The movement of these zeros (as well as the poles) is illustrated in Fig. 12, where

$$g_2^2=\tfrac{1}{4}(\gamma_1+\gamma_2)^2.$$

One finds that the relation (3.8a) is kept satisfied for $g^2\leq g_1^2$, where a_i is the imaginary part of $\tilde{M}_i^2$. This shows that the corresponding point in Fig. 3 leaves the point A and goes up along the circle, which, of course, gets smaller during this movement. Also looking at Fig. 5, one sees that a very remarkable double-peaked structure is likely to be obtained in this model. The part of the circle on the other side in Fig. 3 corresponds to the zeros of S_{22}. The rest of the portion of the curve has no correspondence to this simple model.

For $g^2>g_1^2$, the poles are not along the vertical line. One easily finds that this case provides a simple example of the complex residues. They are, however, not always of the same nature as the complex residues discussed by Goldhaber who analyzed the structure with three peaks observed in the $K\pi\pi$ system.[30] The phases of his residues include, not only the phases of the residues of the scattering amplitudes, but also the contribution from the production matrix element.

[30] G. Goldhaber, Phys. Rev. Letters **19**, 976 (1967).

PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Experimental Tests for Theories of Chiral-Symmetry Breaking

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(Received 1 August 1969)

Assuming a picture of the strong interactions in which the Hamiltonian is the sum of an $SU(3)\otimes SU(3)$-symmetric piece H_0 plus a small symmetry-breaking term ϵH_1, we show how to calculate relations among the corrections of order ϵ to the symmetry limit. Our techniques are purely group-theoretic and involve no extraneous dynamical assumptions, so that our results provide direct experimental tests for various symmetry-breaking schemes. For example, we show that if ϵH_1 belongs to the $(3,\bar{3})\otimes(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$, then there is one sum rule satisfied by the corrections to the generalized Goldberger-Treiman relations for the three decays $n\to p+e+\nu$, $\Sigma\to N+e+\nu$, and $\Lambda\to N+e+\nu$. We also show that the so-called Σ terms, which are closely related to ϵH_1, can be obtained from *on-the-mass-shell* scattering amplitudes (albeit at an unphysical energy point) if terms of order ϵ^2 can be neglected in comparison to terms of order ϵ. The question of whether or not lowest-order calculations of symmetry breaking are meaningful is discussed in some detail.

I. INTRODUCTION

A NUMBER of authors[1–3] have pointed out that the most logical explanation of the successes of the partially conserved axial-vector current (PCAC) hypothesis and current algebra is that the strong interactions possess an approximate $SU(3)\otimes SU(3)$ symmetry. Two major features of such a symmetry, if it were exact, would be the existence of $SU(3)$ multiplets of particles degenerate in mass and eight massless pseudoscalar mesons. It is the appearance of this octet of massless mesons which allows for the conservation of the axial-vector currents without the need for $SU(3)\otimes SU(3)$ multiplets of particles. In the real, broken-symmetry situation it is, in fact, the case that we appear to have approximate $SU(3)$ multiplets of particles and, also, eight low-mass mesons π, K, and η which satisfy an approximate PCAC condition.

The assumption that in the symmetry limit these mesons correspond to the aforementioned octet of Goldstone bosons, is what guarantees that the approximate PCAC condition will hold.

In a recent paper (hereafter called I) by one of us, the basic ideas behind $SU(3)\otimes SU(3)$ symmetry and its breaking were discussed in some detail. The picture of the strong interactions outlined there is one in which

(i) the strong-interaction Hamiltonian can be decomposed into an $SU(3)\otimes SU(3)$-symmetric part H_0, plus a symmetry-breaking term ϵH_1;

(ii) ϵ is assumed small enough so that predictions of the symmetric theory fairly well approximate the real world;

(iii) in the limit $\epsilon\to 0$, the vacuum is taken to be $SU(3)$-symmetrical,[4] but not $SU(3)\otimes SU(3)$-symmet-

* Research supported in part by the National Science Foundation.

[1] The original suggestion that PCAC is related to a slightly broken chiral symmetry is due to Nambu and his collaborators [see Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962) and earlier papers cited therein]. The first paper relating the modern work on current algebra to chiral symmetry seems to be S. Weinberg, Phys. Rev. Letters **16**, 163 (1966). An extensive list of later references on $SU(3)\otimes SU(3)$ is contained in Weinberg's report in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 253.

[2] R. Dashen, Phys. Rev. **183**, 1245 (1969); referred to as Paper I.

[3] R. Dashen and M. Weinstein, Phys. Rev. **183**, 1261 (1969); referred to as Paper II.

[4] This means, in particular, that there is no strange scalar "κ meson" which goes to zero mass when $\epsilon\to 0$. There could still be an octet of scalar mesons which plays a special dynamical role in symmetry breaking.

rical. Thus one is led to predict $SU(3)$ multiplets of particles and an octet of massless pseudoscalar mesons.

We explicitly showed in a second paper (hereafter called II) how these assumptions yield all of the soft-meson theorems usually derived from the combined assumptions of PCAC and current algebra, or by means of effective Lagrangians[3] [which give, in effect, a particular model of $SU(3)\otimes SU(3)$ symmetry]. Strictly speaking, these soft-meson theorems, insofar as they pertain to mass-shell scattering amplitudes, are exact only for $\epsilon=0$. In the actual world they become approximate statements which are to be understood in essentially the same way as the predictions given by $SU(3)$. Clearly, it is desirable to be able to systematically discuss corrections to the symmetric limit. This is the main objective of the present paper.

In order to make any progress towards our goal of developing a systematic way of discussing corrections to $SU(3)\otimes SU(3)$ symmetry, we have to make one further hypothesis which is logically independent of (i)–(iii):

(iv) For the purpose of computing the major deviations from the predictions of the symmetric theory, it is sufficient to work to lowest order in ϵH_1.

While it may appear at first glance that (iv) is a logical consequence of (ii), this is not necessarily true.[5] In fact, in Paper I it was argued that hypothesis (iv) is very unlikely to hold, because if it is valid, there are some rather severe difficulties encountered in trying to understand the electromagnetic masses of mesons[6] and theoretical difficulties with the concept of octet enhancement.

Why, then, are we discussing predictions based upon assumption (iv)? One reason is that one can think of models, as we shall see later, in which relations derived upon the basis of hypothesis (iv) might be expected to hold quite well for processes involving only strong interactions, whereas similarly derived relations for processes involving weak or electromagnetic interactions might not be satisfied at all. While this possibility may have nothing to do with the actual situation, it does point up the fact that there is a dearth of verifiable first-order predictions which one might hope would shed some light on this question. Thus, it is our purpose in this paper to derive several nontrivial relations among observable quantities which should be accurate to order ϵ(i.e., neglecting order ϵ^2). We adopt the point of view that if these relations turn out to be satisfied, then we may assume that calculating to first order in symmetry breaking is, at least sometimes, meaningful. In this case we must try to understand the difficulties raised in Paper I. If, on the other hand, the order-ϵ relations are not satisfied, then we must look for more sophisticated, possibly nonperturbative, ways of computing corrections to the $SU(3)\otimes SU(3)$-symmetric theory.

So much, then, for questions of motivation. Let us now discuss some general features of our approach. First, let us note that in order to make predictions about symmetry breaking, it is, in general, necessary to make some assumption about the transformation properties of the symmetry-breaking part of the Hamiltonian. For example, in $SU(3)$ one usually assumes that the symmetry-breaking term belongs to an octet. One might wonder at this point how we are going to disentangle the general question of the validity of lowest-order calculations from the problem of determining the structure of ϵH_1. A particular answer is that *we know of at least one relation which holds to order* ϵ *and which is independent of any specific assumption on the properties of* ϵH_1. This relation is a theorem on the form factors in K_{l3} decay and has been discussed in a recent letter.[7] Furthermore, if by assuming specific simple transformation properties for ϵH_1 one can obtain several verifiable sum rules which are mutually independent, and all satisfied to 10 or 20%, then one would believe that both the expansion in powers of ϵ and the specific choice for ϵH_1 are correct.

When presenting various results we have, for reasons of clarity of presentation, adopted the suggestion of Gell-Mann, Oakes, and Renner[8] that ϵH_1 is a particular component of the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$. While this is by far the simplest and most elegant choice for ϵH_1, it is by no means necessary for us to restrict attention to this case; our techniques are quite general and can easily be extended to other schemes.

One of our results involves the Goldberger-Treiman relation and its extension to strangeness-changing axial-vector currents. More explicitly, for the decays $\Sigma\rightarrow N+e+\nu$, $\Lambda\rightarrow N+e+\nu$, and $N\rightarrow N+e+\nu$ there are three generalized Goldberger-Treiman relations involving matrix elements of the axial-vector current, the couplings $G_{K\Sigma N}$, $G_{K\Lambda N}$, and $G_{\pi NN}$, and the decay constants f_K and f_π. In a world in which $SU(3)\otimes SU(3)$ were an exact symmetry of the type just described, these generalized Goldberger-Treiman relations would be satisfied identically. Since, in the real world, they are not exact, we can talk about the three deviations from symmetry defined by expressions of the form

[5] The success of the Gell-Mann–Okubo mass formula makes it look as though we are seeing first-order $SU(3)$ breaking. However, this formula merely states that the effective breaking transforms like an octet. There are a number of models (tadpole model, various bootstrap schemes, vector mixing, etc.) which have the property that second and higher orders may be important but end up looking like lowest order in the sense that the net breaking displays an octet pattern. For more details and references see Ref. 2.

