terms of the K^* mass difference:

$$\frac{F_\omega}{G_{\rho^0}}=-\frac{m_\omega^2}{m_{\rho^0}^2}\frac{G_{\rho^0}}{G_\omega}y, \quad (38)$$

where we now have from Eq. (19)

$$y=\frac{2}{\sqrt{3}}\frac{(m_{K^{*0}}-m_{K^{*+}})(m_{K^*}-m_\rho)}{m_\rho^2+m_\omega^2}. \quad (39)$$

Using Eqs. (7) and (28) with $F_\phi=0$, $G_{\phi\pi\pi}=0$, one obtains in this case, without using soft pions, the result Eq. (25), so that the branching ratio is given by

$$\Gamma(\omega\to 2\pi)/\Gamma(\omega\to \text{all}) =0.23y^2\times 10^2\simeq(4\sim 0.2)\times 10^{-5}. \quad (40)$$

Since the expression Eq. (39) for y has an extra damping factor proportional to $m_{K^*}-m_\rho$, the branching ratio (40) is very small and is contradicted by the experimental result (36). Therefore, our model does not seem to be compatible with the Maki-Hara model or the other simple quark models in a tadpole approximation. Of course, it may be that our assumption of the pole dominance or of our neglect of scalar-meson contributions is not a good approximation or that we have to take into account contributions from $\gamma+\rho^0$, $\gamma+\omega$, and $\gamma+\phi$ intermediate states to our sum rules.

In conclusion, it may be stressed that our asymptotic sum rules are in principle more general than any specific models such as the quark models. For example, our formulas could be still valid for the case of field-current identity as in the model of Lee, Weinberg, and Zumino.[17] It may be that somehow we are picking up a solution corresponding to the latter model rather than the quark models. At any rate, a more refined measurement of the $\omega\to 2\pi$ and $\phi\to 2\pi$ branching ratios would be of great interest. We understand that such measurements are in progress.[18]

We would like to thank Dr. W. A. Wenzel for a private communication on the Berkeley data.

[17] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1969).
[18] W. A. Wenzel (private communication).

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Exchange-Degenerate Regge Trajectories and Meson-Nucleon Charge-Exchange Scattering*

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Based on exchange degeneracy and $SU(3)$ symmetry, a Regge-pole model fitting the differential cross sections for pion-nucleon and kaon-nucleon charge exchange and η production as well as polarization in $\pi^-p\to\pi^0n$ is given. Besides the ρ-A_2 trajectory, a conspiring and exchange-degenerate ρ'-A_2' trajectory with the same slope as ρ-A_2 but with an intercept near zero is taken into account. The residues are assumed to have $1/\Gamma(\alpha)$ behavior. A four-parameter fit gives good agreement with the experimental data.

1. INTRODUCTION

A POSSIBLE classification of Regge trajectories according to $SU(3)$ symmetry, exchange degeneracy, and the Lorentz-pole quantum number has been given previously.[1,2] Our aim here is to combine these considerations with the Veneziano-type representation[3] in a phenomenological study of pion-nucleon and kaon-nucleon charge-exchange reactions and η production. The model presented here has numerous other phenomenological implications which are subject to future investigations.

The basic ideas of Refs. 1 and 2 utilized here are the following: The vector and tensor octets of trajectories form an octet of exchange-degenerate trajectories with $SU(3)$ splittings of trajectories determined by the masses of resonances. These trajectories are coupled to mesons and baryons $SU(3)$-symmetrically. Similarly, the octet of pseudoscalar trajectories (to which the pion belongs) together with the octet of axial-vector trajectories (to which the B meson belongs) form an exchange-degenerate octet of conspiring trajectories. The coconspirator of this octet is an exchange-de-

* Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under Grant No. AF-AFOSR-1294-67.
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[1] Akbar Ahmadzadeh, Phys. Rev. Letters **20**, 1125 (1968).
[2] Akbar Ahmadzadeh and Richard J. Jacob, Phys. Rev. **176**, 1719 (1968).
[3] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

generate octet of vector-tensor trajectories. For example, the exchange-degenerate π-B trajectory conspires with an exchange-degenerate ρ'-A_2' trajectory of the same intercept.

