# Dynamical Effects in Overlapping Resonances*

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The dynamical effects of two overlapping resonances are examined. The experimentally observable positions and widths $s_i$, $\Gamma_i$ are related to the isolated quantities $\bar{s}_i$, $\bar{\Gamma}_i$ which would occur if the resonances were widely separated. Dramatic enhancements or depressions of $\Gamma_i/\bar{\Gamma}_i$ are demonstrated. The model is applied to the double-peaked $A_2$ with the results that one of the resonances is quite narrow: $\bar{\Gamma}\lesssim 5$ MeV. Conjectures on exotic resonances are made.

When two resonances occur in the same partial wave, the unitary $S$ matrix, if there is no inelasticity, can be written as[1]

$$S=\frac{(s_1-s+i\Gamma_1)(s_2-s+i\Gamma_2)}{(s_1-s-i\Gamma_1)(s_2-s-i\Gamma_2)}, \tag{1}$$

where $s_i$ and $\Gamma_i$ are interpreted to be the position and width of the $i$th resonance. If only one resonance occurs in a partial wave, the experimentally determined width can be related to a coupling constant. The same statement holds when there are two "isolated" resonances, i.e., resonances in a partial wave separated by many widths; isolated quantities will be denoted by bars. However, if the resonances overlap,[2] there are dynamical effects which greatly alter the isolated $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ (which are directly related to coupling constants and hence the quantities of theoretical interest) from the experimentally determined $\Gamma_1$ and $\Gamma_2$. We examine a quasi-bound-state model for the resonances based on the multichannel $ND^{-1}$ formalism in which we explicitly derive a relationship between $s_i$, $\Gamma_i$ and $\bar{s}_i$, $\bar{\Gamma}_i$ and a parameter $c$. The large effects predicted by this model are illustrated in Fig. 1. Within our model the dynamical parameter $c$ can be estimated for a given reaction.

After presenting the details of our model, we apply it to the $A_2$. Expressions of the form of Eq. (1)[3,4] have been used to fit the double-peak structure of the $A_2$. Using reasonable values for $c$, we find that one of the resonances comprising the $A_2$ is vary narrow, i.e., it has a $\bar{\Gamma}\lesssim 5$ MeV. Some speculative implications of this feature are made concerning exotic resonances.

Our dynamical model for two overlapping elastic resonances consists of the resonances being quasi-bound-states of high-mass closed channels which are coupled to the open channel $a$. We use a three-channel $ND^{-1}$ formalism in which we make a pole approximation for the left-hand cut:

$$B=\begin{bmatrix} O & g_{ab} & g_{ac} \\ g_{ab} & g_{bb} & O \\ g_{ac} & O & g_{cc}\end{bmatrix}\frac{1}{s-s_0}. \tag{2}$$

If we make a subtraction in $D$ at $s=s_0$, then

$$N=B,$$

$$D_{ij}=\delta_{ij}-g_{ij}\varphi_i,$$

$$\varphi_i=\frac{s-s_0}{\pi}P\int_{s_i^T}^{\infty}\frac{\rho_i(s')ds'}{(s'-s_0)^2(s'-s)}+\frac{i\rho_i(s)}{s-s_0}\theta(s-s_i^T), \tag{3}$$

$$(S-1)/2i=\rho^{\frac{1}{2}}ND^{-1}\rho^{\frac{1}{2}},$$

where $s_i^T$ is the threshold for the $i$th channel, $\rho_i$ is a kinematical factor, and $P$ denotes the principal-value integral. In order to obtain two overlapping resonances

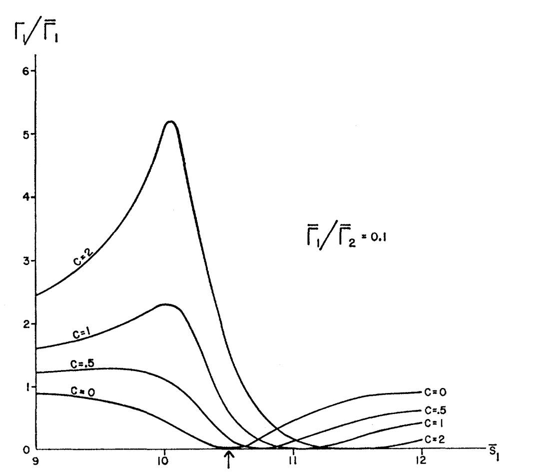


FIG. 1. $\Gamma_1/\bar{\Gamma}_1$ [calculated from Eq. (7)] as a function of $\bar{s}_1$ for various values of $c$. The arrow indicates $\bar{s}_2$. The figure corresponds to the case $\bar{\Gamma}_2=0.5$, $\bar{\Gamma}_1=0.05$.

