PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Additional Relations between Spin-Density Matrix Elements*

ROBERT L. THEWS
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 11 August 1969)

The spin structure of amplitudes for the joint production of a baryon and meson resonance of arbitrary spins has been investigated. Both meson-nucleon and photon-nucleon interactions have been considered. If the production is dominated by the exchange of a single set of quantum numbers, relations between spin-density matrix elements exist in some cases and provide an experimental test for this mechanism. Comparisons with data are given for some reactions.

I. INTRODUCTION

THIS is an extension of previous work[1] on quasi-two-body reactions initiated by pseudoscalar mesons on nucleons with a *single* meson or baryon resonance in the final state. References to this work will be denoted by RT. The more general quasi-two-body reaction is studied here, with both a meson and baryon resonance in the final state. In addition, the same reactions are considered when initiated by photons. A summary of the formalism and notation is presented below. The reader is referred to RT for further details.

Consider the s-channel process $A+B\to C+D$. We look at the decay of particle C or D in the t-channel frame.[2] Then the spin-density matrix elements $\rho_{mm'}$ can be expressed in terms of the helicity amplitudes $F_{\lambda_B\lambda_D,\lambda_A\lambda_C}$ for the crossed- (t-) channel process $A+\bar{C}\to\bar{B}+D$. The relation is

$$\rho_{mm'}{}^{C}=\frac{\sum_{\lambda_A\lambda_B\lambda_D}F_{\lambda_B\lambda_D,\lambda_A m}F_{\lambda_B\lambda_D,\lambda_A m'}{}^{*}}{\sum_{\lambda_A\lambda_B\lambda_C\lambda_D}|F_{\lambda_B\lambda_D,\lambda_A\lambda_C}|^2}\qquad(1)$$

for particle C, with a similar expression for particle D. The independent measurable density matrix elements for a spin-J particle are

$$\rho_{mm},\qquad 0<m\le J$$
$$\mathrm{Re}\rho_{mn},\qquad |n|<m\le J$$

and

$$\rho_{m,-m}\qquad \text{if } J \text{ is an integer.}$$

Relations between spin-density matrix elements follow from relations between the t-channel helicity amplitudes required by conservation laws for the exchange of definite quantum numbers such as parity, isospin, G parity, and, for Regge trajectories, J parity. [$\sigma\equiv P(-1)^J$ for Bosons or $\sigma\equiv P(-1)^{J-1/2}$ for fermions, where P is parity and J is the spin of a particle lying on the trajectory.] The results are summarized in RT (7)–(13) and will be used extensively in the following derivations.

II. $0^-+\frac{1}{2}^+\to V+N^*$

For the meson spin-density matrix elements $\rho_{mm'}$, we look for relations similar to those for single-meson resonance production [RT (36) and (37)]. For a single exchange with J parity σ_E, the results are

$$\mathrm{Re}\rho_{mm'}+\sigma_V\sigma_E(-1)^{m'}\,\mathrm{Re}\rho_{m,-m'}=\sum_{n,\lambda}{}'\,\mathrm{Re}(F_{m,\lambda n}-\sigma_{N^*}\sigma_E(-1)^{n-\lambda}F_{m,-\lambda-n})\times(F_{m',\lambda n}-\sigma_{N^*}\sigma_E(-1)^{n-\lambda}F_{m',-\lambda-n})^{*}\qquad(2)$$

and

$$\mathrm{Re}\rho_{mm'}-\sigma_V\sigma_E(-1)^{m'}\,\mathrm{Re}\rho_{m,-m'}=4\,\mathrm{Re}F_{m,\frac12\frac12}F_{m',\frac12\frac12}{}^{*}+\sum_{n,\lambda}{}'\,\mathrm{Re}(F_{m,\lambda n}+\sigma_{N^*}\sigma_E(-1)^{n-\lambda}F_{m,-\lambda-n})\times(F_{m',\lambda n}+\sigma_{N^*}\sigma_E(-1)^{n-\lambda}F_{m',-\lambda-n})^{*},\qquad(3)$$

where $\sum_{n,\lambda}{}'$ denotes the restricted sum $n>0$, $\lambda=\pm\frac{1}{2}$, $n\neq\lambda$, and the normalization factors have been omitted for simplicity. For a single-meson resonance production the only n value is $+\frac{1}{2}$ and the square of (2) leads directly to the relations RT (36) and (37). However, when $J_{N^*}>\frac{1}{2}$, the sum over n values produces interference terms in the square of (2). These terms prevent the relations RT (36) and (37) from being valid in the general case. However, one can still use the terms with $m=m'$ to derive inequalities involving the diagonal and antidiagonal elements. For this case, all the elements are real, so that the relations will hold for an arbitrary number of exchanges with the same J parity. One writes each t-channel helicity amplitude $F_{m,\lambda n}$ as a product of a residue function and a rotation coefficient $R_{m,\lambda n}(t)d_{n-\lambda,m}{}^{J}(x)$, where x is the cosine of the t-channel center-of-mass scattering angle. Then one uses the parity relations

$$R_{m,-\lambda-m}=\sigma_{N^*}\sigma_E R_{m,\lambda n}\qquad(4)$$

and the relation for rotation coefficients

$$d_{n-\lambda,m}{}^{J}\pm(-1)^{n-\lambda}d_{\lambda-n,m}{}^{J}=[(x+1)^{\tau}\pm(x-1)^{\tau}]f_{m,n-\lambda}{}^{J}(\tau,x)\,,\qquad(5)$$

where $\tau=\min(m,n-\lambda)$ and only leading-order terms in direct-channel energy (s) are kept in f^J. Part of the

---

* Work supported in part by the U. S. Atomic Energy Commission.