[6] In Paper I it is shown that neglecting terms of order $\epsilon\alpha$, one has $m_{\pi^+}^2-m_{\pi^0}^2=m_{K^+}^2-m_{K^0}^2$ which is in violent disagreement with experiment. One difficulty in interpreting this disaster is that the analogous Coleman-Glashow sum rule for electromagnetic mass differences of baryons works very well.

[7] R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969).

[8] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1969).

$(2f_\pi m_N g_A - G_{\pi NN})$ (analogous combinations of the relevant constants are used for the other two cases). We show that in the Gell-Mann–Oakes–Renner theory there is one sum rule which relates the three deviations and which holds up to terms of order ϵ^2. One can think of this result in direct analogy to the Gell-Mann–Okubo mass formula which provides one sum rule among the three mass splittings in the baryon octet.

Another one of our results involves the fact that in the Gell-Mann–Oakes–Renner theory (or any other theory giving a specific model for ϵH_1) one can make definite predictions about the so-called "Σ term" which is given effectively by the commutator $[Q_\alpha^5[Q_\beta^5,\epsilon H_1]]$, where Q^5 stands for an axial-vector charge. One would, of course, like to measure this object and thus obtain a test of this theory. So far, all attempts[9] to do this have been based on one sort or another of off-the-mass-shell extrapolations of various scattering amplitudes. Our contribution to this question is to show that if one is willing to neglect terms of order ϵ^2, then the Σ term can be extracted from the measurable on-mass-shell meson-baryon scattering amplitudes extrapolated to an unphysical point in energy. Although this extrapolation in energy is clearly difficult and requires a careful use of dispersion relations, it is not subject to the host of ambiguities inherent in any off-mass-shell extrapolation procedure.

The techniques used throughout this paper are based on those developed in detail in Paper II. We have, however, attempted to make this paper relatively self-contained and so, in addition to reviewing some of the methods of II, we also summarize a number of rather elementary, but important, points about $SU(3)\otimes SU(3)$ symmetry and its breaking. All proofs which require somewhat more knowledge of the notation and formalism developed in Paper II and which are not vital to the discussion have been included in the Appendix.

We might also mention at this point that we have by no means exhausted the possibilities for calculating interesting corrections to $SU(3)\otimes SU(3)$. On the contrary, the methods developed here and in Paper II should be capable of producing a large number of additional relations among the various deviations from the predictions given by assuming $SU(3)\otimes SU(3)$ symmetry.

Finally, a word of warning: In the above discussion, and through most of this work, we have assumed that the fundamental interaction which breaks $SU(3)\otimes SU(3)$ is a local operator linking hadrons to hadrons. There is no reason why this need be the case. It could be that the basic interaction arises from the coupling of hadrons into some nonhadron. If this is in fact the case, the effective symmetry-breaking effect for processes involving only hadrons is already of second order. That is, if the basic hadron-nonhadron coupling is characterized by a coupling constant g, then ϵ determined in hadronic processes is essentially g^2. A number of the results discussed in this paper will not hold if symmetry breaking is due to an interaction of this type, and we shall return to this point at a later time.

The general plan of this paper is as follows. Section II is basically a review of the essential ideas involved in $SU(3)\otimes SU(3)$ symmetry and its breaking. Section III is devoted to an exposition on general group-theoretic properties which could be assumed for ϵH_1, with special emphasis placed upon the Gell-Mann–Oakes–Renner model. Then Sec. IV gets down to the business of stating and deriving the sum rule for the deviations from the generalized Goldberger-Treiman relations; our result relating the Σ term to on-mass-shell scattering amplitudes is stated and discussed in Sec. V. Finally, Sec. VI is devoted to a discussion of this work, its possible extension, and further theoretical questions which have yet to be resolved.

II. REVIEW OF SYMMETRIC WORLD AND GENERAL PROPERTIES OF SYMMETRY BREAKING

As we pointed out in the Introduction, it is worthwhile for the sake of completeness to briefly review the properties one would expect the world to have in the limit that $SU(3)\otimes SU(3)$ becomes an exact symmetry.

In such a world, the vector and axial-vector charges denoted by

$$Q_\alpha(t)\equiv\int d^3x\,\mathfrak{F}_\alpha{}^0(\mathbf{x},t)\,,\quad Q_\alpha{}^5(t)\equiv\int d^3x\,\mathfrak{F}_\alpha{}^{50}(\mathbf{x},t)\qquad(\alpha=1,2,\cdots,8)\quad(2.1)$$

are time-independent, and under commutation generate the Lie algebra of $SU(3)\otimes SU(3)$. This is, of course, guaranteed by the assumptions

$$\partial_\mu\mathfrak{F}_\alpha{}^\mu(x)=0\,,\quad \partial_\mu\mathfrak{F}_\alpha{}^{5\mu}(x)=0\,,\qquad(2.2\text{a})$$

$$[\mathfrak{F}_\alpha{}^0(x),\mathfrak{F}_\beta{}^\mu(y)]_{x_0=y_0}=i\delta^3(\mathbf{x}-\mathbf{y})f_{\alpha\beta\gamma}\mathfrak{F}_\gamma{}^\mu(x)+\text{S.T.}\,,$$
$$[\mathfrak{F}_\alpha{}^0(x),\mathfrak{F}_\beta{}^{5\mu}(y)]_{x_0=y_0}=i\delta^3(\mathbf{x}-\mathbf{y})f_{\alpha\beta\gamma}\mathfrak{F}_\gamma{}^{5\mu}(x)+\text{S.T.}\,,\qquad(2.2\text{b})$$
$$[\mathfrak{F}_\alpha{}^{50}(x),\mathfrak{F}_\beta{}^{5\mu}(y)]_{x_0=y_0}=i\delta^3(\mathbf{x}-\mathbf{y})f_{\alpha\beta\gamma}\mathfrak{F}_\gamma{}^\mu(x)+\text{S.T.}\,,$$

where S.T. stands for possible Schwinger terms.

As discussed in Paper I, it is perfectly consistent to assume that the octet of vector charges generates an $SU(3)$ symmetry realized in the usual way, by having multiplets of particles which are degenerate in mass and an invariant vacuum state. However, if we wish the single-meson state to be coupled to the vacuum by

[9] See, e.g., F. Von Hipple and Jae Kwan Kim, Phys. Rev. Letters **22**, 740 (1969); C. H. Chan and F. T. Meiere, *ibid.* **22**, 737 (1969).

the axial-vector current, so that we have

$$\langle M_\alpha(q)|\mathfrak{F}_\beta^{5\mu}(0)|0\rangle=(-iq^\mu/2f_\alpha)\delta_{\alpha\beta}, \qquad (2.3)$$

where $(2f_\alpha)^{-1}\neq 0$, then the remainder of the symmetry is realized not by the creation of still larger supermultiplets of particles which are degenerate in mass, but rather by the existence of an octet of massless pseudoscalar mesons (i.e., Goldstone bosons). In this case the vacuum state is not invariant under the larger group, although it remains invariant with respect to the $SU(3)$ group generated by the vector charges. [The fact that the masses of the mesons must be zero follows trivially by taking the divergence of both sides of Eq. (2.3) and making use of Eq. (2.2).] Furthermore, the $SU(3)$ symmetry implies that in this theory the constants f_α are all equal to a single constant which we shall call f_0.

We have already shown in Paper II that as a direct consequence of these assumptions, one has exact generalized Goldberger-Treiman relations and that all of the familiar soft-meson theorems hold exactly for on-mass-shell scattering amplitudes.

It is now natural to inquire as to what happens in such a world if we introduce a "small" symmetry-breaking term ϵH_1 into the total Hamiltonian, that is, if we let

$$H_{\text{tot}}\equiv H_0+\epsilon H_1, \qquad (2.4)$$

where H_0 is $SU(3)\otimes SU(3)$-invariant and $\epsilon\ll 1$.

The first change which obviously must occur is that the masses squared of the pseudoscalar mesons are no longer zero, but rather m_π^2, m_K^2, and m_η^2 all become of order ϵ.[10] What this means (and we wish to emphasize this point) is that if ϵ is small, it is the ratio of meson masses to typical strong interaction masses, such as nucleon masses, which is expected to be small; this does not imply that meson mass differences must be small in comparison to the mean mass of the multiplet. Far from being a difficulty with this point of view, one derives a positive advantage by considering the basic symmetry to be $SU(3)\otimes SU(3)$, since there is no way within the framework of $SU(3)$ alone to explain the fact that the mass differences are comparable to the mean mass. In other words, the fact that in the real world $(m_K^2-m_\pi^2)/m_K^2\sim 1$ is not to be taken as evidence that $SU(3)\otimes SU(3)$-symmetry breaking is large.