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[1] C. Rebbi and R. Slansky, California Institute of Technology report (unpublished); Y. Fujii and M. Kato, this issue, Phys. Rev. 188, 2319 (1969); L. Durand, III, Lecture Notes, Aspen Center of Physics (unpublished).

[2] "Overlapping" can refer to separation of several widths (Γ), i.e., dynamical effects can be appreciable for separation of several widths (see Fig. 1).

[3] See B. French, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 109.

[4] A fit to the data with two incoherent Breit-Wigner distributions or a single broad Breit-Wigner distribution has been essentially ruled out. A successful fit was obtained using a dipole form. See Ref. 3. We do not consider the latter case.

TABLE I. Values of $c$ from Eq. (6) as a function of $s$ and $s_0$ for two choices of the $l=2$ $\pi\rho$ kinematical factor. We use units $m_\pi=1$ ($m_\rho=5.4$); $s=85$ corresponds to the $A_2$ position.

| $s \backslash s_0$ | $\rho=k_{\pi\rho}^5/s^{\frac{3}{2}}$ 0 | $-50$ | $\rho=k_{\pi\rho}^5/s^2$ 0 | $-50$ |
|---|---|---|---|---|
| 70 | 4 0 | 7.6 | 2.4 | 4.0 |
| 80 | 2.7 | 4.8 | 1.6 | 2.6 |
| 90 | 2.1 | 3.4 | 1.2 | 1.9 |
| 100 | 1.7 | 2.7 | 1.0 | 1.5 |
| 300 | 0.2 | 0.7 | $-0.2$ | 0.02 |
| 500 | $-0.03$ | 0.3 | $-0.6$ | $-0.3$ |

in channel $a$, we adjust $g_{bb}$ and $g_{cc}$ to produce zeros in $D_{bb}$ and $D_{cc}$, respectively, at energies $s_b$ and $s_c$ well below the thresholds for $b$ and $c$. The values of $g_{ab}$ and $g_{ac}$ then determine the widths of the resonances.

Using a linear approximation for $D_{bb}$ and $D_{cc}$ in the region of interest,

$$D_{bb}=e_b(s_b-s)\,,\qquad D_{cc}=e_c(s_c-s)\,, \tag{4}$$

we obtain after some algebra the following expression for $S_{aa}$:

$$S=\frac{(\bar{s}_1-s+i\bar{\Gamma}_1)(\bar{s}_2-s+i\bar{\Gamma}_2)-\bar{\Gamma}_1\bar{\Gamma}_2(c-i)^2}{(\bar{s}_1-s-i\bar{\Gamma}_1)(\bar{s}_2-s-i\bar{\Gamma}_2)-\bar{\Gamma}_1\bar{\Gamma}_2(c+i)^2}\,, \tag{5}$$

where $\bar{s}_i$ and $\bar{\Gamma}_i$ are the position and width that the $i$th resonance would have if it were isolated and [5]

$$\begin{aligned}
\bar{\Gamma}_1&=\rho_a\varphi_b g_{ab}^2/e_b(s-s_0)\,,\\
\bar{\Gamma}_2&=\rho_a\varphi_c g_{ac}^2/e_c(s-s_0)\,,\\
c&=\frac{(s-s_0)^2}{\rho_a}\frac{P}{\pi}\int_{s_a{}^T}^{\infty}\frac{\rho_a(s')}{(s'-s_0)^2}\frac{ds'}{s'-s}\,,\\
\bar{s}_1&=s_b-\bar{\Gamma}_1 c\,,\\
\bar{s}_2&=s_c-\bar{\Gamma}_2 c\,.
\end{aligned} \tag{6}$$

Note that the parameter $c$, which depends only on the pole position of the effective left-hand cut (and kinematics), is already present in dynamics of the isolated resonances; i.e., we see in Eq. (6) that $c\bar{\Gamma}_1$ and $c\bar{\Gamma}_2$ are the shift of the bound-state positions $s_b$ and $s_c$ to the resonance position $\bar{s}_i$. We can then rewrite (5) in the form of Eq. (1) by finding the zeros of the denominator of (5). Thus

$$\begin{aligned}
s_1-i\Gamma_1=\tfrac{1}{2}\{&(\bar{s}_1-i\bar{\Gamma}_1)+(\bar{s}_2-i\bar{\Gamma}_2)\\
&-[((\bar{s}-i\bar{\Gamma}_1)-(s_2-i\bar{\Gamma}_2))^2+4\bar{\Gamma}_1\bar{\Gamma}_2(c+i)^2]^{\frac{1}{2}}\}\,,\\
&s_2=\bar{s}_1+\bar{s}_2-s_1\,,\quad \Gamma_2=\bar{\Gamma}_1+\bar{\Gamma}_2-\Gamma_1\,.
\end{aligned} \tag{7}$$

