Faster-Than-Light Intertial Frames and Tachyons

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(Received 11 August 1969)

By means of a mathematical transformation, we introduce a set of reference frames, called superluminal inertial frames, relative to which tachyons in one spatial dimension behave as ordinary particles. One-dimensional processes involving tachyons and photons can be analyzed in the new frames, and the results transformed to the subluminal frames. The mathematical symmetry or duality between subluminal and superluminal frames and particles suggests an extension of the principle of relativity, according to which the totality of physical laws has the same form relative to both subluminal and superluminal frames. One possible consequence of this extended principle of relativity is that charged tachyons might have properties similar to those of magnetic monopoles. Another consequence is that the cross section for the backward scattering of photons by photons should be twice as great as is predicted without taking into account tachyons. The relevance of these results to our three-dimensional world is questionable because it does not appear to be possible to extend the one-dimensional theory to three dimensions. Photon-photon scattering experiments in vacuum can reveal unambiguously whether or not the predictions have physical relevance.

I. INTRODUCTION

AFTER the appearance of special relativity, physicists rejected for a long time the possibility that faster-than-light particles could exist within the context of that theory. Among the grounds for rejection was the disturbing consequence that in some inertial frames such particles would travel backward in time with negative energy. Interest in tachyons finally revived when the reinterpretation principle was introduced.[1] According to that principle, negative-energy tachyons traveling backward in time are to be reinterpreted as positive-energy tachyons moving forward in time with the opposite momentum. The reinterpretation principle has been applied to processes involving tachyons, and questions concerning such topics as causality have been discussed in a number of references.[2]

In this paper, we introduce, for the case of one spatial dimension, a set of reference frames, called superluminal inertial frames, relative to which tachyons behave as ordinary particles. Thus, it is possible to analyze completely in the new reference frames one-dimensional processes involving, for example, interactions among tachyons and photons, and then transform the results to the familiar subluminal inertial frames. Since the tachyons have the properties of ordinary particles, including real proper mass, relative to the superluminal reference systems, a quantum field theory involving one-dimensional interactions among tachyons, and between photons and tachyons, can clearly be introduced in the superluminal frames precisely as for ordinary particles.

The mathematical transformations involved in going between subluminal and superliminal inertial frames are entirely symmetrical with respect to the two kinds of reference systems. For example, subliminal particles behave like tachyons relative to the superluminal frames. Therefore, as a further logical development of the theory, we extend the principle of relativity to include superluminal inertial frames. The *extended principle of relativity* states that the totality of the laws of physics has the same form relative to the superluminal frames as it does relative to the subluminal frames. In general, the laws governing tachyons and subluminal particles will be interchanged in a transformation between a subluminal and a superluminal frame, but the total structure of laws will have the same form. Since photons and other particles traveling at the velocity of light have the same properties in both kinds of frames, the particular laws governing them should be themselves form-invariant under superluminal transformations,[3] rather than be interchanged with other equations. We try to use this extended form invariance of Maxwell's equations to deduce some properties of the electromagnetic field under superluminal transformations. Our results suggest that a charged tachyon may be similar to a magnetic monopole relative to the reference system in which it has infinite velocity.

The extended principle of relativity implies that both superluminal and subluminal particles will interact electromagnetically with photons, and thus with each other. We call that method of interaction via photons the *minimal interaction* between tachyons and subluminal particles. Finally, we suggest a definite experiment involving light-light scattering as a means of testing the physical relevance of the one-dimensional theory. It does not seem to be possible to generalize our theory to three dimensions, so that it may have little

* Work partially supported by the Research Committee of the University of Wisconsin-Milwaukee Graduate School.

[1] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962).

[2] In addition to Ref. 1, see, for example, M. M. Broido and J. G. Taylor, Phys. Rev. **174**, 1606 (1968); G. Feinberg, *ibid.* **159**, 1089 (1967); R. G. Newton, *ibid.* **162**, 1274 (1967); E. C. G. Sudarshan, University of Syracuse Report No. SU-1206-186 (unpublished); Y. G. Terletskii, *Paradoxes in the Theory of Relativity* (Plenum Press, Inc., New York, 1968).

[3] A superluminal transformation is a transformation between a subluminal and superluminal frame. By convention, we call the particles and inertial frames with which we are familiar "subluminal."

if any relevance to constructing a three-dimensional theory of tachyons.

