The results are

$$\frac{4(\mathrm{Re}\rho_{m1})^2}{\rho_{mm}\rho_{11}}=\frac{\{[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]+\eta_1\eta_m^{1/2}\epsilon_1\epsilon_m\}^2}{\eta_1[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]}, \quad (A16)$$

$$\frac{4(\mathrm{Re}\rho_{m,-1})^2}{\rho_{mm}\rho_{11}}=\frac{\{\eta_1\epsilon_1[1+(1-\eta_m\epsilon_m^2)^{1/2}]+\eta_m\epsilon_m[1+(1-\eta_1\epsilon_1^2)^{1/2}]\}^2}{\eta_1\eta_m[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]}. \quad (A17)$$

(iv) $m\geq 3$, $n\geq 2$:

$$\mathrm{Re}\rho_{mn}=(1+a^2)(F_{1m}F_{1n}+F_{-1m}F_{-1n}), \quad (A18)$$

$$\sigma_V(-1)^n\,\mathrm{Re}\rho_{m,-n}=-2a(F_{-1m}F_{1n}+F_{1m}F_{-1n}). \quad (A19)$$

The result is

$$2(\mathrm{Re}\rho_{m,\pm n})^2/\rho_{mm}\rho_{nn}=1+\eta_m\epsilon_m\epsilon_n\pm[(1-\eta_m\epsilon_m^2)(1-\eta_n\epsilon_n^2)]^{1/2}. \quad (A20)$$

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Infinitely Degenerate Leading Baryon Trajectory*

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In the quark model, the leading baryon trajectory is resolved into infinitely many degenerate trajectories. An exchange-degeneracy pattern of periodicity $\Delta\, j=6$ is obtained. At finite physical values of the spin, only a finite number (increasing with the spin) of these infinitely many trajectories support particles. A general hadronic mass formula is proposed.

1. INTRODUCTION

THE absence of exotic hadrons [i.e., baryons other than $SU(3)$ singlets, octets, or decimets, and mesons other than nonets, etc.] that couple very strongly to the usual mesons and baryons is an experimental fact. Channels with exotic quantum numbers can "communicate" with normal channels through crossing (e.g., $K^+p\rightarrow K^+p$ with $K^-p\rightarrow K^-p$). Thus, the absence of very strong resonances in the exotic channel leads to dynamical consequences in normal channels. These consequences take the form of exchange degeneracies between various normal-channel Regge trajectories. For mesonic trajectories, exchange degeneracy has been explored in detail. For baryons, exchange degeneracy has been considered more recently. The difficulty of the problem is due to our lack of knowledge of the detailed baryon spectrum. Following Schmid's[1] proposal of baryonic exchange degeneracy, Capps[2] studied the exchange degeneracy of baryonic $SU(3)$ multiplets. This work, however, is confined to processes involving as external particles only the 36 ground-state mesons and 56 ground-state baryons. He also assumes that the leading baryon trajectories are an even-signature $(56,\ L=\alpha_{56}(s))^+$ trajectory and an odd-signature $(70,\ L=\alpha_{70}(s))^-$ trajectory. The former supports the particle multiplets (roughly equally spaced in mass squared) $(56,\ L=0)^+$, $(56,\ L=2)^+$, $(56,\ L=4)^+$, $\cdots$, while the latter supports $(70,\ L=1)^-$, $(70,\ L=3)^-$, $(70,\ L=5)^-$, $\cdots$. Exchange degeneracy is imposed in the form $\alpha_{56}(s)=\alpha_{70}(s)$, and of certain relations between the residues. In this scheme, the absence of 20-plets is just a consequence of the limitation to **35**-**56** scattering rather than an actual feature of the baryon spectrum. In the processes $MM\rightarrow B\bar{B}$, it requires the presence of exotic resonances. To avoid this undesirable feature, Mandula *et al.*[3] have suggested that an *even*-signature **70** trajectory is degenerate with the even-signature **56**. While this achieves the desired result it also confronts one with the unattractive (and experimentally catastrophic) feature of a low-lying $(70,\ L=0)^+$ supermultiplet. A possible way around this difficulty was proposed by Mandula, Weyers, and Zweig,[4] who suggest that there exists a hierarchy of exchange-degeneracy principles and that the $(56, L=0)^+-(70,\ L=0)^+$ degeneracy is far from the top of this hierarchy and, therefore, is badly broken. Thus, the

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[1] C. Schmid. Nuovo Cimento Letters, **1**, 165 (1969).