[1] G. A. Ringland and R. L. Thews, Phys. Rev. **170**, 1569 (1968).

[2] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

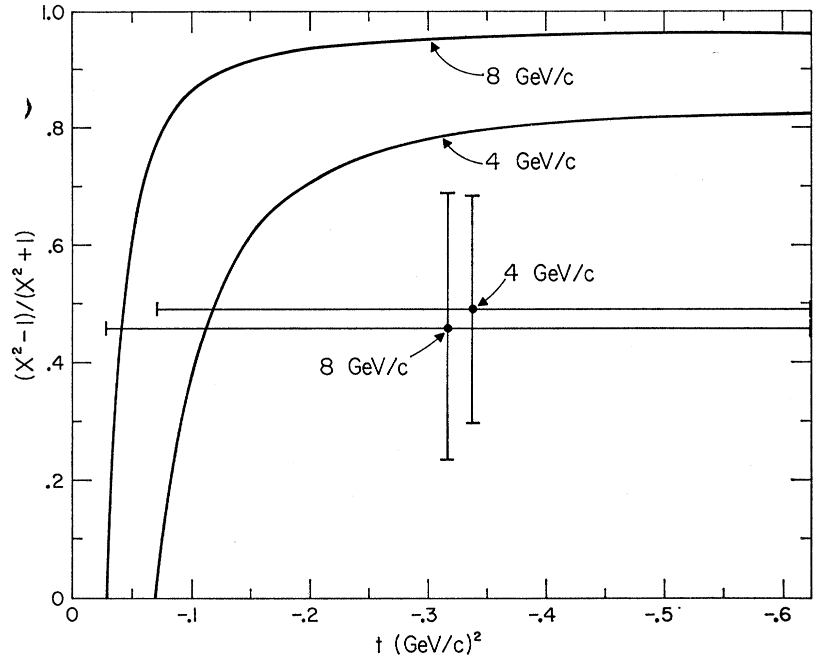

FIG. 1. Lower-bound comparison for $\rho_{1,-1}/\rho_{11}$ in $\pi^+ + p \to \omega^0 + \Delta^{++}$.

summation in (2) can then be substituted in (3) to yield

$$\rho_{mm}-\sigma_V\sigma_E(-1)^m\rho_{m,-m}=4F_{m,\frac{1}{2}\frac{1}{2}}{}^2$$
$$+\left(\frac{(x+1)^m+(x-1)^m}{(x+1)^m-(x-1)^m}\right)^2(\rho_{mm}+\sigma_V\sigma_E(-1)^m\rho_{m,-m})$$
$$+4\sum_{n-\lambda<m}{}'\left(\frac{R_{m,\lambda n}f_{m,\lambda-n}{}^J(\tau,x)}{(x+1)^m-(x-1)^m}\right)^2$$
$$\times(x+1)^{2m}(x^2-1)^{n-\lambda}(\beta^{m-n+\lambda}-1)(\beta^{m+n-\lambda}-1)\,, \quad (6)$$

where $\beta\equiv(x-1)/(x+1)\geq 1$ for the physical s-channel region $x\leq -1$. Since all terms not involving the spin-density matrix elements are positive definite, we get an inequality

$$\sigma_V\sigma_E(-1)^{m+1}\rho_{m,-m}\geq G_m(x)\rho_{mm}\,, \quad (7)$$

where

$$G_m(x)=\frac{2(x^2-1)^m}{(x+1)^{2m}+(x-1)^{2m}}\geq 0\,. \quad (8)$$

Thus we have both a restriction on the sign of $\rho_{m,-m}$ and a lower bound for its magnitude. The result can be stated in the following way. For the production of a meson resonance with J parity σ_V along with a baryon resonance of arbitrary quantum numbers via the exchange of an arbitrary number of trajectories with the same J parity σ_E, the sign of $\rho_{m,-m}$ must be $\sigma_V\sigma_E(-1)^{m+1}$, and the magnitude must satisfy

$$\frac{2(x^2-1)^m}{(x+1)^{2m}+(x-1)^{2m}}\leq\frac{|\rho_{m,-m}|}{\rho_{mm}}\leq 1\,, \quad (9)$$

where the upper bound comes from parity conservation alone. For $m=1$ we see that the lower bound is identical to that in RT (39), where the baryon is restricted to spin $\frac{1}{2}$ but m is arbitrary. It is easily seen that $G_{m+1}/G_m\leq 1$, so that the price of allowing arbitrary-spin baryon production is two weaken the lower bound for high-spin mesons.

