Gauge-Field Chiral Lagrangian

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The $SU(3)\times SU(3)$ chiral Lagrangian with gauge fields is discussed, including a consideration of $SU(3)$-symmetry breaking. In addition to details of the formulation, applications to π-N scattering lengths, K^+-N scattering lengths, and semileptonic decays are treated. The S-wave π-N scattering lengths are found to be independent of the vector-meson-dominance assumption. A possible "improvement" of the calculation by adding the N^* particle is also discussed. The K^+-N S-wave scattering lengths can be calculated in good agreement with experiment when all symmetry breaking (including ω-ϕ mixing) is put into the mass terms of the physical particles and it is assumed that the physical ϕ particle decouples from the nucleons. Alternative $SU(3)$-breaking schemes are investigated and, for a modified mass-term mixing model, the renormalizations to the Cabibbo theory for all semileptonic decays can be correlated in terms of one (known) parameter.

I. INTRODUCTION

In this paper we construct and discuss a chiral $SU(3)\times SU(3)$ Lagrangian of $\frac{1}{2}^+$ baryons, 0^- mesons, and $1^\pm$ gauge particles. The local symmetry is broken initially by mass terms of the gauge particles, the chiral symmetry is broken initially by mass terms of the pseudoscalar mesons, and, finally, the $SU(3)$ symmetry is broken by mass terms of all the particles involved.

Lagrangians of this type[1] yield results in low-order perturbation that (when extrapolated to the appropriate unphysical point) are the same as the highly acclaimed current-algebra (CA) results. Since the Lagrangian results do not require extrapolation, this approach gives, depending on one's point of view, either a physical model for performing the CA extrapolation or a satisfactory model by itself for low-energy phenomena.

Our first application is to the S-wave pion-nucleon scattering lengths. Although this has been widely treated,[1] we show that, contrary to general opinion, the result has nothing to do with vector-meson dominance or the KSRF relation.[2] (These two are, however, essentially equivalent to each other in this case.)

Whereas the numerical values of the S-wave pion-nucleon scattering lengths as predicted by CA are reasonably close to experiment, the CA values of the S-wave K^+-nucleon scattering lengths are significantly worse. Thus K^+-nucleon scattering needs more careful investigation. In a previous paper[3] it was shown that quite good answers could be achieved in a Lagrangian model containing $\frac{1}{2}^+$ baryons and pseudoscalar mesons. These answers reduced to the less satisfactory CA ones when the extrapolation to the CA situation was made. This strengthened our belief in the value of the effective Lagrangian approach to low-energy dynamics. Within the framework of the Lagrangian model, the values of the K^+-nucleon scattering lengths were, however, dependent on the type of $SU(3)$ breaking assumed. We were led, therefore, to the viewpoint that the predicted values of the K^+-nucleon scattering lengths could be considered as a sort of testing ground for the type of $SU(3)$ breaking, in much the same way that the S-wave pion-pion scattering lengths have been considered[4] a testing ground for the breaking of chiral $SU(2)\times SU(2)$. It was found that the only type of symmetry breaking which gave good results was the one induced directly by the mass splitting of the octet baryons. This type does not belong to one of the simple chiral $SU(3)\times SU(3)$ representation that are suggested by the quark model. It is, however, the type of symmetry breaking that permits the various coupling constants to retain their $SU(3)$-symmetric values.

Here we consider the S-wave K^+-nucleon scattering lengths in a theory with gauge fields. This introduces the additional complications of $SU(3)$ mass splitting for the vector and axial-vector mesons and also the ω-ϕ mixing problem. The simplest way to proceed is to

