where j assumes values $j=0, 1, 2, 3$, with $\sigma_0\equiv1$. Expressing $\phi(x)$ and $\xi(x)$ in terms of $q(x)$, one gets

$$\begin{array}{lll}
L_{j\alpha}(x): & \tilde{T}_{4j}{}^{\alpha}(x) & (j=1,2)\\
: & A_3{}^{\alpha}(x) & (j=3)\\
: & -iV_4{}^{\alpha}(x) & (j=0),\\
N_{j\alpha}(x): & A_j{}^{\alpha}(x) & (j=1,2)\\
: & \tilde{T}_{43}{}^{\alpha}(x)\equiv T_{12}{}^{\alpha}(x) & (j=3)\\
: & S^{\alpha}(x) & (j=0),\\
K_{j\alpha}(x): & \tilde{T}_{j3}{}^{\alpha}(x) & (j=1,2)\\
: & iA_4{}^{\alpha}(x) & (j=3)\\
: & -V_3{}^{\alpha}(x) & (j=0),\\
R_{j\alpha}(x): & \epsilon_{3jk}V_k{}^{\alpha}(x) & (j=1,2)\\
: & P^{\alpha}(x) & (j=3)\\
: & -iT_{34}{}^{\alpha}(x)\equiv i\tilde{T}_{12}{}^{\alpha}(x) & (j=0).
\end{array} \qquad \text{(A9)}$$

The statement of the asymptotic $SU(6)_W$ symmetry is now translated to that of

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(K_{j\alpha}(x),K_{i\beta}(y))_+|0\rangle = c\delta_{ij}\delta_{\alpha\beta} \qquad \text{(A10)}$$

for the collinear momentum k of the form Eq. (A3). It is easy to check that this gives Eqs. (20) immediately. Similarly, replacing $K_{j\alpha}$ by $L_{j\alpha}$, we obtain the same result. If we consider

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(L_{j\alpha}(x)N_{i\beta}(y))_+|0\rangle,$$

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(K_{j\alpha}(x)R_{i\beta}(y))_+|0\rangle$$

up to the order $1/k$, we find the asymptotic sum rules (23).

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Asymptotic $SU(6)_W$ Spectral Sum Rules.* II. Applications and Bare Quark Masses

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The sum rules of the preceding paper are investigated in detail in pole dominance. The ratio f_K/f_π is found to be near unity and all nonexotic baryons must satisfy an approximate mass formula $M=a-bY$ with the universal constant $b\simeq m_3-m_1\simeq150$ MeV, where m_1, m_2, and m_3 are masses of bare quarks. Moreover, we compute $m_1\simeq7$ MeV and $m_3\simeq156$ MeV in a model where $SW(3)$ is exact except for the quark mass term.

I. DEFINITION OF COUPLING CONSTANTS

IN the preceding paper[1] (hereafter referred to as I), we have developed several sum rules on the basis of the asymptotic $SU(6)_W$ symmetry. In this paper, we study its applications, saturating the sum rules by pole dominances. To that end, we define various coupling parameters as follows.

(i) Vector:

$$\langle 0|V_\mu{}^{(3)}(0)|\rho^0(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(\rho),$$

$$\langle 0|V_\mu{}^{(8)}(0)|\omega,\phi(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(\omega\text{ or }\phi),$$

$$(1/\sqrt{2})\langle 0|V_\mu{}^{(4-i5)}(0)|K^{*+}(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(K^*),$$

$$(1/\sqrt{2})\langle 0|V_\mu{}^{(4-i5)}(0)|\kappa^+(k)\rangle=(2k_0V)^{-1/2}k_\mu G_V(\kappa),$$

where κ means the 0^+ κ meson.

(ii) Axial vector:

$$\langle 0|A_\mu{}^{(3)}(0)|A_1{}^0(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_A(A_1),$$

$$(1/\sqrt{2})\langle 0|A_\mu{}^{(4-i5)}(0)|K_A{}^+(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_A(K_A),$$

$$\langle 0|A_\mu{}^{(1-i2)}(0)|\pi^+(k)\rangle=(2k_0V)^{-1/2}ik_\mu f_\pi,$$

$$\langle 0|A_\mu{}^{(4-i5)}(0)|K^+(k)\rangle=(2k_0V)^{-1/2}ik_\mu f_K,$$

(iii) Scalar:

$$(1/\sqrt{2})\langle 0|S^{(4-i5)}(0)|\kappa^+(k)\rangle=(2k_0V)^{-1/2}G_S(\kappa),$$

$$(1/\sqrt{2})\langle 0|S^{(1-i2)}(0)|\epsilon^+(k)\rangle=(2k_0V)^{-1/2}G_S(\epsilon),$$

where ϵ is an assumed 0^+ meson with $I=1$ and $Y=0$, which may be[2] $\pi_N(1016)$.

(iv) Pseudoscalar:

$$\langle 0|P^{(3)}(0)|\pi^0(k)\rangle=(2k_0V)^{-1/2}G_P(\pi),$$

$$(1/\sqrt{2})\langle 0|P^{(4-i5)}(0)|K^+(k)\rangle=(2k_0V)^{-1/2}G_P(K).$$

* Work supported in part by the U. S. Atomic Energy Commission.

[1] S. Okubo, preceding paper, Phys. Rev. 188, 2293 (1969).

[2] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. 41, 109 (1969).

(v) Tensor:

$$\langle 0|T_{\mu\nu}^{(3)}(0)|\rho^0(k)\rangle = (2k_0V)^{-1/2}i[k_\nu\epsilon_\mu(k)-k_\mu\epsilon_\nu(k)](1/m_\rho)G_T(\rho),$$

$$(1/\sqrt{2})\langle 0|T_{\mu\nu}^{(4-i5)}(0)|K^{*+}(k)\rangle = (2k_0V)^{-1/2}i[k_\nu\epsilon_\mu(k)-k_\mu\epsilon_\nu(k)](1/m_{K^*})G_T(K^*),$$

$$\langle 0|T_{\mu\nu}^{(3)}(0)|B^0(k)\rangle = (2k_0V)^{-1/2}i\epsilon_{\mu\nu\lambda\rho}\epsilon_\lambda(k)k_\rho(1/m_B)G_T(B),$$

$$(1/\sqrt{2})\langle 0|T_{\mu\nu}^{(4-i5)}(0)|K_A{}^+(k)\rangle = (2k_0V)^{-1/2}i\epsilon_{\mu\nu\lambda\rho}\epsilon_\lambda(k)k_\rho(1/m_{K_A})G_T(K_A),$$

where B represents[2] the B meson with $J^P=1^+$ and $G=+1$. Note that, because of the G-parity conservation, we must have

$$\langle 0|T_{\mu\nu}^{(3)}(0)|A_1{}^0(k)\rangle = 0.$$

Actually, the B meson and A_1 meson will belong to two different $SU(3)$ octets; hence, we may expect to have two types of K_A mesons with $J^P=1^+$, $I=\frac{1}{2}$, and $Y=1$, corresponding to A_1 and B mesons, respectively. Indeed, experimentally[2] we may have such candidates at $M=1240$, 1330, or 1775 MeV. Thus we shall distinguish between two different K_A mesons simply by K_A and K_A' and, accordingly, define respective parameters by adding the primes for the latter.