II. SUPERLUMINAL INERTIAL FRAMES

A. Transformation

Consider a world with only one spatial dimension. We wish to transform from an inertial frame ${}^{\downarrow}S$ with space-time coordinates x,t to another inertial frame ${}^{\uparrow}S$ with space-time coordinates x',t'.[4] The spatial origin $x'=0$ of ${}^{\uparrow}S$ moves along the path $x=vt$ relative to ${}^{\downarrow}S$, with velocity v such that $|v|>c$. We assume that the speed of light is c in both ${}^{\uparrow}S$ and ${}^{\downarrow}S$, and that the transformation between frames is linear.

As a consequence of the previous two assumptions, and the assumption of isotropy, it follows in the usual way that the coordinates of any given event satisfy

$$x^2-c^2t^2=\pm(x'^2-c^2t'^2). \tag{1}$$

For $|v|>c$, the plus sign in Eq. (1) is not acceptable because it leads to the appearance of imaginary numbers in the transformation. Therefore, we are left with

$$x^2-c^2t^2=-(x'^2-c^2t'^2). \tag{2}$$

Equation (2) implies that when $x\pm ct$ vanishes then $x'\pm ct'$ vanishes. The assumed linearity of the transformation then requires that it have one of the following forms:

$$x-ct=-B(v)(x'-ct'), \quad x+ct=B^{-1}(v)(x'+ct') \tag{3}$$

or

$$x-ct=-B(v)(x'+ct'), \quad x+ct=B^{-1}(v)(x'-ct'). \tag{4}$$

Dividing $x-ct$ by $x+ct$ in Eqs. (3) or (4), and putting $x=vt$, $x'=0$ for the coordinates of the spatial origin of ${}^{\uparrow}S$, one finds that

$$B(v)=\pm[(v-c)/(v+c)]^{1/2}. \tag{5}$$

Note that $B(v)$ is real, since $|v|>c$.

The different possibilities above all lead to essentially the same theory except for the signs used in labeling the x' and t' axes. Therefore, we take Eq. (3) with

$$B(v)=[(v-c)/(v+c)]^{1/2} \tag{6}$$

as the transformation from ${}^{\downarrow}S$ to the frame ${}^{\uparrow}S$ moving at relative velocity v, with $|v|>c$.[5] We call such a transformation a superluminal transformation.

For $|v|>c$, we can write

$$B(v)=e^{-\alpha}. \tag{7}$$

Then

$$\begin{aligned}\sinh\alpha&=\tfrac{1}{2}(B^{-1}-B)=(v/|v|)(v^2/c^2-1)^{-1/2},\\ \cosh\alpha&=\tfrac{1}{2}(B^{-1}+B)=(|v|/c)(v^2/c^2-1)^{-1/2},\end{aligned} \tag{8}$$

and

$$\tanh\alpha=c/v. \tag{9}$$

Note that as v goes from $-c$ to $-\infty$ and from $+\infty$ to c, α varies continuously from $-\infty$ to $+\infty$.

Equation (3) can be rewritten in a convenient matrix notation. Let

$$\eta=x-ct, \quad \eta'=x'-ct'$$

and

$$\xi=x+ct, \quad \xi'=x'+ct'. \tag{10}$$

Let

$$Q=\begin{pmatrix}\eta\\ \xi\end{pmatrix}, \quad Q'=\begin{pmatrix}\eta'\\ \xi'\end{pmatrix} \tag{11}$$

and

$$M(\alpha)=\begin{pmatrix}-e^{-\alpha} & 0\\ 0 & e^{\alpha}\end{pmatrix}. \tag{12}$$

Then Eq. (3), with $B(v)$ given by (6), can be written as

$$Q=M(\alpha)Q'. \tag{13}$$

A proper, orthochronous, homogeneous Lorentz transformation in one spatial dimension can be represented in a similar way. Let the frame S', with coordinates x', t', be moving relative to the frame S, with coordinates x,t, at velocity v with $|v|<c$. Then the transformation between the two frames can be written as[6]

$$Q=L(\beta)Q', \tag{14}$$

with Q and Q' given by (11), and

$$L(\beta)=\begin{pmatrix}e^{-\beta} & 0\\ 0 & e^{\beta}\end{pmatrix}, \tag{15}$$

with

$$\tanh\beta=v/c. \tag{16}$$

Some relations satisfied by the superluminal and Lorentz transformation matrices are

$$M(\alpha)M(\beta)=L(\alpha+\beta), \tag{17}$$

$$M(\alpha)L(\beta)=M(\alpha+\beta), \tag{18}$$

$$L(\alpha)L(\beta)=L(\alpha+\beta), \tag{19}$$

$$M^{-1}(\alpha)=M(-\alpha), \tag{20}$$

$$L^{-1}(\alpha)=L(-\alpha). \tag{21}$$

Since the various transformation matrices are diagonal, they commute with one another. While the superluminal transformations alone do not form a group, the Lorentz transformations together with the superluminal

[4] Upward arrows denote superluminal entities and downward arrows denote subluminal entities. The physical meaning of the frame ${}^{\uparrow}S$ becomes clearer later.

