definiteness of the individual contribution to the unitarity condition for forward scattering. Whether ghost eliminating is produced for $t\neq 0$ in the CGL model is unclear, although in simplified versions of this model, ghost eliminating is not achieved automatically.

Work is now under way on possible extensions of our model to include (a) the simultaneous treatment of the bootstrapping of several Regge poles, which is straightforward, but somewhat more complicated than the work that has been discussed here, and (b) the extension to $t\neq 0$, which involves obtaining a satisfactory mechanism for ghost eliminating and treating the problem in the absence of $O(4)$ symmetry.

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Veneziano-Type Representation for πN Amplitudes*

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A Veneziano-type representation for πN amplitudes is written in which the parity partners of the nucleon and the 3-3 resonance are absent. The difficulty in the earlier models of having Δ trajectories in N channel (and vice versa) is avoided. The parameters in our model are determined from low-energy data; the prediction for intermediate-energy resonances and high-energy forward charge-exchange scattering are within a factor of 2 of the experimental values.

FOR the πN scattering amplitude, with the following assumptions, we present a Veneziano-type representation.[1]

I. TRAJECTORIES

(1) The contribution of the Pomeranchuk trajectory is not included.

(2) All trajectories are linear functions of the energy squared variable with a universal slope B. This means that N_α, N_β; N_γ, N_δ; Δ_α, Δ_β; Δ_γ, Δ_δ are pairwise degenerate.

(3) Exchange-degenerate ρ and P' trajectories denoted by α.

(4) Exchange-degenerate N_α, N_γ; Δ_δ, Δ_β trajectories. Thus we can denote all four trajectories with $I=\frac{1}{2}$ as a single trajectory α_N. Similarly for $I=\frac{3}{2}$, we have α_Δ.

$$\begin{aligned}\alpha_N(s)&=\tfrac{1}{2}+B(s-m_N^2)\,,\\ \alpha_\Delta(s)&=\tfrac{3}{2}+B(s-m_\Delta^2)\,,\\ \alpha(t)&=1+B(t-m_\rho^2)\,.\end{aligned}\tag{1}$$

II. A MINIMAL PRINCIPLE

A typical Veneziano term is of the form[2]

$$\frac{\Gamma(m-\alpha_a(x))\Gamma(n-\alpha_b(y))}{\Gamma(m+n-\alpha_a(x)-\alpha_b(y))}P_{m,n}(x,y)+\text{c.s.}\,,\tag{2}$$

where $P_{m,n}(x,y)$ is a polynomial in x and y, and c.s. signifies terms needed to satisfy crossing symmetry [see Eqs. (11)–(14) below]. The degree of $P_{m,n}(x,y)$ can always be chosen so that it is a leading term as far as the asymptotic behavior is concerned. Clearly, we need some principle to limit the number of terms retained in our formula; otherwise we will have an infinite number of parameters. This difficulty of nonuniqueness is inherent in the usual Veneziano-type formalism. We adopt the *ad hoc* principle that the "leading" term is the term with m and n equal to the spin values of the lowest resonances on trajectories α_a and α_b, respectively (e.g , $\frac{1}{2}$ for α_N, $\frac{3}{2}$ for α_Δ, 1 for α), and we only keep the "leading" term.

III. $m_\pi\to 0$ LIMIT AND QUANTIZATION OF MASSES

We assume[3] the width of the resonances $\Gamma\to 0$ as $m_\pi\to 0$; thus a Veneziano-type formula is most applicable in the limit $m_\pi\to 0$. In this limit, the results from partial conservation of axial-vector current (PCAC) become exact; thus we also expect[4] the intercepts of the trajectories to be separated by multiples of $\frac{1}{2}$.

Now we demand that the amplitudes we write down satisfy crossing symmetry under $s\to u$:

$$A^{1/2}(s,t,u)\to\tfrac{4}{3}A^{3/2}(u,t,s)-\tfrac{1}{3}A^{1/2}(u,t,s)\,,$$
$$A^{3/2}(s,t,u)\to\tfrac{1}{3}A^{3/2}(u,t,s)+\tfrac{2}{3}A^{1/2}(u,t,s)\,;$$

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[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

[2] P. E. Kaus (private communication); P. Carruthers and F. Cooper, Cornell University Report (unpublished).

[3] R. T. Poe (private communication).

[4] M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters **22**, 83 (1969).

the superscripts are isospin values. Similar relations hold for the B amplitudes.