[2] R. H. Capps, Phys. Rev. Letters **22**, 215 (1969); and to be published.

[3] J. Mandula, C. Rebbi, R. Slansky, J. Weyers, and G. Zweig, Phys. Rev. Letters **22**, 1147 (1969).

[4] J. Mandula, J. Weyers, and G. Zweig, Phys. Rev. Letters **23**, 266 (1969).

even-signature **70** trajectory exists but way below the **56** trajectory. The corresponding particles are therefore much heavier.

In this paper we wish to expand the scope of these investigations, exploring some features of baryonic Regge trajectories in a quark model. Baryons, being built of three rather than two quarks, have more degrees of freedom than mesons. In particular, there are increasingly many possibilities to form states of maximum orbital angular momentum as the mass of the state increases. This allows an infinite degeneracy of the leading baryon trajectory. It is this feature of baryonic Regge trajectories that we describe in Sec. 2. In Sec. 3 we derive a general hadronic mass formula describing the transition from the quark model's $U(6)\times U(6)\times O(3)$ symmetry to the chiral $U(2)\times U(2)$ symmetry.

2. INFINITELY DEGENERATE LEADING BARYON TRAJECTORY

To describe the possible infinite degeneracy of the leading baryon trajectory, we consider the case of three quarks in a harmonic-oscillator potential. The baryon mass spectrum is then

$$m_n^2=m_0^2+\mu^2 n\,, \tag{1}$$

where n is the radial quantum number (number of oscillator quanta). At mass m_n^2 there is an "accidental" degeneracy. The orbital angular momenta L of the degenerate states range from 0 to n. Each value of L may be occupied more than once The parent (leading Regge trajectory) is given by the equation

$$L=n\,, \tag{2}$$

where [using Eq. (1)]

$$n=(m_n^2-m_0^2)/\mu^2\,. \tag{3}$$

As n increases the value $L=n$ gets occupied by more and more multiplets. We present in Fig. 1 the supermultiplets appearing on the trajectory (2) for $n\leqslant 8$.[5] We see that at $n=0$, there "starts" an even-signature trajectory (marked by the first vertical dashed line in Fig. 1) of 56-plets. At $n=1$, there starts an odd-signature trajectory of 70-plets. At $n=2$, we see the first recurrence of the **56** trajectory that started at $n=0$ and a new even-signature **70** trajectory starts, etc. The general rule is that at[5]

$n=\ 0$, a **56** trajectory of even signature starts;
$=3\nu\neq 0$, a new **56** and a new **20** trajectory of signature $(-1)^{3\nu}$ start;
$=3\nu+1$, a new **70** trajectory of signature $(-1)^{3\nu+1}$ starts;
$=3\nu+2$, a new **70** trajectory of signature $(-1)^{3\nu}$ starts.

[5] This is a straightforward consequence of the group-theoretical arguments of G. Karl and E. Obryk, Nucl. Phys. **B8**, 609 (1968); W. Thirring (private communication).

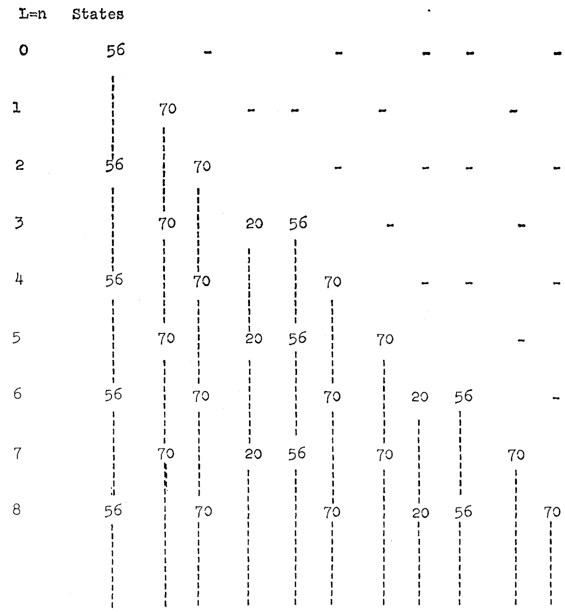

FIG. 1. Supermultiplets appearing on the trajectory $L=n$ for $n\leq 8$.