As an example of the application of these constraints, consider the reaction $\pi^+ + p \to (\rho^0,\omega^0,f^0)+\Delta^{++}$. Data are available at pion lab momenta of 4.0 and 8.0 GeV/c for the ρ^0 and ω^0, and 8.0 GeV/c for the f^0.[3] For ρ^0 and ω^0, the sign of $\rho_{1,-1}$ must equal the J parity of the exchange σ_E, and the inequality

$$\frac{x^2-1}{x^2+1}\leq\frac{|\rho_{1,-1}|}{\rho_{11}}\leq 1 \quad (10)$$

must be satisfied. For ω^0, the sign of $\rho_{1,-1}$ is positive so that if there is a dominant J-parity exchange it must be positive; the ρ, for example. However, the element ρ_{00} only receives contributions from negative J-parity exchange,[2] and it is nonzero in this reaction. As a further check, we plot the lower bound (10) and the data points in Fig. 1. It is seen that the data points fall well below the lower bound, indicating again the necessity of both positive and negative J-parity exchanges. For ρ^0 production, the sign of $\rho_{1,-1}$ is not determined within experimental errors, although the central values are negative. The element ρ_{00} is also nonzero, so that the signs are consistent with a dominant negative J-parity exchange, such as the pion. However, Fig. 2 shows again

[3] Aachen-Berlin-CERN Collaboration, Phys. Letters **22**, 533 (1966).

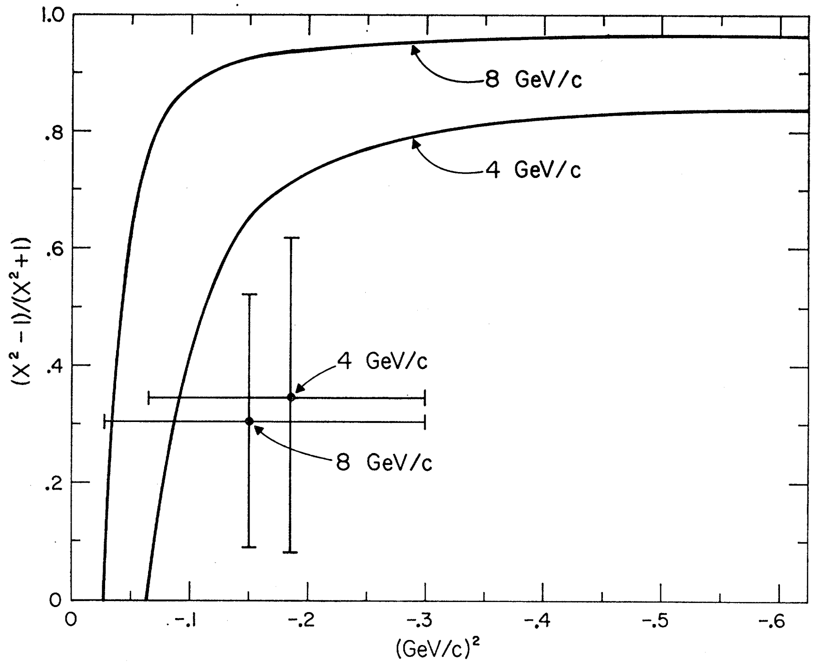

FIG. 2. Lower-bound comparison for $\rho_{1,-1}/\rho_{11}$ in $\pi^+ + p \rightarrow \rho^0 + \Delta^{++}$.

that the lower bound (10) is violated, so that both positive and negative J-parity exchanges are needed.

For f^0 production, we have bounds on the magnitude of $\rho_{2,-2}$ and $\rho_{1,-1}$ as well as prediction for their sign. Unfortunately, the analysis of the angular distribution for f^0 decay indicates a negative value of ρ_{22}. This can come about only when the decaying object is not in a pure spin state, so that the contribution of background events must be substantial. Thus a meaningful interpretation of the spin-density matrix elements for this reaction is not possible.

Data also exist for the processes $K^- + p \rightarrow (\rho,\phi) + Y^*(1385)$ at 4.1 and 5.5 GeV/c.[4] The leading candidates for exchange are the K and K^*. Since they have opposite J parity, the sign and magnitude of $\rho_{1,-1}$ should give information on whether one or the other or both are dominating. For ρ production the sign of $\rho_{1,-1}$ is negative, indicating negative J-parity exchange. But the ratio $|\rho_{1,-1}|/\rho_{11}$ exceeds unity, indicating $J \neq 1$ components in the ρ-decay products, and preventing us from determining if positive J-parity exchanges are also present. For ϕ production the sign of $\rho_{1,-1}$ is not well determined, but the nonzero values of ρ_{00} require some negative J-parity exchange. The ratios are

$$\begin{aligned}\rho_{1,-1}/\rho_{11} &= 0.50_{-0.86}^{+1.17} \quad \text{at} \quad 4.1 \text{ GeV}/c \\ &= -0.23_{-0.41}^{+0.40} \quad \text{at} \quad 5.5 \text{ GeV}/c.\end{aligned}$$

Since average $(x^2-1)/(x^2+1)$ values are typically around 0.6–0.8 at these energies, it does not seem that the lower bound for $|\rho_{1,-1}|$ can be satisfied if it is negative, so that there is an indication of positive J-parity exchange.