In addition, we assume that the trajectories have a universal slope. In the spirit of the Veneziano formula, this simplifying assumption makes extra baryon trajectories in the s and u channels unnecessary. Furthermore, the possible resonances belonging to the coconspirators would be hidden underneath the other (known) resonances. This would explain why they have not been observed experimentally.

For the reactions we consider in this article, only the ρ and A_2 quantum numbers can be exchanged in the t channel. Our model consists of exchange-degenerate ρ-A_2 plus exchange-degenerate and conspiring ρ'-A_2' trajectories. In Sec. 2, the formalism and the results of fitting the parameters are given.

2. FORMULATION AND RESULTS

A phenomenological study of pion-nucleon charge-exchange differential cross section and polarization based on ρ and ρ' trajectories has been given in a separate paper.[4] Here we include the η production and kaon-nucleon charge exchange as well. Because we assume exchange degeneracy and $SU(3)$ symmetry of the couplings, no new parameter is needed.

We define the helicity-nonflip and helicity-flip parts of the ρ-trajectory contribution in meson-nucleon scattering to be of the form

$$A_\rho' = [\beta_\rho{}^n/\Gamma(\alpha_1)]\xi_\rho(as)^{\alpha_1} \tag{1}$$

and

$$B_\rho = [\beta_\rho{}^f/\Gamma(\alpha_1)]\xi_\rho(as)^{\alpha_1-1}, \tag{2}$$

respectively, where $\xi_\rho = (1-e^{-i\pi\alpha_1})/\sin\pi\alpha_1$ and β^n (β^f) is the helicity-nonflip (-flip) residue constant. Equations (1) and (2) can be considered as the leading term (in energy) of a suitable Veneziano-type representation. In fact, Igi[5] has given such a representation for pion-nucleon scattering and his formulas reduce to Eqs. (1) and (2) in the high-energy limit. Furthermore, even if one has to take an infinite number of Veneziano terms (corresponding, perhaps, to a single Lorentz pole), still the leading term in energy will be in the form of Eqs. (1) and (2). The exchange-degenerate ρ-A_2 trajectory passing through $\rho(760)$ and $A_2(1310)$ is given by

$$\alpha_1 \simeq 0.5 + at, \quad a \simeq 0.9\ (\text{GeV})^{-2}. \tag{3}$$

Similarly, the amplitudes for the A_2-trajectory contribution (denoted by the subscript R) are given by

$$A_R' = [\beta_R{}^n/\Gamma(\alpha_1)]\xi_R(as)^{\alpha_1}, \tag{4}$$

$$B_R = [\beta_R{}^f/\Gamma(\alpha_1)]\xi_R(as)^{\alpha_1-1}, \tag{5}$$

where $\xi_R = (1+e^{-i\pi\alpha_1})/\sin\pi\alpha_1$.

[4] Akbar Ahmadzadeh and Jane C. Jackson, Phys. Rev. (to be published).

[5] K. Igi, Phys. Letters **28B**, 330 (1968).