[5] One should not picture ${}^{\uparrow}S$ relative to ${}^{\downarrow}S$ in the same way as another inertial frame of relative velocity less than c. Since the transformation from ${}^{\downarrow}S$ to ${}^{\uparrow}S$ interchanges spacelike and timelike intervals, such a conceptualization is invalid.

[6] W. Rindler, *Special Relativity* (Oliver and Boyd, London, 1966), p. 22, problem 3; L. Parker and G. Schmieg, Am. J. Phys. (to be published).

transformations do form a group, which we call the extended Lorentz group in one spatial dimension.

The interval from the origin to an event corresponding to the column matrix Q in ${}^{\downarrow}S$ or ${}^{\uparrow}S$ can be written as

$$x^2-c^2t^2=\tfrac{1}{2}\tilde{Q}\sigma_1 Q=N(Q)\,, \tag{22}$$

where

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

Using the identities

$$L(\alpha)\sigma_1=\sigma_1 L(-\alpha)$$

and

$$M(\alpha)\sigma_1=-\sigma_1 M(-\alpha)\,,$$

we find that

$$N(LQ)=N(Q)\,,$$

while

$$N(MQ)=-N(Q)\,.$$

As expected, a Lorentz transformation does not change the sign of the interval, whereas a superluminal transformation does change the sign in accordance with Eq. (2).

Given a subluminal frame ${}^{\downarrow}S$, we define the set of subluminal frames as the set of all frames obtained from ${}^{\downarrow}S$ by Lorentz transformations $L(\alpha)$. We define the set of all superluminal inertial frames as the set of all frames obtained from ${}^{\downarrow}S$ by superluminal transformations $M(\alpha)$. From Eq. (18) it is clear that the same set of superluminal frames is obtained regardless of which subluminal frame ${}^{\downarrow}S$ is considered in the definition. As a further consequence of (18), any two superluminal frames are related by a Lorentz transformation, and the entire set of superluminal frames can be obtained from a given frame ${}^{\uparrow}S$ by the Lorentz transformations $L(\alpha)$. Similarly, the set of subluminal frames can be obtained from a given superluminal frame ${}^{\uparrow}S$ by the superluminal transformations $M(\alpha)$. There is complete mathematical symmetry between the two sets of reference systems.

B. Velocity, Momentum, and Energy of Tachyons

We call a particle which moves at velocity less than c relative to a superluminal frame a tachyon or superluminal particle. As we show, such a particle has all the properties attributed to tachyons relative to the subluminal frames, whereas it has the familiar properties of an ordinary particle relative to the superluminal frames.

Consider such a particle, T, at rest at the origin of a superluminal inertial frame ${}^{\uparrow}S_0$. In a frame ${}^{\downarrow}S$ such that ${}^{\downarrow}S=M(\alpha){}^{\uparrow}S_0$,[7] the velocity v of particle T is given by Eq. (9), so that $|v|>c$. In the reference system ${}^{\downarrow}S_0=M(0){}^{\uparrow}S_0$, the tachyon T has velocity $v=\pm\infty$ (the question whether v is $+\infty$ or $-\infty$ is meaningless, since instantaneous motion cannot be assigned a direction). In the frame

$${}^{\downarrow}S=L(\alpha){}^{\downarrow}S_0=L(\alpha)M(0){}^{\uparrow}S_0=M(\alpha){}^{\uparrow}S_0\,,$$

the velocity v of T has the same sign as the parameter α, the change in sign occurring as v goes through $\pm\infty$ or α goes through zero. By considering the motion of T between two space-time points it is easy to show that the change in the sign of v is such that T always moves from the earlier event to the later event. For an event which has coordinates ${}^{\uparrow}x=0$, $c{}^{\uparrow}t>0$ in ${}^{\uparrow}S$, has coordinates ${}^{\downarrow}x=\cosh\alpha(c{}^{\uparrow}t)>0$, and $c{}^{\downarrow}t=\sinh\alpha(c{}^{\uparrow}t)$ in ${}^{\downarrow}S$.