Now, the spectral weights $\rho_{AB}(m)$, etc., can be easily computed by means of Eq. (I8) on the pole approximation. For example, we have, for a, $b=1$, 2, 3,

$$\rho_{ab}(m,P-P)=G_P{}^2(\pi)\delta(m^2-m_\pi{}^2)\delta_{ab},$$

$$\rho_{ab}{}^{(1)}(m,V-V) = \rho_{ab}{}^{(2)}(m,V-V)=G_V{}^2(\rho)\delta(m^2-m_\rho{}^2)\delta_{ab},$$

$$\rho_{ab}{}^{(1)}(m,A-A)=G_A{}^2(A_1)\delta(m^2-m_{A_1}{}^2)\delta_{ab},$$

$$\rho_{ab}{}^{(2)}(m,A-A) = \rho_{ab}{}^{(1)}(m,A-A)+\tfrac{1}{2}f_\pi{}^2m_\pi{}^2\delta(m^2-m_\pi{}^2)\delta_{ab},$$

$$\rho_{ab}{}^{(1)}(m,T-T)=G_T{}^2(B)\delta(m^2-m_B{}^2)\delta_{ab},$$

$$\rho_{ab}{}^{(2)}(m,T-T) = G_T{}^2(B)\delta(m^2-m_B{}^2)\delta_{ab}+G_T{}^2(\rho)\delta(m^2-m_\rho{}^2)\delta_{ab},$$

$$\rho_{ab}(m,V-T)=-(1/m_\rho)G_V(\rho)G_T(\rho)\delta(m^2-m_\rho{}^2)\delta_{ab},$$

$$\rho_{ab}(m,P-A)=-(1/\sqrt{2})f_\pi G_P(\pi)\delta(m^2-m_\pi{}^2)\delta_{ab},$$

$$\rho_{ab}(m,A-\tilde{T})=\rho_{ab}(m,S-V)=0.$$

The last relation follows simply from the G conjugation. Examples of $a=4$, $b=5$ are

$$\rho_{45}(m,S-V)=-G_S(\kappa)G_V(\kappa)\delta(m^2-m_\kappa{}^2),$$

$$\rho_{45}(m,A-\tilde{T}) = (1/m_{K_A})G_T(K_A)G_A(K_A)\delta(m^2-m_{K_A}{}^2) + (1/m_{K_A}')G_T(K_A')G_A(K_A')\delta(m^2-m_{K_A'}{}^2),$$

$$\rho_{45}{}^{(i)}(m,V-V)=\rho_{45}{}^{(i)}(m,A-A) = \rho_{45}{}^{(i)}(m,T-T)=0 \quad (i=1,2).$$

II. SUM RULES

We can now apply the sum rules given in I. First, the zeroth-order $SU(6)_W$ sum rules, Eqs. (I19) and (I20), lead to the results

$$(1/m_\rho{}^2)G_V{}^2(\rho)=(1/m_{K^*}{}^2)G_V{}^2(K^*)+G_V{}^2(\kappa) = (1/m_{A_1}{}^2)G_A{}^2(A_1)+\tfrac{1}{2}f_\pi{}^2 = (1/m_{K_A}{}^2)G_A{}^2(K_A) + (1/m_{K_A'}{}^2)G_A{}^2(K_A')+\tfrac{1}{2}f_K{}^2, \quad (1)$$

$$(1/m_\rho{}^2)G_V{}^2(\rho)=(1/m_\rho{}^2)G_T{}^2(\rho)+(1/m_B{}^2)G_T{}^2(B) = (1/m_{K^*}{}^2)G_T{}^2(K^*)+(1/m_{K_A}{}^2)G_T{}^2(K_A) + (1/m_{K_A'}{}^2)G_T{}^2(K_A). \quad (2)$$

Note that Eq. (1) is nothing but the ordinary first Weinberg sum rule[3] with respect to the $SW(3)$ group.

Next, let us consider the sum rule (I15), which has been obtained from equal-time commutation relations (I14) and corresponds to the first-order broken $SU(6)_W$ symmetry

$$G_S(\kappa)G_V(\kappa) = (1/m_{K_A})G_A(K_A)G_T(K_A) + (1/m_{K_A'})G_A(K_A')G_T(K_A')=-\tfrac{1}{2}\sqrt{3}\xi_8, \quad (3)$$

$$-(1/\sqrt{2})f_\pi G_P(\pi)=(1/m_\rho)G_V(\rho)G_T(\rho) = (\sqrt{\tfrac{2}{3}})\xi_0+(\sqrt{\tfrac{1}{3}})\xi_8, \quad (4)$$

$$-(1/\sqrt{2})f_K G_P(K)=(1/m_{K^*})G_V(K^*)G_T(K^*) = (\sqrt{\tfrac{2}{3}})\xi_0-(\tfrac{1}{2}\sqrt{\tfrac{1}{3}})\xi_8. \quad (5)$$

We may remark that the first relation of Eq. (4), i.e., $-(\sqrt{\tfrac{1}{2}})f_\pi G_P(\pi)=(\sqrt{\tfrac{2}{3}})\xi_0+(\sqrt{\tfrac{1}{3}})\xi_8$, is exact in the soft-pion limit if we recall Eqs. (I14) and (I16). This fact suggests strongly that our pole approximation is a reasonably good one. Similarly, the corresponding relation of Eq. (5) implies that the soft-kaon limit is also reasonable since it leads to the exact validity of the relation.

Eliminating ξ_0 and ξ_8 from Eqs. (3)–(5), we have one more relation:

$$G_S(\kappa)G_V(\kappa)=(1/\sqrt{2})[f_\pi G_P(\pi)-f_K G_P(K)] = -\tfrac{1}{2}\sqrt{3}\xi_8. \quad (6)$$

This relation will be useful in the later applications.