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[1] See, e.g., K. Nishijima, Nuovo Cimento **11**, 698 (1959); G. Kramer, H. Rollnik, and B. Stech, Z. Physik **154**, 564 (1959); F. Gürsey, Ann. Phys. (N. Y.) **12**, 91 (1961); T. K. Kuo and M. Sugawara, Phys. Rev. **151**, 1181 (1966); S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); J. Schwinger, Phys. Letters **24B**, 473 (1967); J. A. Cronin, Phys. Rev. **161**, 1483 (1967); H. S. Mani, Y. Tomozawa, and Y.-P. Yao, Phys. Rev. Letters **18**, 1084 (1967); L. S. Brown, Phys. Rev. **163**, 1802 (1967); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters **19**, 1085 (1967); P. Chang and F. Gürsey, Phys. Rev. **164**, 1752 (1967); T. Shiozaki, Progr. Theoret. Phys. (Kyoto) **39**, 195 (1968); B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968); Y. P. Yao, *ibid.* **171**, 1702 (1968); S. Gasiorowicz and D. A. Geffen, Argonne National Laboratory Report No. ANL/HEP 6809 (unpublished).

[2] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayazuddin, Phys. Rev. **147**, 1071 (1966).

[3] J. Schechter, Y. Ueda, and G. Venturi, Phys. Rev. **177**, 2311 (1969). We shall designate this reference as I. Somewhat different treatments have been given in reports by A. Kumar and R. Ramachandran (unpublished), and by K. Kawarabayashi and S. Kitakado (to be published).

[4] S. Weinberg, Phys. Rev. **166**, 1568 (1968).

introduce, as before, $SU(3)$ violations (including the ω-ϕ mixing) only in the vector and axial-vector mass terms. If this is done, a situation similar to the pion-nucleon case is found, namely, the S-wave scattering lengths are exactly the good ones obtained previously and are independent of vector-meson dominance or the KSRF relation. To obtain this result the additional assumption that the (physical) ϕ meson decouples from the nucleons is needed. This assumption is, of course, in accord with experiment.[5] Alternatively, one may equate the experimental value of the scattering lengths to our expressions and use this to derive the fact that the ϕ decouples from the nucleons.

Other methods of introducing $SU(3)$-symmetry breaking do not seem to lead to very good numerical results for the K^+ scattering lengths. The tentative conclusion is that all $SU(3)$-symmetry breaking in our Lagrangian should be in the physical mass terms. Nevertheless, we also investigate other methods of symmetry breaking for the $1^\pm$ meson system. We cannot strongly rule these out, since there may exist additional sources of symmetry breaking (like the existence of scalar mesons) or additional resonance exchanges which we have not taken into account. In fact, one scheme leads to renormalization of both the baryon and meson vector and axial-vector currents. These renormalizations can be expressed as modifications of the Cabibbo suppression factors and may be tested by data from the semileptonic weak decays. It is interesting that all the "renormalized" Cabibbo angles can be correlated in terms of one (calculable) quantity. This characterization may actually be more general than our specific model, but confirmation depends on improved data for the strangeness-changing leptonic decays.

Throughout this paper, the coupling parameters are obtained by relating them to the weak axial-vector currents through the assumption that these currents are the "Noether" currents of our Lagrangian. Alternatively, the field-current identity could be used, but this approach has been given elsewhere.

The setting up of the Lagrangian is discussed in Sec. II. The application to π-N scattering is given in Sec. III and a discussion of K^+-N scattering in Sec. IV. Section V deals with semileptonic decays and alternative possibilities for $SU(3)$ breaking in the $1^\pm$ meson systems. Equivalence transformations that alter the form but not the predictions of the Lagrangian are discussed in Appendix A. Finally Appendix B contains, for comparison, the K_{l3} form factors which follow from our Lagrangian together with the field-current identity.

II. FORMULATION OF LAGRANGIAN

The $SU(3)\times SU(3)$ chiral Lagrangian of $\frac{1}{2}^+$ baryons and pseudoscalar mesons was written down in I. In this Lagrangian, the baryons were considered to belong to the $[(8,1),(1,8)]$ representation of the chrial group, so that in a representation of the Dirac matrices where γ_5 is diagonal, the baryon spinor could be written as

$$N=\begin{pmatrix} L \\ R \end{pmatrix}, \quad \bar{N}\equiv(\bar{R} \quad \bar{L}), \tag{1}$$

