Violation of the $|\Delta \mathbf{I}| = \frac{1}{2}$ Rule in the $K_{3\pi}$ Decay*

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Recent experimental results reveal the presence of a large $|\Delta\mathbf{I}|=\frac{3}{2}$ amplitude in the $K_{3\pi}$ decay. The relevant information is contained in the four 3-pion ratios $R_1 \equiv \frac{2}{3}\gamma^{000}/\gamma^{+-0}$, $R_2 \equiv \frac{1}{4}\gamma^{++-}/\gamma^{+00}$, $R_3 \equiv \frac{1}{2}\gamma^{+-0}/\gamma^{+00}$, and $R_4 \equiv \gamma^{000}/(\gamma^{++-}-\gamma^{+00})$, which are all equal to 1 in the exact $|\Delta\mathbf{I}|=\frac{1}{2}$ limit (the γ's are partial decay widths divided by phase-space factors, and the superscripts +, −, and 0 indicate positive, negative, and neutral pions, respectively). It is found that the deviations of the ratios R_3 and R_4 from the exact $|\Delta\mathbf{I}|=\frac{1}{2}$ value are of the order of 20%, whereas the deviation is small for the ratios R_1 and R_2. A survey of the experimental situation in nonleptonic weak decays shows that in some cases the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule seems to be almost exact, whereas in others it is clearly violated. One possible theoretical approach assumes that the primary weak interaction can give rise only to $|\Delta\mathbf{I}|=\frac{1}{2}$ transitions if the electromagnetic interactions are discarded, and explains the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitudes by electromagnetic corrections. In this framework, it is shown in this paper that the whole $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitude in the $K_2{}^0 \to 3\pi$ decay may be obtained by virtual electromagnetic corrections. It is shown that the η^0-X^0 mixing plays a rather important role. An order of magnitude estimate of the $\Gamma(X^0 \to 3\pi)$ width is also obtained, which is compatible with the experimental upper limit.

I. INTRODUCTION

The approximate validity of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the nonleptonic weak decays has been known for several years. If we survey the present situation we find both the experiments in which the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule seems to be obeyed rather precisely and the experiments in which the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule appears to be violated (to which the $K \to 3\pi$ decays belong[1]). Below we refer to these experiments as class one and class two, respectively.

Two general theoretical approaches are possible in trying to explain the situation.

(A) In the first approach, one assumes that the primary nonleptonic weak Hamiltonian $H_{\rm NL}{}^W$ contains both the $|\Delta\mathbf{I}|=\frac{1}{2}$ and the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitudes. An example of this type (type A), containing the $|\Delta\mathbf{I}|=\frac{1}{2}$ and $|\Delta\mathbf{I}|=\frac{3}{2}$ amplitudes, is provided by the current-current picture Hamiltonian $H_{\rm NL}{}^W \propto (J_\mu J_\mu{}^\dagger + \text{H.c.})$, where the J_μ are the charged weak currents. These type-A Hamiltonians cannot explain class-one experiments unless some mechanism is discovered which enhances the $|\Delta\mathbf{I}|=\frac{1}{2}$ amplitude relative to the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitude or suppresses the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitudes. The current algebra with soft-pion extrapolation gave some evidence for such a dynamical possibility,[2] which, however, is not yet very compelling. It has also not yet provided a clear explanation for the class-two experiments such as the rate of the $K^+ \to \pi^+\pi^0$ decay, and the violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K \to 3\pi$ decays. Further work, especially a realistic estimate of the terms which vanish in the soft-pion limit and involve the non-$|\Delta\mathbf{I}|=\frac{1}{2}$ amplitude, is important in order to show that the model works.

(B) One may instead assume a primary weak Hamiltonian contributing only to the $|\Delta\mathbf{I}|=\frac{1}{2}$ amplitudes, and attribute the violation of this rule to the virtual electromagnetic corrections (type-B Hamiltonians). In the current-current picture, for example, we can construct such Hamiltonians (which must include the neutral currents); one example is given by $H_{\rm NL}{}^W \propto d_{6ij} J_\mu{}^i J_\mu{}^j + \text{H.c.}$ [where d_{6ij} are the symmetric $SU(3)$ constants and $J_\mu{}^i$ are the octet of weak hadronic currents] which we adopt in the following work, whenever an explicit expression is needed. With the type-B Hamiltonians the experiments of class one are immediately explained; in order to explain the experiments of class two (including the $K_{3\pi}$ decays) we must, however, solve the problem of obtaining the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitudes via the electromagnetic interactions.