Finally, the second-order broken $SU(6)_W$ sum rules (I26) are written as

$$G_S{}^2(\epsilon)+G_P{}^2(\pi)=\tfrac{3}{2}[G_A{}^2(A_1)+G_V{}^2(\rho)]-\tfrac{1}{4}f_\pi{}^2m_\pi{}^2, \quad (7)$$

$$G_S{}^2(\epsilon)-G_P{}^2(\pi)=\tfrac{3}{4}[G_A{}^2(A_1)-G_V{}^2(\rho)] -\tfrac{1}{8}f_\pi{}^2m_\pi{}^2=G_T{}^2(B)-G_T{}^2(\rho), \quad (8)$$

$$G_S{}^2(\kappa)+G_P{}^2(K)=\tfrac{3}{2}[G_A{}^2(K_A)+G_A{}^2(K_A')+G_V{}^2(K^*)] -\tfrac{1}{4}f_K{}^2m_K{}^2-\tfrac{1}{2}m_\kappa{}^2G_V{}^2(\kappa), \quad (9)$$

[3] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

$$G_S^2(\kappa)-G_P^2(K)=\tfrac{3}{4}[G_A^2(K_A)+G_A^2(K_A')-G_V^2(K^*)]-\tfrac{1}{8}f_K^2m_K^2+\tfrac{1}{4}m_\kappa^2G_V^2(\kappa)=G_T^2(K_A)+G_T^2(K_A')-G_T^2(K^*)\,. \quad (10)$$

These are all we derive from our asymptotic $SU(6)_W$ symmetries. Obviously, we do not have a sufficient number of equations to determine all these coupling parameters. Here we assume, in addition, the validity of the ordinary asymptotic $SW(2)$ [but not $SW(3)$] sum rules for vector and axial-vector currents:

$$G_V^2(\rho)=G_A^2(A_1)\,, \quad (11a)$$

$$G_V^2(K^*)=G_A^2(K_A)+G_A^2(K_A')\,. \quad (11b)$$

Notice that we do *not* assume the second $SU(3)$ sum rules such as $G_V^2(K^*)=G_V^2(\rho)$.

Now, first from Eqs. (1), (2), (7), (8), and (11a), we obtain the relations

$$G_P^2(\pi)=\tfrac{3}{2}G_V^2(\rho)-\tfrac{1}{16}f_\pi^2m_\pi^2\,, \quad (12a)$$

$$G_S^2(\epsilon)=\tfrac{3}{2}G_V^2(\rho)-\tfrac{3}{16}f_\pi^2m_\pi^2\,, \quad (12b)$$

$$G_T^2(\rho)=\frac{m_B^2}{m_B^2+m_\rho^2}G_V^2(\rho)+\frac{1}{8}\frac{m_\pi^2m_\rho^2}{m_B^2+m_\rho^2}f_\pi^2, \quad (12c)$$

$$G_T^2(B)=\frac{m_B^2}{m_B^2+m_\rho^2}G_V^2(\rho)-\frac{1}{8}\frac{m_\pi^2m_B^2}{m_B^2+m_\rho^2}f_\pi^2. \quad (12d)$$

The results for $G_T(\rho)$ and $G_T(B)$ differ slightly from those obtained by other authors.[4] This is expected, of course, since our sum rules differ from those used by these authors.

In order to analyze our results, it is convenient to introduce a parameter x by

$$x=G_V(\rho)/m_\rho f_\pi. \quad (13)$$

Note that, if the Kawarabayashi-Suzuki–Riazuddin-Fayazuddin (KSRF) formula[5] is exact, then we would have $x=1$, while from Eqs. (11a) and (11b), together with Eq. (1), we have[3]

$$x^2=\tfrac{1}{2}m_{A_1}^2/(m_{A_1}^2-m_\rho^2)\simeq 1.03\,. \quad (14)$$

Alternatively, the value of x can be directly computed from $\rho^0\to e\bar{e}$ rate by the formula

$$\Gamma(\rho^0\to e\bar{e})=\tfrac{4}{3}\pi(e^2/4\pi)^2(1/m_\rho)f_\pi^2x^2.$$

Using the experimental value $|f_\pi|=130$ MeV from $\pi^+\to\mu^+\nu$ decay rate, we find $x^2\simeq 1.25\pm 0.15$ when we use the experimental value for $\Gamma(\rho^0\to e\bar{e})$ given by Ting.[6] In this note, we regard x as an adjustable parameter with value near unity. Then one can rewrite Eqs. (12) as

$$G_P^2(\pi)=\tfrac{3}{2}G_V^2(\rho)\left[1-\frac{1}{24}\frac{1}{x^2}\left(\frac{m_\pi}{m_\rho}\right)^2\right], \quad (15a)$$

$$G_S^2(\epsilon)=\tfrac{3}{2}G_V^2(\rho)\left[1-\frac{1}{8}\frac{1}{x^2}\left(\frac{m_\pi}{m_\rho}\right)^2\right], \quad (15b)$$

$$G_T^2(\rho)=\frac{m_B^2}{m_B^2+m_\rho^2}G_V^2(\rho)\left[1+\frac{1}{8}\frac{1}{x^2}\left(\frac{m_\pi}{m_B}\right)^2\right], \quad (15c)$$

$$G_T^2(B)=\frac{m_B^2}{m_B^2+m_\rho^2}G_V^2(\rho)\left[1-\frac{1}{8}\frac{1}{x^2}\left(\frac{m_\pi}{m_\rho}\right)^2\right]. \quad (15d)$$

Inserting values of $G_P(\pi)$, $G_T(\rho)$, and $G_T(B)$ given by Eqs. (15) into Eq. (4), we find the sum rule

$$3\left[1-\frac{1}{24}\frac{1}{x^2}\left(\frac{m_\pi}{m_\rho}\right)^2\right]=\frac{4m_B^2}{m_B^2+m_\rho^2}\left[x^2+\frac{1}{8}\left(\frac{m_\pi}{m_B}\right)^2\right]. \quad (16)$$

Using experimental values[2] for m_B and m_ρ, we find

$$x^2=1.05\,, \quad (17)$$

which is close the the previous value, Eq. (14).

We also remark that the exact $U(6,6)$ theory[7] gives $G_P(\pi)=G_V(\rho)$, which differs by 20% from our value Eq. (15a). Actually, we may also obtain $x=-1/\sqrt{2}$ and $G_T(\rho)=G_V(\rho)$ if we follow the prescription given by Sakita and Wali.[7] However, these second relations critically depend upon a form assumed for the mass term in the Bargmann-Wigner equation, as we shall show in the Appendix.