$\Gamma_i$ are the widths which would be observed experimentally, whereas the $\bar{\Gamma}_i$ are the widths which are related to the coupling constants. Huge enhancements or suppressions of one of the resonances can occur depending on the dynamical parameter $c$ and the relative separation of the resonances. These dramatic effects contained in Eq. (7) are illustrated in Fig. 1 for a ratio of $\bar{\Gamma}_1/\bar{\Gamma}_2=0.1$. If $\bar{\Gamma}_1/\bar{\Gamma}_2$ is very narrow, we can expand (7) to obtain the especially simple result

$$\Gamma_1/\bar{\Gamma}_1=(x-c)^2/(1+x^2)\,, \tag{8}$$

where $x=(\bar{s}_1-\bar{s}_2)/\bar{\Gamma}_2$.

Thus we have exhibited a specific dynamical (quasi-bound-state) model in which the presence of two nearby resonances greatly influence each other. This model is easily extended to include more open channels or more resonances. One might consider other types of dynamical models and possibly obtain quantitatively different expressions than (7) relating the $s_i$, $\Gamma_i$ to the $\bar{s}_i$, $\bar{\Gamma}_i$. However, we believe that qualitatively the large effects illustrated in Fig. 1 would be present.

Now we assume that the two overlapping resonances comprising the $A_2$ are dynamically produced as quasi-bound-states of higher-mass channels. Further, we treat only the dominant $\pi\rho$ channel, ignoring the $K\bar{K}$ and $\pi\eta$ decay modes, so that we may directly apply Eq. (7) to the $A_2$ in order to determine the $\bar{s}_i$, $\bar{\Gamma}_i$. In Table I we show how $c$ given by Eq. (6) varies as a function of $s$ for various choices of $s_0$ [6] and asymptotic behavior of the kinematical factor $\rho$ for the $l=2$ $\pi\rho$ channel. We feel that a reasonable choice of $c$ in the region of the $A_2$ ($s\sim 85m_\pi^2$) is $c\gtrsim 1.5$.

Two different solutions of Eq. (1) have been presented by the CERN group in fitting their missing-mass states[3,7]: There is the symmetric solution in which two resonances of equal width (22 MeV) separated by 30 MeV interfere and the asymmetric solution in which a resonance of 90 and one of 12 MeV are at the same

TABLE II. $\bar{\Gamma}_1$, $\bar{\Gamma}_2$, $\bar{M}_1$, and $\bar{M}_2$ (in MeV) determined from inverting Eq. (7) for various values of $c$ by using the two CERN fits to the $A_2$ (Ref. 3) for $\Gamma_1$, $\Gamma_2$, $M_1$, and $M_2$.

| | $\Gamma_1=\Gamma_2=22$, $M_1=1289$, $M_2=1309$ | | | | $\Gamma_1=12$, $\Gamma_2=90$, $M_1=1297$, $M_2=1298$ | | | |
|---|---|---|---|---|---|---|---|---|
| $c$ | $\bar{\Gamma}_1$ | $\bar{M}_1$ | $\bar{\Gamma}_2$ | $\bar{M}_2$ | $\bar{\Gamma}_1$ | $\bar{M}_1$ | $\bar{\Gamma}_2$ | $\bar{M}_2$ |
| 2.5 | 2.7 | 1292 | 41.3 | 1306 | 1.6 | 1293 | 100.4 | 1302 |
| 2.0 | 3.9 | 1290 | 40.1 | 1308 | 2.4 | 1292 | 99.6 | 1303 |
| 1.5 | 5.7 | 1289 | 38.3 | 1309 | 4.0 | 1291 | 98.0 | 1304 |
| 1.0 | 8.9 | 1287 | 35.1 | 1311 | 7.8 | 1289 | 94.2 | 1306 |
| 0.5 | 14.3 | 1285 | 29.7 | 1313 | 19.0 | 1285 | 83.0 | 1310 |
| 0 | 22 | 1284 | 22 | 1314 | 49.8 | 1281 | 52.2 | 1314 |

[5] In the calculation we assume $c=$const over the energy range of interest. This is as good an approximation as assuming $\Gamma$ constant. Note that since $c$ is determined by a principal-value integral, it can be negative. See Table I.