The purpose of this paper is to show that it may be possible to explain the $K \to 3\pi$ data by keeping a primary weak Hamiltonian satisfying the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, by means of virtual electromagnetic transitions. This may sound rather surprising[3] since the violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule appears to be large. Namely, the relevant information concerning the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K_{3\pi}$ decays may be given by using the four 3-pion ratios $R_1 \equiv \frac{2}{3}\gamma^{000}/\gamma^{+-0}$, $R_2 \equiv \frac{1}{4}\gamma^{++-}/\gamma^{+00}$, $R_3 = \frac{1}{2}\gamma^{+-0}/\gamma^{+00}$, and $R_4 \equiv \gamma^{000}/(\gamma^{++-}-\gamma^{+00})$, which are all equal to one in the exact $|\Delta\mathbf{I}|=\frac{1}{2}$ limit (the γ's are partial widths divided by phase space factors, and the superscripts +, −, and 0 indicate positive, negative, and neutral pions, respec-

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[1] See Particle Data Group, Rev. Mod. Phys. 41, 109 (1969), especially p. 187; also, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).
[2] See, for example, H. Sugawara, Phys. Rev. Letters 15, 870 (1965); *ibid.* 15, 997(E) (1965); M. Suzuki, *ibid.* 16, 212 (1966); S. Okubo, R. E. Marshak, and V. S. Mathur, *ibid.* 19, 407 (1967).
[3] See, e.g., M. Gell-Mann, Nuovo Cimento 5, 758 (1957). Also, see S. P. Rosen and S. Pakvasa, in *Advances in Particle Physics*, edited by Cool and Marshak (Wiley-Interscience Inc., New York, 1968), Vol. II, p. 494.

tively).[4] Table II reveals that the discrepancies from the $|\Delta \mathbf{I}| = \frac{1}{2}$ value are of the order of 20% for R_3 and R_4, while they are small and compatible with the exact rule, for R_1 and R_2. Electromagnetic corrections, however, involve $\alpha = 1/137$, the fine structure constant.

We make the assumption (which seems natural and reasonable) that the weak interaction mixes strongly the particles within the same $SU(3)$ multiplet, but mixes weakly particles belonging to different multiplets (so that it is good approximation to neglect this type of mixing in our work). This assumption directly leads to the situation that in the presence of the weak interaction the $K_2{}^0$ will mix appreciably only with the η (and π^0, X^0). Thus, because of the existence of an $\eta^0 \to 3\pi$ transition amplitude of order α, an electromagnetic $K_2{}^0 \to 3\pi$ transition may take place. Also, in order to introduce a minimum of theoretical assumptions, which, in our opinion, would decrease the reliability of the calculation, we use exclusively, as far as possible, information closely related to well-established experiments. For instance, we use, whenever available, effective coupling strengths extracted from experimental widths. Therefore, we will use broken $SU(3)$ sum rules for the sake of consistency since these experimental values include the effect of broken symmetry.

A pseudoscalar-pole model has been used in the past by several authors.[5] The motivation, which is a rather natural one, is to consider the presence of the Feynman diagram $K_2{}^0 \to \eta^0 \to \pi^+\pi^-\pi^0$ $(\pi^0\pi^0\pi^0)$ for the $|\Delta \mathbf{I}| \neq \frac{1}{2}$ amplitude of the $K_2{}^0 \to 3\pi$ decay. The nearness of the mass of the $K_2{}^0$ to that of the η^0 will enhance the importance of this contribution. Since this diagram is not available for the $K^+ \to 3\pi$ decay, we can at the same time explain qualitatively why the ratios R_1 and R_2 are consistant with the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule (a more detailed analysis will confirm this for R_2 only; the apparent lack of violation in the case of R_1 is actually because of the fact that $R_1 \simeq R_4/R_3$ in this model and $R_4 \simeq R_3$). However, in a quantitative discussion, their work met with several difficulties, namely, the large uncertainties in the estimates of the coupling constants (especially $f_{K_2{}^0\eta}$ and $f_{K_2{}^0X^0}$ of the $K_2{}^0 - \eta$ and $K_2{}^0 - X^0$ transitions, which we shall have below, the lack of knowledge of the η-decay width, and of the possible importance of the $\eta^0 - X^0$ mixing).[6] Their estimates gave a value for the $X^0 \to 3\pi$, $|\Delta \mathbf{I}| = \frac{3}{2}$ amplitude smaller than the one observed (see Ref. 1). However, the recently measured[7] rather large $\Gamma(\eta \to 3\pi)$ partial width does suggest that it might indeed be possible to explain the $|\Delta \mathbf{I}| \neq \frac{1}{2}$ effects in the $K \to 3\pi$ decays by means of the above-described mixing mechanism. The $\eta^0 - X^0$ mixing effects might even provide an additional enhancement.

In our approach the problem of explaining the $K_{2\pi}{}^+$ decay by the electromagnetic correction remains unsolved. Nobody has given a convincing argument for the rate of the $K^+ \to \pi^+\pi^0$ decay. We would like also to mention a recent experiment[8] on the $K_1{}^0$ decay, which seems to indicate a very small violation if any, of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. Therefore, we feel that there is enough motivation to study the above-mentioned possibility.