We shall now proceed to the discussion of the K-type sum rules. We have to solve the coupled equations (1), (2), (3), (5), (6), (9), (10), and (11b). This is a very complicated problem and we seek here a numerical solution in which $G_V(\kappa)$ is relatively small and f_K/f_π is of the order of unity. It appears that it is difficult to obtain a consistent solution of these equations when we take $m_{K_A'}=1320$ and $m_{K_A}=1240$ MeV. Hence, we assume $m_{K_A'}=1780$ and $m_{K_A}=1240$ MeV. Then we evaluate

$$f_K/f_\pi\simeq 1.07\,, \quad (18a)$$

$$G_V(\kappa)\simeq -0.14f_\pi\,, \quad (18b)$$

$$G_P(K)\simeq 1.27m_{K^*}f_\pi\,, \quad (18c)$$

$$G_S(\kappa)\simeq 1.24m_{K^*}f_\pi\,, \quad (18d)$$

$$G_P(\pi)\simeq 1.27m_\rho f_\pi\,, \quad (18e)$$

where we assumed $x^2=1.05$, i.e., Eq. (17) with $m_\kappa=1100$ MeV. We remark that values for f_K/f_π, $G_P(K)$, and $G_S(\kappa)$ are relatively insensitive to value used for m_{K_A},

[4] M. Ademollo, G. Longhi, and G. Veneziano, Nuovo Cimento **58A**, 540 (1968); P. A. Cook and G. C. Joshi, Nucl. Phys. **B10**, 253 (1969).

[5] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayazuddin, Phys. Rev. **144**, 1071 (1966).

[6] S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 43.

[7] B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965).

$m_{K_A'}$, and m_κ and hence they are more trustworthy. However, the value $G_V(\kappa)$ is quite sensitive to small variations for these mass values and may vary as much as 50%. It is gratifying[8] to have f_K/f_π near unity. Notice that our value of $G_V(\kappa)$ is somewhat smaller than one obtained by Glashow and Weinberg,[9] who give $G_V(\kappa)/f_\pi| \sim 0.41$ in our notation.

Actually, relative signs among f_π, $G_P(\pi)$, $G_P(K)$, and $G_S(\kappa)$ cannot be determined by our equations. By choosing suitable phase factors for π and ρ state vectors, we can assume hereafter that $f_\pi \simeq 130$ MeV>0 and $x\simeq 1.02>0$ without loss of generality. Then, we have selected appropriate relative signs in Eqs. (18) so that they will be consistent with results of Sec. III.

We can now compute ξ_8 and ξ_0 from Eqs. (3)–(5) to get

$$\xi_0=-1.24 m_\rho f_\pi^2, \quad \xi_8=+0.23 m_\rho f_\pi^2. \tag{19}$$

The fact that $\xi_8/\xi_0 \approx -0.18$ may be interpreted to indicate that the $SU(3)$-violating effect is of the order of 20%.

Finally, we remark that in our approach we need the existence of several octets for 0^{++}, 1^{++}, and 1^{+-} mesons, in addition to the ordinary 0^{--} and 1^{-+} multiplets, so as to saturate our sum rules. For example, if we have no B-meson contribution, we will have a serious difficulty. Similarly, the existence of scalar mesons ϵ and κ, and of two kinds of 1^+ mesons K_A and K_A' is necessary to satisfy our sum rules. It is amusing to observe that we need more particles than required by the linear realization of the $U(6,6)$ group. Indeed, the numbers of our multiplets are somewhat similar to those predicted by the nonlinear realization of the $SL(6,C)\otimes SL(6,C)$ or $SL(12,C)$ group as has been given by Gürsey and Chang.[10]

III. CONSEQUENCES OF PARTIAL CONSERVATION OF VECTOR CURRENT AND MASS FORMULAS

In our derivation of asymptotic sum rules, the underlying assumption was that the $SU(3)$ invariance is broken only by mass difference among quarks. Then, if m_1, $m_2(\equiv m_1)$, and m_3 are bare masses of three quarks, we should have the partially conserved vector current (PCVC) conditions

$$\begin{aligned}\partial_\mu V_\mu{}^{(4-i5)}(x)&=i(m_3-m_1)S^{(4-i5)}(x)\,,\\ \partial_\mu V_\mu{}^{(4+i5)}(x)&=-i(m_3-m_1)S^{(4+i5)}(x)\,.\end{aligned} \tag{20}$$

In this section, we shall explore consequences of Eqs. (20). First, taking a matrix element of both sides with respect to the vacuum and one κ^+ state, one immediately finds

$$(m_3-m_1)G_S(\kappa)=-m_\kappa{}^2 G_V(\kappa)\,. \tag{21}$$

Using values of $G_S(\kappa)$ and $G_V(\kappa)$ given by Eqs. (18b) and (18d), this gives

$$m_3-m_1\simeq 150 \text{ MeV}\,, \tag{22}$$

where we have again assumed $m_\kappa \simeq 1100$ MeV.

Next, let us take a matrix element of both sides of Eqs. (20) between one-π^0 and one-K^+ states:

$$\begin{aligned}i(p_\mu-k_\mu)\langle\pi^0(k)|V_\mu{}^{(4-i5)}(0)|K^+(p)\rangle\\ =i(m_3-m_1)\langle\pi^0(k)|S^{(4-i5)}(0)|K^+(p)\rangle.\end{aligned} \tag{23}$$

Now we take the standard soft-pion limit $k_\mu \to 0$ on both sides of Eq. (23), with

$$\lim_{k\to 0}\langle\pi^0(k)|V_\mu{}^{(4-i5)}(0)|K^+(p)\rangle =(4k_0p_0V^2)^{-1/2}\left(-\frac{1}{\sqrt{2}}\frac{f_K}{f_\pi}\right)p_\mu\,, \tag{24}$$

$$\lim_{k\to 0}\langle\pi^0(k)|S^{(4-i5)}(0)|K^+(p)\rangle =(4k_0p_0V^2)^{-1/2}\frac{1}{f_\pi}G_P(K)\,. \tag{25}$$

Inserting these results into Eq. (23), one obtains

$$(m_3-m_1)G_P(K)=(1/\sqrt{2})f_K m_K{}^2\,. \tag{26}$$

Because of Eqs. (18a) and (18c), we have

$$m_3-m_1\simeq 160 \text{ MeV}\,, \tag{27}$$

which is very close to the previous value Eq. (22). Actually, if we use Eqs. (21) and (26), together with sum rules of Sec. II, we could have computed the mass of the κ meson to be around $m_\kappa \simeq 1100$ MeV.