The momentum ${}^{\uparrow}P_0$ and the energy ${}^{\uparrow}E_0$ of the particle T with respect to the system ${}^{\uparrow}S_0$ are given by the ordinary expressions for a particle at rest, namely,

$${}^{\uparrow}P_0=0\,,\quad {}^{\uparrow}E_0=m_0c^2\,, \tag{23}$$

where m_0 is the proper mass of the particle. The quantity m_0 is real and positive. Under homogeneous Lorentz transformations, the quantities P and E/c transform like x and ct, respectively. We assume that also under superluminal transformations P and E/c transform like x and ct. We call a quantity which transforms like x and ct under both homogeneous Lorentz and superluminal transformations an extended Lorentz vector.

Let

$$\lambda=P-E/c\,,\quad \sigma=P+E/c\,. \tag{24}$$

Also let

$$\Lambda=\begin{pmatrix}\lambda\\ \sigma\end{pmatrix}. \tag{25}$$

Then the transformation between ${}^{\uparrow}\Lambda$ in ${}^{\uparrow}S$, and ${}^{\downarrow}\Lambda$ in ${}^{\downarrow}S=M(\alpha){}^{\uparrow}S$, has the same form as Eq. (13):

$${}^{\downarrow}\Lambda=M(\alpha){}^{\uparrow}\Lambda\,. \tag{26}$$

The momentum ${}^{\downarrow}P_0$ and energy ${}^{\downarrow}E_0$ of T in the frame ${}^{\downarrow}S_0=M(0){}^{\uparrow}S_0$, obtained by means of Eqs. (23)–(26), are

$${}^{\downarrow}P_0=m_0c\,,\quad {}^{\downarrow}E_0=0\,. \tag{27}$$

It will be recalled that T has infinite velocity in ${}^{\downarrow}S_0$. Equation (27) agrees with the energy and momentum generally attributed to such a tachyon. The reinterpretation principle leaves ambiguous the sign of the momentum which an observer in ${}^{\downarrow}S_0$ will attribute to the tachyon T, since the energy in ${}^{\downarrow}S_0$ vanishes. However, if one considers a tachyon carrying momentum instantaneously between two particles, it becomes evident that the sign of the momentum attributed to the tachyon depends on the direction of motion, which, as we pointed out before, is ambiguous when $|v|=\infty$. Hence, the ambiguity in the sign of the attributed momentum in ${}^{\downarrow}S_0$ merely reflects the physical situation.

[7] The notation ${}^{\downarrow}S=M(\alpha){}^{\uparrow}S$ means that ${}^{\downarrow}Q=M(\alpha){}^{\uparrow}Q$, where ${}^{\downarrow}Q$ and ${}^{\uparrow}Q$ are the coordinates of a given event in ${}^{\downarrow}S$ and ${}^{\uparrow}S$, respectively. Similarly, $S=L(\alpha)S'$ means $Q=L(\alpha)Q'$.

In a frame ${}^{\downarrow}S=L(\alpha){}^{\downarrow}S_0$, we have

$$\begin{pmatrix} {}^{\downarrow}\lambda \\ {}^{\downarrow}\sigma \end{pmatrix} = L(\alpha)\begin{pmatrix} {}^{\uparrow}\lambda_0 \\ {}^{\uparrow}\sigma_0 \end{pmatrix} = L(\alpha)\begin{pmatrix} m_0c \\ m_0c \end{pmatrix}. \tag{28}$$

Hence,

$${}^{\downarrow}P=\tfrac{1}{2}({}^{\downarrow}\lambda+{}^{\downarrow}\sigma)=(m_0c)\cosh\alpha \tag{29}$$

and

$${}^{\downarrow}E=\tfrac{1}{2}c({}^{\downarrow}\sigma-{}^{\downarrow}\lambda)=(m_0c^2)\sinh\alpha. \tag{30}$$