II. $|\Delta \mathbf{I}| = \frac{3}{2}$ AMPLITUDES

A. Feynman Graphs

The processes which contribute to the $|\Delta \mathbf{I}| = \frac{3}{2}$ $K_2{}^0 \to 3\pi$ decay amplitude in the mechanism discussed in the Introduction are shown by the Feynman diagram given in Fig. 1. Figure 2 shows the graphs contributing to the $K_2{}^0 \to 2\gamma$ decay in the same mechanism. We consider the intermediate particles as stable particles.

We explain the notation in detail for the η-pole diagrams, as there are only obvious changes for the others (π^0 and X^0). We take the following as invariant couplings for the $K_2{}^0 - \eta$, $\eta \to \pi^+\pi^-\pi^0$, $\eta \to \pi^0\pi^0\pi^0$, and $\eta \to \gamma\gamma$ vertices, respectively:

$$f_{K_2{}^0\eta} m_p{}^2, \quad g_{\eta\pi^+\pi^-\pi^0}, \quad g_{\eta\pi^0\pi^0\pi^0}, \quad \text{and} \quad (f_{\eta 2\gamma}/m_\eta)\epsilon_{\alpha\beta\gamma\delta}\epsilon_\alpha{}^{(2)} k_\beta{}^{(2)} p_\gamma{}^{(\eta)} \epsilon_\delta{}^{(1)}. \tag{1}$$

The f's and g's are dimensionless effective coupling constants, m_p is the proton mass, m_η is the η mass, and $\epsilon_{\alpha\beta\gamma\delta}$ is the totally antisymmetric unit tensor density of rank 4. Finally, $\epsilon_\delta{}^{(1)}$ and $\epsilon_\alpha{}^{(2)}$ are covariant polarization vector components of the γ_1 and γ_2; and $k_\beta{}^{(2)}$ and $p_\gamma{}^{(\eta)}$ are four-momentum vector components of the γ_2 and η particle, respectively. We also assume time-reversal invariance.

It is important to realize[9] that only a $|\Delta \mathbf{I}| = \frac{3}{2}$ amplitude is contributing to the $K \to 3\pi$ amplitude from the η^0 and X^0 pole if one assumes, consistently with present evidence,[10] that the 3π final states are symmetric ($I=1$)

[4] See Ref. 1 for further discussion.

[5] C. Bouchiat, J. Nuyts, and J. Prentki, Phys. Letters **3**, 156 (1963); S. Oneda and S. Hori, Phys. Rev. **132**, 1800 (1963); S. Oneda, Y. S. Kim, and D. Korff, *ibid.* **136**, B1066 (1964); S. Oneda and J. C. Pati, *ibid.* **155**, 1621 (1967); J. C. Pati and S. Oneda, *ibid.* **136**, B1097 (1964).

[6] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); A. Baracca and A. Bramon, *ibid.* **51A**, 873 (1967).

[7] One actually measures the $\Gamma(\eta \to 2\gamma)$ width, and the $\eta \to 3\pi$ branching ratios. See, for instance, G. Goldhaber and S. Goldhaber, in *Advances in Particle Physics*, edited by Cool and Marshak (Wiley-Interscience Inc., New York, 1968), Vol. II, p. 32. We take $\Gamma(\eta \to 2\gamma) = 0.88 \pm 0.19$ keV.

[8] G. Morphin and D. Sinclair, Bull. Am. Phys. Soc. **14**, 518 (1969). These authors gave the $(K_1{}^0 \to \pi^+\pi^-)/(K_1{}^0 \to \pi^0\pi^0)$ branching ratio as 2.09 ± 0.07.

[9] This point has not been explicitly stated in the past. Actually, C. Bouchiat *et al.* (Ref. 5) state that the amplitude is a mixture of $|\Delta \mathbf{I}| = \frac{1}{2}$ and $\frac{3}{2}$. If this were the case, any estimate would give only the upper bound of the pseudoscalar-pole contribution to the $|\Delta \mathbf{I}| = \frac{3}{2}$ amplitude.

[10] See, for instance, S. P. Rosen and S. Pakvasa, Ref. 3, p. 506; G. Goldhaber and S. Goldhaber, in Ref. 4. Also, see Jewan Kim, Nevis Laboratory Report No. 170, 1969 (unpublished). This measurement gives $(\eta^0 \to 3\pi^0)/(\eta^0 \to \pi^+\pi^-\pi^0) = 1.58 \pm 0.25$, in good agreement with the assumption that the 3-pion final state has $I=1$. If the value of the review of particle properties compilation (Ref. 1) is taken for this branching ratio, then a small admixture of $|\Delta \mathbf{I}| \neq \frac{3}{2}$ amplitude might be present in the pseudoscalar-pole-model $K_2{}^0 \to 3\pi$ amplitude.