Similarly, we compute

$$I=\int d^4x\langle 0|(S^{(4-i5)}(x),P^{(4+i5)}(0))_+|\pi^0(k)\rangle\,. \tag{28}$$

We can evaluate this expression in two different ways. First, we use the PCVC conditions (20) to get

$$I=(2k_0V)^{-1/2}(-i)G_P(\pi)/m_3-m_1\,. \tag{29}$$

On the other hand, we can calculate Eq. (28) by taking the soft-pion limit $k_\mu \to 0$. Then, comparing the result with Eq. (29), we derive the sum rule

$$\sqrt{2}(m_3-m_1)\int_0^\infty dm^2\frac{1}{m^2}[\rho_{44}(m,P-P)-\rho_{44}(m,S-S)] =f_\pi G_P(\pi)\,. \tag{30}$$

[8] H. T. Nieh [Phys. Rev. Letters **19**, 43 (1967)] computes f_K/f_π to be 1.17 on the basis of the ordinary $SW(3)$ Weinberg sum rules. However, he uses only one K_A ($M=1330$ MeV) with the exact KSRF relation, i.e., $x=1$, and neglects the κ-meson contribution.

[9] See, e.g., S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 253.

[10] F. Gürsey and P. Chang, Phys. Letters **26B**, 520 (1968); F. Gürsey, paper presented at the Symposium on Hadron Spectroscopy, Keszthely, 1968 (unpublished).

However, in the pole approximation, this equation turned out to give Eq. (26) again, if we use Eqs. (6) and (21).

Next, let us take matrix elements of both sides of Eqs. (20) with respect to one-baryon states. For example, we have

$$(p_\mu-p_\mu')\langle P(p')|V_\mu{}^{(4+i5)}(0)|\Lambda(p)\rangle = -(m_3-m_1)\langle P(p')|S^{(4+i5)}(0)|\Lambda(p)\rangle. \quad (31)$$

For small momentum transfer, one can set

$$\langle P(p')|V_\mu{}^{(4+i5)}(0)|\Lambda(p)\rangle \simeq i(m_\Lambda m_P/p_0p_0'V^2)^{1/2}G_{P\Lambda}\bar{u}(p')\gamma_\mu u(p), \quad (32a)$$

$$\langle P(p')|S^{(4+i5)}(0)|\Lambda(p)\rangle \simeq (m_\Lambda m_P/p_0p_0'V^2)^{1/2}S_{P\Lambda}\bar{u}(p')u(p), \quad (32b)$$

which leads to

$$(m_\Lambda-m_N)G_{P\Lambda}\simeq(m_3-m_1)S_{P\Lambda}. \quad (33)$$

The Ademollo-Gatto theorem[11] will give

$$G_{P\Lambda}\simeq-\tfrac{1}{2}\sqrt{6}, \quad (34)$$

although we need not use it for most of our applications except for the fact that $G_{P\Lambda}$ is not identically zero.

Now, if we appeal to the $U(6,6)$ theory,[7] we have

$$S_{P\Lambda}\approx G_{P\Lambda}. \quad (35)$$

Together with Eq. (33), this gives

$$m_\Lambda-m_N\simeq m_3-m_1. \quad (36)$$

Actually, without using the $U(6,6)$, we can obtain Eqs. (35) and (36) as follows: In the quark model, all baryons are supposed to consist of three quarks. Since they do not contain any antiquark, one may replace $\bar{q}$ by q^+ in the static limit. Hence, in this limit, one can approximate

$$S^{(4\pm i5)}(x)\approx -iV_4{}^{(4\pm i5)}(x). \quad (37)$$

This immediately reproduces Eq. (35), and thus Eq. (36), in the static limit. Of course Eq. (37) is not Lorentz-invariant and we have to use it only in the rest system. Actually, this naive approach immediately gives us the relations

$$m_\Sigma\simeq m_\Lambda, \quad (38a)$$

$$m_\Xi-m_\Lambda\simeq m_\Lambda-m_N\simeq m_3-m_1, \quad (38b)$$

$$m(\Omega)-m(\Xi^*)\simeq m(\Xi^*)-m(Y_1^*) \simeq m(Y_1^*)-m(N^*)\simeq m_3-m_1. \quad (38c)$$

Alternatively, these can be derived if we appeal to the $U(6,6)$, so that matrix elements of $S^{(4+i5)}(x)$ and $V_\mu{}^{(4+i5)}(x)$ are the same as in Eq. (35). From Eqs. (38), one can estimate m_3-m_1 again by

$$m_3-m_1\simeq\tfrac{1}{2}(m_\Xi-m_N)\simeq187 \text{ MeV}, \quad (39)$$

$$m_3-m_1\simeq M(Y_1^*)-M(N^*)\simeq150 \text{ MeV}. \quad (39')$$

[11] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1965).

These are again in reasonable agreement with the previous estimates Eqs. (22) and (27), in both sign and magnitude. Actually, we can improve the estimate Eq. (39) as follows: If we use the standard $SU(3)$-perturbation mass formula in the first order, we obtain

$$(m_3-m_1)S_{P\Lambda} \simeq -\tfrac{1}{2}(\sqrt{6})[\tfrac{1}{2}(m_\Xi-m_N)-\tfrac{1}{4}(m_\Sigma-m_\Lambda)]. \quad (40)$$

Now, together with Eqs. (34) and (35), this gives

$$m_3-m_1\simeq\tfrac{1}{2}(m_\Xi-m_N)-\tfrac{1}{4}(m_\Sigma-m_\Lambda)\simeq168 \text{ MeV}, \quad (39'')$$

which is closer to other values.

Our approximation (37) will not hold for mesons, since mesons contain antiquarks, so that the replacement of $\bar{q}$ by q^+ is by no means justified. We remark also that analogs of the relations (38) should be valid for all other nonexotic baryon multiplets, which are, by definition, bound states of three-quarks. This simple rule may be useful to phenomenological $SU(3)$ classifications of higher baryon resonances. For example, $N^*(1518)$, $Y_0^*(1690)$, $Y_1^*(1670)$, and $\Xi^*(1820)$ excellently obey the analog of our mass formulas (38a) and (38b) with $m_3-m_1\simeq150$ MeV. Therefore, they may form an octet with $J^P=\frac{3}{2}^-$, although the spin-parity assignment of the $\Xi^*(1820)$ is not yet known.[2] Similarly, $N^*(1550)$ and $Y_0^*(1670)$ could be forming a part of a $\frac{1}{2}^-$ octet by our rule. A large deviation from Eqs. (28), if it happens, must be attributed to a presence of a sizable antiquark component in the multiplet under consideration. Our method also indicates that possible configuration mixing between two baryon multiplets does not affect our sum rule, indicating that its effect must somehow be reduced. This may account for apparent small mixing between $Y_0^*(1520)$ and $Y_0^*(1690)$, or between $N(940)$ and $N(1460)$.

