Asymptotic $SU(6)_W$ Spectral Sum Rules.* I. Formulation

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Spectral sum rules on the basis of asymptotic $SU(6)_W$ symmetry are given. It is shown that the $SU(6)_W$ group is likely to be a reasonably good asymptotic symmetry as far as the two-point function is concerned.

I. INTRODUCTION

In recent years, we have seen several interesting applications of so-called Weinberg sum rules.[1] While many of these sum rules can be derived on the basis of the algebra of currents, the physics underlying them is also not difficult to understand. As Lehmann and Källén have observed[2] some years ago, the Fourier transform of two-point causal Green's functions of, say, two scalar mesons has exactly the same asymptotic form as that of the corresponding bare Green's functions in the infinite-momentum limit. This fact may be interpreted to imply physically that the kinematical term will dominate eventually in the high-energy region over all mass and interaction Hamiltonians. If this reasoning is correct, we can easily understand the reason for the validity of the Weinberg sum rules. By the same logic, in the infinite-momentum limit, we should expect[3] that the kinematical part of our Lagrangian will dominate over all others, so that the causal two-body Green's functions should manifest symmetries possessed by the kinematical part in that limit. Thus, we should have validities of asymptotic $SU(3)$ and $SW(3)$ symmetries, which immediately give the first Weinberg sum rules for these groups. As for second Weinberg sum rules, we may probably see some effects of the mass and interaction Lagrangian in the asymptotic symmetry, which may violate these groups. It is then necessary for us to consider at least the first-order breaking of the $SU(3)$ or $SW(3)$ group in the study of the second sum rules.[4] Note that such behavior can be demonstrated in many models, especially the cases of ordinary Lehmann-Källén sum rules, as has been emphasized elsewhere.[4]

It is clear from the above argument that the ordinary static $SU(6)$ group cannot become an asymptotic symmetry because the free-quark Lagrangian is not invariant under it. However, the so-called $SU(6)_W$ group[5] can, in contrast, be a candidate for the asymptotic symmetry, since in a sense it leaves the kinematical part of the Lagrangian invariant. As a matter of fact, one can construct[6] a Lagrangian invariant under a much larger group $SL(12,C)$ if we admit a nonlinear realization. As we shall see shortly, we can show that the $SU(6)_W$ appears to be a reasonably good asymptotic symmetry for the two-point function, at least up to its first-order breaking.

On the other hand, we remark that the simple idea of asymptotic symmetry must be somewhat modified for scattering problems, first because the interaction Hamiltonian is now responsible for the scattering and, secondly, because the energy-momentum must satisfy the mass-shell condition $K_\mu^2=-m^2$ so that one cannot take both spatial and time components of the four-momentum k_μ to be independently large. Nevertheless, even for this case, it is tempting to apply a similar argument. Suppose, for example, that both kinematical and interacting Lagrangians are invariant under the $SU(3)$ group and that it is only a part of the mass term that violates the $SU(3)$ symmetry. Then, if one can find a suitable experimental condition in which the part of the mass term violating the $SU(3)$ would give a relatively minor correction in comparison to the rest, we will have an approximate $SU(3)$ symmetry. Such a situation hopefully may be operating in the high-energy large-angle scattering region $s\gg|t|\gg\mu^2$. In contrast, for the forward high-energy scattering, mass differences between π and K (or ρ and K^*) are certainly not negligible, as we may see from the peripheral or Regge exchange mechanism. Hence, we may hope for the validity of asymptotic $SU(3)$, or even $SW(3)$ symmetries, only for the extreme high-energy large-angle scattering.[7] As a matter of fact, the experiment shows that the $SU(3)$ is reasonably well satisfied[8] at such a region for reactions $\gamma p\rightarrow\pi^+n$, $K^+\Lambda$, and $K^+\Sigma_0$, but not at the forward direction. Also, we note that one can

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[1] For a comprehensive account of this subject, see S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[2] See, e.g., P. Roman, *Introduction to Quantum Field Theory* (John Wiley & Sons, Inc., New York, 1968), Chap. II.

[3] This idea is essentially an extension of the so-called Källén's conjecture to internal symmetries.

[4] S. Okubo, Lecture Notes at University of Islamabad, 1967 (unpublished); in *Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, New York, 1967*, edited by C. R. Hagen *et al.* (Wiley-Interscience, Inc., New York, 1968).

[5] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965); K. J. Barnes, *ibid.* **14**, 798 (1965).

[6] P. Chang and F. Gürsey, Phys. Letters **26B**, 520 (1968), F. Gürsey, a paper presented at the Symposium on Hadron Spectroscopy, Keszthely, 1968 (unpublished).

[7] Y. Hara [Progr. Theoret. Phys. (Kyoto) **39**, 1020 (1968)] has given some relations on the basis of asymptotic $SW(2)$ symmetries for large-angle scattering for $NN\rightarrow NN$ reactions.

[8] B. Richter, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

apply a similar idea to three-point functions. Indeed, some applications[9] of it suggest reasonable success.

Unfortunately, then, we cannot apply the idea of asymptotic $SU(6)_W$ symmetry for the scattering problems, since we have to consider only the forward or backward scattering in the $SU(6)_W$ group. However, under a certain situation, it is possible that one can apply it even for forward scattering. For example, we know that asymptotic $SW(2)$ symmetry for the Fourier transform of forward amplitude

$$\langle\pi(p)|(A_\mu^{(\alpha)}(x)A_\nu^{(\beta)}(y))_+|\pi(p)\rangle - \langle\pi(p)|(V_\mu^{(\alpha)}(x)V_\nu^{(\beta)}(y))_+|\pi(p)\rangle$$

holds because of the Fubini–Dashen–Gell-Mann sum rule.[10] This case has been extensively investigated by some authors[11] with success. Similarly, we note reasonable validity of the Johnson-Treiman relation,[12] and, recently, Meshkov and Ponzini[13] have shown that they can roughly explain the high-energy photoproduction of vector mesons on the basis of $SU(6)_W$ symmetry. These, together with another ambitious attempt[14] at a Reggeized $SU(6)_W$ theory, indicate that the $SU(6)_W$ group may be a reasonably good symmetry at the high-energy limit. Of course, this statement must be taken with some caution because it automatically implies the validity of $SU(3)$ even for the forward direction. Presumably, it is rather the $SU(4)_W$ group which may become a good asymptotic symmetry in the high-energy region. Then, we have to take into account the effects of first-order $SU(3)$ breaking on all applications of $SU(6)_W$ symmetry to scattering problems. This problem will be treated elsewhere.