All these trajectories are *degenerate*. Trajectories that start at a certain value of n, extend below that value of n, but they do not support particles (the spaces marked by blanks in Fig. 1) with $L<n$. Thus, the leading baryon trajectory is *infinitely degenerate*. Yet at any finite physical value of the spin, only a finite number (increasing with spin) out of these infinitely many degenerate trajectories support particles.

Now, let us see the way in which the exchange degeneracy of Regge residues ensures the absence of exotic resonances in the crossed channel. A trajectory that "starts" at certain value L must have zeros in its residues for $L<n$ that cannot be present in trajectories that start at $L'\leqslant L-2$. Therefore, the exchange-degeneracy pattern should be such that cancellations occur only among trajectories "starting" in neighboring (i.e., differing by 1) values of L. The "natural" pattern is therefore

$$(56)^+\leftrightarrow(70)^-;\quad (70)^+\leftrightarrow\begin{pmatrix}56\\20\end{pmatrix}^-;\quad (70)^+\leftrightarrow(70)^-;$$
$$\begin{pmatrix}56\\20\end{pmatrix}^+\leftrightarrow(70)^-;\quad (70)^+\leftrightarrow\begin{pmatrix}56\\20\end{pmatrix}^-;(70)^+\leftrightarrow(70)^-;\quad \cdots, \tag{4a}$$

where the sign indicates the "orbital" signature (which is equal to the parity). We see that the pattern is periodic. It repeats itself every sixth value of L. From (4) the leading baryon trajectories will follow the periodic exchange-degeneracy pattern

$$(10)^+\leftrightarrow(8)^-;\ (8)^+\leftrightarrow(10)^-;\ (8)^+\leftrightarrow(8)^-;\ \cdots. \tag{4b}$$

The next-to-leading trajectories obviously also follow a periodic pattern:

$$(8)^+ \leftrightarrow \begin{bmatrix}10\\8\\1\end{bmatrix}^-, \quad \begin{bmatrix}10\\8\\1\end{bmatrix}^+ \leftrightarrow (8)^-, \quad \begin{bmatrix}10\\8\\1\end{bmatrix}^+ \leftrightarrow \begin{bmatrix}10\\8\\1\end{bmatrix}^-;$$

$$(8)^- \leftrightarrow (8)^+, \quad (10)^- \leftrightarrow (8)^+, \quad (8)^- \leftrightarrow \cdots. \quad (4c)$$

We have checked (see Appendix A) the patterns (4b) and (4c) in **35**-**56** scattering and **35**-**70** scattering. The D/F ratios and other Clebsch-Gordan coefficients predicted by $U(6)_W \times O(2)_{L_z}$ are such that the absence of exotic baryons in both s and u channels is simultaneously implemented (some of these results are contained in Ref. 3). In **35**-**56** scattering, of course, all **20**'s decouple. Observe that we require cancellation among the leading trajectories [Eq. (4b)] and next to leading trajectories [Eq. (4c)] separately. We use only the $SU(6)$-vertex predictions but not any collinear four-point predictions. As such, our results are not sensitive to mass splittings within $SU(6)$ multiplets.

In **35**-**20** scattering, the **56**'s decouple. This requires a shifting of the pattern by one unit for this case since the first ($n=0$) 56 does not have a 20 partner. Thus, for **35**-**20** scattering, we have

$$(70)^- \leftrightarrow (70)^+, \quad (20)^- \leftrightarrow (70)^+, \quad (70)^- \leftrightarrow (20)^+, \quad \cdots, \quad (5)$$

and similar shifts in the patterns of leading trajectories. In this case the D/F ratio of $+1$ for the coupling of the $J=L+\frac{3}{2}$ octet of the **70** to the $J=L+\frac{1}{2}$ octet of the **20** and the 0^- octet mesons is just that required by the $(20)^- \leftrightarrow (70)^+$ and $(70)^- \leftrightarrow (20)^+$ links in the pattern (5). Thus, this periodic (again with period $\Delta L=6$) pattern is consistent for **35**-**20** scattering as well.

We have implemented in all these cases the absence of exotic resonances in all baryon-number-one channels. In the $B\bar{B} \to MM$ channel, we have to invoke ideas of the type advanced in Ref. 4 in order to rid it of exotic states. Thus, somewhere at higher masses we have to find the **70** $L=0$ multiplet and its recurrences, daughters, etc.