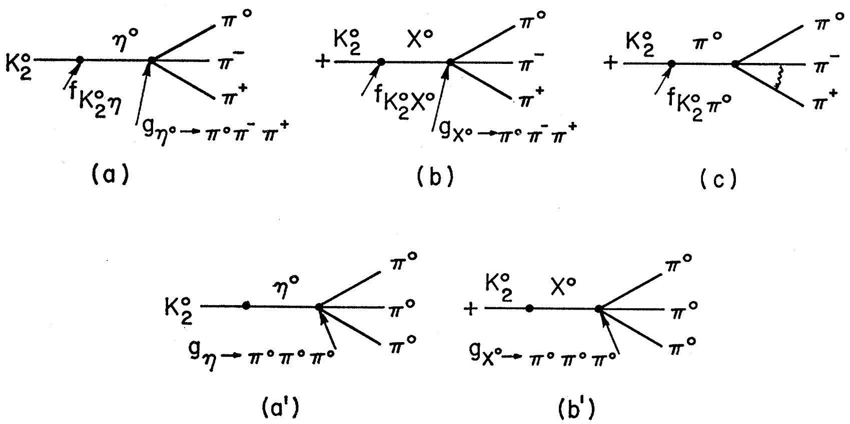

FIG. 1. Feynman diagrams for the $|\Delta\mathbf{I}|=\frac{3}{2}$ amplitudes. Diagram (c) is assumed to be negligible, according to our estimate.

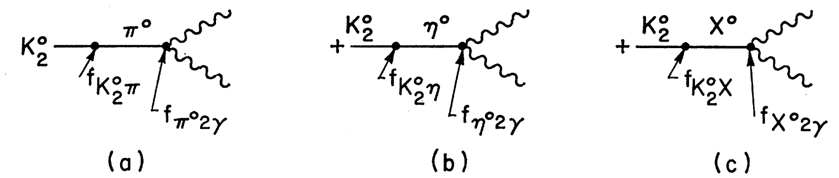

FIG. 2. Feynman diagrams for the $K_2^0 \to 2\gamma$ decay.

states. This can be proved by writing the interaction as $g(K_2^0 S)(T\cdot\phi)(\phi\cdot\phi)$, where S and T are the spinor and vector spurion, respectively, and ϕ is the pseudoscalar field.

B. x Violation Parameters

We explain our work for R_3 first. We introduce, following Oneda and Hori,[11] the violation parameter x_3. It is defined by assuming an effective Hamiltonian as

$$H(K_2^0 \to \pi^+\pi^-\pi^0) = g_{K_2^0\pi^+\pi^-\pi^0} K_2^0(\pi^+\pi^-\pi^0) = g_{K_2^0\pi^+\pi^-\pi^0}^{(1/2)}(1-x_3)K_2^0(\pi^+\pi^-\pi^0)\,, \quad (2)$$

where $g_{K_2^0\pi^+\pi^-\pi^0}^{(1/2)}$ is the effective coupling constant satisfying the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule (x_3 vanishes in the exact $|\Delta\mathbf{I}|=\frac{1}{2}$ limit).

The parameter x_3 can be related to the experimentally measured three-pion ratio, since

$$(1-x_3)^2 = \left|\frac{g_{K_2^0\pi^+\pi^-\pi^0}}{g_{K_2^0\pi^+\pi^-\pi^0}^{(1/2)}}\right|^2 \simeq \frac{\Gamma(K_2^0 \to \pi^+\pi^-\pi^0)}{\Gamma_{|\Delta I|=1/2}(K_2^0 \to \pi^+\pi^-\pi^0)} = \frac{\gamma^{+-0}}{2\gamma^{+00}} \equiv R_3\,. \quad (3)$$

The γ's are reduced rates, defined as partial widths divided by phase-space factors. We use the nonuniform Dalitz plot (NUDP) phase-space factors.[12]

From our model we also obtain

$$x_3 = \frac{m_p^2}{g_{K_2^0\pi^+\pi^-\pi^0}^{(1/2)}} \times \left(\frac{f_{K_2^0\eta}\, g_{\eta^0\pi^+\pi^-\pi^0}}{m_\eta^2 - m_{K_2^0}^2} + \frac{f_{K_2^0X^0}\, g_{X^0\pi^+\pi^-\pi^0}}{m_{X^0}^2 - m_{K_2^0}^2}\right). \quad (4)$$

We neglect the pion-pole contribution.[13] For the model to be convincing, all observed R ratios must be reproduced. To obtain R_4 and x_4, we use the equations analogous to (3) and (4). Concerning R_1, the model gives the interesting relation $R_1 \simeq R_4/R_3$, which indicates that the observed lack of violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule for this ratio may be only apparent and due to the circumstance that $R_4 \simeq R_3$. Furthermore, this relation puts a rather stringent test on the model. Finally, R_2 obviously is equal to 1 in the model.