The exchange-degenerate ρ-A_2 trajectory is assumed to have the same slope as the ρ'-A_2' trajectory. This simplifying assumption is made in order to avoid introducing new baryon trajectories in the s and u channels. Namely, one can imagine a Veneziano-type formula in which the same baryon trajectories coexist with ρ-A_2 as with ρ'-A_2'. Furthermore, as pointed out earlier,[4] this model would predict the existence of a particle with the same mass as the $B(1220)$ meson and with the ρ quantum numbers. This is because in our scheme the ρ'-A_2' trajectory coincides with the π-B trajectory. This coincidence would be a realization of chiral symmetry. The same coincidence should occur for higher recurrences. On the other hand, we assume that the A_2' trajectory chooses nonsense[6] at $\alpha=0$. Therefore we do not expect a scalar particle at the pion mass. This scheme is consistent with the quark model; namely, if as in Ref. 2 we assume that all trajectories are coupled to a quark-antiquark system, then the point $\alpha=0$ is a nonsense point for the A_2' trajectory. Note that the A_2' trajectory couples to the spin triplet of $q\bar{q}$ system. The ρ'-A_2' trajectory, based on the masses of the pion and the B meson, is given by

$$\alpha_2 \simeq -0.02 + 0.9t. \tag{6}$$

The ρ' contribution is given by

$$A_{\rho'}' = [t\beta_{\rho'}{}^n/\Gamma(\alpha_2)]\xi_{\rho'}(as)^{\alpha_2} \tag{7}$$

and

$$B_{\rho'} = [\beta_{\rho'}{}^f/\alpha_2\Gamma(\alpha_2)]\xi_{\rho'}(as)^{\alpha_2-1}. \tag{8}$$

Similarly, for the A_2' trajectory we have

$$A_{R'}' = [t\beta_{R'}{}^n/\Gamma(\alpha_2)]\xi_{R'}(as)^{\alpha_2}, \tag{9}$$

$$B_{R'} = [\beta_{R'}{}^f/\alpha_2\Gamma(\alpha_2)]\xi_{R'}(as)^{\alpha_2-1}. \tag{10}$$

The factor of t in Eqs. (7) and (9) is included because ρ' and A_2' are assumed to be conspiring trajectories (see Ref. 7, for example). Note the factor α in the denominators of the right-hand sides of Eqs. (8) and (10). Due to this factor there is a pole in the flip amplitude of the A_2' contribution at $\alpha_2=0$. This is a nonsense pole and thus we are assuming the existence of a compensating trajectory.[6] In Ref. 4, this factor of α was not included in the flip amplitude of the ρ'. It turns out that this extra factor improves our fit for polarization in $\pi^- p \to \pi^0 n$.

Now using the exchange degeneracy of the residues, we have

$$\beta_1{}^n \equiv \beta_\rho{}^n = \beta_R{}^n, \quad \beta_1{}^f \equiv \beta_\rho{}^f = \beta_R{}^f, \tag{11}$$

and, similarly,

$$\beta_2{}^n \equiv \beta_{\rho'}{}^n = \beta_{R'}{}^n, \quad \beta_2{}^f \equiv \beta_{\rho'}{}^f = \beta_{R'}{}^f. \tag{12}$$

Note that, since the trajectories are fully determined by Eqs. (3) and (6), we are left with only four parameters

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$\beta_1{}^n$, $\beta_1{}^f$, $\beta_2{}^n$, and $\beta_2{}^f$. Defining the new amplitudes $\mathcal{A}$ and $\mathcal{B}$ as

$$\mathcal{A}_\rho = A_\rho{}' + A_{\rho'}{}', \quad \mathcal{A}_R = A_{R'}{}' + A_{R'}{}', \qquad \mathcal{B}_\rho = B_\rho + B_{\rho'}, \quad \mathcal{B}_R = B_R + B_{R'}, \tag{13}$$

from $SU(3)$ symmetry of the couplings, we have[8]

$$\begin{aligned} A'(\pi^- p \to \pi^0 n) &= \sqrt{2}\,\mathcal{A}_\rho, \\ A'(K^- p \to \bar{K}^0 n) &= \mathcal{A}_\rho - \mathcal{A}_R, \\ A'(\pi^- p \to \eta n) &= -(\tfrac{2}{3})^{1/2}\mathcal{A}_R, \\ A'(K^+ n \to K^0 p) &= -\mathcal{A}_\rho - \mathcal{A}_R, \end{aligned} \tag{14}$$