where L and R transform according to the left and right chiral subgroups, respectively. For convenience, we shall adopt a matrix notation so that each object in (1), for example, is actually a 3×3 matrix. The pseudoscalar mesons, following Nishijima,[1] Gürsey,[1] and Cronin,[1] were considered to transform nonlinearly under the chiral group in such a way that the auxiliary matrix function $M(\phi)$ satisfying

$$M(\phi)M^\dagger(\phi)=M^\dagger(\phi)M(\phi)=1\,, \tag{2}$$

transforms according to the $(3,3^*)$ representation. The Hermitian conjugate matrix $M^\dagger(\phi)$ transforms according to $(3^*,3)$. The expansion of $M(\phi)$, as a power series in ϕ, is given by

$$M(\phi)=1+2if\phi-2f^2\phi^2+\cdots\,, \tag{3a}$$

$$M^\dagger(\phi)=1-2if\phi-2f^2\phi^2+\cdots\,, \tag{3b}$$

where f is an arbitrary constant that gets identified as an (unrenormalized) pion-decay constant. The coefficients of the first three terms given in (3) follow from the unitarity restriction (2). Furthermore, all predictions of the Lagrangian are the same for different choices of M (see Appendix A).

The Lagrangian density was then written as a sum of several terms:

$$\mathfrak{L}=\mathfrak{L}_{\text{kin}}+\mathfrak{L}_1+\mathfrak{L}_2+\mathfrak{L}_{\text{viol}}+\mathfrak{L}_{\Delta m}\,. \tag{4}$$

In (4), $\mathfrak{L}_{\text{kin}}$ stands for the baryon and pseudoscalar-meson kinetic terms, $\mathfrak{L}_1$ stands for the chiral-invariant term that generates the baryon mass and multilinear meson-nucleon interactions, $\mathfrak{L}_2$ stands for the pseudovector coupling term that adjusts the D and F values of the axial-vector current, $\mathfrak{L}_{\text{viol}}$ stands for the meson mass term that breaks the chiral symmetry, and, finally, $\mathfrak{L}_{\Delta m}$ stands for the baryon mass-splitting term. The Lagrangian represented by (4) differs from those of various other authors[1] in various ways, the most consistent difference being the form of the $\mathfrak{L}_{\Delta m}$ term. Also, it is generally more popular to make an equivalence transformation on the baryon fields to a new situation where $\mathfrak{L}_{\text{kin}}$ acquires a pseudovector-type Yukawa interaction and $\mathfrak{L}_1$ becomes just a baryon mass term. This procedure is discussed in Appendix A. Now, (4) is invariant under *constant* $SU(3)\times SU(3)$ transformations. In this paper we modify (4) by adding vector and axial-vector gauge fields according to the Yang-Mills[6] prescription. This esthetic procedure guarantees the

[5] H. Sugawara and F. von Hippel, Phys. Rev. **145**, 1331 (1966).

[6] C. N. Yang and F. Mills, Phys. Rev. **96**, 191 (1954); R. Utiyama, *ibid.* **101**, 1597 (1956); M. Gell-Mann and S. Glashow, Ann. Phys. (N. Y.) **15**, 437 (1961); J. J. Sakurai, *ibid.* **11**, 1 (1960).

invariance of $\mathcal{L}$ under *local* chiral $SU(3)\times SU(3)$ transformations. The local symmetry is, however, broken by the vector and axial-vector mass terms. It is interesting to note that all symmetry violations in our Lagrangian are in the mass terms.

The required modification of (4) consists of adding the free spin-1 meson Lagrangian and replacing every derivative ∂_μ by a "gauge derivative" D_μ. The vector octet V_μ and the axial-vector octet A_μ are more conveniently given as linear combinations of "left" and "right" fields,

$$V_\mu=l_\mu+r_\mu\,, \tag{5a}$$

$$A_\mu=l_\mu-r_\mu\,. \tag{5b}$$

Under the parity transformation, $\mathbf{l}\leftrightarrow-\mathbf{r}$. Under an infinitesimal left-handed gauge transformation, the various fields transform as:

$$\begin{aligned}&l_\mu\to l_\mu-(i/g)\partial_\mu E_l-[l_\mu,E_l]\,,\\&M\to M+E_lM\,,\\&M^\dagger\to M^\dagger-M^\dagger E_l\,,\\&L\to L+[E_l,L]\,,\\&R\to R\,,\quad r_\mu\to r_\mu\,,\end{aligned} \tag{6}$$

where E_l is an infinitesimal matrix function of space-time satisfying

$$E_l(x)=-E_l^\dagger(x)\,,$$

and g is a constant which gets identified with the vector-meson coupling constant. Similarly, under a right-handed gauge transformation we have

$$\begin{aligned}&r_\mu\to r_\mu-(i/g)\partial_\mu E_r-[r_\mu,E_r]\,,\\&M\to M-ME_r\,,\\&M^\dagger\to M^\dagger+E_rM^\dagger\,,\\&R\to R+[E_r,R]\,,\\&L\to L\,,\quad l_\mu\to l_\mu\,,\end{aligned} \tag{7}$$

where $E_r(x)=-E_r^\dagger(x)$.

The gauge derivative replacement is given by

$$\begin{aligned}\partial_\mu M&\to D_\mu M=\partial_\mu M-igl_\mu M+igMr_\mu\,,\\\partial_\mu M^\dagger&\to D_\mu M^\dagger=\partial_\mu M^\dagger-igr_\mu M^\dagger+igM^\dagger l_\mu\,,\\\partial_\mu L&\to D_\mu L=\partial_\mu L-ig[l_\mu,L]\,,\\\partial_\mu R&\to D_\mu R=\partial_\mu R-ig[r_\mu,R]\,.\end{aligned} \tag{8}$$

It is easily verified that the substitutions above guarantee the *local* invariance of the chiral-symmetric part of (4).

The Yang-Mills term $\mathcal{L}_{\rm YM}$ which is to be added to (4) is

$$\mathcal{L}_{\rm YM}=-\tfrac{1}{2}\,{\rm Tr}(F_{\mu\nu}{}^rF_{\mu\nu}{}^r+F_{\mu\nu}{}^lF_{\mu\nu}{}^l)\,, \tag{9}$$

with

$$\begin{aligned}F_{\mu\nu}{}^l&=\partial_\mu l_\nu-\partial_\nu l_\mu-ig[l_\mu,l_\nu]\,,\\F_{\mu\nu}{}^r&=\partial_\mu r_\nu-\partial_\nu r_\mu-ig[r_\mu,r_\nu]\,.\end{aligned}$$

We must also add the spin-1 meson mass term that breaks the local invariance:

$$\begin{aligned}\mathcal{L}_{\rm YM}'&=-m_0^2\,{\rm Tr}(l_\mu l_\mu+r_\mu r_\mu)\\&=-\tfrac{1}{2}m_0^2\,{\rm Tr}(V_\mu V_\mu+A_\mu A_\mu)\,.\end{aligned} \tag{10}$$

The effects of $SU(3)$ mass splitting and ω-ϕ mixing on these mesons will be discussed later.

Before extracting the effective interactions that correspond to interesting physical processes, it is necessary to effect a redefinition of the pseudoscalar field and of the axial-vector field. The reason is that, by changing ∂_μ to D_μ in the meson kinetic term, we have introduced a bilinear term of the form ${\rm Tr}(\partial_\mu\phi A_\mu)$. This requires a diagonalization of the bilinear terms in order that the pseudoscalar and axial-vector fields be the physical ones. The meson kinetic term has the expansion

$$\begin{aligned}\mathcal{L}_{\rm kin}{}^M&=-(1/8f^2)\,{\rm Tr}(D_\mu M^\dagger D_\mu M)\\&=-\tfrac{1}{2}\,{\rm Tr}(\partial_\mu\phi\partial_\mu\phi)+(g/2f)\,{\rm Tr}(\partial_\mu\phi A_\mu)\\&\qquad-(g^2/8f^2)\,{\rm Tr}(A_\mu A_\mu)+\cdots\,,\end{aligned}$$

where higher than bilinear terms have not been written. The portion of the Lagrangian that is to be diagonalized includes a piece from (10) and is

$$-\tfrac{1}{2}\,{\rm Tr}[\partial_\mu\phi\partial_\mu\phi-(g/f)\partial_\mu\phi A_\mu+(m_0^2+g^2/4f^2)A_\mu A_\mu]\,. \tag{11}$$