[4] J. Mott *et al.*, Phys. Rev. **177**, 1966 (1969).

For the baryon density matrix elements $\rho_{nn'}$, one can write the expressions (assuming single trajectory exchange, so that all amplitudes have equal phase)

$$(\mathrm{Re}\rho_{nn'})^2 = \sum_{m,\lambda,m',\lambda'} F_{m,\lambda n} F_{m,\lambda n'} F_{m',\lambda' n} F_{m',\lambda' n'} \quad (11)$$

and

$$(\mathrm{Re}\rho_{n,-n'})^2 = \sum_{m,\lambda,m',\lambda'} (-1)^{\lambda-\lambda'} \times F_{m,\lambda n} F_{-m,-\lambda n'} F_{m',\lambda' n} F_{-m',-\lambda' n'}. \quad (12)$$

If *no meson resonance is produced*, we have $m = m' = 0$, and the terms with $\lambda \neq \lambda'$ in (12) cancel, so that we can write

$$(\mathrm{Re}\rho_{nn'})^2 + (\mathrm{Re}\rho_{n,-n'})^2 = \rho_{nn}\rho_{n'n'},$$

which is RT(19). However, when we allow m to be nonzero, the terms with different values of m interfere, and no general relation is evident. One cannot *a priori* restrict the sum to $m=0$ values, since an exchange that couples to $m=0$ will in general also couple to $m \neq 0$. But if $\sigma_V \sigma_E = +1$, the $m=0$ amplitude will not be present,[2] and we have only $m \neq 0$. This may be useful in the production of vector and axial-vector mesons where only $m = \pm 1$ values are present, and one can be related to the other by parity conservation. The relations are

$$F_{-1,\lambda n} = -aF_{1,\lambda n}, \qquad n-\lambda \geq 1 \quad (13a)$$

$$F_{-1,\lambda n} = -(1/a)F_{1,\lambda n}, \qquad n-\lambda \leq -1 \quad (13b)$$

$$F_{-1,\lambda n} = F_{1,\lambda n}, \qquad n-\lambda = 0 \quad (13c)$$

$$a = (1-x)/(1+x), \quad (14)$$

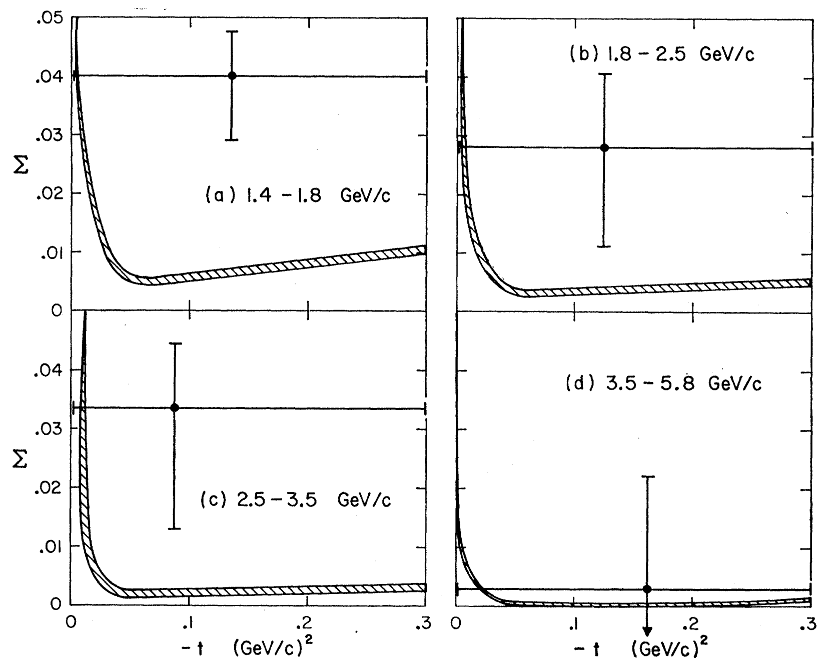

FIG. 3. Upper bound (21) for the reaction $\gamma+p\rightarrow\pi^-+\Delta^{++}$.

where x is the usual cosine of the t-channel scattering angle, and we have dropped lower-order terms in the direct-channel energy (s). The sum in (11) and (12) is over $\lambda=\pm\frac{1}{2}$, so that if we restrict $n,n'>\frac{1}{2}$, case (13c) will never occur. Then it is easy to derive the following relation:

$$4a^2(\mathrm{Re}\rho_{nn'})^2+(1+a^2)^2(\mathrm{Re}\rho_{n,-n'})^2=(1+a^2)^2\rho_{nn}\rho_{n'n'}\,. \quad (15)$$