A second change which is of some importance is that in Eq. (2.3) the constants f_α no longer need to be equal. Rather, one now has the condition that

$$\langle M_\alpha(q)|\mathfrak{F}_\beta^{5\mu}(0)|0\rangle\equiv(-iq^\mu/2f_\alpha)\delta_{\alpha\beta}, \qquad (2.5)$$

where

$$f_\alpha/f_\beta=1+O(\epsilon) \quad \text{when} \quad \alpha\neq\beta.$$

There is one further point worth emphasizing at this stage of our discussion, and that is that the equation

$$\langle M_\alpha(q)|\partial_\mu\mathfrak{F}_\beta^{5\mu}(0)|0\rangle\equiv(m_\alpha^2/2f_\alpha)\delta_{\alpha\beta} \qquad (2.6)$$

is a kinematic identity when m_α^2 is nonzero, and places no constraints upon the $SU(3)$ transformation properties of the divergence, $\partial_\mu\mathfrak{F}_\beta^{5\mu}(0)$. In particular, it does not require that the divergence transform as a member of an $SU(3)$ octet. Moreover, Eq. (2.6) makes it clear that if one wishes to work to lowest order in ϵ, the correct version of Eq. (2.6) is

$$\langle M_\alpha(q)|\partial_\mu\mathfrak{F}_\beta^{5\mu}(0)|0\rangle\equiv(m_\alpha^2/2f_0)\delta_{\alpha\beta}+O(\epsilon^2). \qquad (2.7)$$

The preceding statements summarize those aspects of symmetry breaking which we will use extensively throughout this paper. Note that these are all model-independent statements, since in the next section we will discuss those aspects of symmetry breaking which are not independent of the model one chooses for ϵH_1.

III. GROUP-THEORETIC STATEMENTS ABOUT ϵH_1

As we mentioned in the Introduction, the main purpose of this paper is to show how one can derive verifiable sum rules from any specific model for $SU(3)\otimes SU(3)$-symmetry breaking. Before we do so, however, there are several important points we must discuss. These are facts which follow from any assumption of transformation properties for ϵH_1 and which have nothing to do with the assumption that one is calculating effects to lowest order in symmetry breaking. This section is devoted to a statement of these general results and a detailed discussion of the case in which ϵH_1 belongs to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$. Since the points covered here are important to an understanding of the rest of this paper, we include them even though they have been completely treated elsewhere.[11]

If, as in Sec. II, we let $Q_\alpha(t)$ and $Q_\alpha^5(t)$ denote the vector and axial-vector charges, respectively, then it is well known that the chiral combinations

$$Q_\alpha^\pm(t)\equiv\tfrac{1}{2}[Q_\alpha(t)\pm Q_\alpha^5(t)] \qquad (3.1)$$

satisfy the commutation relations of two commuting $SU(3)$ subalgebras, namely,

$$[Q_\alpha^+(t),Q_\beta^-(t)]=0,$$
$$[Q_\alpha^\pm(t),Q_\beta^\pm(t)]\equiv if_{\alpha\beta\gamma}Q_\gamma^\pm(t). \qquad (3.2)$$

Moreover, if we denote the parity operator for the strong interactions by $\mathcal{P}$, then

$$\mathcal{P}Q^\pm\mathcal{P}^{-1}\equiv Q^\mp. \qquad (3.3)$$

[10] That it is m_α^2 rather than m_α which is of order ϵ follows from the fact that in the limit of exact symmetry $m_\alpha^2=0$. Thus in order to get the change in energy of such a zero-mass state, we note that $E_0+\delta E\equiv\langle\mathbf{p}|H_0+\epsilon H|\mathbf{p}\rangle\equiv(p^2+m_\alpha^2)^{1/2}\equiv(p^2)^{1/2}+m_\alpha^2/(p^2)^{1/2}+\cdots$; and since $E_0=(p^2)^{1/2}$ and δE is of order ϵ, it follows that m_α^2 is of order ϵ. Note that $\mathbf{p}$ cannot be taken to be zero because the eigenstates of H_0 are massless mesons.

[11] M. Gell-Mann, Physics 1, 63 (1964).

Adopting these conventions, and assuming that the symmetry-breaking term ϵH_1 is given by

$$\epsilon H_1(t) \equiv \int d^3x\, \mathcal{H}_1(\mathbf{x},t)\,, \tag{3.4}$$

one then has the following general result, in the case that $\epsilon\mathcal{H}_1(\mathbf{x},t)$ contains no derivatives:

$$\begin{aligned} \partial_\mu \mathfrak{F}_\alpha{}^\mu(\mathbf{x},t) &= i\epsilon[Q_\alpha(t),\mathcal{H}_1(\mathbf{x},t)]\,,\\ \partial_\mu \mathfrak{F}_\alpha{}^{5\mu}(\mathbf{x},t) &= i\epsilon[Q_\alpha{}^5(t),\mathcal{H}_1(\mathbf{x},t)]\,. \end{aligned} \tag{3.5}$$

The importance of this statement is that once one assumes that $\epsilon\mathcal{H}_1$ belongs to a particular representation of $SU(3)\otimes SU(3)$, then Eqs. (3.5) tells us that the divergences of the currents belong to the same representation. Much of what we shall do in the sections devoted to the derivation of sum rules which are correct to lowest order in ϵ will make use of this fact.[12]

Equations (3.5) immediately suggest many interesting possible approaches which one could follow in pursuing the question of calculating corrections to $SU(3)\otimes SU(3)$ symmetry. All of these can be classified by the group-theoretical way one chooses to describe the symmetry-breaking term ϵH_1. For example, one could choose to decompose ϵH_1 with respect to its transformation properties with respect to the chiral $SU(2)\otimes SU(2)$ subgroup of $SU(3)$. That is, one could assume that

$$\epsilon H_1 \equiv \epsilon' H' + \epsilon'' H''\,, \tag{3.6}$$

where $\epsilon' H'$ breaks $SU(3)$ but not $SU(2)\otimes SU(2)$, and $\epsilon'' H''$ is the piece of the Hamiltonian which breaks $SU(2)\otimes SU(2)$ and gives the pions a mass. Physically this would be an interesting procedure to follow if one wished to argue that $SU(2)\otimes SU(2)$ was a better symmetry than either $SU(3)$ or $SU(3)\otimes SU(3)$, and that the breaking of $SU(2)\otimes SU(2)$ has a different physical origin than the breaking of $SU(3)$ or $SU(3)\otimes SU(3)$. That is, one argues that $\epsilon'' \ll \epsilon'$ and the structures of H' and H'' are unrelated. If, however, one would like to argue that the breaking of $SU(3)$ is related to the mechanism which breaks $SU(2)\otimes SU(2)$, then this is not as interesting a thing to do. For example, as we shall see later, if one assumes that ϵH_1 belongs to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$, then there is a definite relationship between the amount of $SU(3)$ breaking and $SU(2)\otimes SU(2)$ breaking which is present in any specific model. In this case, it is more interesting to classify ϵH_1 as to its transformation properties under the usual $SU(3)$ subgroup of $SU(3)\otimes SU(3)$. Since it is the second case which leads us to the largest number of verifiable sum rules (for reasons which will become clear) we shall fix our attention upon this classification scheme.

[12] In fact, our actual results will only involve the fact that $\partial_\mu \mathfrak{F}_\alpha{}^\mu(q) \equiv \int d^4x\, e^{+iq\cdot x}\partial_\mu \mathfrak{F}_\alpha{}^\mu(x)$ belongs to this representation when $q=0$. This result is true under more general assumptions, but we shall not pursue this point further.

Let us now go on to see how the general relation given in Eq. (3.4) works in the particular case that ϵH_1 belongs to a Gell-Mann–Oakes–Renner type of model. Adopting the notation introduced by these authors we note that if we decompose this representation with respect to the usual $SU(3)$ subgroup generated by the vector charges, we get an even- and odd-parity singlet and octet. We shall denote a complete set of even-parity operators as U_0, U_i ($i=1,\cdots,8$) and corresponding odd-parity operators as V_0, V_i ($i=1,\cdots,8$). These operators then satisfy the commutation relations

$$\begin{aligned} &[Q_\alpha,U_\beta]\equiv if_{\alpha\beta\gamma}U_\gamma \quad\text{for}\quad \alpha,\beta,\gamma\equiv 1,\cdots,8\,,\\ &[Q_\alpha,V_\beta]\equiv if_{\alpha\beta\gamma}V_\gamma\,,\\ &[Q_\alpha{}^5,U_\beta]\equiv -id_{\alpha\beta\gamma}V_\gamma - i(\sqrt{\tfrac{2}{3}})\delta_{\alpha\beta}V_0\,,\\ &[Q_\alpha{}^5,V_\beta]\equiv -id_{\alpha\beta\gamma}U_\gamma - i(\sqrt{\tfrac{2}{3}})\delta_{\alpha\beta}U_0\,,\\ &[Q_\alpha,U_0]\equiv[Q_\alpha,V_0]\equiv 0\,,\\ &[Q_\alpha{}^5,U_0]\equiv -i(\sqrt{\tfrac{2}{3}})V_\alpha\,,\\ &[Q_\alpha{}^5,V_0]\equiv -i(\sqrt{\tfrac{2}{3}})U_\alpha\,. \end{aligned} \tag{3.7}$$

Moreover, remembering that ϵH_1 must conserve both isospin and hypercharge, we see that the most general form for $\epsilon\mathcal{H}_1(\mathbf{x},t)$ is

$$\epsilon\mathcal{H}_1(\mathbf{x},t) \equiv \epsilon[C_0U_0(\mathbf{x},t)+C_8U_8(\mathbf{x},t)]\,. \tag{3.8}$$