with similar expressions for the B amplitudes. In what follows, we consider the first three reactions in Eqs. (14). We are not including the K^+n charge-exchange case because of the meager high-energy experimental data and because of the complications due to the Glauber corrections for deuteron target. Note, however, that with the four parameters determined below, our model makes an unambiguous prediction of the K^+n charge-exchange reaction. The differential cross section is given by[9]

$$d\sigma/dt = (1/64\pi q^2 s)\{(4m^2-t)|A'|^2 + [t/(4m^2-t)] \times [4\mu^2 m^2 - ts - (s-m^2-\mu^2)^2]|B|^2\}, \tag{15}$$

where q is the center-of-mass momentum, m is the nucleon mass, μ is the meson mass, and s and t are the usual invariant energy and momentum transfer. Note that in η production we should use unequal-mass kinematics. However, at the energies we are considering, this extra complication is not necessary. The polarization is given[9] by

$$P = \frac{-\sin\theta\,\mathrm{Im}(A'B^*)}{16\pi s^{1/2}(d\sigma/dt)}, \tag{16}$$

where θ is the scattering angle in the center-of-mass system.

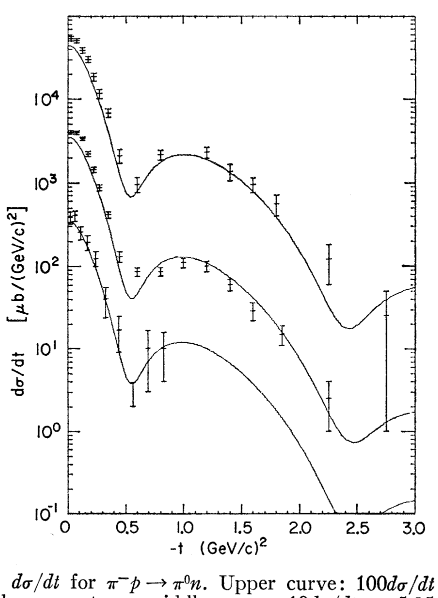

FIG. 1. $d\sigma/dt$ for $\pi^- p \to \pi^0 n$. Upper curve: $100 d\sigma/dt$ at 4.83 GeV/c lab momentum; middle curve: $10 d\sigma/dt$ at 5.85 GeV/c; lower curve: $d\sigma/dt$ at 6.0 GeV/c.

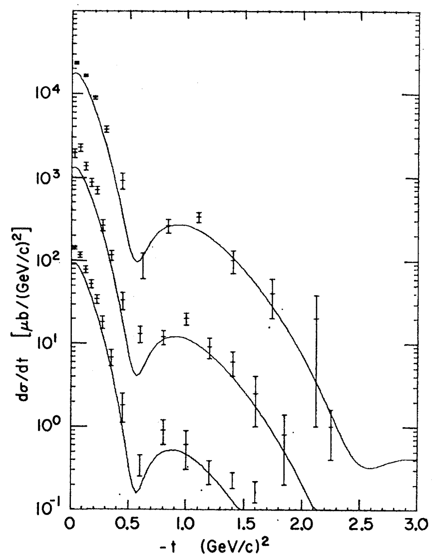

FIG. 2. $d\sigma/dt$ for $\pi^- p \to \pi^0 n$. Upper curve: $100 d\sigma/dt$ at 10 GeV/c; middle curve: $10 d\sigma/dt$ at 13.3 GeV/c; lower curve: $d\sigma/dt$ at 18.2 GeV/c.

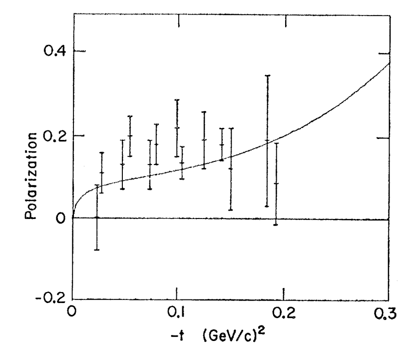

FIG. 3. Polarization in $\pi^- p \to \pi^0 n$ at 5.9 GeV/c.