Note that as $s\rightarrow\infty$, $x\rightarrow-\infty$ *away from the forward direction*, so that $a\rightarrow-1$ and (15) becomes identical to RT(19). However, close to the forward direction x remains finite and the correction terms in (15) are quite important, even at high energy. When we allow $n'=\frac{1}{2}$, the relations become more complicated:

$$\mathrm{Re}\rho_{n\frac{1}{2}}=(1-a)AB+\sigma_E\sigma_{N^*}[(1+a^2)/a]CD\,, \quad (16)$$

$$\mathrm{Re}\rho_{n,-\frac{1}{2}}=2AD+\sigma_E\sigma_{N^*}(1-a)BC\,, \quad (17)$$

$$\rho_{nn}=(1+a^2)(A^2+C^2)\,, \quad (18)$$

$$\rho_{\frac{1}{2}\frac{1}{2}}=2B^2+[(1+a^2)/a^2]D^2\,, \quad (19)$$

where

$$A=F_{1,\frac{1}{2}n}\,,\quad B=F_{1,\frac{1}{2}\frac{1}{2}}\,,$$
$$C=F_{1,-\frac{1}{2}n}\,,\quad D=F_{1,\frac{1}{2}-\frac{1}{2}}\,.$$

It can be seen that no combination of the off-diagonal elements can be expressed as some function of the diagonal elements alone, so that an expression of the type (15) is not possible. One can get an upper bound for a similar expression, using the following procedure. First, the correction terms are expressed in terms of the amplitudes:

$$\rho_{nn}\rho_{\frac{1}{2}\frac{1}{2}}-(\mathrm{Re}\rho_{n\frac{1}{2}})^2-(\mathrm{Re}\rho_{n,-\frac{1}{2}})^2=\frac{(1+a)^2}{2a}B^2+(1+a)^2\rho_{\frac{1}{2}\frac{1}{2}}A^2-\frac{(1+a)^4}{2a}A^2B^2-\frac{(1+a)^2}{2a}(\mathrm{Re}\rho_{n,-\frac{1}{2}})^2. \quad (20)$$

The right-hand side of (20) is then maximized with respect to A^2 and B^2, using the upper bounds implied by (18) and (19). The result is

$$\rho_{nn}\rho_{\frac{1}{2}\frac{1}{2}}-(\mathrm{Re}\rho_{n\frac{1}{2}})^2-(\mathrm{Re}\rho_{n,-\frac{1}{2}})^2\leq\frac{(1+a)^2}{1+a^2}\rho_{nn}\rho_{\frac{1}{2}\frac{1}{2}}-\frac{(1+a)^2}{2a}(\mathrm{Re}\rho_{n,-\frac{1}{2}})^2. \quad (21)$$

Note that the expressions (20), (21), and RT(19) are useful for determining if more than one exchange is present only when the J parity of the exchange has already been determined by looking at the meson spin-density matrix elements. If $\rho_{00}=0$, and the meson has $J=1$, we can use (19) or (20) to determine if more than one exchange with $\sigma_E=\sigma_V$ is present. If $\rho_{00}=1$, we can use RT(19) for any meson spin to determine if more than one exchange with $\sigma_E=-\sigma_V$ is present. Potential candidates for this type of analysis are the reactions $\pi N\rightarrow(\rho,\omega,f)\Delta$ and $KN\rightarrow K^*\Delta$. At presently available energies, all of these are in the intermediate region as far as the preceding tests are concerned, in that ρ_{00} is not either exactly zero or 1. In addition, analysis of the meson density matrix indicates that these reactions probably require exchanges of both J parities (see Sec. I).

TABLE I. Test of relation (24) for the reaction $\gamma+p \rightarrow (\rho^0,\omega^0)+p$.

Reaction	E_γ (GeV/c)	$\cos\theta_{c.m.}$	Σ
$\gamma+p\rightarrow\rho^0+p$	2.5–3.5	0.975–1.0	$0.029_{-0.013}^{+0.011}$
		0.95 –0.975	$0.020_{-0.015}^{+0.012}$
		0.9 –0.95	$0.024_{-0.015}^{+0.012}$
		0.7 –0.9	$0.009_{-0.012}^{+0.012}$
	3.5–5.8	0.975–1.0	$0.027_{-0.010}^{+0.011}$
		0.95 –0.975	$0.005_{-0.013}^{+0.012}$
		0.9 –0.95	$0.022_{-0.018}^{+0.015}$
		0.7 –0.9	$-0.046_{-0.032}^{+0.026}$
$\gamma+p\rightarrow\omega^0+p$	1.4–2.5	0.95 –1.0	$0.018_{-0.017}^{+0.014}$
		0.9 –0.95	$-0.0002_{-0.026}^{+0.020}$
		0.8 –0.9	$0.047_{-0.018}^{+0.015}$
		0.6 –0.8	$0.038_{-0.027}^{+0.020}$
	2.5–5.8	0.95 –1.0	$0.014_{-0.019}^{+0.015}$
		0.8 –0.95	$0.006_{-0.026}^{+0.023}$

III. $\gamma+N\rightarrow V+N$

We divide this section into separate consideration of exchanges according to their coupling to nucleons.