Now, ${}^{\downarrow}S=L(\alpha){}^{\downarrow}S_0=L(\alpha)M(0){}^{\uparrow}S_0=M(\alpha){}^{\uparrow}S_0$, so that the velocity v of the tachyon relative to ${}^{\downarrow}S$ is given by Eq. (9). It follows from (8) that

$${}^{\downarrow}P=\frac{m_0|v|}{(v^2/c^2-1)^{1/2}} \tag{31}$$

and

$${}^{\downarrow}E=\frac{v}{|v|}\,\frac{m_0c^2}{(v^2/c^2-1)^{1/2}}. \tag{32}$$

According to the reinterpretation principle, when ${}^{\downarrow}E$ is negative, the momentum ${}^{\downarrow}P_a$ and energy ${}^{\downarrow}E_a$ attributed to the tachyon by an observer in ${}^{\downarrow}S$ are $-{}^{\downarrow}P$ and $-{}^{\downarrow}E$, respectively. Thus, the attributed momentum and energy are

$${}^{\downarrow}P_a=\frac{m_0v}{(v^2/c^2-1)^{1/2}} \tag{33}$$

and

$${}^{\downarrow}E_a=\frac{m_0c^2}{(v^2/c^2-1)^{1/2}}. \tag{34}$$

Therefore, the particle T at rest at the origin of ${}^{\uparrow}S_0$ has the properties generally attributed to a tachyon in any subluminal frame ${}^{\downarrow}S$. Since all superluminal frames are related to ${}^{\uparrow}S_0$ by Lorentz transformations, it follows that particles traveling at velocities less than c relative to the superluminal frames appear as tachyons relative to the subluminal frames. The dual theorem, with the words "subluminal" and "superluminal" interchanged (along with the upward and downward arrows), is clearly also valid.

C. Conservation Laws

In this section we show that if the laws of conservation of energy and momentum hold in superluminal frames, then those laws are also valid in subluminal frames. The converse theorem then follows immediately from the symmetry or duality between the two kinds of frames. The proof depends on the reinterpretation principle,[8] and on the fact that the energy and momentum form an extended Lorentz vector.

[8] Since superluminal transformations can change the sign of the energy of subluminal particles and photons, as well as superluminal particles, we extend the reinterpretation principle to apply to all cases in which the sign of the energy becomes negative. In Ref. 1, the application of the reinterpretation principle to negative-energy photons was mentioned.

Consider a collision in a frame ${}^{\uparrow}S$ involving particles with speeds smaller than c, equal to c, and greater than c. To be specific, suppose that there are three incoming particles labeled 1, 2, and 3 and four outgoing particles labeled 4, 5, 6, and 7, all having positive energies. Then conservation of energy and momentum in ${}^{\uparrow}S$ can be written as

$${}^{\uparrow}\Lambda_1+{}^{\uparrow}\Lambda_2+{}^{\uparrow}\Lambda_3={}^{\uparrow}\Lambda_4+{}^{\uparrow}\Lambda_5+{}^{\uparrow}\Lambda_6+{}^{\uparrow}\Lambda_7, \tag{35}$$

where Λ is given by Eqs. (24) and (25). It follows from Eq. (26) that, in the frame ${}^{\downarrow}S=M(\alpha){}^{\uparrow}S$,

$${}^{\downarrow}\Lambda_1+{}^{\downarrow}\Lambda_2+{}^{\downarrow}\Lambda_3={}^{\downarrow}\Lambda_4+{}^{\downarrow}\Lambda_5+{}^{\downarrow}\Lambda_6+{}^{\downarrow}\Lambda_7. \tag{36}$$

Long after the collision, the time in ${}^{\uparrow}S$ or ${}^{\downarrow}S$ will clearly be positive, and long before the collision it will be negative. Consider a space-time point, with time coordinate ${}^{\uparrow}t$, on the world line of one of the incoming or outgoing particles in ${}^{\uparrow}S$. The quantity $c^2\,{}^{\uparrow}t$ transforms like the energy ${}^{\uparrow}E$ of the particle having the world line on which the point lies. Therefore, when the sign of ${}^{\downarrow}E$ is opposite to that of ${}^{\uparrow}E$, the sign of ${}^{\downarrow}t$ will be opposite to that of ${}^{\uparrow}t$. In such a case, an incoming or outgoing world line in ${}^{\uparrow}S$ will become the opposite kind of world line in ${}^{\downarrow}S$. Also, since ${}^{\uparrow}E$ is positive, ${}^{\downarrow}E$ will be negative, so that the attributed momentum and energy in ${}^{\downarrow}S$ will be ${}^{\downarrow}P_a=-{}^{\downarrow}P$, and ${}^{\downarrow}E_a=-{}^{\downarrow}E$, in accordance with the reinterpretation principle. Therefore, in such a case ${}^{\downarrow}\Lambda_a=-{}^{\downarrow}\Lambda$.