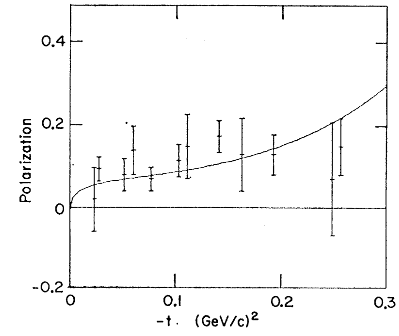

FIG. 4. Polarization in $\pi^- p \to \pi^0 n$ at 11.2 GeV/c.

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[9] V. Singh, Phys. Rev. 129, 1889 (1963).

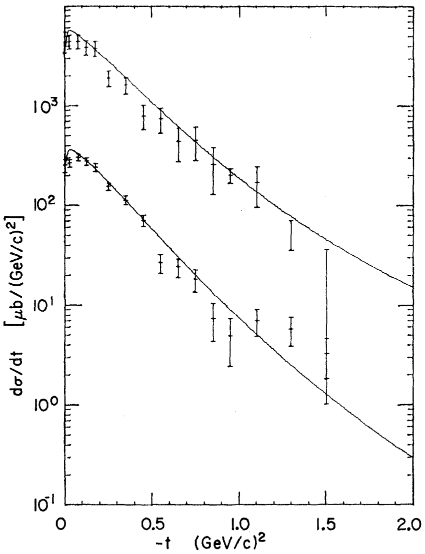

FIG. 5. $d\sigma/dt$ for $K^-p \to \bar{K}^0 n$. Upper curve: $10d\sigma/dt$ at 5 GeV/c; lower curve: $d\sigma/dt$ at 7.1 GeV/c.

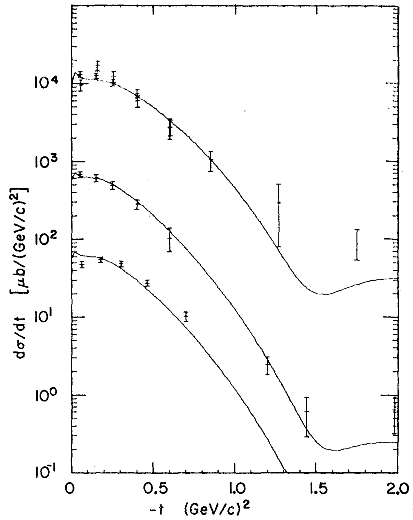

FIG. 7. $d\sigma/dt$ for $\pi^-p \to \eta n$. Upper curve: $100d\sigma/dt$ at 5.9 GeV/c; middle curve: $10d\sigma/dt$ at 10 GeV/c; lower curve: $d\sigma/dt$ at 9.8 GeV/c.

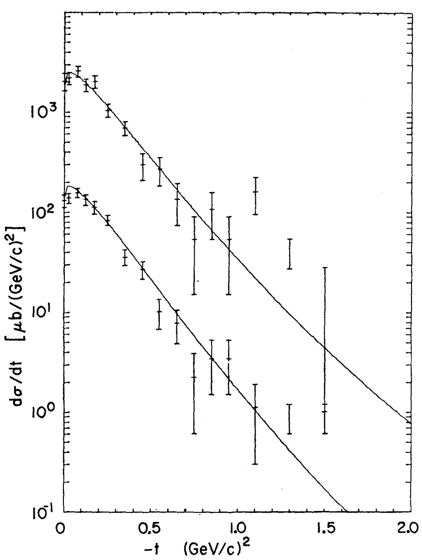

FIG. 6. $d\sigma/dt$ for $K^-p \to \bar{K}^0 n$. Upper curve: $10d\sigma/dt$ at 9.5 GeV/c; lower curve: $d\sigma/dt$ at 12.3 GeV/c.