Equations (3.5) therefore imply that the divergences of the vector and axial-vector currents can be written as

$$\begin{aligned} \partial_\mu \mathfrak{F}_\alpha{}^\mu(\mathbf{x},t) &\equiv \epsilon C_8 \sum_{\gamma=1}^{8} f_{\alpha 8\gamma}U_\gamma(\mathbf{x},t)\,,\\ \partial_\mu \mathfrak{F}_\alpha{}^{5\mu}(\mathbf{x},t) &\equiv -\epsilon[(\sqrt{\tfrac{2}{3}})\delta_{\alpha 8}C_8V_0 + C_0(\sqrt{\tfrac{2}{3}})V_\alpha + \sum_{\gamma=1}^{8} C_8 d_{8\alpha\gamma}V_\gamma]\,. \end{aligned} \tag{3.9}$$

If one notes that the matrix $(d_8)_{\alpha\beta}$ is diagonal, it then follows that we can write the divergence of the axial-vector current as

$$\partial_\mu \mathfrak{F}_\alpha{}^{5\mu}(\mathbf{x},t) \equiv \epsilon[C_\alpha V_\alpha(\mathbf{x},t) + (\sqrt{\tfrac{2}{3}})\delta_{\alpha 8}C_8V_0(\mathbf{x},t)]\,. \tag{3.10}$$

Moreover, Eq. (2.6) implies that

$$\begin{aligned} \langle M_\alpha|\,\partial_\mu \mathfrak{F}_\beta{}^{5\mu}(0)\,|0\rangle &\equiv (m_\alpha{}^2/2f_\alpha)\delta_{\alpha\beta}\\ &\equiv \epsilon C_\alpha\langle M_\alpha|\,V_\beta(0)\,|0\rangle \equiv \epsilon C_\alpha\delta_{\alpha\beta}\|V\| \end{aligned} \tag{3.11}$$

(to lowest order in ϵ), where $\|V\|$ is a reduced matrix element independent of α and β. Thus we have

$$\epsilon C_\alpha \equiv m_\alpha{}^2/2f_\alpha\|V\|\,. \tag{3.12}$$

A similar analysis can be performed for the Σ term, which is formally equal to

$$\Sigma_{\beta\alpha}(\mathbf{x},t) \equiv -\epsilon[Q_\alpha{}^5(t),[Q_\beta{}^5(t),\mathcal{H}_1(\mathbf{x},t)]]\,. \tag{3.13}$$

In terms of the Gell-Mann–Oakes–Renner model for $\epsilon\mathcal{H}_1$, this gives

$$\begin{aligned} \Sigma_{\beta\alpha}(\mathbf{x},t) \equiv \epsilon\{&[\tfrac{2}{3}C_0 + \delta_{\alpha\beta} + (\sqrt{\tfrac{2}{3}})C_8d_{8\alpha\beta}]U_0\\ &+[C_0(\sqrt{\tfrac{2}{3}})d_{\alpha\beta\gamma} + C_8d_{8\beta\sigma}d_{\sigma\alpha\gamma} + \tfrac{2}{3}\delta_{\beta 8}\delta_{\alpha\gamma}]U_\gamma\}\,, \end{aligned} \tag{3.14}$$

where we have used the summation convention. As was shown in a previous paper by one of the authors, one can, if one works to lowest order in ϵ, relate the Σ term to the meson mass matrix, the specific relation being[13]

$$(m^2)_{\alpha\beta}=4f_0^2\langle 0|\Sigma_{\beta\alpha}(0)|0\rangle. \quad (3.15)$$

Using this result and the specific formula for $\Sigma_{\beta\alpha}$ given in Eq. (3.14), we get

$$(m^2)_{\alpha\beta}\equiv+4f_0^2\epsilon[\tfrac{2}{3}C_0\delta_{\alpha\beta}+(\sqrt{\tfrac{2}{3}})C_8d_{8\alpha\beta}]\langle 0|U_0|0\rangle. \quad (3.16)$$

The important thing to note at this point is that besides giving us a relationship between the Σ term and the mass matrix, Eq. (3.16) makes it clear that we can conserve $SU(2)\otimes SU(2)$ symmetry (leaving the pions massless) only if there is a definite relation between C_0 and C_8, namely,

$$C_0=-(1/\sqrt{2})C_8. \quad (3.17)$$

Moreover, the deviation from Eq. (3.17) being satisfied gives, in any Gell-Mann–Oakes–Renner model, a specific relation between the type of $SU(3)$ breaking and the amount of $SU(2)\otimes SU(2)$ violation which occurs. Certainly, since there is no reason why such a relationship must occur in the real world, the Gell-Mann–Oakes–Renner model, while convenient to work with, is far from general.

We shall discuss at a later point how the fact that $SU(3)\otimes SU(3)$ and $SU(3)$ can be broken without breaking $SU(2)\otimes SU(2)$ affects the interpretation of some of our sum rules. The model defined by Eq. (3.17) will be very useful in our discussion of this point.

IV. SUM RULES FROM GENERALIZED GOLDBERGER-TREIMAN RELATIONS

In this section we discuss in some detail the derivation of identities relating the meson-baryon coupling constants to the constants which describe the coupling of the axial-vector current to the same baryon states. First we present the derivation of the relevant identity in the case H_1 that belongs to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$; we then discuss how one might experimentally check this result.

We shall explicitly carry out the derivation of this identity in order to introduce the necessary notation. The theorem follows from the basic equation

$$\int d^4x\, e^{+iq\cdot x}\langle B'(p')|\partial_\mu\mathfrak{F}_\alpha{}^{5\mu}(x)|B(p)\rangle = iq_\mu\int d^4x\, e^{+iq\cdot x}\langle B'(p')|\mathfrak{F}_\alpha{}^{5\mu}(x)|B(p)\rangle, \quad (4.1)$$

where $|B'(p')\rangle$ and $|B(p)\rangle$ denote arbitrary baryon states of momentum p and $p'=p+q$. We can now write the matrix element for the axial-vector current between baryons as

$$\langle B'(p')|\mathfrak{F}_\alpha{}^{5\mu}(0)|B(p)\rangle\equiv\bar{u}_{B'}(p')\{\tfrac{1}{2}[\gamma^\mu\gamma_5 g_{B'B\alpha}(q^2)+q^\mu\gamma_5 h_{B'B\alpha}(q^2)]\}u_B(p), \quad (4.2)$$

and taking the divergence of Eq. (4.2) gives

$$\langle B'(p')|\partial_\mu\mathfrak{F}_\alpha{}^{5\mu}(0)|B(p)\rangle \equiv i\bar{u}_{B'}(p')\{\gamma_5\tfrac{1}{2}\partial_{B'B\alpha}(q^2)\}u_B(p), \quad (4.3)$$

where

$$\partial_{B'B\alpha}(q^2)\equiv(m_{B'}+m_B)g_{B'B\alpha}(q^2)+q^2h_{B'B\alpha}(q^2). \quad (4.4)$$

If we write $\partial_{B'B\alpha}(q^2)$ as a meson pole term plus a remainder,

$$\partial_{B'B\alpha}(q^2)\equiv G_{B'B\alpha}m_\alpha^2/f_\alpha(m_\alpha^2-q^2)-\delta_{B'B\alpha}(q^2), \quad (4.5)$$

where $G_{B'B\alpha}$ is the relevant meson-baryon coupling constant [in Eq. (4.5) $\delta_{B'B\alpha}$ is defined to be $\partial_{B'B\alpha}$ minus the meson pole term (it is important to remember that it is $\delta_{B'B\alpha}(q^2)$ which is of order ϵ and not $\partial_{B'B\alpha}(q^2)$, which is of order $G_{B'B\alpha}f_\alpha^{-1}$)] and one now observes that the form factor $h_{B'B\alpha}(q^2)$ also has a one-meson pole term in it, we can rewrite Eq. (4.4) and Eq. (4.5) at $q^2=0$ as

$$G_{B'B\alpha}/f_\alpha=(m_{B'}+m_B)g_{B'B\alpha}+\delta_{B'B\alpha}(0). \quad (4.6)$$

At this point we refer to the observation made in Sec. III that if H_1 belongs to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$, then the divergence of the axial-vector current transforms as an $SU(3)$ singlet plus an $SU(3)$ octet under the algebra of vector charges [Eqs. (3.7)–(3.9)]. This means that matrix elements of the divergence of the axial-vector current between meson states are, to first order in ϵ, given by two independent reduced matrix elements, since once the one meson-pole term is removed, the divergence of the axial-vector current is explicitly of order ϵ. That is, the term $\delta_{B'B\alpha}(0)$ in Eq. (4.6) can be replaced by the general expression

$$G_{B'B\alpha}/f_\alpha=(m_{B'}+m_B)g_{B'B\alpha}+b[(1-a)f_{B'B\alpha}+iad_{B'B\alpha}](m_\alpha^2/2f_\alpha), \quad (4.7)$$

where $f_{B'B\alpha}$ and $d_{B'B\alpha}$ are the usual "f" and "d" symbols for $SU(3)$.

Equation (4.7) is the basic result which we wished to obtain in this section. The rest of the section is devoted to a discussion of how one might test this result experimentally and what special points must be considered in its application.

