	0.0	0.0	1.5039	1.3281	1.0156	0.3711
0.277	0.0	0.0	0.0	1.6016	1.4160	1.1328	0.4883
0.377	0.0	0.0	0.0	1.7383	1.5137	1.2500	0.6445
0.477	0.0	0.0	0.0	1.9141	1.6113	1.3770	0.8105
0.577	0.0	0.0	0.0	2.1484	1.7187	1.5137	0.9766
0.677	0.0	0.0	0.0	2.5000	1.8457	1.6504	1.1523
0.777	0.0	0.0	0.0	2.9687	1.9775	1.7920	1.3281
0.877	0.0	0.0	0.0	3.5937	2.1289	1.9434	1.5137
0.977	0.0	0.0	0.0	4.4141	2.2998	2.1045	1.7139
1.077	0.0	0.0	0.0	5.4687	2.5049	2.2754	1.9189
1.177	0.0	0.0	0.0	6.8164	2.7588	2.4609	2.1387
1.277	0.0	0.0	0.0	8.5156	3.1006	2.6611	2.3657
1.377	0.0	0.0	0.0	0.0	3.5840	2.8809	2.6074
1.477	0.0	0.0	0.0	0.0	4.2432	3.1250	2.8589

TABLE VII. The function $f(\phi,\mu,\theta)$ as a function of μ for $\theta=1$. The chemical potential depends on the temperature and density through Table V. Each group of values of μ and f correspond to one column of Table V, starting with the right column of Table V.

μ	f
0.156	3.685
0.173	3.715
0.224	3.804
0.265	3.878
0.300	3.942
0.361	4.054
0.412	4.152
0.500	4.321
0.625	4.570
0.818	4.976

μ	f
0.039	1.235
0.173	1.369
0.224	1.422
0.265	1.466
0.300	1.504
0.361	1.572
0.439	1.663
0.575	1.826
0.781	2.091
1.045	2.455
1.313	2.848
1.597	3.283
1.890	3.752
2.195	4.262

μ	f
0.156	0.491
0.173	0.503
0.176	0.504
0.224	0.537
0.273	0.574
0.352	0.634

μ	f
0.469	0.734
0.605	0.863
0.781	1.048
0.947	1.237
1.123	1.446
1.309	1.666
1.499	1.881
1.699	2.086
1.904	2.272
2.126	2.452

μ	f
0.234	0.151
0.313	0.182
0.430	0.238
0.547	0.309
0.625	0.365
0.742	0.465
0.840	0.563
0.957	0.697
1.074	0.845
1.211	1.025
1.338	1.187
1.475	1.339
1.621	1.461
1.768	1.536
1.929	1.575
2.090	1.588

μ	f
0.703	0.128
0.781	0.170
0.820	0.196
0.898	0.257

μ	f
0.977	0.334
1.074	0.455
1.152	0.570
1.250	0.733
1.357	0.916
1.465	1.072
1.582	1.174
1.689	1.192
1.826	1.139
1.963	1.061
2.124	0.988
2.300	0.948

μ	f
0.898	0.062
0.938	0.078
0.977	0.097
1.055	0.148
1.133	0.225
1.211	0.336
1.309	0.538
1.445	0.880
1.641	1.022
1.875	0.679
2.227	0.560
2.695	0.597
3.320	0.776
4.102	1.114
5.117	1.722
6.406	2.701

μ	f
0.977	0.019
1.016	0.026

μ	f
1.035	0.031
1.074	0.040
1.133	0.057
1.211	0.092
1.309	0.216
1.445	0.712
1.621	0.936
1.875	0.299
2.227	0.359
2.695	0.371
3.320	0.493
4.102	0.702
5.117	1.083
6.406	1.704

μ	f
1.016	0.009
1.035	0.012
1.055	0.015
1.094	0.023
1.132	0.029
1.211	0.017
1.426	0.346
1.602	0.894
1.875	0.111
2.227	0.225
2.695	0.225
3.320	0.327
4.102	0.460
5.117	0.675
6.406	1.079
8.008	1.566

For magnetic fields much lower than $\theta=0.1$, the numerical computations become prohibitively hard; the function $f(\phi,\mu,\theta)$ has discontinuities whenever $\mu^2-a_{n'}^2=0$. Take as an example $\theta=10^{-2}$; we would then have $a_n^2=1+0.02n'$. If μ is even only of the order of 3, the upper limit n' would be higher than 3×10^2. For the case $\theta=0.1$, we stopped the computation for $\mu\simeq5$ because of the tremendous number of discon-

TABLE VIII. Same as in Table VII for $\theta=0.1$. The chemical potential depends on the temperature and density through Table VI.