It is the purpose of this paper to investigate the validity of the asymptotic $SU(6)_W$ symmetry in a much more simple case of the two-point Green's functions. Indeed, we shall see that it appears to be very good; the detailed consequences will be given in the following paper.

It may not be trivial to emphasize the incompatibility of $SU(6)_W$ or $U(6,6)$ with the $SW(3)$ group in the framework of pure quark Lagrangian models. Indeed, only the scalar $\bar{q}(x)q(x)$ is invariant under the former, while only the vector $\bar{q}(x)\gamma_\mu q(x)$, or axial vector $\bar{q}(x)\gamma_\mu\gamma_5 q(x)$, is invariant under the $SW(3)$ group. This situation is rather puzzling since both groups seem to be good symmetries. However, it appears that the $SW(3)$ group prefers nonlinear realizations, while the $U(6,6)$ chooses linear representations. If this is so, the pure quark four-fermion model may be a bit misleading, or it may be that the reality is very complicated and we have to seek[6] nonlinear representations of a group larger than both the $U(6,6)$ and the $SW(3)$ group.

Finally, we remark that the idea of asymptotic symmetry is consistent with the notion of the generalized algebra of currents[15] on the light cone, if relevant commutators on the light cone show the required symmetry properties under the group in question. Although this approach is in principle better, we have unfortunately no reliable way to calculate singularities at the light cone except for cases of two-point Green's functions.

II. LEHMANN-KÄLLÉN REPRESENTATIONS AND ASYMPTOTIC $SU(6)_W$ SUM RULES

Throughout this paper we shall assume the quark model, at least to establish notations.

We set

$$\begin{aligned}
S^a(x)&=\tfrac{1}{2}\bar{q}(x)\lambda_a q(x)\,,\\
P^a(x)&=\tfrac{1}{2}i\bar{q}(x)\lambda_a\gamma_5 q(x)\,,\\
V_\mu{}^a(x)&=\tfrac{1}{2}i\bar{q}(x)\lambda_a\gamma_\mu q(x)\,,\\
A_\mu{}^a(x)&=\tfrac{1}{2}i\bar{q}(x)\lambda_a\gamma_\mu\gamma_5 q(x)\,,\\
T_{\mu\nu}{}^a(x)&=\tfrac{1}{4}i\bar{q}(x)\lambda_a[\gamma_\mu,\gamma_\nu]q(x)\,,
\end{aligned}\tag{1}$$

where λ_a ($a=0,1,\ldots,8$) are the standard 3×3 matrices with $\lambda_0=\sqrt{\tfrac{2}{3}}$ and we adopt the convention that Latin and Greek indices refer to $SU(3)$ and Lorentz spaces, respectively. Also, we use the standard Dyson notation with $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. Sometimes it is convenient to introduce the conjugate of $T_{\mu\nu}$ by

$$\tilde{T}_{\mu\nu}{}^a(x)=-\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}T_{\alpha\beta}{}^a(x)=\tfrac{1}{4}i\bar{q}(x)\lambda_a[\gamma_\mu,\gamma_\nu]\gamma_5 q(x)\,.\tag{2}$$

Now, let Q_A represent any of $\frac{1}{2}\lambda_a$, $\frac{1}{2}\lambda_a\gamma_5$, $\frac{1}{2}i\lambda_a\gamma_\mu$, $\frac{1}{2}i\lambda_a\gamma_\mu\gamma_5$, $\frac{1}{4}i\lambda_a[\gamma_\mu,\gamma_\nu]$, and $\frac{1}{4}i\lambda_a[\gamma_\mu,\gamma_\nu]\gamma_5$, and set

$$\langle A,B\rangle\equiv i\int d^4x\, e^{-ik(x-y)}\{\langle0|(\bar{q}(x)Q_Aq(x),\bar{q}(y)Q_Bq(y))_+|0\rangle-\langle0|\bar{q}(x)Q_Aq(x)|0\rangle\langle0|\bar{q}(y)Q_Bq(y)|0\rangle\}\,.\tag{3}$$

[9] J. Schechter and G. Venturi, Phys. Rev. Letters **19**, 276 (1967); S. Okubo, Ann. Phys. (N.Y.) **47**, 351 (1968); L. H. Chan, L. Clavelli, and R. Torgeson, Phys. Rev. **185**, 1754 (1969).

[10] S. Fubini, Nuovo Cimento **43A**, 475 (1966); R. F. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966*, edited by A. Perlmutter, J. Wujtzszek, G. Sudarshan, and B. Kurşunoğlu (W. H. Freeman and Co., San Francisco, 1966).

[11] D. S. Narayan, R. P. Srivastava, and R. P. Saxena, Phys. Rev. **167**, 1379 (1968); V. S. Mathur and R. N. Mohapatra, *ibid.* **173**, 1668 (1968).

[12] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 1178 (1965); J. C. Carter, J. J. Coyne, and S. Meshkov, *ibid.* **14**, 523 (1965); **14**, 850(E) (1965).

[13] S. Meshkov and R. Ponzini, Phys. Rev. **175**, 2030 (1968).

[14] R. Delbourgo and A. Salam, Phys. Letters **28B**, 497 (1969).

[15] S. Okubo, Physics **3**, 165 (1967); in *Proceedings of the Fourth Coral Gables Conferences on Symmetry Principles at High Energies, University of Miami, 1967*, edited by A. Perlmutter and B. Kurşunoğlu (W. H. Freeman and Co., San Francisco, 1967).