We also demand the correct asymptotic behavior:

$$\begin{aligned} &A \to s^{\alpha(t)}, \quad B \to s^{\alpha(t)-1} \quad \text{for } t \text{ fixed}, \quad s \to \infty \\ &A \to t^{\alpha(s)-1/2}, \quad B \to t^{\alpha(s)-1/2} \quad \text{for } s \text{ fixed}, \quad t \to \infty \\ &A \to s^{\alpha(u)-1/2}, \quad B \to s^{\alpha(u)-1/2} \quad \text{for } u \text{ fixed}, \quad s \to \infty . \end{aligned} \tag{3}$$

The physical amplitudes $f_1(w,t)$ and $f_2(w,t)$ are related to A and B as follows:

$$\begin{aligned} f_1(w) &= [(E+m)/8\pi w][A+(w-m_N)B] \\ &= \sum a_{J-1/2}P_{J+1/2}' - \sum a_{J+1/2}P_{J-1/2}', \\ f_2(w) &= [(E-m)/8\pi w][-A+(w+m_N)B] = -f_1(-w) \\ &= \sum a_{J+1/2}P_{J+1/2}' - \sum a_{J-1/2}P_{J-1/2}'. \end{aligned}$$

The partial-wave amplitudes are given by

$$a_{l=J\mp 1/2} = \frac{1}{2}\int_{-1}^{1} dz\, [f_1 P_{J\mp 1/2}(z) + f_2 P_{J\pm 1/2}(z)]. \tag{4}$$

The Veneziano representation satisfying the above properties is

$$A_N = \frac{\Gamma(\frac{1}{2}-\alpha_N(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(s)-\alpha(t))}(v_1+v_2 s)+\text{c.s.}, \tag{5}$$

$$B_N = \frac{\Gamma(\frac{1}{2}-\alpha_N(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(s)-\alpha(t))}v_3+\text{c.s.}, \tag{6}$$

where the subscript N stands for the $I=\frac{1}{2}$ state. Similarly, for the $I=\frac{3}{2}$ amplitudes,

$$A_\Delta = \frac{\Gamma(\frac{3}{2}-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(s)-\alpha(t))} \times(u_1+u_2 s+u_3 t+u_4 st)+\text{c.s.}, \tag{7}$$

$$B_\Delta = \frac{\Gamma(\frac{3}{2}-\alpha_\Delta(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(s)-\alpha(t))}(u_5+u_6 t)+\text{c.s.}. \tag{8}$$

We notice here that the experimental fact that the nucleon and the 3-3 resonance do not have parity partners can be imposed on the representation very simply. Using (4), all we have to demand is

$$v_1+v_2 m_N^2=0, \tag{9}$$

$$u_3+u_4 m_\Delta^2-u_6(m_\Delta+m_N)=0. \tag{10}$$

These will also give rise to positive residues for the leading N and Δ trajectories.

We can now write down the complete form including the c.s. terms of the invariant amplitudes without the N and the Δ parity partners:

$$\begin{aligned} A_N = {} & w_1\frac{\Gamma(\frac{3}{2}-\alpha_N(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(s)-\alpha(t))} - \tfrac{1}{3}w_1\frac{\Gamma(\frac{3}{2}-\alpha_N(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(u)-\alpha(t))} \\ & + \tfrac{4}{3}[\delta_1+\delta_2 u+\delta_3(u-m_\Delta^2)t+\delta_4(m_\Delta+m)t]\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(u)-\alpha(t))}, \end{aligned} \tag{11}$$

$$\begin{aligned} A_\Delta = {} & \tfrac{2}{3}w_1\frac{\Gamma(\frac{3}{2}-\alpha_N(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(u)-\alpha(t))} \\ & + [\delta_1+\delta_2 s+\delta_3(s-m_\Delta^2)t+\delta_4(m_\Delta+m)t]\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(u)-\alpha(t))} \\ & + \tfrac{1}{3}[\delta_1+\delta_2 u+\delta_3(u-m_\Delta^2)t+\delta_4(m_\Delta+m)t]\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(u)-\alpha(t))}, \end{aligned} \tag{12}$$

$$B_N = w_2\frac{\Gamma(\frac{1}{2}-\alpha_N(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(s)-\alpha(t))} + \tfrac{1}{3}w_2\frac{\Gamma(\frac{1}{2}-\alpha_N(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(u)-\alpha(t))} - \tfrac{4}{3}(\delta_5+\delta_4 t)\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(u)-\alpha(t))}, \tag{13}$$

$$B_\Delta = -\tfrac{2}{3}w_2\frac{\Gamma(\frac{1}{2}-\alpha_N(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{3}{2}-\alpha_N(u)-\alpha(t))} + (\delta_5+\delta_4 t)\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(s))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(s)-\alpha(t))} - \tfrac{1}{3}(\delta_5+\delta_4 t)\frac{\Gamma(\frac{3}{2}-\alpha_\Delta(u))\Gamma(1-\alpha(t))}{\Gamma(\frac{5}{2}-\alpha_\Delta(u)-\alpha(t))}. \tag{14}$$