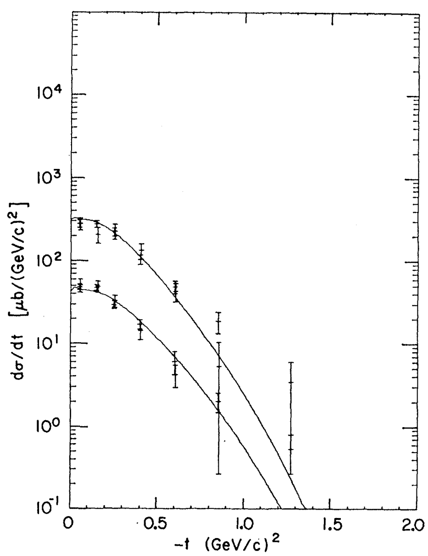

FIG. 8. $d\sigma/dt$ for $\pi^-p \to \eta n$. Upper curve: $10d\sigma/dt$ at 13.3 GeV/c; lower curve: $d\sigma/dt$ at 18.2 GeV/c.

The experimental data[10–13] are fitted using the four free parameters. The minimum χ^2 is obtained for

$$\beta_1{}^n = 9.81 \pm 1\,, \quad \beta_1{}^f = 119.0 \pm 9\,, \qquad \beta_2{}^n = -36 \pm 15\,, \quad \beta_2{}^f = 38 \pm 15\,. \tag{17}$$

For a total of 223 data points, we have obtained $\chi^2 = 369$ which we consider to be rather reasonable especially in view of ignoring the other (nonleading) terms. One could, of course, break $SU(3)$ and/or exchange degeneracy and obtain a smaller χ^2. But as long as we do not know how to take all other possible terms into account, there is very little information to be gained.

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[12] P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. H. West, P. Zanella, W. Beusch, W. Fischer, B. Gobbi, M. Pepin, and E. Polgar, Phys. Letters **23**, 396 (1966).

[13] O. Guisan, J. Kirz, P. Sonderegger, A. V. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Vairant, B. Amblard, C. Caverzasio, J. P. Guillaud, and M. Yvert, Phys. Letters **18**, 200 (1965); M. A. Wahlig and I. Mannelli, Phys. Rev. **168**, 1515 (1968).

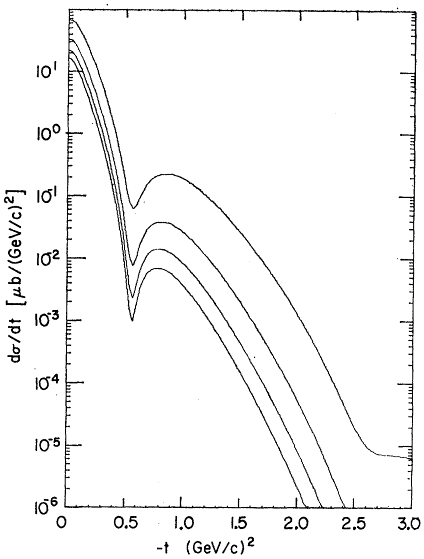

FIG. 9. Predicted $d\sigma/dt$ for $\pi^- p \to \pi^0 n$. Lab momentum from top to bottom is 25, 50, 75, 100 GeV/c, respectively.

The errors given in Eqs. (17) indicate the change in each parameter necessary to increase χ^2 by 10%. The large uncertainty in $\beta_2{}^n$ and $\beta_2{}^f$ reflects the lack of extensive polarization data, to which these parameters are sensitive. We are looking forward to more polarization data in these reactions, especially at higher momentum transfers, to determine these parameters with better confidence. One feature of our model worth mentioning is that it predicts zero polarization in $\pi^- p$ charge exchange at $t \simeq -0.6$ (GeV/c)^2, where $\alpha_\rho = 0$, and also predicts a zero in the $\pi^- p \to \eta n$ polarization at $t \simeq -1.6$ (GeV/c)^2, where $\alpha_R = -1$.