Besides the implications for the baryon spectrum, an important consequence of the infinite degeneracy of the leading baryon trajectory concerns near-backward meson-baryon scattering. In this region one can parametrize the meson-baryon scattering amplitudes in terms of a few Regge poles with smoothly varying residues. If there are infinitely many degenerate trajectories, then these smoothly varying residues are actually some effective residues obtained from *all* these trajectories. Then, it would not be surprising if, upon extrapolation of these smooth residues to the mass of the first physical particle [say, the $\Delta(1238)$ on the Δ trajectory], they would not match the value of the residue obtained from the elastic width (i.e., from *one* trajectory). Such a mismatch seems indeed to occur for the Δ trajectory.[6]

[6] R. Amann (private communication).

An important experimental test of our proposal is the following. We predict that at $L=2$ along with the Regge recurrence of the **56** we should have also a (70, 1; $L=2$) multiplet. This means in particular an octet of $J^P=\frac{7}{2}^+$. This octet contains an $I=0$ $Y=0$ baryon $\Lambda_{7/2}{}^+$. *If such a $\Lambda_{7/2}{}^+$ is found in the 2-GeV mass region (i.e., in the $L=2$ mass region), this will be strong evidence in favor of our scheme*, as opposed, say, to a scheme where all even L multiplets are **56**'s and all odd L multiplets are **70**'s.

In this paper we have confined ourselves to leading (parent) trajectories. Satellite (daughter) trajectories can be discussed along the same lines and will be also infinitely degenerate. The details, however, will be different for second and higher daughters. For first daughters in the harmonic-oscillator model, the sequence of appearance of new trajectories is the same as for the parent.

3. GENERAL HADRONIC MASS FORMULA

Our discussion so far has kept within the quark model and the $U(6)\times U(6)\times O(3)$ classification of hadrons that it entails. Experimentally, the chiral $U(3)\times U(3)$ [or, more accurately, $U(2)\times U(2)$] classification is more realistic for classifying Regge trajectories. Indeed, we have $\alpha_\Delta(s)-\alpha_N(s)=\alpha_\rho(s)-\alpha_\pi(s)=\frac{1}{2}$ and not 1 as expected from $U(6)\times U(6)\times O(3)$. We therefore ask ourselves whether chiral $U(2)\times U(2)$ can be obtained by a suitable breaking mechanism of $U(6)\times U(6)\times O(3)$. The clue to this problem is that in the $U(2)\times U(2)$ limit there are still a number of unwanted degeneracies[7] like

$$m_B{}^2=m_{A_1}{}^2, \quad m_\eta{}^2=m_\pi{}^2=0, \quad (6a)$$

along with the desirable relations such as

$$m_{A_2}{}^2 : m_{A_1}{}^2 : m_\sigma{}^2 : m_\rho{}^2 = 3:2:1:1,$$

$$m_\Delta{}^2-m_N{}^2=m_\rho{}^2, \quad m_f{}^2=mA_2{}^2, \quad m_\omega{}^2=m_\rho{}^2, \text{ etc.}, \quad (6b)$$

and

(the equality of the slopes of *all* hadronic Regge trajectories). (6c)

The fact that the B and A_1 mesons are degenerate, along with Eq. (6c) in this limit excludes the possibility of an $\mathbf{L}\cdot\mathbf{S}$ force producing the bulk of the mass splittings. We first classify all hadrons according to $U(6)\times U(6)\times O(3)$. Mesons belong into $(6, \bar{6}; L=a+bt)$ representations and baryons into $(56,1; L=c+dt)$, $(70,1; L=e+ft)$, etc., representations. Let us label each hadron H by the following quantum numbers: $\nu=$No. of quarks+No. of antiquarks in H, $B=$baryon number of H, $\nu_\lambda=$No. of λ+No. of $\bar{\lambda}$ in H, $L=$total orbital angular momentum of quarks in H, $S=$total spin-angular

[7] M. Ademollo, G. Veneziano and S. Weinberg, Phys. Rev. Letters **22**, 83 (1969); P. G. O. Freund and E. Schonberg, Phys. Letters **28B**, 600 (1969).

momentum of quarks in H, J = total angular momentum of H, n = genealogic radial quantum number defined such that $n=0$ for particles on parent trajectory, $n=i$ for ith daughters, and X = all other quantum numbers such as $SU(6)$ multiplet, $SU(3)$ multiplet, isospin, etc. We now write down a mass formula that depends on these quantum numbers in such a way that the initial $U(6)\times U(6)\times O(3)$ symmetry is broken down to $U(2)\times U(2)$ and the relations (6) hold. This formula is (see Appendix B)

$$m^2(\nu,B,\nu_\lambda,L,J,n,S,X)=m_\rho^2(L+J+2n) +m_\rho^2[\alpha|B|+(1-\alpha)(\nu-2)]+\tfrac{1}{2}m_K^2\nu\nu_\lambda. \quad (7)$$