For the purposes of our discussion let us limit our considerations to experimental information which one might expect to have available in the near future. This limits us to $g_{N'N\pi}$, $g_{\Sigma NK}$, and $g_{\Lambda NK}$, all of which one might hope to determine correctly to better than twenty per cent (from the baryon leptonic decays) within the next few years. One might also hope that

[13] It is worth noting that the Σ term is not symmetric in α and β. Rather, it is simple to show using the Jacobi identity that in general $\Sigma_{\beta\alpha}(\mathbf{x},t)-\Sigma_{\alpha\beta}(\mathbf{x},t)\equiv i\epsilon f_{\alpha\beta\gamma}[Q_\gamma(t),H_1(\mathbf{x},t)]$. However, the vacuum-expectation value of the antisymmetric part vanishes, since $Q_\alpha(t)$ annihilates the vacuum.

better data on kaon-nucleon scattering will allow us to determine (at least to 20%) the meson-baryon coupling constants $G_{N\Sigma K}$, and $G_{N\Lambda K}$ (of course, $G_{NN\pi}$ is already known to this accuracy).

Knowing these three pieces of information allows us to check the sum rule obtained by eliminating b and a from the equations

$$\left(\frac{G_{np\pi}}{f_\pi}-(m_n+m_p)g_{np\pi}\right)=\frac{bm_\pi^2}{2f_\pi}, \quad (4.8)$$

$$\left(\frac{G_{\Sigma nK}}{f_K}-(m_\Sigma+m_n)g_{\Sigma nK}\right)=\frac{b(2a-1)m_K^2}{2f_K}, \quad (4.9)$$

$$\left(\frac{G_{\Lambda pK}}{f_K}-(m_\Lambda+m_p)g_{\Lambda pK}\right)=\frac{b(2a-3)m_K^2}{(\sqrt{6})2f_K}. \quad (4.10)$$

At this juncture, there are two important points to be made. The first is that if $SU(2)\otimes SU(2)$ remains an exact symmetry, then the three pieces of experimental information which we have listed do not give a sum rule. This is clear from an inspection of Eq. (4.8), since in this case, $m_\pi^2=0$ and the Goldberger-Treiman relation is exact; thus, we have only two nontrivial equations by which to determine the unknowns a and b. Such a problem does not arise if we think that the real world conforms to the type we have been discussing, namely, one in which H_1 belongs to a $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$, since the Goldberger-Treiman relation is known to be true only to 15%. However, as we have already said, one could imagine that the breaking of $SU(3)\otimes SU(3)$ and $SU(3)$ is entirely unrelated to the breaking of $SU(2)\otimes SU(2)$; that is, the ϵH_1 is of the form given by Eqs. (3.5). In such a case the deviations from the generalized Goldberger-Treiman relations for the strangeness-nonchanging currents are of order ϵ'' and clearly one cannot hope to treat the results for the strangeness-changing currents on the same footing. This means that if one wants to derive verifiable sum rules for such a world, one should concentrate only on Goldberger-Treiman relations obtained for interactions in which strangeness is changed. This means that in order to get a sum rule one would have to include Ξ^- decay which requires knowledge of the hard-to-determine coupling constant $G_{K\Lambda\Xi}$. However, when neutrino-production experiments are available so that one can determine axial-vector-current matrix elements between stable and unstable particles, then a whole new set of sum rules are possible.

A second point which is of interest is that these generalized Goldberger-Treiman relations can be quite badly violated for matrix elements of the current between opposite-parity states such as a nucleon and Y_0^*. Since the discussion of this point comes up most naturally after discussion of the theorems that can be proved for meson-baryon scattering, we shall defer it to the end of Sec. V.

Before concluding this section, one remark worth making is that this technique is obviously easily extended so as to enable one to make predictions about any process involving one soft meson. For example, one could use this to get some predictions pertaining to photoproduction amplitudes. The next section is devoted to a discussion of things to be learned from processes involving two soft mesons.

V. MESON-BARYON SCATTERING

This section is devoted to a discussion of an interesting theorem that can be derived for meson-baryon scattering amplitudes. The theorem provides a way of determining the value of the Σ term at zero momentum transfer by means of extrapolating experimentally determined scattering amplitudes to an unphysical energy using fixed-t on-the-mass-shell dispersion relations. Since the structure of the Σ term is determined by the group-theoretic properties assumed for ϵH_1, this prediction, like those for the generalized Goldberger-Treiman relations, provides us with a fair number of verifiable sum rules.

Since the actual derivation of this theorem requires the use of some notation which is developed in Paper II, we have included it in the Appendix. At this point we will only state the simplest and most useful version of the theorem; in order to do so, however, we must first define what we mean by "spin-averaged amplitudes."

We turn our attention to processes of the form $B+M_\alpha\rightarrow B'+M_\beta$, where B and B' denote two arbitrary members of the baryon octet with four-momenta p and p', respectively, and M_α and M_β denote two arbitrary members of the pseudoscalar meson octet with four-momenta q and k. We can then write the amplitude for such a process in its most general form as

$$\langle B'(p'),M_\beta(k)|T|B(p),M_\alpha(q)\rangle \equiv\bar{u}_{B'}(p')[A_{\beta\alpha}(\nu,x)+(\boldsymbol{k}+\boldsymbol{q})D_{\beta\alpha}(\nu,x)]u_B(p), \quad (5.1)$$

where

$$\nu\equiv\tfrac{1}{2}(p'+p)\cdot(q+k), \quad x\equiv q\cdot k. \quad (5.2)$$

Using this, we define the "spin-averaged amplitude" to be

$$\mathfrak{M}_{\alpha\beta}(\nu,x)\equiv\tfrac{1}{4}\,\mathrm{Tr}\left[\left(\frac{\boldsymbol{p}'+m_{B'}}{2m_{B'}}\right)(A_{\beta\alpha}(\nu,x) +(\boldsymbol{k}+\boldsymbol{q})D_{\beta\alpha}(\nu,x))\left(\frac{\boldsymbol{p}+m_B}{2m_B}\right)\right]. \quad (5.3)$$

Similarly, the Σ term can be most generally written as

$$\langle B'(p')|\Sigma_{\alpha\beta}(0)|B(p)\rangle\equiv\bar{u}_{B'}(p')u_B(p)\chi_{\alpha\beta}(t), \quad (5.4)$$

where $t=(p'-p)^2$.

We then define a "spin-averaged Σ term" by the equation

$$\sigma_{\alpha\beta}(t)=\frac{1}{4}\,\mathrm{Tr}\left[\left(\frac{p'+m_{B'}}{2m_{B'}}\right)\left(\frac{p+m_B}{2m_B}\right)\right]\chi_{\alpha\beta}(t)\,. \quad (5.5)$$

With these kinematic preliminaries out of the way, our result can be stated as follows:

Theorem: The spin-averaged amplitude for the process $B+M_\alpha\to B'+M_\beta$ at the point $\nu=0$ and $x=0$, is given, neglecting terms of order ϵ^2, by the formula

$$[1/(2f_\alpha)(2f_\beta)]\mathfrak{M}_{\alpha\beta}(0,0)=(\text{poles})_{\alpha\beta}-\tfrac{1}{2}i[\sigma_{\alpha\beta}(0)+\sigma_{\beta\alpha}(0)]+O(\epsilon^2)\,, \quad (5.6)$$

where the term $(\text{poles})_{\alpha\beta}$ is given (also to order ϵ^2) by

$$-(g_A)^2[(m_{B'}+m_B)(m_{B'}^2-m_B^2)^2/16m_Bm_{B'}]\times\left(\sum_{B''}\frac{[cd_{B''B\alpha}+i(1-c)f_{B''B\alpha}][cd_{B'B''\beta}+i(1-c)f_{B'B''\beta}]}{\frac{1}{2}(m_{B'}^2+m_B^2)-m_{B''}^2}+(\beta\leftrightarrow\alpha)\right), \quad (5.7)$$

where $g_A\approx 1.2$ is the usual axial-vector coupling constant and $(1-c)/c$ gives the f to d ratio for the process in question which we can determine by determining the best value of c such that

$$G_{B'B\alpha}\cong G_{NN\pi}[cd_{B'B\alpha}+i(1-c)f_{B'B\alpha}].$$

Clearly, $[\sigma_{\alpha\beta}(0)+\sigma_{\beta\alpha}(0)]$ can be readily evaluated once one chooses a specific model for $\epsilon\mathcal{H}_1$. If, for example one chooses $\epsilon\mathcal{H}_1$ to belong to a Gell-Mann–Oakes–Renner model, $\sigma_{\beta\alpha}$ can be easily determined by applying Eq. (3.13). Before concluding this section, there is an important point to be made about difficulties which might arise in the applications of the theorems just stated.

As we pointed out in the Introduction and also in the beginning of this section, the results that we derive depend very strongly upon the use of an expansion to lowest order in ϵ. Clearly, if this is generally invalid, then the theorems are untrue; however, it is possible for the theorems to fail to apply even if, in general, an expansion in powers of ϵ is meaningful. The way this can occur is if the point $\nu=0$ and $q\cdot k=0$ lies close to a resonance, since overly simple perturbation theory in the neighborhood of such a point cannot be expected to apply (due to the presence of small energy denominators). More precisely, resonances can be expected to be a problem whenever either of the following conditions hold:

$$\tfrac{1}{2}(m_{B'}^2+m_B^2)\approx m^2(\text{baryon-resonance}) \quad (5.8)$$

or

$$m_\alpha^2+m_\beta^2\approx m^2(\text{meson-resonance})\,. \quad (5.9)$$

For example, the Y_0^* can possibly make important contributions to the ϵ^2 terms in processes like $\bar{K}N\to\pi\Sigma$ or $\bar{K}N\to K\Xi$. We show in the Appendix, that if the generalized Goldberger-Treiman relations for $G_{\bar{K}NY_0^*}$, $G_{\bar{K}N\Xi}$, and $G_{\pi\Sigma Y_0^*}$ have fractional deviations (a term which we shall define precisely) that are small in comparison to 1, then one can safely ignore the effects of the Y_0^*.