Then, one can write the Lehman-Källen (LK) representation for $I_{AB}(k)$:

$$I_{AB}(k)\equiv\langle A,B\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}\left(\rho_{AB}{}^{(1)}(m)+ik_\mu\rho_{AB,\mu}{}^{(2)}(m)+\frac{k_\mu k_\nu}{m^2}\rho_{AB,\mu\nu}{}^{(3)}(m)\right)-\int_0^\infty dm^2\frac{1}{m^2}\rho_{AB,44}{}^{(3)}(m)\,, \quad (4)$$

where the last term corresponds to the so-called Schwinger term.[16] To determine $\rho_{AB}{}^{(1)}(m)$ unambiguously, we impose the traceless condition

$$\rho_{AB,\mu\mu}{}^{(3)}(m)=0\,. \quad (5)$$

From the definition, it is obvious that we have

$$\rho_{AB}{}^{(1)}(m)=\rho_{BA}{}^{(1)}(m)\,,\quad \rho_{AB,\mu}{}^{(2)}(m)=-\rho_{BA,\mu}{}^{(2)}(m)\,,\quad \rho_{AB,\mu\nu}{}^{(3)}(m)=\rho_{BA,\mu\nu}{}^{(3)}(m)=\rho_{AB,\nu\mu}{}^{(3)}(m)\,. \quad (6)$$

Corresponding to Eqs. (3) and (4), we have also the LK representation for the commutator[17]:

$$\langle 0|[\bar{q}(x)Q_Aq(x),\bar{q}(y)Q_Bq(y)]|0\rangle=\int_0^\infty dm^2\left(\rho_{AB}{}^{(1)}(m)+\rho_{AB,\mu}{}^{(2)}(m)\frac{\partial}{\partial x_\mu}-\frac{1}{m^2}\rho_{AB,\mu\nu}{}^{(3)}(m)\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right)\Delta(x-y,\,m)\,. \quad (7)$$

Also, the spectral weights are defined by

$$\sum_{n\neq 0}\langle 0|\bar{q}(0)Q_Aq(0)|n\rangle\langle n|\bar{q}(0)Q_Bq(0)|0\rangle\delta^{(4)}(k-p_n)$$
$$=\frac{1}{(2\pi)^3}\theta(k_0)\left(\delta_{\mu\nu}\rho_{AB}{}^{(1)}(m)+ik_\mu\rho_{AB,\mu}{}^{(2)}(m)+\frac{k_\mu k_\nu}{m^2}\rho_{AB,\mu\nu}{}^{(3)}(m)\right)\,,\quad k^2=-m^2<0\,. \quad (8)$$

For practical applications, it is more convenient to define LK representations for individual components:

$$\langle S^a,S^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,S-S)\,,$$

$$\langle P^a,P^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,P-P)\,,$$

$$\langle V_\mu{}^a,V_\nu{}^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}\left[\delta_{\mu\nu}\rho_{ab}{}^{(1)}(m,\,V-V)+\frac{1}{m^2}k_\mu k_\nu\rho_{ab}{}^{(2)}(m,\,V-V)\right]-n_\mu n_\nu\int_0^\infty dm^2\frac{1}{m^2}\rho_{ab}{}^{(2)}(m,\,V-V)\,,$$

$$\langle A_\mu{}^a,A_\nu{}^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}\left(\delta_{\mu\nu}\rho_{ab}{}^{(1)}(m,\,A-A)+\frac{1}{m^2}k_\mu k_\nu\rho_{ab}{}^{(2)}(m,\,A-A)\right)-n_\mu n_\nu\int_0^\infty dm^2\frac{1}{m^2}\rho_{ab}{}^{(2)}(m,\,A-A)\,,$$

$$\langle T_{\mu\nu}{}^a,T_{\alpha\beta}{}^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}[(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha})\rho_{ab}{}^{(1)}(m,\,T-T)$$
$$+(\delta_{\mu\alpha}k_\nu k_\beta+\delta_{\nu\beta}k_\mu k_\alpha-\delta_{\mu\beta}k_\nu k_\alpha-\delta_{\nu\alpha}k_\mu k_\beta)(1/m^2)\rho_{ab}{}^{(2)}(m,\,T-T)]$$
$$-(\delta_{\mu\alpha}n_\nu n_\beta+\delta_{\nu\beta}n_\mu n_\alpha-\delta_{\mu\beta}n_\nu n_\alpha-\delta_{\nu\alpha}n_\mu n_\beta)\int_0^\infty dm^2\frac{1}{m^2}\rho_{ab}{}^{(2)}(m,\,T-T)\,, \quad (9)$$

$$\langle S^a,V_\mu{}^b\rangle=ik_\mu\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,S-V)\,,$$

$$\langle P^a,A_\mu{}^b\rangle=ik_\mu\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,P-A)\,,$$

$$\langle V_\mu{}^a,T_{\alpha\beta}{}^b\rangle=i(\delta_{\mu\alpha}k_\beta-\delta_{\mu\beta}k_\alpha)\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,V-T)\,,$$

$$\langle A_\mu{}^a,\tilde{T}_{\alpha\beta}{}^b\rangle=(\delta_{\mu\alpha}k_\beta-\delta_{\mu\beta}k_\alpha)\int_0^\infty dm^2\frac{1}{k^2+m^2}\rho_{ab}(m,\,A-\tilde{T})\,,$$

[16] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959); T. Goto and T. Imamura, Progr. Theoret. Phys. (Kyoto) **14**, 396 (1955).
[17] Notations are the same as in S. Okubo, Nuovo Cimento **44A**, 1015 (1966).

where we set $n_\mu=\delta_{\mu 4}$ for simplicity. Note that we did not add the factor i in the right-hand side of the last equation. The rest of the combinations are either zero or derivable from these listed above. For example, one can also write

$$\langle \tilde{T}_{\mu\nu}{}^a,\tilde{T}_{\alpha\beta}{}^b\rangle=\int_0^\infty dm^2\frac{1}{k^2+m^2}[(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha})\rho_{ab}{}^{(1)}(m,\tilde{T}-\tilde{T}) + (\delta_{\mu\alpha}k_\nu k_\beta+\delta_{\nu\beta}k_\mu k_\alpha-\delta_{\mu\beta}k_\nu k_\alpha-\delta_{\nu\alpha}k_\mu k_\beta)(1/m^2)\rho_{ab}{}^{(2)}(m,\tilde{T}-\tilde{T})] - (\delta_{\mu\alpha}n_\nu n_\beta+\delta_{\nu\beta}n_\mu n_\alpha-\delta_{\mu\beta}n_\nu n_\alpha-\delta_{\nu\alpha}n_\mu n_\beta)\int_0^\infty dm^2\frac{1}{m^2}\rho_{ab}{}^{(2)}(m,\tilde{T}-\tilde{T}), \quad (10a)$$

with

$$\rho_{ab}{}^{(1)}(m,\tilde{T}-\tilde{T})=\rho_{ab}{}^{(1)}(m,T-T)-\rho_{ab}{}^{(2)}(m,T-T),$$
$$\rho_{ab}{}^{(2)}(m,\tilde{T}-\tilde{T})=-\rho_{ab}{}^{(2)}(m,T-T). \quad (10b)$$