To be specific, suppose that ${}^{\downarrow}E_3$ is negative. Then particle 3, which is incoming in ${}^{\uparrow}S$, will be an outgoing particle in ${}^{\downarrow}S$ with the attributed momentum ${}_{\uparrow}P_{3a}=-{}^{\downarrow}P_3$ and attributed energy ${}^{\downarrow}E_{3a}=-{}^{\downarrow}E_3$, so that ${}_{\uparrow}\Lambda_{3a}=-{}^{\downarrow}\Lambda_3$. Suppose that ${}^{\downarrow}E_j$ for the other particles is positive, so that ${}^{\downarrow}\Lambda_{ja}={}^{\downarrow}\Lambda_j$. An observer in ${}^{\downarrow}S$ will naturally write the conservation law in terms of the attributed quantities. Since particle 3 is an outgoing particle in ${}^{\downarrow}S$, he will write

$${}^{\downarrow}\Lambda_{1a}+{}^{\downarrow}\Lambda_{2a}={}^{\downarrow}\Lambda_{3a}+{}^{\downarrow}\Lambda_{4a}+{}^{\downarrow}\Lambda_{5a}+{}^{\downarrow}\Lambda_{6a}+{}^{\downarrow}\Lambda_{7a}. \tag{37}$$

However, this equation is clearly equivalent to Eq. (36), which followed from Eq. (35). Therefore, conservation in ${}^{\uparrow}S$ implies conservation in ${}^{\downarrow}S$. We also note in passing that, to conserve charge, a particle must clearly be replaced by its antiparticle when the incoming or outgoing character of its world line is changed.

III. EXTENDED RELATIVITY PRINCIPLE

A. Statement of the Principle

Because of the symmetry or duality between superluminal frames and subluminal frames, we are led to introduce the following postulate, which we call the *extended principle of relativity:* The totality of the laws of physics has the same form relative to the superluminal frames as it does relative to the subluminal frames. In general, the laws governing tachyons and subluminal particles will be interchanged in a transformation between a subluminal and a superluminal frame, but the total structure of the laws will have the same form. Consequently, the laws governing superluminal particles relative to superluminal frames (in which those particles appear like ordinary particles) are the same as the laws governing subluminal particles relative to subluminal frames. Since photons and other particles traveling at the velocity of light have the same properties in both kinds of frames, the particular equations governing them should be themselves form-invariant under superluminal transformation, rather than be interchanged with other equations. We call such a property extended form invariance.

B. Electromagnetic Field

The simplest superluminal transformation corresponds to the matrix $M(0)$.[9] It is easy to show that the attributed energy and momentum of a photon are unchanged under the transformation $M(0)$, when the reinterpretation principle is applied. Consequently, the transformation corresponding to $M(0)$ must leave $\mathbf{E}\times\mathbf{B}$ and $\mathbf{E}^2+\mathbf{B}^2$ unchanged, where $\mathbf{E}$ and $\mathbf{B}$ are the electromagnetic fields corresponding to a photon moving in the $\pm x$ direction. For such a photon $\mathbf{E}$ and $\mathbf{B}$, which are perpendicular to one another, depend only on x and t; and E_x and B_x vanish in both frames. The transformation of $\mathbf{E}$ and $\mathbf{B}$ must also leave form-invariant Maxwell's free-space equations, since those equations must satisfy extended form invariance.

The identity transformation of the fields satisfies the above requirements. However, the identity transformation of the fields already corresponds to the transformation $L(0)$.[10] If we want a transformation of the field which is unique (to within a rotation about the x axis) to correspond to each of the matrices of the extended Lorentz group in one dimension, then we must exclude the identity transformation of the field as corresponding to $M(0)$.

With the identity excluded, the previous requirements imply that under the transformation corresponding to $M(0)$, the above photon's electric and magnetic fields change according to the duality transformation ${}^{\downarrow}\mathbf{E}=-{}^{\uparrow}\mathbf{B}$, ${}^{\downarrow}\mathbf{B}={}^{\uparrow}\mathbf{E}$ to within a rotation about the x axis.[11] Since E_x and B_x are zero, the above considerations do not tell us how they transform, but they do suggest that the y and z components of $\mathbf{E}$ and $\mathbf{B}$ would transform under $M(0)$ as indicated, for any electromagnetic field.