C. Coupling Strengths

We need the coupling constants to be introduced into formulas (3) and (4) (and their analogs for the calculation of R_4).

We obtain $|g_{K_2^0\pi^+\pi^-\pi^0}|^2$ from the measured

$$\Gamma(K_2^0 \to \pi^+\pi^-\pi^0)$$

partial width, and $g_{K_2^0\pi^+\pi^-\pi^0}^{(1/2)}$ from the measured

[11] S. Oneda and S. Hori (Ref. 5).

[12] T. J. Devlin and S. Barshay, Phys. Rev. Letters **19**, 881 (1967) and Ref. 1.

[13] No one has ever succeeded in making a rigorous calculation of this contribution. However, if one attempts an estimate of the radiative corrections to the $\pi\pi$ amplitude, based on either electromagnetic mixing or tadpole models, one can easily convince oneself that the amplitudes are at least of the second order in α.

TABLE I. List of possible assignments for $\sin\theta_{\eta X}=-10.24°$.

	$f_{\pi^0\gamma\gamma}$	$f_{\eta\gamma\gamma}$	$f_{X^0\gamma\gamma}$	$f_{K_2{}^0\pi}$	$f_{K^0\eta}$	$f_{K_2{}^0X}$	$g_\eta/g_{K^0}{}^{1/2}$	x_3(theor)	x_4(theor)[a]
1	$+0.665\times10^{-2}$	$+0.995\times10^{-2}$	-3.14×10^{-2}	2.17×10^{-8}	1.108×10^{-8}	0.88×10^{-8}	4.55×10^{5}	8.15×10^{-2}	9.15×10^{-2}
2	-0.665×10^{-2}	-0.995×10^{-2}	$+3.14\times10^{-2}$	2.17×10^{-8}	0.635×10^{-8}	3.47×10^{-8}	-4.55×10^{5}	$4.7\ \times10^{-2}$	5.35×10^{-2}
3	$+0.665\times10^{-2}$	$+0.995\times10^{-2}$	-3.14×10^{-2}	-2.17×10^{-8}	-0.635×10^{-8}	-3.47×10^{-8}	-4.55×10^{5}	$4.7\ \times15^{-2}$	5.35×10^{-2}
4	-0.665×10^{-2}	-0.995×10^{-2}	$+3.14\times10^{-2}$	-2.17×10^{-8}	-1.108×10^{-8}	-0.88×10^{-8}	$+4.55\times10^{5}$	8.15×10^{-2}	9.15×10^{-2}

[a] This Table gives the x_3 and x_4 obtained by neglecting the X^0-η^0 pole in the ($K_2{}^0\rightarrow 3\pi$) amplitude. However, it is important to consider it in the ($K_0{}^2\rightarrow 2\gamma$) amplitude. By including the X^0 contribution in the calculation of the x, it is possible to improve further the agreement with the experiment.

$\Gamma(K_{\tau'}{}^+)$ and the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.[14] We obtain $|g_{\eta\pi^+\pi^-\pi^0}|^2$ from the experimentally known width. In the first approximation we neglect the X^0 contribution to the x_3 and x_4 parameters in formula (4). (This will be found to be of the order of 10%.) We have at our disposal three equations for the three unknowns $f_{K_2{}^0\eta}$, $f_{K_2{}^0X^0}$, and $f_{X^02\gamma}$, namely, the two broken-$SU(3)$ sum rules,[15]

$$f_{\pi^02\gamma}-\sqrt{3}\cos\theta_{X\eta}\,f_{\eta2\gamma}+\sqrt{3}\sin\theta_{X\eta}\,f_{X^02\gamma}\simeq0\,, \tag{5a}$$

$$f_{K_2{}^0\pi}-\sqrt{3}\cos\theta_{X\eta}\,f_{K_2{}^0\eta}+\sqrt{3}\sin\theta_{X\eta}\,f_{K_2{}^0X^0}=0\,, \tag{5b}$$

and the expression for the $\Gamma(K_2{}^0\rightarrow 2\gamma)$ in the pseudoscalar-pole model[16]:

$$\Gamma(K_2{}^0\rightarrow 2\gamma)=(1/16\pi)(m_{K^0}/m_\eta)^2m_{K^0}\times[f_{K_2{}^0}f_{\pi^02\gamma}D(\pi)m_\pi{}^2+\eta^0,\ X^0\text{ terms}]^2\,, \tag{5c}$$

where $D(\pi)=m_p{}^2/(m_\pi{}^2-m_{K_2{}^0}{}^2)$. We obtain $|f_{\pi^02\gamma}|$, $|f_{\eta2\gamma}|$, and the $\Gamma(K_2{}^0\rightarrow 2\gamma)$ from the experiments.[17] We take for the X^0-η^0 $SU(3)$ mixing angle[18] the value $\theta_{X^0\eta^0}=\pm10.24°$. We relate the $f_{K_2{}^0\pi^0}$ to the $f_{K^+\pi^+}$ strength, which is related to the $\Gamma(K_1{}^0\rightarrow\pi^+\pi^-)$ width, as follows.