Also, the positivity condition implies[17]

$$\rho_{aa}{}^{(2)}(m,B-B)\geq\rho_{aa}{}^{(1)}(m,B-B)\geq 0 \quad \text{(no summation on } a) \quad (11)$$

for all $B\equiv A$, V, and T. Similarly for S and P, we have

$$\rho_{aa}(m,S-S)\geq 0,\quad \rho_{aa}(m,P-P)\geq 0 \quad \text{(no summation on } a). \quad (12)$$

The upper limit $\rho_{aa}{}^{(2)}\equiv\rho_{aa}{}^{(1)}$ is attainable if and only if we have conservation laws $\partial_\mu V_\mu{}^a(x)=0$, $\partial_\mu A_\mu{}^a(x)=0$, $\partial_\mu T_{\mu\nu}{}^a(x)=0$.

After these preliminaries, one can now derive some spectral sum rules. For this, we note the following special cases of Eq. (7):

$$\langle 0|[S^a(x),V_\mu{}^b(y)]|0\rangle=\int_0^\infty dm^2\rho_{ab}(m,S-V)\frac{\partial}{\partial x_\mu}\Delta(x-y,m),$$
$$\langle 0|[P^a(x),A_\mu{}^b(y)]|0\rangle=\int_0^\infty dm^2\rho_{ab}(m,P-A)\frac{\partial}{\partial x_\mu}\Delta(x-y,m), \quad (13)$$
$$\langle 0|[V_\mu{}^a(x),T_{\alpha\beta}{}^b(y)]|0\rangle=\int_0^\infty dm^2\rho_{ab}(m,V-T)\left(\delta_{\mu\alpha}\frac{\partial}{\partial x_\beta}-\delta_{\mu\beta}\frac{\partial}{\partial x_\alpha}\right)\times\Delta(x-y,m),$$
$$\langle 0|[A_\mu{}^a(x),\tilde{T}_{\alpha\beta}{}^b(y)]|0\rangle=-i\int_0^\infty dm^2\rho_{ab}(m,A-\tilde{T})\left(\delta_{\mu\alpha}\frac{\partial}{\partial x_\beta}-\delta_{\mu\beta}\frac{\partial}{\partial x_\alpha}\right)\times\Delta(x-y,m).$$

Also, we have the following equal-time commutation relations for $x_0=y_0$:

$$[S^a(x),V_4{}^b(y)]=-f_{abc}S^c(x)\delta^{(3)}(x-y),$$
$$[V_\mu{}^a(x),T_{4\nu}{}^b(y)]=-[d_{abc}\delta_{\mu\nu}S^c(x)+f_{abc}T_{\mu\nu}{}^c(x)]\delta^{(3)}(x-y)\quad(\nu\neq 4),$$
$$[P^a(x),A_4{}^b(y)]=-d_{abc}S^c(x)\delta^{(3)}(x-y), \quad (14)$$
$$[A_\mu{}^a(x),\tilde{T}_{4\nu}{}^b(y)]=i[f_{abc}\delta_{\mu\nu}S^c(x)-d_{abc}T_{\mu\nu}{}^c(x)]\delta^{(3)}(x-y)\quad(\nu\neq 4),$$

where the repeated indices c implies automatical summation over $c=0,1,\ldots,8$, with $f_{ab0}=0$ and $d_{ab0}=\delta_{ab}\sqrt{\tfrac{2}{3}}$. Now, setting $x_0=y_0$ in Eq. (13), and comparing it with Eq. (14), we obtain the sum rules

$$\int_0^\infty dm^2\rho_{ab}(m,S-V)=-\int_0^\infty dm^2\rho_{ab}(m,A-\tilde{T})=\xi_8 f_{8ab}, \quad (15a)$$

$$\int_0^\infty dm^2\rho_{ab}(m,P-A)=-\int_0^\infty dm^2\rho_{ab}(m,V-T)=(\sqrt{\tfrac{2}{3}})\xi_0\delta_{ab}+\xi_8 d_{8ab}, \quad (15b)$$

where we have set

$$\xi_0=\langle 0|S^0(0)|0\rangle,\quad \xi_8=\langle 0|S^8(0)|0\rangle \quad (16)$$

and used the fact that $\langle 0|S^a(x)|0\rangle\equiv 0$, except for $a=0$ and $a=8$ because of the isotopic-spin invariance and the conservation of the hypercharge. We remark that, in the equal-time commutation relation (14) we should not have any Schwinger term or, if we have, then it must be a q-number Schwinger term with zero vacuum expectation value, since otherwise it will lead to a contradiction with the LK representation (13). Therefore, our sum rules, Eqs. (15), must be independent of the ambiguity of the Schwinger term. This fact may be compared with other cases that require the existence of Schwinger terms so as to avoid possible contradictions[17] with LK representations.

We notice that Eqs. (15) are examples of the validity of the asymptotically broken $SU(3)$ symmetry that we mentioned in the Introduction. We shall prove, in Sec. III, that the sum rules (15) also represent those corresponding to the asymptotically broken $SU(6)_W$ symmetry.

Now, let us assume that the theory becomes asymptotically $SU(6)_W$-invariant when we let $k \to \infty$. Although we could do this in a purely $SU(6)_W$ framework, as we shall show in the Appendix, here we shall utilize the notation of $U(6,6)$ symmetry,[18] which is a group of 12×12 matrices with U satisfying the condition $U^\dagger \gamma_4 U = \gamma_4$. However, in order to keep the free kinematical term invariant, we must impose an additional constraint $U^{-1}(i\gamma k)U = (i\gamma k)$, i.e., $i\gamma k$ must be taken[19] to be formally invariant. Then for $k \to \infty$, the statement of the asymptotic $SU(6)_W$ symmetry is equivalent to[19]

$$\lim_{k\to\infty} I_{AB}(k) = a_1 \operatorname{tr}(Q_A Q_B) + a_2 \operatorname{tr}(Q_A)\operatorname{tr}(Q_B) + a_3(1/k^2)\operatorname{tr}[Q_A(i\gamma k)Q_B(i\gamma k)] + a_4(1/k^2)\operatorname{tr}(Q_A i\gamma k)\operatorname{tr}(Q_B i\gamma k), \quad (17)$$