It is interesting to note, however, that these fractional deviations might be large even though ϵ is small. We shall therefore conclude this section with a discussion of this point.

Let us recall the form of the Goldberger-Treiman relation for pion-nucleon scattering that reads

$$G_{\pi NN}=f_\pi(2m_N)(g_A)_{\pi NN}+\delta_{\pi NN}\,, \quad (5.10)$$

where the coupling constants and the correction term $\delta_{\pi NN}$ are defined in Sec. III. Most importantly, the term $\delta_{\pi NN}$ is defined to be

$$\langle N'|\,\partial_\mu\mathfrak{F}_\pi^{5\mu}(q)\,|N\rangle\equiv\bar{u}_N(p')\gamma^5u_N(p)\delta_{\pi NN}f_\pi^{-1}+\text{pion pole} \quad (5.11)$$

and as such has the scale of a typical matrix element of the divergence of the axial-vector current with its pion-pole removed. Experimentally, $\delta_{\pi NN}$ is known to be on the order of 1 and $\delta_{\pi NN}G_{\pi NN}^{-1}\approx 0.1$.

Now let us consider similar Goldberger-Treiman relations for such coupling constants as $G_{KNY_0^*}$ or $G_{\pi\Sigma Y_0^*}$ which, because the Y_0^* is a scalar resonance, read

$$G_{KNY_0^*}=f_K(m_{Y_0^*}-m_N)g_{\bar{K}NY_0^*}+\delta_{\bar{K}NY_0^*}\,, \quad (5.12)$$

and

$$G_{\pi\Sigma Y_0^*}=f_\pi(m_{Y_0^*}-m_\Sigma)g_{\pi\Sigma Y_0^*}+\delta_{\pi\Sigma Y_0^*}\,, \quad (5.13)$$

where $\delta_{\bar{K}NY_0^*}$ and $\delta_{\pi\Sigma Y_0^*}$ are defined in analogy to Eq. (5.11) as

$$\langle Y_0^*|\,\partial_\mu\mathfrak{F}_K^{5\mu}\,|N\rangle=\bar{u}_{Y_0^*}(p')u_N(p)\delta_{\bar{K}NY_0^*}f_K^{-1}+\text{kaon pole}\,,$$
$$\langle Y_0^*|\,\partial_\mu\mathfrak{F}_\pi^{5\mu}\,|\Sigma\rangle=\bar{u}_{Y_0^*}(p')u_\Sigma(p)\delta_{\pi\Sigma Y_0^*}f_K^{-1}+\text{pion pole}. \quad (5.14)$$

From Eq. (5.11) we see that there is no *a priori* reason why $\delta_{\bar{K}NY_0^*}$ and $\delta_{\pi\Sigma Y_0^*}$ should not be of the same order as $\delta_{\pi NN}$, that is, of order 1. This has important consequences since it is experimentally known that $G_{KNY_0^*}$ and $G_{\pi\Sigma Y_0^*}$ are also of order 1, which means that the fractional deviations defined by

$$\Delta_{\bar{K}NY_0^*}=\delta_{\bar{K}NY_0^*}G_{\bar{K}NY_0^*}^{-1}\,,$$
$$\Delta_{\pi\Sigma Y_0^*}=\delta_{\pi\Sigma Y_0^*}G_{\pi\Sigma Y_0^*}^{-1} \quad (5.15)$$

would be of order 1 as opposed to $\Delta_{\pi NN}\approx 0.1$. Clearly, one reason for this could be that $G_{\bar{K}NY_0^*}$ and $G_{\pi\Sigma Y_0^*}$ are suppressed in magnitude because of the fact that in Eqs. (5.12) and (5.13) the factors $g_{\bar{K}NY_0^*}$ and $g_{\pi\Sigma Y_0^*}$ are multiplied by baryon mass differences. Thus, small values for $G_{\bar{K}NY_0^*}$ and $G_{\pi\Sigma Y_0^*}$ are consistent with values of g of order 1, which is not the case for example in Eq. (5.10).

Although no conclusive evidence exists that would enable us to make a decision as to whether or not $\Delta_{\bar{K}NY_0^*}$ and $\Delta_{\pi\Sigma Y_0^*}\ll 1$, it is interesting to note that taking the ratios of the expressions for $G_{\bar{K}NY_0^*}$ and $G_{\pi\Sigma Y_0^*}$ and assuming that the fractional deviations are negligible and that $g_{\bar{K}NY_0^*}\approx g_{\pi\Sigma Y_0^*}$, as predicted by $SU(3)$, we get

$$\frac{G_{\bar{K}NY_0^*}}{G_{\pi\Sigma Y_0^*}}\approx\frac{f_K(m_{Y_0^*}-m_N)}{f_\pi(m_{Y_0^*}-m_\Sigma)}\approx 2. \tag{5.16}$$

The experimental situation seems to be consistent with Δ's on the order of $0.2\sim 0.3$.[14]

Regardless of whether or not the fractional deviations for the Goldberger-Treiman relations in these specific cases are small, the fact that they may not be small is of real significance. We strongly suggest that those readers who might be interested in actually using the theorems quoted in this section read the discussion in the Appendix, where we show precisely how the size of $\Delta_{KNY_0^*}$, etc., affects the applicability of these results.

VI. MESON-MESON SCATTERING AMPLITUDES

Since we feel it is highly unlikely that anyone will measure general meson-meson scattering lengths in the near future, we shall not discuss the theorems which can be proven in this case in any great detail. Rather, we merely note that in Paper II we show that if one defines on an off-mass-shell continuation of the meson-meson scattering amplitude by

$$\begin{aligned}(2\pi)^4\delta^4(q_1+q_2+q_3+q_4)i\mathcal{T}_{\alpha\beta\gamma\delta}(q_1,q_2,q_3,q_4)\\ =(q_1^2-m_\alpha^2)(q_2^2-m_\beta^2)(q_3^2-m_\gamma^2)(q_4^2-m_\delta^2)\\ \times(2f_\alpha)(2f_\beta)(2f_\delta)(2f_\gamma)\left\langle 0\left|T\left(\frac{\partial_\mu\mathfrak{F}_\alpha^{5\mu}(q_1)}{m_\alpha^2}\right.\right.\right.\\ \left.\left.\left.\times\frac{\partial_\nu\mathfrak{F}_\beta^{5\nu}(q_2)}{m_\beta^2}\frac{\partial_\rho\mathfrak{F}_\gamma^{5\rho}(q_3)}{m_\gamma^2}\frac{\partial_\sigma\mathfrak{F}_\delta^{5\sigma}(q_4)}{m_\delta^2}\right)\right|0\right\rangle,\end{aligned} \tag{6.1}$$

then this continuation has the property that as we let ϵ go to zero (so that m_α^2, m_β^2, m_γ^2 and m_δ^2 go to zero), this amplitude goes to the scattering amplitude in the symmetric theory. Furthermore, we showed that if we go to the point

$$\nu=\tfrac{1}{2}(q_1+q_4)\cdot(q_2+q_3)=0 \quad\text{and}\quad x=q_2\cdot q_3=0,$$

the amplitude goes to a constant $(\epsilon A_1)_{\alpha\beta\gamma\delta}$ which can be evaluated from the formula

$$\begin{aligned}(\epsilon A_1)_{\alpha\beta\gamma\delta}\equiv\{&\langle 0|[Q_\delta^5,[Q_\gamma^5,[Q_\alpha^5,[Q_\beta^5\epsilon\mathcal{H}_1]]]]|0\rangle\\ &+\langle 0|[Q_\delta^5,[Q_\beta^5,[Q_\alpha^5,[Q_\gamma^5,\epsilon\mathcal{H}_1]]]]|0\rangle\\ &+\langle 0|[Q_\gamma^5,[Q_\beta^5,[Q_\alpha^5,[Q_\delta^5,\epsilon\mathcal{H}_1]]]]|0\rangle\},\end{aligned} \tag{6.2}$$

where $\epsilon\mathcal{H}_1$ stands for the Hamiltonian density at $\mathbf{x}=0$, $t=0$ and all commutators are equal-time commutators.

Clearly, this is a very general expression and can only be evaluated if one picks a particular model for $\epsilon\mathcal{H}_1$. For example, if $\epsilon\mathcal{H}_1$ is chosen to belong to a $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$, then Eq. (6.2) is easily evaluated using the formulas given in Sec. III.

VII. DISCUSSION

Having now presented a detailed statement of some of the results obtainable by the methods developed in Paper II, we use this section to review what we have discussed, raise what we feel to be important theoretical questions, suggest possible avenues for meaningful extension of this work, and emphasize what we think to be the most interesting way in which to view these results.

The first point is that to the best of our knowledge these results, together with the results presented in our letter on K_{l3} decays,[7] are quite different from those previously obtained. By this we mean that we have obtained them by exploiting a single idea, that of expanding in powers of a parameter ϵ, and have not invoked non-group-theoretical dynamical concepts such as supposing that particular particles dominate various dispersion relations. This seems to be a definite improvement in the state of the art, in that it greatly reduces the number of unmotivated technical assumptions used in obtaining a particular result. Moreover, the techniques which we have developed here and in Paper II suggest many possible ways in which one can hope to obtain additional interesting relations. The single most important idea to keep in mind when searching for such relations is that in a Goldstone-boson type of symmetry, which is the kind we are talking about in $SU(3)\otimes SU(3)$, it is the deviations from soft-meson theorems which are to be calculated. Once one has fixed this point fimrly in one's mind, many possibilities suggest themselves.