Via the relation[19]

$$A(K^+\rightarrow\pi^+)=F_\pi A(K_1{}^0\rightarrow\pi^+\pi^-)(\times\tfrac{1}{16})\times(3m_{\pi^0}{}^2+4m_{K_1{}^0}{}^2+m_\eta{}^2)/(m_{K^0}{}^2-m_{\pi^0}{}^2)\,,$$

we obtain $A(K^+\rightarrow\pi^+)=1.91\times10^{-2}$ MeV^2 and $f_{K^+\pi^+}=A(K^+\rightarrow\pi^+)/m_p{}^2=2.17\times10^{-8}$. The $f_{K_2{}^0\pi^0}$ strength is related to the $f_{K^+\pi^+}$ by means of the Hermitian and CP-invariant interaction $i[(\bar{K}\boldsymbol{\tau}S)\cdot\boldsymbol{\phi}-(\bar{S}\boldsymbol{\tau}K)\cdot\boldsymbol{\phi}]$ as $f_{K_2{}^0\pi^0}=f_{K^+\pi^+}=2.17\times10^{-8}$.

We now give an outline and discussion of the calculation. We consider the graph of Fig. 2. We have normalized the $f_{\pi^02\gamma}$, $f_{\eta^02\gamma}$, and $f_{X^02\gamma}$ strengths by use of $\Gamma(\pi^0\rightarrow2\gamma)=(1/16\pi)|f_{\pi^02\gamma}|^2m_\pi{}^3/m_\eta{}^2$ and analogous expressions for the η^0 and X^0 case according to Eq. (1). From (6) we obtain $f_{\eta2\gamma}=\pm0.955\times10^{-2}$ using the experimental widths (Ref. 1). Inserting these numerical values into Eqs. (5a)–(5c), we obtain two different solutions (up to the sign) for the $f_{X^02\gamma}$. We discard the largest one since it leads to a very large value of the $\Gamma(X^0\rightarrow2\gamma)$ width, namely, 5.10 MeV, compared with the experimental upper limit of 4 MeV. The value we retain gives $\Gamma(X^0\rightarrow2\gamma)=0.935$ MeV, in qualitative agreement with the one obtained by Baracca and Bramon[20] who use different inputs.

We consider first the assignments with $\sin\theta<0$. There is some ambiguity in the solutions, because of sign ambiguities in the input. The solutions are given in Table I. We note that if we use only the π and η pole in the calculation of the $K_2{}^0\rightarrow2\gamma$ amplitudes, we obtain $f_{K_2{}^0\eta}\simeq0.53\times10^{-8}$ and $x_3\simeq5.3\times10^{-2}$, which is smaller than the experimental value $9\pm2.5\times10^{-2}$. Since the X^0 propagator is less than 10% of the η^0 propagator, and the coupling constants involving the three pseudoscalar mesons are of the same order of magnitude, one may reasonably expect that the $f_{K_2{}^0\eta}$ will not be drastically changed by the inclusion of the X^0 pole. Therefore we chose the first or last assignments as the most likely ones. Both lead to a value of $x_3\simeq8.25\times10^{-2}$, in good agreement with experiment.[21] We obtain this value by neglect of the X^0 pole in form (4). It is interesting to note that while it is important to keep the X^0 pole in the $K\,{}_2{}^0\rightarrow2\gamma$ amplitude (we thus obtain a more reliable estimate of $f_{K_2{}^0\eta}$, and an estimate for $f_{K_2{}^0X^0}$), this pole gives only a small contribution to the x_3 value. This is to be traced to the larger $\Gamma(X_2{}^0\rightarrow\gamma\gamma)$ in this approach, and to the fact that we can adjust the $X^0\rightarrow3\pi$ amplitude as to have x_3(expt)$=x_3$(theor).

We assume by analogy with the $\eta^0\rightarrow3\pi$ decay an $I=1$ final state. We obtain, in standard way,

[14] See Ref. 1. We neglect the off-mass-shell corrections. As pointed out at the end of Sec. II, this does not appear to change significantly our conclusions concerning the $|\Delta\mathbf{I}|\neq\frac{1}{2}$ amplitude. It might, however, change significantly the numerical estimate of the X^0 width, which, anyhow, has a large theoretical uncertainty.

[15] S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967) and Ref. 22, where it is shown that (5a) is approximately satisfied. Equation (5b) holds also in broken $SU(3)$.