where a_1, a_2, a_3, and a_4 are some constants. Comparing this with Eq. (4), and noting the condition equations (5) and (6), we obtain the sum rule

$$\int_0^\infty dm^2 \frac{1}{m^2} \rho_{AB,\mu\nu}{}^{(3)}(m) = c_1[\operatorname{tr}(Q_A\gamma_\mu Q_B\gamma_\nu) - \tfrac{1}{4}\delta_{\mu\nu}\operatorname{tr}(Q_A\gamma_\lambda Q_B\gamma_\lambda)] + c_2[\operatorname{tr}(Q_A\gamma_\mu)\operatorname{tr}(Q_B\gamma_\nu) - \tfrac{1}{4}\delta_{\mu\nu}\operatorname{tr}(Q_A\gamma_\lambda)\operatorname{tr}(Q_B\gamma_\lambda)] + (A \leftrightarrow B). \quad (18)$$

Note that terms such as $\delta_{\mu\nu}\operatorname{tr}(Q_AQ_B)$ do not appear because of the traceless condition equation (5).

Equation (18) is equivalent to

$$\int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, V-V) = -8c_1\delta_{ab} - 8c_2 \operatorname{tr}(\lambda_a)\operatorname{tr}(\lambda_b), \quad (19a)$$

$$\int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, A-A) = \int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, T-T) = -8c_1\delta_{ab}. \quad (19b)$$

It is interesting to notice that if $a \neq 0$ or $b \neq 0$, then this gives

$$\int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, V-V) = \int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, A-A) = \int_0^\infty dm^2 \frac{1}{m^2} \rho_{ab}{}^{(2)}(m, T-T) = -8c_1\delta_{ab} \quad (a\neq 0 \text{ or } b \neq 0). \quad (20)$$

It is remarkable[20] that the first relation of Eq. (20) is nothing but the first Weinberg sum rule for the asymptotic $SW(3)$ symmetry. As we have remarked in the Introduction, we should keep in mind the fact that the $U(6,6)$ group and $SW(3)$ group have no relation to each other, since the scalar $\bar{q}(x)q(x)$ is the only invariant under $U(6,6)$,while the vector $\bar{q}(x)\gamma_\mu q(x)$ and axial vector $\bar{q}(x)\gamma_\mu\gamma_5 q(x)$ are sole invariants of the $SW(3)$ group. The fact that we reproduce the $SW(3)$ sum rule from $SU(6)_W$ is highly encouraging.

The equivalence of the tensor and vector spectral sum rule in Eq. (20) has also been obtained by some other authors.[21] It can also be obtained from the Jacobi identity

$$\langle 0|[T_{4k}{}^a(x),[V_4{}^b(y),T_{\mu\nu}{}^c(z)]]|0\rangle + \langle 0|[V_4{}^b(y),[T_{\mu\nu}{}^c(z),T_{4k}{}^a(x)]]|0\rangle + \langle 0|[T_{\mu\nu}{}^c(z),[T_{4k}{}^a(x),V_4{}^b(y)]]|0\rangle = 0,$$

as has been noted,[21] if the Schwinger terms are all c numbers, and if we use the equal-time commutation relations

$$[V_4{}^a(x),T_{\mu\nu}{}^b(y)]_{x0=y0} = -f_{acb}T_{\mu\nu}{}^c(x)\delta^{(3)}(x-y) + (\text{Schwinger term}),$$

$$[T_{4k}{}^a(x),T_{\mu\nu}{}^b(y)]_{x0=y0} = id_{abc}\epsilon_{k\mu\nu\lambda}A_\lambda{}^c(x)\delta^{(3)}(x-y) + f_{abc}[\delta_{\mu k}V_\nu{}^c(x) - \delta_{\nu k}V_\mu{}^c(x)]\delta^{(3)}(x-y) + (\text{Schwinger term}) \quad (k\neq 4).$$

The proof, then, essentially goes in the same way as the ordinary proof of the first Weinberg sum rule.[22] However, the formal use of a Jacobi identity is rather suspect.[23] Indeed, a similar identity among spatial vector currents leads[24] to a contradiction, unless the Schwinger term is a q number. Similarly, a Jacobi identity among $S^a(x)$, $S^b(y)$, and $V_\mu{}^c(z)$ $(\mu\neq 4)$ is easily verified to give a contradiction. Of course it may be that a Jacobi identity holds only when we consider quantities having fourth-component indices such as $V_4{}^a$ or $T_{4k}{}^b$. Anyway, we take the view that we may use a Jacobi identity unless we encounter a manifest contradiction with known facts.

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[18] B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965).

[19] This is equivalent to the inclusion of the so-called irregular terms. For an extensive review of $U(6,6)$ or $\tilde{U}(12)$ theory, see review articles by H. Ruegg, W. Rühl, and T. S. Santhanam [Helv. Phys. Acta **40**, 9 (1967)] and by R. Delbourgo, M. A. Raschid, A. Salam, and J. Strathdee [International Atomic Energy Agency Report, Vienna, 1965 (unpublished)].

[20] This fact has been already noted in Ref. 4.

[21] M. Ademollo, G. Longhi, and G. Veneziano, Nuovo Cimento **58A**, 540 (1968); P. A. Cook and G. C. Joshi, Nucl. Phys. **B10**, 253 (1969).

[22] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

[23] K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. **37–38**, 74 (1966); G. Konishi and K. Yamamoto, *ibid.* **37**, 1314 (1967).

[24] F. Buccella, G. Veneziano, R. Gatto, and S. Okubo, Phys. Rev. **149**, 1268 (1966).

Another way of deriving Eq. (20) is to calculate the Schwinger terms for equal-time commutators directly by the limiting procedure, as has been indicated in Ref. 4. In this way, one obtains Eq. (20) again, even including the case with $a=b=0$. At any rate, we believe that the validity of Eq. (20) is reasonable and that there are some theoretical grounds for it. Together with the other sum rules, Eqs. (15), which will be shown to be also derivable from the asymptotically broken $SU(6)_W$, these indicate, indeed, that the $SU(6)_W$ group may be a reasonably good asymptotic symmetry in the high-energy limit, as far as the two-point functions are concerned.