[9] Under $M(0)$ each space-time event is reflected with respect to the line $x-ct=0$. Since we have not specified the behavior of y and z under superluminal transformation, the considerations in this section are essentially heuristic in nature.

[10] Also rotations about the x axis correspond to $L(0)$, since we have not specified the transformations of y and z. Therefore, along with the identity transformations of the field we include transformations induced by rotations about the x axis. Also for the case of the transformation of the field corresponding to $M(0)$, we cannot exclude transformations induced by rotations about the x axis, or even more general kinds of transformations involving y and z.

[11] We use Heaviside-Lorentz units.

Thus, for example, a charged superluminal particle at rest at the origin of a frame ${}^{\uparrow}S$, would, in the frame ${}^{\downarrow}S=M(0){}^{\uparrow}S$, have the y and z components of its electromagnetic field equal to those of a magnetic monopole moving at infinite speed. The fields in other subluminal frames are related by Lorentz transformations.

The above conclusions are also supported by the following considerations. The transformation of the y and z components of the electromagnetic field under the Lorentz transformation $L(\alpha)$ can be written in the form

$$\begin{aligned} E_z-B_y&=e^{\alpha}(E_z'-B_y'),\\ E_z+B_y&=e^{-\alpha}(E_z'+B_y'),\\ E_y+B_z&=e^{\alpha}(E_y'+B_z'),\\ E_y-B_z&=e^{-\alpha}(E_y'-B_z'), \end{aligned} \tag{38}$$

where $S'=L(\alpha)S$. The matrix $L(\alpha)$ is converted into $M(\alpha)$ by replacing $e^{-\alpha}$ by $-e^{-\alpha}$. Hence, we might expect that the transformation of the y and z components of the field, corresponding to $M(\alpha)$, would be

$$\begin{aligned} {}^{\uparrow}E_z-{}^{\uparrow}B_y&=e^{\alpha}({}^{\downarrow}E_z-{}^{\downarrow}B_y),\\ {}^{\uparrow}E_z+{}^{\uparrow}B_y&=-e^{-\alpha}({}^{\downarrow}E_z+{}^{\downarrow}B_y),\\ {}^{\uparrow}E_y+{}^{\uparrow}B_z&=e^{\alpha}({}^{\downarrow}E_y+{}^{\downarrow}B_z),\\ {}^{\uparrow}E_y-{}^{\uparrow}B_z&=-e^{-\alpha}({}^{\downarrow}E_y-{}^{\downarrow}B_z), \end{aligned} \tag{39}$$

where ${}^{\downarrow}S=M(\alpha){}^{\uparrow}S$.

One can confirm directly that the transformation (39), together with the coordinate transformation corresponding to $M(\alpha)$, does indeed leave Maxwell's free-space equations form-invariant in the case when the fields depend only on x and t. For fields which depend on y and z, as well as x and t, Maxwell's free-space equations remain form-invariant provided that, in addition, the transformations of y, z, B_x, and E_x leave effectively unchanged the combinations of those quantities which appear in Maxwell's equations. When $\alpha=0$, Eq. (39) reduces to

$$\begin{gathered} {}^{\downarrow}E_y={}^{\uparrow}B_z,\qquad {}^{\downarrow}E_z=-{}^{\uparrow}B_y\\ \text{and}\\ {}^{\downarrow}B_y=-{}^{\uparrow}E_z,\quad {}^{\downarrow}B_z={}^{\uparrow}E_y. \end{gathered} \tag{40}$$

These equations are equivalent to the y and z components of the duality transformation ${}^{\downarrow}\mathbf{E}=-{}^{\uparrow}\mathbf{B}$, ${}^{\downarrow}\mathbf{B}={}^{\uparrow}\mathbf{E}$ to within a rotation about the x axis of one of the frames. Owing to the fact that the three-dimensional superluminal transformation apparently does not exist, the considerations in this section are merely suggestive, and are by no means conclusive.

C. Interactions

The extended principle of relativity implies that the laws governing the existence of elementary particles relative to the superluminal frames are the same as the corresponding laws relative to the subluminal frames. It follows that the same kinds of elementary particles can exist relative to the superluminal frames (they will be tachyons in the subluminal frames), and the same interactions can occur.