[6] It is possible that the cut in $B_{12}$ can begin at $s>s_a{}^T$. We assume that the effective cut position is at $s_0\lesssim 0$. If the higher-mass channel producing the resonance is a quark-antiquark channel, a calculation of the Regge trajectory of the $\rho$ meson treated as a quark-antiquark bound state supports this assumption: P. Coulter, Phys. Rev. **179**, 1592 (1969).

[7] The fit is done using total energy as the variable. In Eq. (1), $s\to E$, $s_i\to M_i$, $\Gamma_i\to\frac{1}{2}\Gamma_i$.

position. In Table II we show the results of inverting Eq. (7) to find the isolated values of the positions[7] $\bar{M}_i$ and widths $\bar{\Gamma}_i$ for these two solutions for different values of $c$. Combining our above estimate of $c \gtrsim 1.5$, we see from Table II that, for both of the CERN solutions, we are led to a very asymmetric-type isolated situation with one narrow resonance with a $\bar{\Gamma} \lesssim 5$ MeV interfering with a broad resonance to produce an asysmmetric double peak.

Now we want to conjecture[8] on the consequences of the above result that one of the resonances which we will denote by $e$ comprising the $A_2$ has an isolated width $\bar{\Gamma}_e \lesssim 5$ MeV. Since the $A_2$ is not associated with any threshold effects, $\bar{\Gamma}_e$ is small due to an intrinsically small coupling constant[9] $g_e^2 \propto \bar{\Gamma}_e/\rho_e$. In a production experiment the number of $e$'s produced is proportional to $g_e^2$ and thus it would not be copiously produced. This is in contrast to a resonance, which is narrow owing to a small kinematical factor such as the $\omega$. Here the coupling $g_{\omega\rho\pi}{}^2$ is large so that the $\omega$ is copiously produced even though the small kinematical factor in the $\pi\rho$ channel leads to a narrow width for the $\omega$. A narrow resonance (narrow compared to experimental resolution) is easily seen if it is copiously produced. On the other hand, the $e$ would not be detected with present experimental resolution if it were not dynamically enhanced by its overlap with the broad resonance in the $A_2$.

Consider now the $SU(3)$ analogs of the $A_2$. If the counterpart of the intrinsically narrow $e$ does not overlap the broad resonance in, e.g., the $K^*(1420)$, or even if it does and is dynamically narrowed by being on the high side [as noted from Fig. 1 or Eq. (8), since $c$ would be positive], it would not be detected. Thus the odds are against the analogs of the $e$ being observed with present resolution.

Finally, we conjecture that just as all the known resonances can be fitted into the simple quark model where the boson resonances are quasi-bound-states of $q\bar{q}$ (real or mathematical) with large intrinsic couplings to open channels, there exist multiquark boson resonances which are $2q2\bar{q}$ (or $3q3\bar{q}$) quasi-bound-states with much smaller intrinsic couplings to the open channels [possibly proportional to $SU(3)$ breaking]. The $e$ may be the first of these narrow, moderately produced multiquark states that has been seen. The degeneracies of these multiquark levels become vary large and thus there may exist very many of these narrow resonances at moderately high energies. It is clear that it is worthwhile to look with high resolution $\sim 5$ MeV for these narrow resonances—in particular, in channels with exotic quantum numbers,[10] or in situations similar to the $A_2$ where they might interfere with one of the usual broad resonances.

To summarize, we stress the following points:

(1) Quantitative calculations based on our model indicate that large changes in the ratio $\Gamma_i/\bar{\Gamma}_i$ can result from coupling two resonances. The model implies that one of the resonances in the $A_2$ would have a width of about 5 MeV if it were isolated.

(2) Thus we suggest that the $A_2$ structure is already evidence for a narrow (due to a small coupling constant) resonance which could not be detected with present experimental resolution if it were isolated. The existence of one narrow resonance, which does not easily fit into a quark-antiquark model, is a strong incentive for a concentrated effort to look for other narrow resonances.

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[8] Some of these remarks were motivated by discussion at the Topical Conference on Resonances, University of Oregon, Eugene, Ore. (unpublished).

[9] The coupling constant could also be small because of a selection rule; e.g., a partial width could be suppressed by $SU(3)$, but it is unlikely that all the decay channels are suppressed.

[10] It is possible that the exotic resonances are pair-produced more strongly than they are singly produced.