[16] Following the philosophy stated in the Introduction, we assume that weak-interaction mixing between different multiplets is negligible. Further, we use consistently broken-$SU(3)$ and η^0-X^0 mixing. For a different approach see D. G. Greenberg, Nuovo Cimento **56A**, 597 (1968). It seems to us that the reliability of his calculation is questionable because of the possibility of at least partial double counting in the diagrams of Fig. 6, which leads to a very large contribution (in his paper). Similar remarks apply to an estimate, also by D. G. Greenberg [Phys. Rev. **178**, 2190 (1969)], of the $|\Delta\mathbf{I}|=\frac{3}{2}$, $K_1{}^0\rightarrow2\pi$ amplitude, where the $A_1{}^{(8)}$ (which belongs to a different multiplet than $K_2{}^0$ and is not firmly established) is assumed to mix with the $K_2{}^0$; the X^0-η^0 mixing is not included there. [*Note added in manuscript.* See also R. Rockmore, Phys. Rev. **182**, 1512 (1969)].

[17] See. Ref. 1. We take $\Gamma(K_2{}^0\rightarrow2\gamma)=9.95\times10^3$ sec^{-1}.

[18] We use the following conventions: $\eta=\cos\theta_{X\eta}\,\eta_8+\sin\theta_{X\eta}\,\eta_1$ and $X^0=-\sin\theta_{X\eta}\,\eta_8+\cos\theta_{X\eta}\,\eta$.

[19] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).

[20] A. Baracca and A. Bramon, Ref. 6.

[21] The presence of X^0 pole can enhance the $f_{K_2{}^0\eta}$ strength, if the X^0 contribution interferes destructively with the $\eta\rightarrow2\gamma$ amplitude. In any case, the existence of an $X^0\rightarrow3\pi$ amplitude permits adjustment to the $K_2{}^0\rightarrow3\pi$, $|\Delta\mathbf{I}|=\frac{3}{2}$ amplitude further. In this sense we can see that the X^0 pole plays a rather important role.

TABLE II. Summary of results. The values in parentheses in the "η^0-X^0 mixing" column are obtained by keeping the X^0 pole in the $K_2^0 \to 2\gamma$ amplitude and neglecting it in the $K_2^0 \to 3\pi$ amplitude, because of the unknown $X^0 \to 3\pi$ amplitude.

	Experiments		Theory		
	Vienna[a] (NUDP)	Part. Data Group[b] (NUDP)	No η^0-X^0 mixing	η^0-X^0 mixing	Exact $\lvert\Delta \mathbf{I}\rvert=\frac{1}{2}$
R_1[c]	0.99 ± 0.04	1.00 ± 0.04	1.12	0.97 (0.985)	1
R_2	1.00 ± 0.03	0.99 ± 0.03	1	1	1
R_3	0.82 ± 0.04	0.79 ± 0.04	0.95	0.82 (0.845)	1
R_4	0.79 ± 0.03	0.80 ± 0.04	0.91	0.79 (0.83)	1
x_1[d]	0	0	≈ -0.06	≈ 0.008	0
x_2	0	0	0	0	0
x_3	0.09 ± 0.025	0.11 ± 0.025	0.0398	0.09 (0.0815)	0
x_4	0.11 ± 0.02	0.105 ± 0.025	0.0445	0.11 (0.102)	0
$X^0 \to$ all		<4 MeV		≈ 0.935 MeV	
$X^0 \to 2\gamma$		<240 keV		≈ 60 keV	
$X^0 \to 3\pi$		<280 keV		≈ 30 keV	

[a] Reference 1.
[b] Particle Data Group, Rev. Mod Phys. **41**, 109 (1969).
[c] The three-pion ratios R_1, R_2, R_3, and R_4 are defined as follows: $R_1 \equiv \frac{2}{3}\gamma^{000}/\gamma^{+-0}$, $R_2 \equiv \frac{1}{4}\gamma^{++0}/\gamma^{+00}$, $R_3 \equiv \frac{1}{2}\gamma^{+-0}/\gamma^{+00}$, and $R_4 \equiv \gamma^{000}/(\gamma^{++-}-\gamma^{+00})$. The γ's are partial decay widths divided by phase-space factors, and the superscripts $+$, $-$, and 0 indicate the pionic charge.
[d] Violation parameters x_1, x_2, x_3, and x_4 may be obtained from $R \simeq (1-x)^2$.

$\Gamma(X^0 \to 3\pi) \simeq 27$ keV, with a very large theoretical uncertainty (between 0 and 210 keV), which can be traced to the 25% experimental error in x_{expt}. This estimate is below the experimental upper limit $\Gamma(X^0 \to 3\pi) \leq 65$ keV. In the process of estimating the $\Gamma(X^0 \to 3\pi)$ width, we could identify as the most likely assignment the first one of Table I, based on the expectation $g_{X^0\to 3\pi} \simeq g_{\eta \to 3\pi}$.[22] A similar analysis for $\sin\theta > 0$ leads to the same results for the violation parameters. We can obtain x_4 in a similar way; the results are given in Table II.