Our results [Eqs. (38)] can be rewritten in the compact expression

$$m=M(I,Y)\simeq M_0-aY \quad (41)$$

for particle masses with the isospin I and the hypercharge Y, where M_0 is a constant independent of I and Y and a is the universal constant $a\simeq(m_3-m_1)\simeq150$ MeV. Equation (41) is the mass formula that must be valid for all nonexotic baryon multiplets. Also, it must be emphasized that our derivation does not depend upon a perturbation method with respect to the $SU(3)$-violating interaction. If the perturbation is used, Eq. (41) can be easily obtained, since in the first-order perturbation the mass shift will be given by

$$\Delta M_i=\langle i|H_I|i\rangle,$$

with

$$H_I=-(2/\sqrt{3})(m_3-m_1)\int d^3x\, S^{(8)}(x).$$

Replacing $S^{(8)}(x)$ by $-iV_4{}^{(8)}(x)$ according to the same reasoning, and noticing that the hypercharge operator

Y is given by

$$Y=(-i)(2/\sqrt{3})\int d^3x\, V_4{}^{(8)}(x)\,,$$

we get $\Delta M(i)=-(m_3-m_1)Y$ and, hence, we reproduce Eq. (41).

As we indicated, one cannot use Eq. (37) for mesons, so that we cannot derive relations like Eqs. (38). The use of the $U(6,6)$ theory is also not reliable in this case, since many relations so obtained critically depend upon the form assumed for the mass term in the Bargmann-Wigner equation, because of large mass differences among the 35-dimensional multiplet. However, one can nevertheless find some relations that are less dependent on assumed forms of the mass term (see Appendix). Then, together with the PCVC condition Eq. (20), one may obtain (see Appendix) the relations

$$4m_K{}^2=m_\pi{}^2+3m_\eta{}^2\,, \tag{42a}$$

$$m_\omega{}^2=m_\rho{}^2\,, \tag{42b}$$

$$m_\phi{}^2-m_{K^*}{}^2=m_{K^*}{}^2-m_\rho{}^2\,, \tag{42c}$$

$$m_{K^*}{}^2-m_\rho{}^2=m_K{}^2-m_\pi{}^2\,. \tag{42d}$$

These are nothing but the ordinary $SU(6)$ mass formulas.

At any rate, we find that our results are mutually consistent with the value $m_3-m_1\simeq 150$ MeV.

To conclude this section, we make the following two comments. First, the PCVC relations (20) give, among spectral weights,

$$(m_3-m_1)^2\rho_{44}(m,\,S-S) =m^2[\rho_{44}{}^{(2)}(m,\,V-V)-\rho_{44}{}^{(1)}(m,\,V-V)]\,, \tag{43a}$$

$$(m_3-m_1)\rho_{44}(m,\,S-S)=m^2\rho_{44}(m,\,S-V)\,. \tag{43b}$$

Then, the sum rule Eq. (I15) leads to

$$(m_3-m_1)\int_0^\infty dm^2\frac{1}{m^2}\rho_{44}(m,\,S-S)=\tfrac{1}{2}\sqrt{3}\,\xi_8\,. \tag{44}$$

It may be worthwhile to emphasize that the validity of this relation is independent of our asymptotic $SU(6)_W$ sum rules. Noticing the positiveness condition [Eqs. (I11) and (I12)] for the spectral weights, an assumption $\xi_8=0$ implies the exact conservation law $\partial_\mu V_\mu{}^{(4)}(x)=0$. Together with the validity of $SU(2)$ and hypercharge conservation, this implies the exact validity of the $SU(3)$ group. Therefore, it is impossible to have $\xi_8=\langle 0|S^{(8)}(0)|0\rangle\equiv 0$. This conclusion is independent of asymptotic $SU(6)_W$ symmetry, and it will be relevant to the theory of Gell-Mann, Oakes, and Renner,[12] who show that ξ_8 must be very small.

[12] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

Secondly, let us consider the partially conserved axial-vector current (PCAC) conditions

$$\begin{aligned}
\partial_\mu A_\mu{}^{(\alpha)}(x)&=(1/\sqrt{2})m_\pi{}^2 f_\pi \pi_\alpha(x) &&(\alpha=1,\,2,\,3)\,,\\
\partial_\mu A_\mu{}^{(\alpha)}(x)&=(1/\sqrt{2})m_K{}^2 f_K K_\alpha(x) &&(\alpha=4,\,5,\,6,\,7)\,,\\
\partial_\mu A_\mu{}^{(8)}(x)&=(1/\sqrt{2})m_8{}^2 f_8 \eta_8(x) &&(\alpha=8)\,.
\end{aligned} \tag{45}$$

In our approach, all mesons are supposed to be bound states of a quark-antiquark system. Thus, Eqs. (45) must be interpreted to define the fields $\pi_\alpha(x)$, $K_\alpha(x)$, and $\eta_8(x)$. Then, it is by no means obvious whether these form an octet operator or not. If we suppose they do, we must have, for instance, a relation such as

$$i\left[\int d^3x\, V_4{}^{(4-i5)}(x),K_{4+i5}(y)\right] =(1/\sqrt{2})\pi_3(y)+\tfrac{1}{2}(\sqrt{6})\eta_8(y)\quad (x_0=y_0)\,. \tag{46}$$

Together with Eqs. (20) and (45), this will give us nontrivial relations. To see this, let us consider the expression

$$I=i\langle 0|\int d^4x (S^{(4-i5)}(x),K_{4+i5}(y))_+|\pi_3(k)\rangle\,. \tag{47}$$

One can estimate this in two different ways. First, let us use the PCVC conditions (20) and integrate in part to obtain

$$I=(2k_0V)^{-1/2}(1/\sqrt{2})1/(m_3-m_1)\,,$$

where we used Eq. (46). On the other hand, Eq. (47) can be computed by means of Eqs. (45) when we integrate in part with respect to y rather than x. In this way, we get

$$I=(2k_0V)^{-1/2}G_P(\pi)/f_K m_K{}^2\,,$$

where we have neglected a term proportional to $k^2=-m_\pi{}^2$ (i.e., soft-pion limit). Equating both expressions, we find

$$(m_3-m_1)G_P(\pi)=(1/\sqrt{2})f_K m_K{}^2\,.$$

Comparing with Eq. (26), this gives

$$G_P(K)=G_P(\pi)\,, \tag{48a}$$

which in turn can be reexpressed as

$$\sqrt{2}\xi_0(f_K-f_\pi)=-\xi_8(f_K+\tfrac{1}{2}f_\pi)\,, \tag{48b}$$

if we use Eqs. (4) and (5). Unfortunately these relations are not so well satisfied by our numerical solution, although the defect may be due to the soft-pion assumption. The negative answer may imply that Eq. (46) is probably not valid and that $\pi_\alpha(x)$, $K_\alpha(x)$, and $\eta_8(x)$ defined by Eqs. (45) do not form an octet operator under the $SU(3)$ group. We note that this conclusion is manifest in the theory of Gell-Mann, Oakes, and Renner,[12] as we shall see in the next section.