Note also that our formalism can be used to calculate the differential cross section and polarization in the $\pi^+ p \to K^+ \Sigma^+$ reaction. In this case only the d/f ratios remain as free parameters, for the K^*-K^{**} and $K^{*\prime}$-$K^{**\prime}$ trajectories are determined from the masses in the same way as our ρ-A_2 and ρ'-$A_2{}'$ trajectories. The residues in this case are of course related to those of Eqs. (17).

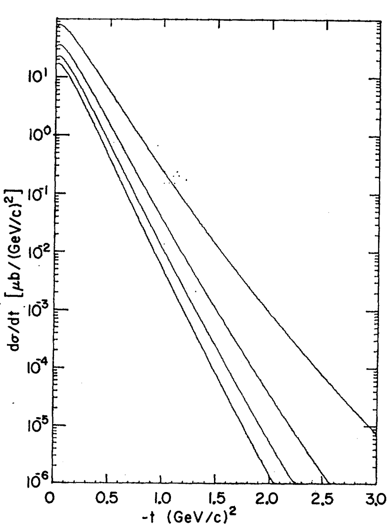

FIG. 10. Predicted $d\sigma/dt$ for $K^- p \to \bar{K}^0 n$, at same momenta as in Fig. 8.

Figures 1–8 show the experimental data as well as our theoretical curves. The error bars shown on these curves are statistical only. In calculating the χ^2, systematic errors in the data are also taken into account when given in the experimental articles. In the η-production data we have assumed a branching ratio[14] of 0.381.

Based on our model and the parameters obtained here, predictions are made for future data up to 100 GeV/c. Figures 9–11 show the results of our prediction.

To summarize, we have obtained a four-parameter model fitting the high-energy meson-nucleon charge-exchange and η-production data. The model gives a natural explanation of polarization using exchange-degenerate Lorentz poles—and without necessitating the existence of cuts.[15]

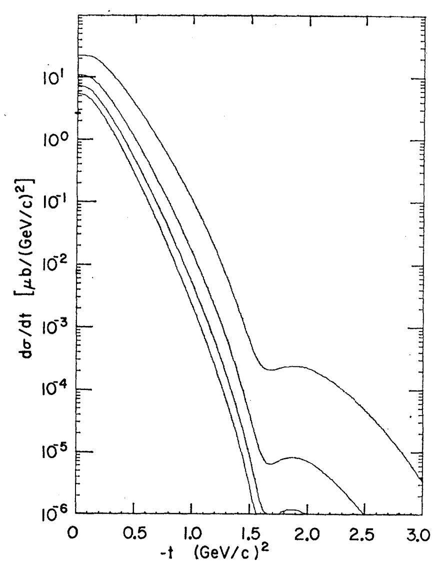

FIG. 11. Predicted $d\sigma/dt$ for $\pi^- p \to \eta n$, at same momenta as in Fig. 8.

ACKNOWLEDGMENT

We thank Professor Geoffrey F. Chew for his hospitality at the Lawrence Radiation Laboratory, where this work was done.

[14] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

[15] Numerous models incorporating Regge cuts with absorption have been given by various authors. See, for example, G. Cohen-Tannoudji, A. Morel, and H. Navelet, Nuovo Cimento **48**, 1075 (1967); M. L. Blackmon and G. Goldstein, Phys. Rev. **179**, 1480 (1969); F. Henyey, G. L. Kane, Jon Pumplin, and M. H. Ross, *ibid.* **182**, 1529 (1969). We are, of course, taking the point of view that once all the Regge contributions are taken into account no absorptive corrections should be made. It is well known that such "corrections" also have the unpleasant feature of double counting. Our model avoids this.