We have also studied the effect of possible changes in the input, in particular of the Hara-Nambu[19] correction of the $f_{K^+\pi^+}$, and of the $f_{\eta 2\gamma}$ strength, which is after all based on only one experiment. Therefore we have also repeated the calculation by a slightly different approach, which consists in using, instead of Eq. (5a) relating the $f_{\pi^0 2\gamma}$, $f_{\eta^0 2\gamma}$, and $f_{X^0 2\gamma}$ coupling strengths, a broken-$SU(3)$ sum rule, obtained by estimating the previously neglected terms[23] on the right side of (5a). This is essentially equivalent to lowering the $\lvert f_{\eta 2\gamma}\rvert$. Thus we obtain instead of (5a),

$$(f_{\eta^0 2\gamma}/f_{\pi^0 2\gamma}) \simeq 1.04 \quad \text{and} \quad (f_{X^0 2\gamma}/f_{\pi^0 2\gamma}) \simeq \pm 1.36 \qquad (5a')$$

(where the upper sign corresponds to the positive choice for the sign of $\theta_{X^0\eta^0}$). [This method also provides a more consistent check of the broken-$SU(3)$ approximation.]

In this alternative we obtain eight possible assignments for $f_{K_2^0\pi^0}$, $f_{K_2^0\eta^0}$, and $f_{K_2^0 X^0}$. Two of them have to be rejected because they give a very large $\Gamma(X^0 \to 3\pi)$; the rest give smaller $f_{K_2^0\eta}$ ranging from 0.27×10^{-8} to 0.72×10^{-8}. Using the X^0-pole contribution we can still obtain $x(\text{theor}) = x(\text{expt})$; the resulting $\Gamma(X^0 \to 3\pi)$ are generally small and of the same order of magnitude as obtained before. If one takes the $f_{K^+\pi^+}$ without the Hara-Nambu correction, one gets somewhat larger values of the $f_{K_2^0\eta}$ and $f_{K^0\eta}$ and one still obtains the observed values of x_3 and x_4, with $\Gamma(X^0 \to 3\pi)$ widths of the same order as before.[22]

III. CONCLUDING REMARKS

We conclude that our estimates of the $\lvert\Delta \mathbf{I}\rvert = \frac{3}{2}$ amplitudes show that it may be possible to explain their origin by virtual electromagnetic transitions. We believe that our approximations are reasonable: In particular, in the case of the $f_{K^0\pi^0}$ strength a comparable value was obtained by Loebbaka and Pati in the study of hyperon decays.[24] Work is in progress to study the detailed effect of this choice and of $\Gamma(\pi^0 \to 2\gamma)$ on the numerical values of the $f_{K_2^0\eta}$ and $f_{K_2^0 X^0}$ strengths. However, we can already confidently state that our main result, namely, the possibility of obtaining the $\lvert\Delta \mathbf{I}\rvert = \frac{3}{2}$ amplitude by means of virtual $\eta^0 \to 3\pi$ *and* $X^0 \to 3\pi$ transitions, is quite insensitive to the exact numerical values of the input, if the input is chosen either compatible with present experimental information, or according to a chiral $SU(3)\times SU(3)$ theory, which uses $H_{\text{NL}}{}^W \propto d_{6ij} J_\mu{}^i J_\mu{}^j + \text{H.c.}$ This latter part of the work can probably be considered as an argument for the existence of a weak nonleptonic Hamiltonian of this form, including neutral currents.

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[22] We are further investigating the effect of the input value in both alternatives.

[23] S. Matsuda and S. Oneda, Phys. Rev. **186**, 2107 (1969). This sum rule gives $\langle\gamma\gamma|\pi\rangle - \sqrt{3}\cos\theta_{X\eta}\langle\gamma\gamma|\eta\rangle + \sqrt{3}\sin\theta_{X\eta}\langle\gamma\gamma|X^0\rangle = XY$ with $X = [1 - (m_\rho m_\phi/m_8{}^2)\cos^2\alpha - (m_\rho m_\omega/m_1{}^2)\sin^2\alpha]$ and

$$Y = \langle\gamma\gamma^V|\pi\rangle - \sqrt{3}\cos\alpha\langle\gamma\gamma^V|\eta\rangle + \sqrt{3}\sin\alpha\langle\gamma\gamma^V|X^0\rangle,$$

where α is the $\omega-\phi$ mixing angle.

[24] D. Loebbaka and J. C. Pati, Phys. Rev. **147**, 1047 (1966).