A. Pion-Type Exchanges

Pion-type exchanges contribute only to t-channel amplitudes with equal nucleon-antinucleon helicities. One can then write the usual combination of density matrix elements:

$$\begin{aligned}\mathrm{Re}\rho_{mn}\pm\sigma_V(-1)^n\,\mathrm{Re}\rho_{m,-n}&=2\,\mathrm{Re}(F_{\frac12\frac12,1m}\pm F_{\frac12\frac12,-1m})(F_{\frac12\frac12,1n}\pm F_{\frac12\frac12,-1n})^*\\&\equiv 2\,\mathrm{Re}A_m{}^{\pm}A_n{}^{\pm*}.\end{aligned}\tag{22}$$

For a single-pion-type exchange, all amplitudes will have the same phase, so that we can write

$$(\mathrm{Re}A_m{}^{\pm}A_n{}^{\pm*})^2=|A_m{}^{\pm}|^2|A_n{}^{\pm}|^2.\tag{23}$$

This can be translated into an expression for the off-diagonal matrix elements in terms of the diagonal ones (ρ_{mm}) and antidiagonal ones ($\rho_{m,-m}$). It is

$$\frac{2(\mathrm{Re}\rho_{m,\pm n})^2}{\rho_{mm}\rho_{nn}}=1+\epsilon_m\epsilon_n\pm[(1-\epsilon_m{}^2)(1-\epsilon_n{}^2)]^{1/2},\tag{24}$$

where

$$\epsilon_m\equiv(-1)^m\rho_{m,-m}/\rho_{mm}.\tag{25}$$

Because of the presence of both ±1 helicity photons, there are no lower-bound inequalities for the ϵ_m. Of course, the upper bound is still unity from parity conservation alone.

B. A_1-Type Exchanges

A_1-type exchanges couple only to unequal nucleon helicities. Due to the presence of spin flip, the analysis is a little more complicated, and the details are presented in the Appendix. The results are helicity-dependent, but all of the expressions reduce to (24) in the limit of high energy for nonforward scattering.

C. ρ-Type Exchanges

ρ-type exchanges contribute to all helicity amplitudes. In the square of the expression for the off-diagonal matrix elements, there are interference terms between amplitudes with equal and unequal nucleon helicity. These cannot be canceled by any combination of diagonal and antidiagonal elements. Relations of the type just discussed for pion- and A_1-type exchanges do not exist for ρ-type exchanges. Lower bounds for ϵ_m also do not exist because of the presence of both ±1 helicity photons.

In summary, for the reaction $\gamma+N\rightarrow V+N$ the only tests possible are for the presence of a single π-type or a single A_1-type exchange. We can examine the reactions $\gamma+p\rightarrow(\rho^0,\omega^0)+p$. The necessary data exist up to 5.8-GeV/c photon energy.[5] To test for π-type exchange, we use (24) with $m=1$, and $n=0$, and define

$$\Sigma\equiv\tfrac12\rho_{00}(\rho_{11}-\rho_{1,-1})-(\mathrm{Re}\rho_{10})^2.\tag{26}$$

With the energy- and momentum-transfer values used, the x values are large enough so that (A5), valid for A_1-type exchange, is essentially identical to (24). Thus Σ should be zero for either a single π-type or a single A_1-type exchange. Of course, it may be accidentally zero for some other mechanism, so that we can only perform a negative test. If Σ is nonzero, we can be sure that a single π-type or A_1-type exchange is not dominating.

The Σ values for ρ^0 and ω^0 production are shown in Table I. For ρ^0 production, almost all of the Σ values are more than one standard deviation from zero. This is to be expected, since the energy dependence of this reaction indicates a diffractive mechanism,[5] or Pomeranchon exchange, which is a ρ-type trajectory. For ω^0 production, some of the Σ values are consistent with zero, indicating that pion exchange may be dominating. The largest violations come at low energy and high momentum transfer, where other mechanisms might be expected to be more important.

IV. $\gamma+N\rightarrow V+N^*$

Since all exchanges couple to t-channel amplitudes with arbitrary N and N^* helicities, the results of Sec. III indicate that no relations of the usual type exist for the meson density matrix elements. Similarly, the results of Sec. II indicate that no relations of the usual type exist for the N^* density matrix elements as long as the meson spin is nonzero. Thus we are left with only one case to consider—the reaction $\gamma+N\rightarrow P+N^*$, where P has zero spin, and consider the N^* density matrix elements. Since the photon has only helicities ±1 and P has only zero helicity, this is equivalent (in an exchange model) to the reaction $\pi+N\rightarrow V+N^*$ where the meson has only helicity ±1. This reaction

[5] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Rev. **175**, 1669 (1968).

was considered in Sec. II [Eqs. (13)–(21)], and the results derived there apply as well to this case. For matrix elements with all helicities $\geq\frac{3}{2}$, we get the relation (15). For matrix elements involving helicity $\frac{1}{2}$, we get the inequality (21). Note that for the photoproduction reaction we do not have to restrict the J parity of the exchange, since the ± 1 helicity states are automatically the only ones populated for real photons.