We achieve the diagonalization[7] by defining

$$A_\mu=\tilde{A}_\mu+(g/2m_0^2f_r)\partial_\mu\tilde{\phi}+\cdots\,, \tag{12a}$$

$$\phi=(1/Z)\tilde{\phi}\,, \tag{12b}$$

with

$$Z=(1+g^2/4m_0^2f^2)^{-1/2} \tag{13a}$$

and

$$f_r=(1/Z)\times f\,. \tag{13b}$$

In (12), $\tilde{\phi}$ and $\tilde{A}_\mu$ stand for the *physical* fields. Later we shall see that f_r corresponds to the *physical* pion-decay constant. Then (11) becomes the diagonal form

$$-\tfrac{1}{2}\,{\rm Tr}(\partial_\mu\tilde{\phi}\partial_\mu\tilde{\phi})-\tfrac{1}{2}(m_0/Z)^2\,{\rm Tr}(\tilde{A}_\mu\tilde{A}_\mu)\,. \tag{14}$$

It is evident that in (12a) we are free to let the three dots stand for any suitable higher-than-linear term we like. For simplicity, however, we shall retain the equation without any additions.

From (10) we identify m_0 with the vector-meson mass m_V, and then from (14) we find the following relation between the vector and axial-vector meson masses:

$$m_V{}^2=Z^2m_A{}^2\,. \tag{15}$$

Furthermore, we may manipulate (13a) into the useful form

$$g^2/4m_0^2f_r{}^2=1-Z^2\,. \tag{16}$$

As is well known,[1,7] the choice $Z^2=\frac{1}{2}$ converts (15) into the Weinberg relation and (16) into the KSRF relation, so that these two are correlated but not derived in the Lagrangian formulation.

[7] Here we follow the notation of Y. Nambu, University of Chicago Report No. EFI 68-11 (unpublished).

Now we are in a position to write down the interesting terms in our Lagrangian. This involves making the substitutions indicated by (12) as well as using (3) and (8) wherever necessary. The kinetic spin-1 meson term (9) expands into

$$\begin{aligned}\mathcal{L}_{\mathrm{YM}}=&-\tfrac{1}{4}\operatorname{Tr}(F_{\mu\nu}{}^{V}F_{\mu\nu}{}^{V}+F_{\mu\nu}{}^{A}F_{\mu\nu}{}^{A})\\&+\tfrac{1}{2}ig\operatorname{Tr}[F_{\mu\nu}{}^{V}(V_\mu V_\nu+\tilde{A}_\mu\tilde{A}_\nu)]\\&+\tfrac{1}{2}ig\operatorname{Tr}(F_{\mu\nu}{}^{A}[V_\mu,\tilde{A}_\nu])\\&+\tfrac{1}{2}ig(g/2f_rm_0{}^2)^2\operatorname{Tr}(F_{\mu\nu}{}^{V}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi})\\&+\tfrac{1}{2}ig(g/2f_rm_0{}^2)\operatorname{Tr}(F_{\mu\nu}{}^{A}[V_\mu,\partial_\nu\tilde{\phi}]\\&\quad+F_{\mu\nu}{}^{V}[\tilde{A}_\mu,\partial_\nu\tilde{\phi}])+(\text{quadrilinear terms})\,,\end{aligned}\tag{17}$$

where

$$F_{\mu\nu}{}^{V}=\partial_\mu V_\nu-\partial_\nu V_\mu\,,\quad F_{\mu\nu}{}^{A}=\partial_\mu\tilde{A}_\nu-\partial_\nu\tilde{A}_\mu\,.$$