For example, consider a Gell-Mann–Oakes–Renner model for $SU(3)\otimes SU(3)$ and its breaking. It is then easy to show, using only $SU(3)$, that there are six unknown parameters which specify the first-order corrections to $G_{B'BM}$ and $g_{B'BM}$, that is, 12 parameters in all. Two of these are related to the $SU(3)$ singlet part of the symmetry-breaking Hamiltonian, and the other 10 to the octet piece. If we use only our knowledge of the $SU(3)$ structure of the theory, this does not relate the 10 parameters coming from the octet part of symmetry breaking in any way. However, if one uses

[14] This fact was observed earlier by Gell-Mann, Oakes, and Renner, Ref. 8.

the sum rules derived from generalized Goldberger-Treiman relations treated in Sec. IV, one finds that the first-order corrections to combinations of the form

$$G_{B'BM}/f_M-(m_{B'}+m_B)g_{B'BM}$$

are defined in terms of only two unknown parameters. This clearly means that up to the specification of these two parameters, the first-order corrections to $g_{B'BM}$ are given once those for $G_{B'BM}$ are specified. In other words, one has a reduced number of parameters specifying the corrections to $SU(3)$ symmetry at the soft-meson point. Clearly, one obtains a similar reduction in the large number of unknown parameters needed to specify the general first-order corrections to $SU(3)$ symmetry for meson-baryon scattering if one uses the idea of an approximate $SU(3)\otimes SU(3)$ symmetry and works at the soft-meson point.

Additional examples of interesting results that can be obtained by these methods might be provided by an investigation of photoproduction or electroproduction amplitudes at the soft-meson point, since first-order $SU(3)$-symmetry breaking at this point should be specifiable by a smaller number of independent parameters than is generally thought from arguments based upon $SU(3)$ alone.

There is one further avenue of possible future investigation which we feel is interesting enough to mention. Thus, we conclude our discussion of suggestions for future work by noting that there seems to be room for extending the results we obtained for K_{l3} decay to other weak and electromagnetic processes. In particular, there seems to be room for significantly improving the treatments given to date of processes like $K\to 3\pi$, $\eta\to 3\pi$, and K_{l4} decays.

We turn our attention to some more general questions associated with the question of the relevance of calculations to order ϵ. In particular, we list what we see as the possible alternatives to be decided among by experiment. These alternatives are threefold and can be summarized as follows.

Case I: It could be that all of the results derivable by these methods or at least a preponderant number of them, work to order 10–20%. In that case, we are faced with the difficulties of explaining the electromagnetic masses of the pseudoscalar mesons as pointed out in Paper I. Of course, if this turns out to be the only anomaly, then one could have recourse to accident and say that purely by chance a lowest-order calculation fails to work for this special case. Another possibility is that this mass difference is not purely electromagnetic in origin.

Case II: There is nothing much to be said about Case II, since by this we mean the case in which no calculation of order ϵ is reliable. Assuming that the symmetrical predictions continue to be in fair agreement with experiment, then one would conclude that the idea of an approximate symmetry is correct, but that calculating to lowest order in symmetry breaking is not useful. See Paper I for a discussion of this point.

Case III: This case could be thought of as a special subcase of the first. Namely, it is one in which the predictions involving purely strong-interaction processes work, but those involving weak or electromagnetic processes fail. Since there is trouble with the electromagnetic mass differences of mesons, and since the possibility exists that there might be difficulty with the prediction for the ξ parameter in K_{l3} decay, this might be the case in the real world. Such a situation suggests an interesting possibility, that is, that the weak and electromagnetic currents are not exactly the currents of $SU(3)\otimes SU(3)$, even in the symmetrical limit. For example, $J_{\rm em}$ could be given by

$$J_{\rm em}{}^{\mu}\equiv\mathfrak{F}_3{}^{\mu}+\sqrt{3}\mathfrak{F}_8{}^{\mu}+K^{\mu}\,, \qquad (7.1)$$

where the charge associated with K^{μ} is zero. In such a theory there is no obvious way to make predictions about processes involving the weak and/or electromagnetic interactions.

Finally, we discuss several theoretical points which are related to the whole question of the structure of the symmetric theory. The first point is that recently a number of authors[15] have discussed anomalous terms arising when one pulls derivatives through a time-ordered product of current operators. Since our discussion of the symmetric theories involve such manipulations, there is the possibility that there exists no symmetrical world consistent with an $SU(3)\otimes SU(3)$ symmetry of the type which we have discussed. More pertinent, however, than the question of whether or not formal Ward identities hold, is the question of whether or not there exists a world in which all of the usual soft-meson theorems hold in the form in which they were derived in Paper II. Clearly, if no such symmetrical world can exist, it is extremely difficult to understand why one sees any traces of an approximate symmetry of the type which we have been discussing, unless for some reason the anomalous terms are always small.

It is an extreme but interesting possibility that something like this does happen. In this case, one might not need an explicit ϵH_1 to break the symmetry, since the anomalous terms account for the fact that the symmetrical predictions are only in approximate agreement with the world.

A less extreme possibility is that symmetry breaking is responsible for the introduction of anomalous terms. For example, adding electromagnetism might give rise to an anomalous commutator which would invalidate the result of Paper I on meson electromagnetic mass

[15] S. Adler, Phys. Rev. **177**, 2426 (1969); S. Adler and W. A. Bardeen, *ibid.* (to be published); S. Adler and W. K. Tung, Phys. Rev. Letters **22**, 978 (1969); S. Adler and D. Boulware, Phys. Rev. **184**, 1740 (1969); R. Jackiw and G. Preparata, Phys. Rev. Letters **22**, 975 (1969).

differences. In this connection, it should be noted that the anomalous terms found by Adler[15] do not effect electromagnetic mass differences in order α but only in order α^2.

Obviously, the question of whether or not one must consider any or all of these alternatives to the simplest notion of an approximate $SU(3)\otimes SU(3)$ symmetry can only be resolved by the accumulation of experimental data which would allow us to test a large number of results of the type which we have discussed in the preceding sections.

ACKNOWLEDGMENT

We wish to thank Professor W. A. Bardeen for many helpful discussions.

APPENDIX

This appendix is devoted to outlining the proof of the theorem on meson-baryon scattering amplitudes stated in Sec. V. The notation and formulas used in this proof are studied for the first time in Paper II of this series and will be only briefly reviewed here.

Our starting point is the basic identity proven in Paper II,

$$\begin{aligned}
[1/(2f_\alpha)(2f_\beta)]\langle B'+M_\beta(q)|S|B+M_\alpha(k)\rangle
&= q_\mu k_\nu\langle B'|T(\mathfrak{F}_\beta{}^{5\mu}(q)\mathfrak{F}_\alpha{}^{5\nu}(-k))|B\rangle^\star \\
&+(q_\mu/2f_\alpha)\langle B'|\mathfrak{F}_\beta{}^{5\mu}(q)|B+M_\alpha(k)\rangle^\star \\
&-(k_\nu/2f_\beta)\langle B'+M_\beta(q)|\mathfrak{F}_\alpha{}^{5\nu}(-k)|B\rangle^\star \\
&-\tfrac{1}{2}(q_\mu+k_\mu)f_{\alpha\beta\rho}\langle B'|\mathfrak{F}_\rho{}^\mu(q-k)|B\rangle \\
&+i\langle B'|\Sigma_{\alpha\beta}(q-k)|B\rangle \\
&+\langle B'|T(\partial_\beta(q)\partial_\alpha(-k))|B\rangle^\star, \qquad \text{(A1)}
\end{aligned}$$

where we have defined

$$\begin{aligned}
\langle B'|\mathfrak{F}_\alpha{}^{5\mu}(q)|B\rangle &\equiv \int d^4x\, e^{+iq\cdot x}\langle B'|\mathfrak{F}_\alpha{}^{5\mu}(x)|B\rangle, \\
\langle B'|\partial_\alpha(q)|B\rangle &\equiv \int d^4x\, e^{+iq\cdot x}\langle B'|\partial_\mu\mathfrak{F}_\alpha{}^{5\mu}(x)|B\rangle, \qquad \text{(A2)} \\
+i\langle B'|\Sigma_{\beta\alpha}(q-k)|B\rangle &\equiv \int d^4x d^4y\, e^{+iq\cdot x}e^{-ik\cdot y} \\
&\times\delta(x_0-y_0)\langle B'|[\partial_\mu\mathfrak{F}_\alpha{}^{5\mu}(y),\mathfrak{F}_\beta{}^{50}(x)]|B\rangle,
\end{aligned}$$

and where by the star notation

$$[\text{e.g., } \langle B'|T(\mathfrak{F}_\beta{}^\mu(q)\mathfrak{F}_\alpha{}^\nu(k))|B\rangle^\star, \text{ etc.}]$$

we mean that one should write out the most general expression for the time-ordered product of operator matrix elements in question in terms of invariant amplitudes [written as functions of the variables q^2, k^2 $x\equiv q\cdot k$, and $\nu=\frac{1}{2}(p'+p)(q+k)$] and remove from the relevant invariant amplitudes the poles which they have at $q^2=m_\beta{}^2$ and $k^2=m_\alpha{}^2$. In particular, if one modifies Eq. (A1) so that all matrix elements are replaced by their spin-averaged amplitudes as defined in Sec. V, there are several invariant amplitudes to study. More explicitly, we have