It contains only one unknown parameter: α. This parameter α fixes the dependence of m^2 on ν and should be measurable once exotic resonances are firmly established. The over-all m_ρ^2 factor in the second term has been adjusted so that the empirical formula

$$m_N^2=\tfrac{3}{2}m_\rho^2 \quad (8)$$

is obeyed. This corresponds to $\alpha_\Delta(0)=\frac{1}{4}$, $\alpha_N(0)=-\frac{1}{4}$. It is interesting that with formula (7), *all* nonstrange hadronic Regge trajectories (mesonic *and* baryonic) with $\nu\leqslant 3$ become equally spaced. Their zero intercepts $\alpha(0)=+\frac{1}{2}, +\frac{1}{4}, 0, -\frac{1}{4}, -\frac{1}{2}, \cdots$. The last term has been arranged to implement the quark-model mass formula[8]

$$(m_K^2-M_\pi^2)/(m_\Xi^2-M_N^2)=\tfrac{1}{3} \quad (9)$$

and the analog formulas for $\nu>3$. The full $U(6)\times U(6)\times O(3)$ symmetry breaking in (6) originates in the terms proportional to J and ν_λ. This formula should be useful in future discussions of the relation of $U(6)$ and chiral-type symmetries in strong interactions.

4. CONCLUSIONS

To sum up, in this paper we have shown that because of their qqq structure, *all* baryon trajectories including the leading (parent) baryon trajectory are likely to be *infinitely degenerate*, while supporting a *finite* number of particles at each finite physical value of the spin. We have presented a specific mass formula [Eq. (6)] that allows the transition from the supersymmetric $U(6)\times U(6)\times O(3)$ case to the more realistic chiral $U(2)\times U(2)$ case to be made.

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APPENDIX A

The method for checking baryonic exchange degeneracy is well known. Here we simply give a brief derivation of one of our new results. Consider the case of **35**-**20** scattering and, specifically, the scattering of the 0^- octet on the $^2L_{L+1/2}$ octet of the **20**. Only **70**'s and **20**'s can contribute, since the **56**-**35**-**20** coupling is forbidden. The leading $J=L+\frac{3}{2}$ trajectories of the **70** and **20** are, respectively, an octet and a singlet. For an octet and a singlet to cancel in all exotic channels [using the well-known $8\times 8\rightarrow 8\times 8$ $SU(3)$ crossing matrix], one finds that the octet has to couple with $D/F=+1$. This is precisely the D/F ratio predicted by $U(6)_W\times O(2)_{L_z}$. Our other checks can be made along the same lines.

APPENDIX B

We present here our argument in favor of the mass formula (7). The fact that *all* trajectories must be straight lines means that m^2 must be of the form

$$m^2(\nu,B,\nu_\lambda,L,J,n,S,X)=a+bL+cJ+2d\mathbf{L}\cdot\mathbf{S} +en+fS(S+1), \quad (B1)$$

where all coefficients a–f can be functions of ν, B, ν_λ, and X.

The chiral mass formula $m_B^2=m_{A_1}^2$ implies that

$$d=f. \quad (B2)$$

The equality of the slopes of *all* Regge trajectories then requires that

$$d=f=0 \text{ and } b+c=\text{const independent of } \nu,\ \nu_\lambda,\ B,\ X. \quad (B3)$$

The relations

$$m^2(2,0,0,0,1,0,1,Y=0,I=1)=m_\rho^2,$$
$$m^2(2,0,0,0,0,0,0,Y=0,I=1)=m_\pi^2=0,$$

and

$$m^2(2,0,0,1,1,0,1,Y=0,I=1)=m_{A_1}^2=2m_\rho^2$$

then require that

$$a|_{B=0,\ \nu=2}=0, \quad b=c=m_\rho^2, \quad (B4)$$

and predict $m_{A_2}^2=3m_\rho^2$.

To ensure that at the A_2 mass we have an $L=0$, $J^P=1^-$ "daughter," we require that $e=2m_\rho^2$. The B, ν, and ν_λ dependence of a is explained in the main text. This concludes our argument for Eq. (7).

[8] P. G. O. Freund, Nuovo Cimento **39**, 769 (1965).