IV. BARE MASSES OF QUARKS

In Sec. III we have computed the difference m_3-m_1. We can compute m_1 and m_3 individually if we further assume that not only $SU(3)$ but also $SW(3)$ group is exact except for the quark-mass term. Such a theory has been proposed and investigated by many authors.[13–16] However, several interesting results have been recently derived by Gell-Mann, Oakes, and Renner[12] on the basis of this model.

Setting, as usual,

$$\epsilon_0=(\sqrt{\tfrac{2}{3}})(2m_1+m_3)\,,\quad \epsilon_8=(2/\sqrt{3})(m_1-m_3)\,,\qquad(49)$$

our hypothesis implies

$$\partial_\mu A_\mu{}^{(a)}(x)=(\sqrt{\tfrac{2}{3}})\epsilon_0 P^{(a)}(x)+\epsilon_8 d_{8ab}P^{(b)}(x)\qquad(50)$$

in addition to Eqs. (20). In terms of components, this is rewritten as

$$\partial_\mu A_\mu{}^{(a)}(x)=(m_1+m_2)P^{(a)}(x)\quad(a=1,2,3)\,,\qquad(51\text{a})$$

$$\partial_\mu A_\mu{}^{(a)}(x)=(m_1+m_3)P^{(a)}(x)\quad(a=4,5,6,7)\,,\qquad(51\text{b})$$

$$\partial_\mu A_\mu{}^{(8)}(x)=\tfrac{2}{3}(m_1+2m_3)P^{(8)}(x)+\tfrac{2}{3}\sqrt{2}(m_1-m_3)P^{(0)}(x)\quad(a=8)\,,\qquad(51\text{c})$$

$$\partial_\mu A_\mu{}^{(0)}(x)=\tfrac{2}{3}(2m_1+m_3)P^{(0)}(x)+\tfrac{2}{3}\sqrt{2}(m_1-m_3)P^{(8)}(x)\quad(a=0)\,.\qquad(51\text{d})$$

Comparing these with Eqs. (45), it is obvious that $\pi_\alpha(x)$, $K_\alpha(x)$, and $\eta_8(x)$, so defined, will not form a octet operator unless $m_1=m_3$, as has been mentioned in Sec. III.

Taking matrix elements of both sides of Eqs. (51a) and (51b) between the vacuum and the π or K state, one gets

$$f_\pi m_\pi{}^2=\sqrt{2}(m_1+m_2)G_P(\pi)\,,\qquad(52\text{a})$$

$$f_K m_K{}^2=\sqrt{2}(m_1+m_3)G_P(K)\,.\qquad(52\text{b})$$

Using Eqs. (18), we calculate

$$m_1=m_2\simeq 7\text{ MeV}\,,\quad m_3\simeq 156\text{ MeV}\,,\qquad(53)$$

which is consistent with $m_3-m_1\simeq 150$ MeV obtained in Sec. III. Actually, Eqs. (52) give Eq. (26) in the soft-pion limit $m_\pi{}^2=0$. Our estimates [Eqs. (53)] are in rough accord with $m_1/m_3\simeq 0.042$ obtained by Gell-Mann, Oakes, and Renner. Similarly, small values for m_1 and m_3 have been commented on by several authors.[15,17,18] In particular, Tomozawa[18] has recently derived a high-energy theorem on the basis of the algebra of currents. He finds that nonzero values of m_1 and m_3 may be related to a possible breakdown of the Pomeranchuk theorem at high energy and, by using experimental data on high-energy $\sigma(\pi^+p)-\sigma(\pi^-p)$ and $\sigma(K^+p)-\sigma(K^-p)$ cross sections, he estimates $m_1\simeq 3$ or 9 MeV and $m_3\simeq 70$ MeV. These values are reasonably close to our values [Eqs. (53)]. Similarly, our estimates, Eqs. (34) and (35), or the analogous relations for similar quantities, are, if we use Eq. (53), in rough agreement with experimental scattering lengths of meson-baryon scatterings, as has been shown by Hippel and Kim.[19] Although these facts are, indeed, very encouraging, we may have some difficulties with this model.

From Eqs. (51c) and (51d), we obtain

$$\partial_\mu A_\mu{}^{(8)}(x)+\sqrt{2}\partial_\mu A_\mu{}^{(0)}(x)=2m_1[P^{(8)}(x)+\sqrt{2}P^{(0)}(x)]$$
$$=(1/\sqrt{3})(\sqrt{2}\epsilon_0+\epsilon_8)[P^{(8)}(x)+\sqrt{2}P^{(0)}(x)]\,.\qquad(54)$$

Therefore, in the $SW(2)$ limit $m_1\rightarrow 0$, we have

$$\partial_\mu A_\mu{}^{(8)}(x)=-\sqrt{2}\partial_\mu A_\mu{}^{(0)}(x)\,,\qquad(55)$$

which gives relations such as

$$f_\eta=-\sqrt{2}\sigma_\eta\,,\qquad(56)$$

where f_η and σ_η are defined by

$$\sqrt{2}\langle 0|A_\mu{}^{(8)}(0)|\eta(k)\rangle=(2k_0V)^{-1/2}ik_\mu f_\eta\,,$$
$$\sqrt{2}\langle 0|A_\mu{}^{(0)}(0)|\eta(k)\rangle=(2k_0V)^{-1/2}ik_\mu\sigma_\eta\,.\qquad(57)$$

Unfortunately, Eq. (56) is rather difficult to accept from the $SU(3)$ viewpoint, since, ordinarily, the η meson is expected to be dominantly an octet so that we should have $|\sigma_\eta|\ll|f_\eta|$, in contrast to Eq. (56). Thus, if we accept the Gell-Mann–Oakes–Renner model, we must be prepared for a large $SU(3)$ violation. Actually, a large discrepancy between $\pi^0\rightarrow 2\pi$ and $\eta\rightarrow 2\gamma$ decay ratio may be due to such a mechanism. Similarly, if the E meson is a 0^- meson, then the abnormally large decay[2] $E\rightarrow K^*\bar{K}$ might be explained by a large $SU(3)$ violation, provided that the E meson is a unitary singlet.