III. ASYMPTOTICALLY BROKEN $SU(6)_W$ SYMMETRY

In the previous section, we derived an asymptotic $SU(6)_W$ sum rule in Eqs. (19) and (20). Here let us consider sum rules involving $\rho_{AB,\mu}{}^{(2)}$ and $\rho_{AB}{}^{(1)}$. For these, we have to consider terms of the order $1/k$ and $1/k^2$, respectively, when we let $k\to\infty$ for $I_{AB}(k)$. As we have emphasized in the Introduction, it is then likely that we have to consider some effects of symmetry violations. For example, consider the LK representation for the Green's function $\langle P,A\rangle$, and apply the idea of asymptotic $SU(3)$ symmetry. When $k\to\infty$, then it decreases as $1/k$, as we see from Eq. (9), and hence we should expect an asymptotically broken $SU(3)$ symmetry in the first order. Indeed, this is precisely what is happening as we see from the sum rule Eq. (15b). Analogously, for $SU(6)_W$, we have to take into account the first-order breaking of $SU(6)_W$ symmetry. Now, taking the quark Lagrangian model as a guide, we suppose that it is the mass term $\delta m\bar{q}(x)\lambda_8 q(x)$ that violates both $SU(3)$ and $SU(6)_W$ groups. Then, our argument suggests that we expect to have

$$\begin{aligned}\lim_{k\to\infty} I_{AB}(k) &= O(1)+\frac{i}{k^2}\sum_{i,j}\alpha_{ij}\\ &\quad\times\{\mathrm{tr}[Q_A\lambda_iQ_B\lambda_j(\gamma k)]-(A\leftrightarrow B)\}+\frac{i}{k^2}\sum_{i,j}\beta_{ij}\\ &\quad\times\{\mathrm{tr}(Q_A\lambda_i)\,\mathrm{tr}[Q_B\lambda_j(\gamma k)]\\ &\qquad -(A\leftrightarrow B)\}+O(1/k^2)\end{aligned}\tag{21}$$

for some constants α_{ij} and β_{ij}, where the term $O(1)$ is given by Eq. (17), and the summation over i and j runs over only $i=j=0$, $i=0$, $j=8$, and $i=8$, $j=0$ corresponding to the first-order $SU(3)$ violation. Then Eq. (21) leads to the sum rule

$$\begin{aligned}\int_0^\infty dm^2\rho_{AB,\mu}{}^{(2)}(m) &= \sum_{i,j}\alpha_{ij}[\mathrm{tr}(Q_A\lambda_iQ_B\lambda_j\gamma_\mu)-(A\leftrightarrow B)]\\ &\quad+\sum_{i,j}\beta_{ij}[\mathrm{tr}(Q_A\lambda_i)\,\mathrm{tr}(Q_B\gamma_\mu\lambda_j)-(A\leftrightarrow B)].\end{aligned}\tag{22}$$

Furthermore, because of the charge-conjugation invariance of the theory, it is not difficult to show that we must have $\beta_{ij}=0$ in Eq. (22). Then, in terms of components, Eq. (22) is rewritten as

$$\begin{aligned}\int_0^\infty dm^2\rho_{ab}(m,S-V) &= -\int_0^\infty dm^2\rho_{ab}(m,A-\tilde{T})\\ &= -4(\sqrt{\tfrac{2}{3}})(\alpha_{08}-\alpha_{80})f_{8ab}\,,\end{aligned}\tag{23a}$$

$$\begin{aligned}\int_0^\infty dm^2\rho_{ab}(m,P-A) &= -\int_0^\infty dm^2\rho_{ab}(m,V-T)\\ &= 4[\tfrac{2}{3}\alpha_0\delta_{ab}+(\alpha_{08}+\alpha_{80})(\sqrt{\tfrac{2}{3}})d_{8ab}].\end{aligned}\tag{23b}$$

It is remarkable that these relations are essentially the same as Eqs. (15a) and (15b) if we have $\alpha_{08}=0$ with the identification $\xi_0=4(\sqrt{\tfrac{2}{3}})\alpha_{00}$, $\xi_8=4(\sqrt{\tfrac{2}{3}})\alpha_{80}$. The fact that our procedure essentially reproduces Eqs. (15) may be taken as other evidence of the correctness of the idea of broken asymptotic $SU(6)_W$ symmetry.

Encouraged by this fact, we may consider the sum rule up to the order $1/k^2$. We must now consider $SU(6)_W$ violations up to the second order. Thus, we may write

$$\lim_{k\to\infty} I_{AB}(k)=O(1)+O\left(\frac{1}{k}\right)+I_{AB}{}^{(2)}(k)+O\left(\frac{1}{k^3}\right),$$

$$\begin{aligned}I_{AB}{}^{(2)}(k) &= \frac{1}{k^2}\sum_{i,j}g_{ij}[\mathrm{tr}(Q_A\lambda_iQ_B\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\frac{1}{k^2}\sum_{i,j}f_{ij}[\mathrm{tr}(Q_A\lambda_i)\,\mathrm{tr}(Q_B\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\frac{1}{k^4}\sum_{i,j}h_{ij}[\mathrm{tr}(Q_A\gamma k\lambda_iQ_B\gamma k\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\frac{1}{k^4}\sum_{i,j}l_{ij}[\mathrm{tr}(Q_A\gamma k\lambda_i)\,\mathrm{tr}(Q_B\gamma k\lambda_j)+(A\leftrightarrow B)],\end{aligned}\tag{24}$$

where the summation over i and j runs only over 0 and 8, and g_{ij}, f_{ij}, h_{ij}, and l_{ij} are some constants. Equation (24) gives two sum rules for

$$\int_0^\infty dm^2\rho_{AB,\mu\nu}{}^{(3)}(m)\quad\text{and}\quad\int_0^\infty dm^2\rho_{AB}{}^{(1)}(m).$$