The sum of $\mathcal{L}_{\mathrm{YM}}'$ and $\mathcal{L}_{\mathrm{kin}}{}^{M}$ is

$$\begin{aligned}\mathcal{L}_{\mathrm{YM}}'+\mathcal{L}_{\mathrm{kin}}{}^{M}=&-\tfrac{1}{2}m_V{}^2\operatorname{Tr}(V_\mu V_\mu)-\tfrac{1}{2}m_A{}^2\operatorname{Tr}(\tilde{A}_\mu\tilde{A}_\mu)\\&-\tfrac{1}{2}\operatorname{Tr}(\partial_\mu\tilde{\phi}\partial_\mu\tilde{\phi})+\tfrac{1}{2}ig\operatorname{Tr}(V_\mu\tilde{\phi}\overleftrightarrow{\partial_\mu}\tilde{\phi})\\&+(ig^2/4f_rZ^2)\operatorname{Tr}(\tilde{A}_\mu[\tilde{\phi},V_\mu])+\cdots.\end{aligned}\tag{18}$$

In (18) the quadrilinear and all higher terms were not written. We note that both (17) and (18) give contributions to the decay of a vector meson into two pseudoscalar mesons.

The baryon kinetic term becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{kin}}{}^{B}&=-\operatorname{Tr}(\bar{L}\sigma_\mu D_\mu L+\bar{R}\tilde{\sigma}_\mu D_\mu R)\\&=-\operatorname{Tr}(\bar{N}\gamma_\mu\partial_\mu N)\\&\quad+\tfrac{1}{2}ig\operatorname{Tr}(\bar{N}\gamma_\mu[V_\mu,N]+\bar{N}\gamma_\mu\gamma_5[\tilde{A}_\mu,N])\\&\quad\quad+i(1-Z^2)f_r\operatorname{Tr}(\bar{N}\gamma_\mu\gamma_5[\partial_\mu\tilde{\phi},N]).\end{aligned}\tag{19}$$

Equation (19) contains the whole vector-meson–baryon interaction as well as parts of the axial-vector–baryon and pseudoscalar-baryon interactions.

The chiral-invariant "baryon mass" term $\mathcal{L}_1$ is the same as in I:

$$\begin{aligned}\mathcal{L}_1&=-m\operatorname{Tr}(\bar{L}MRM^\dagger+\bar{R}M^\dagger LM)\\&=-m\operatorname{Tr}(\bar{N}N)+2imf_r\operatorname{Tr}(\bar{N}\gamma_5[\tilde{\phi},N])\\&\quad+2mf_r{}^2\operatorname{Tr}(\bar{N}[\tilde{\phi}^2,N]_+-2\bar{N}\tilde{\phi}N\tilde{\phi})+\cdots,\end{aligned}\tag{20}$$

where m is the nucleon mass.

$\mathcal{L}_2$, the term which adjusts the D and F values of the axial-vector current, now becomes

$$\begin{aligned}\mathcal{L}_2=&2\alpha\operatorname{Tr}(\bar{L}\sigma_\mu D_\mu MM^\dagger L+\bar{R}\tilde{\sigma}_\mu D_\mu M^\dagger MR)\\&+2\beta\operatorname{Tr}(\bar{L}\sigma_\mu LD_\mu MM^\dagger+\bar{R}\tilde{\sigma}_\mu RD_\mu M^\dagger M)\\=&4if_rZ^2\operatorname{Tr}(\alpha\bar{N}\gamma_\mu\gamma_5\partial_\mu\tilde{\phi}N+\beta\bar{N}\gamma_\mu\gamma_5N\partial_\mu\tilde{\phi})\\&-2ig\operatorname{Tr}(\alpha\bar{N}\gamma_\mu\gamma_5\tilde{A}_\mu N+\beta\bar{N}\gamma_\mu\gamma_5N\tilde{A}_\mu)\\&-4f_r{}^2Z^2\operatorname{Tr}(\alpha\bar{N}\gamma_\mu[\tilde{\phi},\partial_\mu\tilde{\phi}]N\\&\quad+\beta\bar{N}\gamma_\mu N[\tilde{\phi},\partial_\mu\tilde{\phi}])+\cdots.\end{aligned}\tag{21}$$

The constants α and β will be fixed by the axial-vector current.