Similarly, the one-dimensional quantum field theory of tachyons interacting with one another and with photons will clearly be precisely the same relative to the superluminal frames, as the field theory of ordinary particles and photons in the subluminal frames. In order to construct a field theory involving superluminal particles, subluminal particles, and photons it is necessary to transform the field theory of tachyons from the superluminal frames to the subluminal frames, and to join it with the field theory of ordinary particles. In view of the difficulties involved in incorporating tachyons into quantum field theory, it would be interesting to explicitly carry out such a program.[12] However, we will not go further into such matters in this paper, although we will apply the results of field theory in the superluminal frames to a process involving tachyons and photons.

The extended relativity principle implies that both superluminal and subluminal particles will interact electromagnetically with photons, and thus indirectly with each other. We call that interaction via photons the minimal interaction between tachyons and subluminal particles. We would also expect weaker interactions of tachyons with neutrinos and gravitons to exist.

In thinking of an experiment to detect tachyons,[13] it seems safest to assume that the density of free tachyons in space is very low, or effectively zero. Thus, for example, we would not expect to detect a process which required the presence of an incoming tachyon in the superluminal frames (where according to the extended principle of relativity the familiar physical laws can be applied to predict the probability of the process). It can be shown that processes such as the annihilation of a photon into a tachyon-antitachyon pair, or the creation of a tachyon-antitachyon pair in the head on collision of two photons in a subluminal frame, require the presence of an incoming tachyon in the superluminal frames. In that way processes which would at first seem rather probable if charged tachyons interacting electromagnetically with photons existed can be excluded.

A higher-order process in which the electromagnetic interaction of tachyons with photons could be detected is photon-photon scattering. A process in which two incoming photons collide head on and are scattered in the backward direction will appear the same, as far as the incoming and outgoing photons are concerned, in both the superluminal and subluminal frames. Therefore, if the probability of that process as calculated in the subluminal frames not taking into account tachyons is P, the probability as calculated in the superluminal frames not taking into account subluminal particles will also be P. Assuming that the contribution to that process resulting from tachyon-antitachyon pairs does not interfere with the contribution from ordinary particle-antiparticle pairs, the total probability of the process, taking into account both subluminal and superluminal particles, will be $2P$. Thus, we predict that if two photon beams collide head on, the probability of photon scattering near the backward direction will be twice as great as would be predicted without taking into account tachyons. For other angles of scattering we can make no definite predictions because our theory is only one-dimensional. Photon-photon scattering is discussed, and references to the literature are given in a paper by Csonka.[14] Experiments involving direct photon-photon scattering may be feasible in the near future, and would serve as an important test of quantum electrodynamics.

IV. CONCLUSIONS

The purposes of this work were to present a transformation which makes tachyons into ordinary particles in one dimension, and to suggest the extended relativity principle, which specifies the dynamics of one-dimensional processes involving tachyons, and can be subjected to experimental test. Even if the results of photon-photon scattering experiments were to contradict the predictions made here, the part of this paper on superluminal transformations would remain valid, and might continue to be useful in formalistic considerations, such as those connected with a one-dimensional quantum field theory of tachyons. A negative result would show that the one-dimensional theory presented here was not relevant to the three-dimensional world, and would tend to support the suggestion of Dhar and Sudarshan[12] that charged tachyons do not exist.[15]

ACKNOWLEDGMENT

It is a pleasure to thank my colleagues for stimulating discussions.

[12] Field theories involving tachyons have been proposed in G. Feinberg (Ref. 2); M. Arons and E. C. G. Sudarshan, Phys. Rev. **173**, 1622 (1968); J. Dhar and E. C. G. Sudarshan, *ibid.* **174**, 1808 (1968). Further discussion appears in M. M. Broido and J. G. Taylor (Ref. 2).

[13] Experiments aimed at detecting tachyons have been reported in T. Alvager and P. Erman, Nobel Institute Report, 1966 (unpublished); T. Alvager and N. M. Kreisler, Phys. Rev. **171**, 1357 (1968). Dhar and Sudarshan (Ref. 12) also suggest possible ways of experimentally detecting tachyons.

[14] P. L. Csonka, Phys. Letters **B24**, 625 (1967).

[15] Since the completion of this work, Dr. K. H. Mariwalla informed me that he has considered, in unpublished report, a three-dimensional transformation which reduces to the superluminal transformation considered here in one dimension. He also pointed out that a number of kinematic aspects of the one-dimensional superluminal transformation were considered in R. T. Jones, J. Franklin Inst. **275**, 1 (1963).