From Eq. (1) and discussions in Sec. III, we take $B=m_N=1$ and $m_\pi=0$; thus,

$$\begin{aligned} \alpha_N(s) &= -0.5+s, \\ \alpha_\Delta(s) &= s, \\ \alpha(t) &= 0.5+t. \end{aligned} \tag{15}$$

We now use the experimental information on the low-energy resonances in the s and t channels, and the Adler condition, to determine the seven parameters w_1, w_2, δ_1, δ_2, δ_3, δ_4, and δ_5. Comparing the nucleon and 3-3 residues with their experimental values, we obtain

$$w_2 = 3Bg_r^2, \tag{16}$$

$$\delta_4 = 12\pi B\gamma_{33}, \tag{17}$$

where g_r^2 $(=4\pi\times 14.6)$ is the πN coupling constant, and γ_{33} is proportional to the width of the 3-3 resonance.

For the two ρ residues we obtain

$$w_1 B + \delta_2 + \delta_3 m_\rho^2 = -18\pi B\gamma_2, \tag{18}$$

$$\delta_5 - w_2 + \delta_4 m_\rho^2 = 18\pi B(\gamma + 2m\gamma_2), \tag{19}$$

where γ_1 and γ_2 are proportional to the vector and tensor couplings, respectively, of the ρ meson to the $N\bar{N}$ system. We use the values obtained by Ball and Wong.[5] The Adler condition gives

$$\tfrac{2}{3}w_1 + \tfrac{4}{3}\pi(\delta_1 + \delta_2 m^2) = g_r^2 K(0)/m, \tag{20}$$

where $K(0)$ is the form factor whose value we assumed to be unity. Finally, in order that the widths of all the resonances (e.g., all resonances with $l=J-\frac{1}{2}$) on the *leading* trajectory have a positive sign, we must have

$$w_1 = 0, \tag{21}$$

$$\delta_3 = 0. \tag{22}$$

The relations (16)–(22) determine the seven parameters.

We compare our predicted partial widths of some of the intermediate-energy resonances with those given by experiment. The values are given in Table I. We notice that the predictions of the model are not quantitatively accurate; our predictions are, however, within a factor of 2 of the experimental values.

The S-wave scattering lengths depend critically on the value of m_π. In our formula in the limit $m_\pi = 0$, both the $I=\frac{1}{2}$ and the $I=\frac{3}{2}$ S-wave scattering lengths vanish.

We now compare our results with the high-energy behavior of the πN charge-exchange scattering. (Since our model does not incorporate the Pomeranchuk trajectory, this is the only meaningful reaction where comparisons can be made.) First let us define the quantities $a(t)$ and $b(t)$ as follows (we take $B=m_N=1$):

$$A^{(-)}(s,t) \to a^{(-)}(t)\left(\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha(t)},$$

$$B^{(-)}(s,t) \to b^{(-)}(t)\left(\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha(t)-1},$$

[5] J. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963); B. R. Desai, *ibid.* **142**, 1255 (1966).

TABLE I. The predicted and experimental values[a] of the partial widths of some of the intermediate resonances (we choose $B=m_N=1$).

Resonances	Partial widths Predicted	Experimental
$N(1688)\ \frac{5}{2}^+$	0.051	0.080
$N(1680)\ \frac{5}{2}^-$	0.19	0.069
$N(1518)\ \frac{3}{2}^-$	0.045	0.061
$\Delta(1950)\ \frac{7}{2}^+$	0.11	0.089
$\Delta(2420)\ \frac{11}{2}^+$	0.061	0.036

[a] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

where

$$A^{(-)} = \tfrac{1}{3}(A_N - A_\Delta),$$
$$B^{(-)} = \tfrac{1}{3}(B_N - B_\Delta).$$

We obtain the following:

$$a^{(-)}(0) = -27 \text{ (pred.)}$$
$$= -35 \text{ (expt)},$$
$$b^{(-)}(0) = 125 \text{ (pred.)}$$
$$= 79 \text{ (expt)}.$$

By "expt" we mean the values obtained by the phenomenological fits of Rarita *et al.*[6] Here again we observe that our predictions are not quantitatively accurate but give values which are within a factor of 2 of the experimental values.[7]

Our results differ from those of earlier works on the πN problem[8] in that we assume the expression (2) as the starting point for writing the Veneziano representation. Unlike others, we have been successful in eliminating the nucleon and the 3-3 parity partners. Furthermore, in previous papers the N (Δ) channel contained trajectories which were related to Δ (N) trajectories, while we do not have any such difficulty.

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[6] W. Rarita *et al.*, Phys. Rev. **165**, 1615 (1968).

[7] For backward πN scattering, because N_γ is exchange-degenerate with $N\alpha$, we do not have the dip at $\alpha_N = -\frac{1}{2}$. The fits to the data will not be good.

[8] See, for example, K. Igi, Phys. Letters **28B**, 330 (1968).