$$\begin{aligned}
&q_\mu k_\nu\langle B'|T(\mathfrak{F}_\beta{}^{5\mu}(q)\mathfrak{F}_\alpha{}^{5\nu}(-k))|B\rangle^\star{}_{\text{spin av}}(q^2,k^2,\nu,x) \\
&\equiv q_\mu k_\nu[P^\mu P^\nu F_1(q^2,k^2,\nu,x)+P^\mu q^\nu F_2(q^2,k^2,\nu,x) \\
&+P^\nu q^\mu F_3(q^2,k^2,\nu,x)+P^\mu k^\nu F_4(q^2,k^2,\nu,x) \\
&+P^\nu k^\mu F_5(q^2,k^2,\nu,x)+q^\mu k^\nu F_6(q^2,k^2,\nu,x) \\
&+q^\nu k^\mu F_7(q^2,k^2,\nu,x)+q^\mu q^\nu F_8(q^2,k^2,\nu,x) \\
&+k^\mu k^\nu F_9(q^2,k^2,\nu,x)+g^{\mu\nu}F_{10}(q^2,k^2,k^2,\nu,x)] \qquad \text{(A3)}
\end{aligned}$$

and

$$\begin{aligned}
&q_\mu\langle B'|\mathfrak{F}_\beta{}^{5\mu}(q)|B+m_\alpha(k)\rangle^\star{}_{\text{spin av}}(q^2;\nu,x) \\
&\equiv q_\mu[P^\mu G_1(q^2;\nu,x)+k^\mu G_2(q^2;\nu,x) \\
&\qquad +q^\mu G_3(q^2;\nu,x)], \qquad \text{(A4)} \\
&k_\nu\langle B'+M_\beta(q)|\mathfrak{F}_\alpha{}^{5\nu}(-k)|B\rangle^\star{}_{\text{spin av}}(k^2;\nu,x) \\
&\equiv k_\nu[P^\nu H_1(k^2;\nu,x)+q^\nu H_2(k^2;\nu,x) \\
&\qquad +k^\nu H_3(k^2;\nu,x)],
\end{aligned}$$

where $P^\mu\equiv\frac{1}{2}(p'^\mu+p^\mu)$ and where there are obvious relations imposed by crossing symmetry between the functions G and H. Of course, as noted before, the various invariant amplitudes in this expression are defined to be regular at the points $q^2\equiv m_\beta{}^2$ and $k^2\equiv m_\alpha{}^2$.

Examination of Eq. (A3), using the identities

$$\begin{aligned}
(q+k)\cdot P &\equiv \nu, \\
(q-k)\cdot P &\equiv \tfrac{1}{2}(m_{B'}{}^2-m_{B'}{}^2)\approx O(\epsilon), \qquad \text{(A5)}
\end{aligned}$$

shows us that only the coefficient of the $P^\mu P^\nu$ term gives a nonvanishing contribution to the amplitude at the point $q^2=k^2=\nu=x=0$. Furthermore, this contribution is explicitly of order ϵ^2, and thus *appears* to be unimportant when calculating terms of order ϵ. Similarly, if one examines Eqs. (A4), we see that only the coefficients of P^μ give rise to a nonvanishing contribution at the point in question. While this term is explicitly of order ϵ, one can argue that it is in fact of order ϵ^2. The argument goes as follows: If one considers the functions $G_1(0;\nu,x)$ and $H_1(0;\nu,x)$ expanded in powers of ϵ, we see immediately that only the form factors which are zeroth order in ϵ contribute terms of order ϵ to the scattering amplitudes. However, if one looks at Paper II one sees that in the symmetric theory G_1 and H_1 must, by crossing symmetry, be odd functions of ν and thus vanish at $\nu=0$. Therefore, $G_1(0,0,0)$ and $H_1(0,0,0)$ are themselves of order ϵ. Combining these arguments, we are *apparently* led to the conclusion that the terms involving $\mathfrak{F}^5$'s do not contribute to the order-ϵ part of the meson-baryon scattering amplitude. We emphasize the word apparently, however, since the explicit power of ϵ is not really enough. One must exercise some care because of the fact that there can be terms in the invariant amplitude which have denominators which are themselves of order ϵ and one must be careful to keep such terms. In Sec. V we called

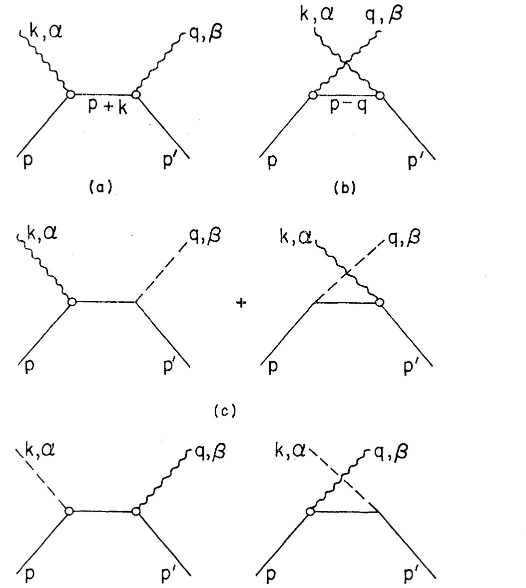


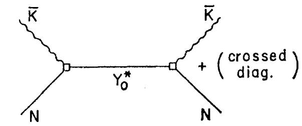
FIG. 1. Representation of the terms called $(\text{poles})_{\alpha\beta}$ in the text. Note that wavy lines attaching to circles stand for the axial-vector current $\mathfrak{F}^5$, and dashed lines for mesons.

such terms "pole terms" and they arise because of the possibility of diagrams such as those depicted in Figs. 1(a)–1(d). These diagrams are easily evaluated if one realizes that one only wants the coefficient of the term $P^\mu P^\nu$ in the term containing two $\mathfrak{F}$'s and P^μ in the terms containing one $\mathfrak{F}^5$. Straightforward calculation gives the result quoted in Sec. V.

Besides the terms just discussed [which, following the convention introduced in Sec. V, we call $(\text{poles})_{\beta\alpha}$], we need only keep the Σ term and remember that the term $\langle B'|\partial\partial|B\rangle^\star$ is explicitly of order ϵ^2. (Moreover, explicit calculation of its pole term show that they do not contribute to order ϵ.) Once one observes that crossing implies that the part of the amplitude antisymmetric in α and β vanishes at the point in question, one is immediately led to Eq. (5.6).

Having completed the formal proof of the theorem, we now discuss the point raised in Sec. V concerning the fractional deviations in the generalized Goldberger-Treiman relations. The point here is that one must also

FIG. 2. Typical diagram that can be enhanced if $\Delta_{KNY_0^*}$ is large in comparison to 1. Wavy lines attaching to boxes stand for axial-vector divergences ∂.



be careful in asserting that since the term

$$\langle B|\partial_\beta(q)\partial_\alpha(-k)|B\rangle^\star$$

is of order ϵ^2 it is negligible, as it might have contributions from low-mass S-wave resonances such as the Y_0^*, as shown in Fig. 2. Now, although the vertices in the diagram in Fig. 2 are each of order ϵ, the fact that the denominator is of the form $[\frac{1}{2}(m_{B'}^2+m_B^2)-m_{Y_0^*}^2]^{-1}$ tells us that if this difference happens to be small (accidentally), the effect of this term can be greatly enhanced. In order to see how this works in somewhat more detail, consider the process $\bar{K}N\to\bar{K}N$ and let $\Delta_{\bar{K}NY_0^*}$ be defined as in Eq. (5.15), so that the $\bar{K}NY_0^*$ vertices are of the order $\Delta_{\bar{K}NY_0^*}G_{\bar{K}NY_0^*}$. It is easy to convince oneself that the diagram in Fig. 2 is then of the order

$$(\Delta_{\bar{K}NY_0^*}G_{\bar{K}NY_0^*})^2[1/(m_N^2-m_{Y_0^*}^2)] \qquad \text{(A6)}$$

at the point $\nu=x=q^2=k^2=0$. Using the formula

$$(1-\Delta_{\bar{K}NY_0^*})G_{\bar{K}NY_0^*}=(2f_k)(m_{Y^*}-m_N)g_{\bar{K}NY_0^*},$$

we have Eq. (A6) equivalent to

$$\frac{\Delta_{\bar{K}NY_0^*}{}^2}{(1-\Delta_{\bar{K}NY_0^*})}\,\frac{G_{\bar{K}NY_0^*}(2f_k)g_{\bar{K}NY_0^*}}{(m_N+m_{Y_0^*})}. \qquad \text{(A7)}$$

Equation (A7) makes clear that the size of the parameter $\Delta_{\bar{K}NY_0^*}$ is what is important in determining whether or not such pole terms in $\langle B'|\partial_\beta(q)\partial_\alpha(-k)|B\rangle^\star$ make large contributions to meson-baryon scattering amplitudes at the unphysical point.

This example clearly exhibits the fact that the presence of low-mass S-wave resonances can seriously affect the usefulness of our theorem. However, it is interesting to note that, at least sometimes, one can show that if their effects are important in scattering amplitudes, then the fractional deviation of the appropriate generalized Goldberger-Treiman relation must be large.