μ	f
0.137	0.900
0.151	0.920
0.166	0.939
0.181	0.960
0.205	0.995
0.234	1.038
0.293	1.129
0.371	1.261
0.488	1.486
0.644	1.840
0.810	2.294
0.977	2.833
1.152	3.509
1.328	4.303
1.514	5.273

μ	f
0.313	0.214
0.391	0.257
0.508	0.337
0.625	0.439
0.703	0.521
0.820	0.669

μ	f
0.918	0.819
1.016	0.994
1.133	1.243
1.250	1.538
1.377	1.908
1.514	2.372
1.650	2.899
1.792	3.513
1.943	4.234
2.104	5.073
2.275	6.029
2.461	7.130
2.661	8.369
2.881	9.791

μ	f
0.820	0.176
0.898	0.228
0.938	0.259
1.016	0.329
1.094	0.414
1.172	0.514
1.250	0.631
1.328	0.765

μ	f
1.416	0.937
1.514	1.154
1.611	1.399
1.719	1.699
1.846	2.093
1.977	2.540
2.129	3.089
2.300	3.749
2.505	4.570
2.759	5.622
3.101	7.076
3.584	9.251

μ	f
1.055	0.137
1.094	0.162
1.152	0.203
1.211	0.252
1.269	0.307
1.328	0.369
1.406	0.463
1.504	0.601
1.602	0.758
1.738	1.011

μ	f
1.914	1.388
2.148	1.956
2.500	2.867
2.969	4.107
3.594	5.870

μ	f
2.187	1.291
2.812	2.328
3.437	3.420
4.062	4.620

μ	f
2.187	0.814
2.812	1.470
3.437	2.158
4.062	2.919

μ	f
2.187	0.513
2.812	0.928
3.437	1.363
4.062	1.843

tinuities already involved in this interval. Unlike the free case, we have to use very small intervals for μ, otherwise we lose the discontinuities. Tables I and II do not include the ion-ion correlation, i.e., $\Phi\to 1$. In Table III we compare σ_H with σ_0, the conductivity in the case of zero magnetic field.[1] The ratio still shows the undulating behavior of σ_H and the general conclusion is that σ_H is much higher than σ_0. This phenomenon is known as negative magnetoresistance. This conclusion remains unchanged when the ion-ion correlation is included. As explained in a previous paper,[1] the inclusion of the function Φ causes many difficulties, especially in the present problem where it becomes a function of the indices n and n'. As explained in Ref. 1, the strength of the ion-ion correlation is determined by the parameter $\Gamma\equiv(Z^2e^2/kT)[\frac{4}{3}\pi(N_i/\Omega)]^{+1/3}$, which is the ratio of the potential energy to the kinetic energy. We have computed one case, namely, $Z=20$, and $\Gamma=100$ which is the highest set of values for which the function Φ is known. The results are given in Table IV. The general effect is that the conductivity increases as in the case of zero magnetic field and it reaches almost a value of 2 for $\log_{10}\rho_6\simeq 1.28$. The tendency is that $\sigma_H(\Phi)/\sigma_H\,(\Phi=1)$ increases with density. For the nondegenerate case, the situation is much more complicated in the following two respects. First, the integral on E in Eq. (49) cannot be easily performed as in the degenerate case where $\partial f/\partial E=-\delta(E-\mu)$, but it has to be performed exactly. A second problem arises because of the implicit relation between μ, ρ_6, and $\phi\equiv(kT/mc^2)$. They are related via the equation

$$\rho_6=3\theta\left[\tfrac{1}{2}C_4(\phi,\mu)+\sum_{n=1}^{s}a_nC_4\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right)\right],\qquad(63)$$

where

$$a_nC_4\left(\frac{\phi}{a_n}\frac{\mu}{a_n}\right)=\int_0^\infty(1+e^{(\epsilon-\mu)/\phi})^{-1}dx.\qquad(64)$$

In Tables V and VI we give μ as a function of ρ_6, ϕ for $\theta=1$ and 0.1. These values of μ are then used to compute $f(\phi,\mu,\theta)$ and the results are quoted in Tables VII and VIII. Figure 3 shows the function $f(\phi,\mu,\theta)$ as a function of μ for $\theta=1$ and $\phi=1$.

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APPENDIX

Given the two integrals

$$I=\int_1^\infty G(E)dE\,,\quad R_\pm=\int_0^\infty f(s)ds\,,$$

the following steps were followed. The $R_\pm$ were computed by use of a 16-point Gaussian quadrature formula.[9]

Because of the jump discontinuities of $G(E)$ at $E^2=1+2(N-1)$ for all positive integers $N>1$, the computation of I required separate integration over each panel (E_j,E_{j+1}), $j=1$, k, where k was chosen so that three-place convergence was obtained for I. A five-point open-type quadrature formula was applied over each panel.

The order of the quadrature formulas used has been increased until three-place convergence was obtained in the final results.

As for the computer program to evaluate I, we found that in order to prevent "overflow" and "underflow" it was necessary to make use of Stirling's formula and logarithms. It was also necessary to vary the order of arithmetical computations in evaluating the integrand $f(s)$ of $R_\pm$ according to whether s was less than or greater than 1.

To speed up the program, we found that we could truncate series at $n=18$ without affecting the three-place significance of our final results.

[9] This computation was made by N. Rushfield, Computer Applications Inc., New York, N. Y.