However, the simultaneous validities of both sum rules are shown to lead to the relation

$$\int_0^\infty dm^2[\rho_{ab}{}^{(2)}(m,A-A)-\rho_{ab}{}^{(1)}(m,A-A)]=\int_0^\infty dm^2\rho_{ab}{}^{(1)}(m,V-V)\,,$$

which is very undesirable since in the soft-pion limit the left-hand side vanishes identically. However, the sum rule involving $\rho_{AB,\mu\nu}^{(3)}(m)$ is probably too strong, since a similar one is already given by Eq. (18). As a matter of fact, a similar difficulty arises even for asymptotic $SW(3)$ or $SU(3)$ symmetries for $\langle V,V\rangle$ or $\langle A,A\rangle$ Green's functions when we consider higher-order effects. Presumably, for the sum rule involving $\rho_{AB,\mu\nu}^{(3)}(m)$, we must also consider possible $SU(6)_W$-violating interaction Hamiltonians. Hence, we take an attitude that the second-order asymptotic sum rule, Eq. (24), must be valid for the one involving $\rho_{AB}^{(1)}(m)$, but not $\rho_{AB,\mu\nu}^{(3)}(m)$, i.e., we should only use

$$\begin{aligned}\int_0^\infty & dm^2\rho_{AB}^{(1)}(m)\\ &=\sum_{i,j} g_{ij}[\mathrm{tr}(Q_A\lambda_iQ_B\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\sum_{i,j} f_{ij}[\mathrm{tr}(Q_A\lambda_i)\,\mathrm{tr}(Q_B\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\tfrac{1}{4}\sum_{i,j} h_{ij}[\mathrm{tr}(Q_A\gamma_\mu\lambda_iQ_B\gamma_\mu\lambda_j)+(A\leftrightarrow B)]\\ &\quad+\tfrac{1}{4}\sum_{i,j} l_{ij}[\mathrm{tr}(Q_A\gamma_\mu\lambda_i)\,\mathrm{tr}(Q_B\gamma_\mu\lambda_j)+(A\leftrightarrow B)]. \qquad (25)\end{aligned}$$

Incidentally, our philosophy would be correct if we have a purely vector four-quark interaction of the form $(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$, as the $SU(6)_W$-violating interaction, and if we have to add the first-order breaking effect of such a term to the right-hand side of Eq. (24). Then we would still have Eq. (25) but not the sum rule involving $\rho_{AB,\mu\nu}^{(3)}(m)$. The presence of such interaction implies that we may have a specific $SU(6)_W$-breaking interaction corresponding to a sum of $35\oplus 280\oplus \overline{280}\oplus 189 \oplus 405$ dimensional representations.

At any rate, Eq. (25) gives, for cases $a\neq 0$, 8 and $b\neq 0$, 8, the relations

$$\begin{aligned}\int_0^\infty & dm^2[\rho_{ab}(m,S-S)+\rho_{ab}(m,P-P)]\\ &=2\int_0^\infty dm^2[\rho_{ab}^{(1)}(m,A-A)+\rho_{ab}^{(1)}(m,V-V)\\ &\qquad-\tfrac{1}{4}\rho_{ab}^{(2)}(m,A-A)-\tfrac{1}{4}\rho_{ab}^{(2)}(m,V-V)]\\ &=2\sum_{i,j} h_{ij}[\mathrm{tr}(\lambda_a\lambda_i\lambda_b\lambda_j)+(a\leftrightarrow b)]\quad (a,b\neq 0,8), \qquad (26a)\end{aligned}$$

$$\begin{aligned}\int_0^\infty & dm^2[\rho_{ab}(m,S-S)-\rho_{ab}(m,P-P)]\\ &=\int_0^\infty dm^2[\rho_{ab}^{(1)}(m,A-A)-\rho_{ab}^{(1)}(m,V-V)\\ &\qquad-\tfrac{1}{4}\rho_{ab}^{(2)}(m,A-A)+\tfrac{1}{4}\rho_{ab}^{(2)}(m,V-V)]\\ &=\int_0^\infty dm^2[2\rho_{ab}^{(1)}(m,T-T)-\rho_{ab}^{(2)}(m,T-T)]\\ &=2\sum_{i,j} g_{ij}[\mathrm{tr}(\lambda_a\lambda_i\lambda_b\lambda_j)+(a\leftrightarrow b)]\quad (a,b\neq 0,8). \qquad (26b)\end{aligned}$$

If f_{88} and l_{88} are small, i.e., if we can neglect the second-order $SU(3)$-breaking effects, then Eqs. (26) will be valid also for $a=b=8$.

Our sum rules, Eqs. (26), differ from and are more complicated than ones suggested by other authors.[21] Applications of our sum rules will be given in the following paper.

APPENDIX

Here we consider a classification of S, V, A, T, and P operators on the basis of the $SU(6)_W$ group.[5] To that end, we first note that the $U(6,6)$ group is defined as a group consisting of 12×12 matrices satisfying the condition

$$U^\dagger\gamma_4U=\gamma_4. \qquad (A1)$$

Now, the $SU(6)_W$ subgroup of $U(6,6)$ is defined by the further restrictions on U,

$$U^{-1}(i\gamma k)U=i\gamma k\,,\quad \det U=1\,, \qquad (A2)$$

for all collinear four-momenta k. For simplicity, let k_μ be a momentum along the z axis, i.e.,

$$k_\mu=(0,0,k,ik_0)\,. \qquad (A3)$$

Then Eq. (A2) gives

$$\gamma_4U=U\gamma_4\,,\quad \gamma_3U=U\gamma_3\,. \qquad (A4)$$

If we use the Dirac representation for γ matrices, then Eq. (A4) implies that U must have the form

$$U=\begin{pmatrix}V & 0\\ 0 & \sigma_3V\sigma_3\end{pmatrix}, \qquad (A5)$$

where V is a 6×6 unitary unimodular matrix belonging to the $SU(6)$ group. Corresponding to Eq. (A5), let us decompose $q(x)$ into

$$q(x)=\begin{pmatrix}\phi(x)\\ \sigma_3\xi(x)\end{pmatrix}, \qquad (A6)$$

where $\phi(x)$ and $\sigma_3\xi(x)$ are six-component spinors. Then, when we let $q(x)\to Uq(x)$, we have

$$\phi(x)\to V\phi(x)\,,\quad \xi(x)\to V\xi(x)\,. \qquad (A7)$$

Hence, if we write these components as $\phi_A(x)$ and $\xi_A(x)$ $(A=1,\ldots,6)$, then ϕ_A and ξ_A are covariant spinors in the $SU(6)_W$ group. Thus, one can form the following four types of 36-dimensional $SU(6)_W$ tensors out of ϕ and ξ:

$$\begin{aligned}L_{j\alpha}(x)&=\tfrac{1}{2}[\varphi^\dagger(x)\lambda_\alpha\sigma_j\varphi(x)+\xi^\dagger(x)\lambda_\alpha\sigma_j\xi(x)],\\ N_{j\alpha}(x)&=\tfrac{1}{2}[\varphi^\dagger(x)\lambda_\alpha\sigma_j\varphi(x)-\xi^\dagger(x)\lambda_\alpha\sigma_j\xi(x)],\\ K_{i\alpha}(x)&=\tfrac{1}{2}[\varphi^\dagger(x)\lambda_\alpha\sigma_j\xi(x)+\xi^\dagger(x)\lambda_\alpha\sigma_j\varphi(x)],\\ R_{j\alpha}(x)&=-\tfrac{1}{2}i[\varphi^\dagger(x)\lambda_\alpha\sigma_j\xi(x)-\xi^\dagger(x)\lambda_\alpha\sigma_j\varphi(x)],\end{aligned} \qquad (A8)$$

where j assumes values $j=0, 1, 2, 3$, with $\sigma_0\equiv 1$. Expressing $\phi(x)$ and $\xi(x)$ in terms of $q(x)$, one gets

$$
\begin{aligned}
L_{j\alpha}(x):\ & \tilde{T}_{4j}{}^{\alpha}(x) && (j=1,2)\\
:\ & A_3{}^{\alpha}(x) && (j=3)\\
:\ & -iV_4{}^{\alpha}(x) && (j=0),\\
N_{j\alpha}(x):\ & A_j{}^{\alpha}(x) && (j=1,2)\\
:\ & \tilde{T}_{43}{}^{\alpha}(x)\equiv T_{12}{}^{\alpha}(x) && (j=3)\\
:\ & S^{\alpha}(x) && (j=0),\\
K_{j\alpha}(x):\ & \tilde{T}_{j3}{}^{\alpha}(x) && (j=1,2)\\
:\ & iA_4{}^{\alpha}(x) && (j=3)\\
:\ & -V_3{}^{\alpha}(x) && (j=0),\\
R_{j\alpha}(x):\ & \epsilon_{3jk}V_k{}^{\alpha}(x) && (j=1,2)\\
:\ & P^{\alpha}(x) && (j=3)\\
:\ & -iT_{34}{}^{\alpha}(x)\equiv i\tilde{T}_{12}{}^{\alpha}(x) && (j=0).
\end{aligned}
\tag{A9}
$$

The statement of the asymptotic $SU(6)_W$ symmetry is now translated to that of

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(K_{j\alpha}(x),K_{i\beta}(y))_+|0\rangle = c\delta_{ij}\delta_{\alpha\beta} \tag{A10}$$

for the collinear momentum k of the form Eq. (A3). It is easy to check that this gives Eqs. (20) immediately. Similarly, replacing $K_{j\alpha}$ by $L_{j\alpha}$, we obtain the same result. If we consider

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(L_{j\alpha}(x)N_{i\beta}(y))_+|0\rangle,$$

$$\lim_{k\to\infty} i\int d^4x\, e^{-ik(x-y)}\langle 0|(K_{j\alpha}(x)R_{i\beta}(y))_+|0\rangle$$

up to the order $1/k$, we find the asymptotic sum rules (23).

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Asymptotic $SU(6)_W$ Spectral Sum Rules.* II. Applications and Bare Quark Masses

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The sum rules of the preceding paper are investigated in detail in pole dominance. The ratio f_K/f_π is found to be near unity and all nonexotic baryons must satisfy an approximate mass formula $M=a-bY$ with the universal constant $b\simeq m_3-m_1\simeq 150$ MeV, where m_1, m_2, and m_3 are masses of bare quarks. Moreover, we compute $m_1\simeq 7$ MeV and $m_3\simeq 156$ MeV in a model where $SW(3)$ is exact except for the quark mass term.

I. DEFINITION OF COUPLING CONSTANTS

IN the preceding paper[1] (hereafter referred to as I), we have developed several sum rules on the basis of the asymptotic $SU(6)_W$ symmetry. In this paper, we study its applications, saturating the sum rules by pole dominances. To that end, we define various coupling parameters as follows.

(i) Vector:

$$\langle 0|V_\mu{}^{(3)}(0)|\rho^0(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(\rho),$$

$$\langle 0|V_\mu{}^{(8)}(0)|\omega,\phi(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(\omega\text{ or }\phi),$$

$$(1/\sqrt{2})\langle 0|V_\mu{}^{(4-i5)}(0)|K^{*+}(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_V(K^*),$$

$$(1/\sqrt{2})\langle 0|V_\mu{}^{(4-i5)}(0)|\kappa^+(k)\rangle=(2k_0V)^{-1/2}k_\mu G_V(\kappa),$$

where κ means the 0^+ κ meson.

(ii) Axial vector:

$$\langle 0|A_\mu{}^{(3)}(0)|A_1{}^0(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_A(A_1),$$

$$(1/\sqrt{2})\langle 0|A_\mu{}^{(4-i5)}(0)|K_A{}^+(k)\rangle=(2k_0V)^{-1/2}\epsilon_\mu(k)G_A(K_A),$$

$$\langle 0|A_\mu{}^{(1-i2)}(0)|\pi^+(k)\rangle=(2k_0V)^{-1/2}ik_\mu f_\pi,$$

$$\langle 0|A_\mu{}^{(4-i5)}(0)|K^+(k)\rangle=(2k_0V)^{-1/2}ik_\mu f_K,$$

(iii) Scalar:

$$(1/\sqrt{2})\langle 0|S^{(4-i5)}(0)|\kappa^+(k)\rangle=(2k_0V)^{-1/2}G_S(\kappa),$$

$$(1/\sqrt{2})\langle 0|S^{(1-i2)}(0)|\epsilon^+(k)\rangle=(2k_0V)^{-1/2}G_S(\epsilon),$$

where ϵ is an assumed 0^+ meson with $I=1$ and $Y=0$, which may be[2] $\pi_N(1016)$.

(iv) Pseudoscalar:

$$\langle 0|P^{(3)}(0)|\pi^0(k)\rangle=(2k_0V)^{-1/2}G_P(\pi),$$

$$(1/\sqrt{2})\langle 0|P^{(4-i5)}(0)|K^+(k)\rangle=(2k_0V)^{-1/2}G_P(K).$$

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[1] S. Okubo, preceding paper, Phys. Rev. 188, 2293 (1969).

[2] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. 41, 109 (1969).