The inequality (21) has been compared with the data on $\gamma+p\rightarrow\pi^-+\Delta^{++}$, and the results are shown in Fig. 3. The upper bound [right-hand side of (21)] is plotted as a continuous function of momentum transfer, using exact x values and the t-averaged density matrix elements. The data points for the left-hand side of (21) denoted by Σ, are plotted with calculated errors. It is seen that Σ exceeds the upper bound by a substantial amount for all energy intervals except the highest, indicating that perhaps a single exchange is becoming dominant at high energy. In any event, it is clear that a single exchange is not adequate to explain the spin structure of this reaction at the lower energies.

V. SUMMARY

It may be useful to restate the most important results.

(1) The relations derived here are useful to test the compatibility of data with an exchange model before performing a comprehensive data fitting.

(2) Only the negative results say something definite. Positive results merely indicate a compatibility.

(3) The relations are basically of two types. The inequalities for the diagonal and antidiagonal meson elements depend on equal J parity of all exchanges. The relations involving off-diagonal elements for both mesons and baryons depend on a single exchange but arbitrary quantum numbers.

(4) These relations are by no means exhaustive. For arbitrary high-spin resonance production, almost all of the off-diagonal elements must be determined by the diagonal ones, since the number of elements grows like J^2 while the number of exchange amplitudes grows only like J. The form of these additional relations is not evident, so that presumably they must be computed on an individual basis.

APPENDIX

A_1-type exchanges couple only to t-channel amplitudes with unequal nucleon helicity. Since the helicity flip at the nucleon vertex is always ± 1, the coefficient which relates positive- to negative-helicity amplitudes depends on the helicity flip at the photon vertex, which in turn depends on the meson helicity. We must consider four separate cases. We suppress the nucleon helicities for convenience and consider a single A_1-type exchange so that all amplitudes will have the same phase. The parameter a defined in (14) is also used.

(i) $m=1$, $n=0$:

$$\mathrm{Re}\rho_{10}=(F_{10}/a)[(a-1)F_{11}-\sigma_V(a^2+1)F_{-11}], \quad \text{(A1)}$$

$$\rho_{00}=(2F_{10}{}^2/a^2)(a^2+1), \quad \text{(A2)}$$

$$\rho_{11}=2F_{11}{}^2+(1+a^2)F_{-11}{}^2, \quad \text{(A3)}$$

$$\rho_{1,-1}=2\sigma_V(a-1)F_{11}F_{-11}. \quad \text{(A4)}$$

These can be converted into an expression for $\mathrm{Re}\rho_{10}$:

$$\frac{4(\mathrm{Re}\rho_{10})^2}{\rho_{00}\rho_{11}}=\frac{[\eta_1\epsilon_1+1+(1-\eta_1\epsilon_1{}^2)^{1/2}]^2}{\eta_1[1+(1-\eta_1\epsilon_1{}^2)^{1/2}]}, \quad \text{(A5)}$$

where ϵ_1 is defined by (25),

$$\eta_1\equiv 2(a^2+1)/(a-1)^2=(1+x^2)/x^2, \quad \text{(A6)}$$

and x is the usual cosine of the t-channel scattering angle. From the reality condition, we also get an upper bound for ϵ_1 which is more stringent than that implied by parity conservation alone:

$$|\epsilon_1|=|\rho_{1,-1}|/\rho_{11}\leq|x|/(1+x^2)^{1/2}. \quad \text{(A7)}$$

(ii) $m\geq 2$, $n=0$:

$$\mathrm{Re}\rho_{m0}=F_{10}\{2F_{1m}-\sigma_V[(1+a^2)/a]F_{-1m}\}, \quad \text{(A8)}$$

$$\rho_{mm}=(1+a^2)(F_{1m}{}^2+F_{-1m}{}^2), \quad \text{(A9)}$$

$$\rho_{m,-m}=4a\sigma_V(-1)^{m+1}F_{-1m}F_{1m}. \quad \text{(A10)}$$

Again we can solve for the off-diagonal elements.

$$\frac{4(\mathrm{Re}\rho_{m0})^2}{\rho_{00}\rho_{mm}}=\frac{[1+(1-\eta_m\epsilon_m{}^2)^{1/2}+\eta_m\epsilon_m]^2}{\eta_m[1+(1-\eta_m\epsilon_m{}^2)^{1/2}]}, \quad \text{(A11)}$$

where

$$\eta_m\equiv\frac{(a^2+1)^2}{4a^2}=\left(\frac{1+x^2}{1-x^2}\right)^2. \quad \text{(A12)}$$

Also, we get another upper bound for ϵ_m, $m\geq 2$:

$$|\epsilon_m|=\frac{|\rho_{m,-m}|}{\rho_{mm}}\leq\frac{x^2-1}{x^2+1}. \quad \text{(A13)}$$

(iii) $m\geq 2$, $n=1$:

$$\mathrm{Re}\rho_{m1}=(1-a)F_{1m}F_{11}+(1+a^2)F_{-1m}F_{-11}, \quad \text{(A14)}$$

$$\sigma_V\,\mathrm{Re}\rho_{m,-1}=(a-1)F_{-1m}F_{11}+2aF_{1m}F_{-11}. \quad \text{(A15)}$$