Finally, it should be emphasized that the smallness of the bare masses of quarks will not necessarily imply smallness of physical quark masses because of strong interactions. Of course, it is very dangerous to believe in the literal existence of quarks and in the Lagrangian formalism we are more or less utilizing. Our hope is that at least some features of our results may survive in the future theory, as did Bohr's semiclassical theory of the hydrogen atom even after the establishment of the quantum mechanics.

APPENDIX

Here we investigate in some detail the $U(6,6)$ version of the Bargmann-Wigner equation for mesons. It can

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be written as

$$\tfrac{1}{2}[\gamma_\lambda,(\partial/\partial x_\lambda)\Phi(x)]+M\Phi(x)=0\,, \qquad \text{(A1)}$$

where we regard $\Phi(x)$ to be a 12×12 matrix. The most general form of the mass operator M is obtained by assuming

$$M\Phi(x)=m_S\Phi+m_P(\gamma_5\Phi\gamma_5)+m_V(\gamma_\lambda\Phi\gamma_\lambda)+m_A(i\gamma_\lambda\gamma_5)\Phi(i\gamma_\lambda\gamma_5)+m_T\sum_{\lambda>\rho}(i\gamma_\lambda\gamma_\rho)\Phi(i\gamma_\lambda\gamma_\rho)\,, \qquad \text{(A2)}$$

where m_S, m_P, m_V, m_A, and m_T are independent of γ matrices, and operate on the $SU(3)$ space by the formula

$$m_Q\Phi=m_Q{}^{(1)}\Phi+m_Q{}^{(2)}(\delta\Phi+\Phi\delta)\,. \qquad \text{(A3)}$$

In Eq. (A3), $m_Q{}^{(1)}$ and $m_Q{}^{(2)}$ ($Q=S, P, V, A, T$) are c numbers and the 3×3 matrix δ is given by

$$\delta=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}. \qquad \text{(A4)}$$

Now, following Sakita and Wali,[7] we expand Φ into

$$\Phi(x)=S'(x)+i\gamma_5P'(x)+i\gamma_\mu V_\mu'(x)+i\gamma_5\gamma_\mu A_\mu'(x)+\tfrac{1}{4}i[\gamma_\mu,\gamma_\nu]T_{\mu\nu}'(x)\,, \qquad \text{(A5)}$$

where S', P', V', A', and T' are 3×3 matrices independent of γ matrices. We added primes on these quantities so as to avoid a possible confusion with those used in I.

Inserting Eq. (A5) into Eq. (A1), we obtain

$$\begin{aligned}
&(m_S+m_P+4m_V+4m_A+6m_T)S'(x)=0\,,\\
&(m_S-m_P-4m_V-4m_A+6m_T)P'(x)=-(\partial/\partial x_\mu)A_\mu'(x)\,,\\
&(m_S-m_P-2m_V+2m_A)V_\mu'(x)=(\partial/\partial x_\nu)T_{\mu\nu}'(x)\,,\\
&(m_S-m_P+2m_V-2m_A)A_\mu'(x)=-(\partial/\partial x_\mu)P'(x)\,,\\
&(m_S+m_P-2m_T)T_{\mu\nu}'(x)=(\partial/\partial x_\nu)V_\mu'(x)-(\partial/\partial x_\mu)V_\nu'(x)\,.
\end{aligned} \qquad \text{(A6)}$$

From these equations, we find

$$\begin{aligned}
(\Box-M_P{}^2)P'(x)&=0\,,\\
(\Box-M_V{}^2)V_\mu'(x)&=0\,,
\end{aligned} \qquad \text{(A7)}$$

where

$$\begin{aligned}
M_P{}^2&=(m_S-m_P+2m_V-2m_A)\times(m_S+m_P-4m_V-4m_A+6m_T)\,,\\
M_V{}^2&=(m_S+m_P-2m_T)(m_S-m_P-2m_V+2m_A)\,.
\end{aligned} \qquad \text{(A8)}$$

Only experimentally determinable quantities are mass matrices $M_P{}^2$ and $M_V{}^2$. Clearly, we have insufficient numbers of equations to determine all of m_S, m_P, m_V, m_A, and m_T.

Now, writing Φ as $\Phi_B{}^A$ ($A, B=1, 2, \ldots, 12$) in the tensor notation, we can compute

$$\langle 0|\bar{q}_A(x)q_B(x)|\Phi\rangle\propto\Phi_B{}^A$$

in the exact $U(6,6)$ limit. Thus, one can calculate $G_P(\pi)$ and $G_V(\rho)$ from Eqs. (A5) and (A6), to get

$$G_P(\pi)=G_V(\rho)=G_P(K)=G_V(K^*)\,. \qquad \text{(A9)}$$

However, the relation between $G_V(\rho)$ and f_π is dependent upon special choices for m_Q. Sakita and Wali choose essentially $m_S{}^{(1)}$, $m_S{}^{(2)}$, and $m_P{}^{(2)}\neq0$, with all other m_Q being zero, and this gives $x=-1/\sqrt{2}$, which is not so good experimentally. Similarly, the ordinary $U(6,6)$ relations between coupling constants for VVP and VPP vertices are dependent upon the choices of m_Q.

We next consider the matrix element

$$\langle\Phi|\bar{q}_A(x)q_B(x)|\Phi\rangle.$$

In the exact $U(6,6)$ limit, this is proportional to the matrix element of $\Phi_C{}^A\Phi_B{}^C$. Again, we find that most of the relations so obtained depend upon the special choice of m_Q. However, one can obtain some relations that are independent of the choice. Some such relations are

$$\begin{aligned}
\langle\rho^0|S^{(4-i5)}(0)|K^{*+}\rangle&=\langle\omega|S^{(4-i5)}(0)|K^{*+}\rangle\\
&=-1/\sqrt{2}\langle\phi|S^{(4-i5)}(0)|K^{*+}\rangle\\
&=\langle\pi^0|S^{(4-i5)}(0)|K^+\rangle\\
&=-\sqrt{3}\langle\eta|S^{(4-i5)}(0)|K^+\rangle.
\end{aligned} \qquad \text{(A10)}$$

Together with the Ademollo-Gatto theorem for the matrix elements of $V_\mu{}^{(4-i5)}(x)$, Eqs. (A10) and (20) give the desired relations (42).