The results are

$$\frac{4(\mathrm{Re}\rho_{m1})^2}{\rho_{mm}\rho_{11}}=\frac{\{[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]+\eta_1\eta_m^{1/2}\epsilon_1\epsilon_m\}^2}{\eta_1[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]}, \tag{A16}$$

$$\frac{4(\mathrm{Re}\rho_{m,-1})^2}{\rho_{mm}\rho_{11}}=\frac{\{\eta_1\epsilon_1[1+(1-\eta_m\epsilon_m^2)^{1/2}]+\eta_m\epsilon_m[1+(1-\eta_1\epsilon_1^2)^{1/2}]\}^2}{\eta_1\eta_m[1+(1-\eta_m\epsilon_m^2)^{1/2}][1+(1-\eta_1\epsilon_1^2)^{1/2}]}. \tag{A17}$$

(iv) $m\geq 3$, $n\geq 2$:

$$\mathrm{Re}\rho_{mn}=(1+a^2)(F_{1m}F_{1n}+F_{-1m}F_{-1n}), \tag{A18}$$

$$\sigma_V(-1)^n\,\mathrm{Re}\rho_{m,-n}=-2a(F_{-1m}F_{1n}+F_{1m}F_{-1n}). \tag{A19}$$

The result is

$$2(\mathrm{Re}\rho_{m,\pm n})^2/\rho_{mm}\rho_{nn}=1+\eta_m\epsilon_m\epsilon_n\pm[(1-\eta_m\epsilon_m^2)(1-\eta_n\epsilon_n^2)]^{1/2}. \tag{A20}$$

PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Infinitely Degenerate Leading Baryon Trajectory*

PETER G. O. FREUND AND RONALD WALTZ†
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637

(Received 5 July 1969)

In the quark model, the leading baryon trajectory is resolved into infinitely many degenerate trajectories. An exchange-degeneracy pattern of periodicity $\Delta\, j=6$ is obtained. At finite physical values of the spin, only a finite number (increasing with the spin) of these infinitely many trajectories support particles. A general hadronic mass formula is proposed.

1. INTRODUCTION

THE absence of exotic hadrons [i.e., baryons other than $SU(3)$ singlets, octets, or decimets, and mesons other than nonets, etc.] that couple very strongly to the usual mesons and baryons is an experimental fact. Channels with exotic quantum numbers can "communicate" with normal channels through crossing (e.g., $K^+p\to K^+p$ with $K^-p\to K^-p$). Thus, the absence of very strong resonances in the exotic channel leads to dynamical consequences in normal channels. These consequences take the form of exchange degeneracies between various normal-channel Regge trajectories. For mesonic trajectories, exchange degeneracy has been explored in detail. For baryons, exchange degeneracy has been considered more recently. The difficulty of the problem is due to our lack of knowledge of the detailed baryon spectrum. Following Schmid's[1] proposal of baryonic exchange degeneracy, Capps[2] studied the exchange degeneracy of baryonic $SU(3)$ multiplets. This work, however, is confined to processes involving as external particles only the 36 ground-state mesons and 56 ground-state baryons. He also assumes that the leading baryon trajectories are an even-signature $(56,\ L=\alpha_{56}(s))^+$ trajectory and an odd-signature $(70,\ L=\alpha_{70}(s))^-$ trajectory. The former supports the particle multiplets (roughly equally spaced in mass squared) $(56,\ L=0)^+$, $(56,\ L=2)^+$, $(56,\ L=4)^+$, $\cdots$, while the latter supports $(70,\ L=1)^-$, $(70,\ L=3)^-$, $(70,\ L=5)^-$, $\cdots$. Exchange degeneracy is imposed in the form $\alpha_{56}(s)=\alpha_{70}(s)$, and of certain relations between the residues. In this scheme, the absence of 20-plets is just a consequence of the limitation to **35**-**56** scattering rather than an actual feature of the baryon spectrum. In the processes $MM\to B\bar{B}$, it requires the presence of exotic resonances. To avoid this undesirable feature, Mandula *et al.*[3] have suggested that an *even*-signature **70** trajectory is degenerate with the even-signature **56**. While this achieves the desired result it also confronts one with the unattractive (and experimentally catastrophic) feature of a low-lying $(70,\ L=0)^+$ supermultiplet. A possible way around this difficulty was proposed by Mandula, Weyers, and Zweig,[4] who suggest that there exists a hierarchy of exchange-degeneracy principles and that the $(56, L=0)^+-(70,\ L=0)^+$ degeneracy is far from the top of this hierarchy and, therefore, is badly broken. Thus, the

* Work supported in part by the U. S. Atomic Energy Commission.
† National Science Foundation predoctoral fellow.
[1] C. Schmid. Nuovo Cimento Letters, **1**, 165 (1969).
[2] R. H. Capps, Phys. Rev. Letters **22**, 215 (1969); and to be published.
[3] J. Mandula, C. Rebbi, R. Slansky, J. Weyers, and G. Zweig, Phys. Rev. Letters **22**, 1147 (1969).
[4] J. Mandula, J. Weyers, and G. Zweig, Phys. Rev. Letters **23**, 266 (1969).