All the terms written up to now are $SU(3)\times SU(3)$-symmetric. The term that breaks chiral symmetry is taken to lowest order to be

$$\begin{aligned}\mathcal{L}_{\mathrm{viol}}&=-\tfrac{1}{2}\mu^2\operatorname{Tr}(\tilde{\phi}\tilde{\phi})+(\mu^2-\mu_K{}^2)\operatorname{Tr}(\tilde{\phi}\tilde{\phi}S)+\cdots\\&=-\tfrac{1}{2}\mu^2(\pi^0\pi^0+2\pi^+\pi^-)\\&\quad-\mu_K{}^2(\bar{K}^+K^++\bar{K}^0K^0)+\cdots,\end{aligned}\tag{22}$$

where μ is the pion mass, μ_K the kaon mass, and S the matrix

$$S=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}.$$

The question of which higher terms should be added to (22) is interesting, but, since it does not affect our applications, we will not consider it here.

Finally the baryon mass-splitting term is

$$\begin{aligned}\mathcal{L}_{\Delta m}=&(m-m_\Sigma)\operatorname{Tr}(\bar{N}N-\bar{N}NS)\\&\quad+(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{N}SN)\\=&-(m_\Sigma-m)\bar{\Sigma}\Sigma-(m_\Lambda-m)\bar{\Lambda}\Lambda-(m_\Xi-m)\bar{\Xi}\Xi.\end{aligned}\tag{23}$$

As pointed out in I, this term transforms according to the (8,8) representation of $SU(3)\times SU(3)$. This seems a bit unusual from the standpoint of the quark model, which suggests transformation properties like either $(3,3^*)+(3^*,3)$ or $(8,1)+(1,8)$, but it gives much better results for the S-wave K^+-nucleon scattering lengths. It may be reconcilable with the quark model if we assume that the mass splitting comes from a higher iteration of the quark-splitting term.

The total Lagrangian density is the sum of Eqs. (17)–(23). Still to be added are the spin-1 meson $SU(3)$-breaking terms; these will be discussed later. If all the masses of the physical particles are taken as input, the only unknown parameters are the quantities f_r, α, and β. These, however, will be determined below by comparison with the weak axial-vector currents. One question that remains is what additions to our Lagrangian must be made in order that it give a good description of all strong interactions, not just very low-energy phenomena. Here we can take two points of view. The first is simply that the Lagrangian we have written is just a mnemonic, useful for keeping track of the various particles and symmetries of our theory. According to this point of view, it is necessary to add new terms corresponding to any new particles or anomalous interactions that we care to describe. The second point of view is that (even though a quark-type substructure may actually be fundamental) the Lagrangian we have written is effectively responsible for a good portion of observed strong-interaction physics. According to this point of view, the N^* resonance, for example, would come about as an iteration of our basic Lagrangian, as would the "magnetic" coupling of vector mesons to baryons. Deciding between these points of view is obviously a difficult task and, at present, does not seem possible. Some more discussion, however, will be given in connection with the pion-nucleon scattering problem.

One additional term, which is considered[8,9] useful for explaining the A_1 width, is an anamolous pseudoscalar meson coupling to the spin-1 mesons:

$$-i\zeta\,\mathrm{Tr}(D_\mu M D_\nu M^\dagger F_{\mu\nu}{}^l + D_\mu M^\dagger D_\nu M F_{\mu\nu}{}^r)$$
$$= -i\zeta\{4f_r{}^2Z^4\,\mathrm{Tr}(\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}F_{\mu\nu}{}^V) + g^2\,\mathrm{Tr}(\tilde{A}_\mu\tilde{A}_\nu F_{\mu\nu}{}^V)$$
$$-2Z^2gf_r\,\mathrm{Tr}([\partial_\mu\tilde{\phi},A_\nu]F_{\mu\nu}{}^V) + \cdots\}. \quad (24)$$

From the expressions for $V\phi\phi$ and $VA\phi$ coupling, we can identify ζ in terms of the usual parameter δ.[9] $V\phi\phi$ coupling, for instance, is given by

$$\mathcal{L}_{V\phi\phi} = \tfrac{1}{2}ig[\mathrm{Tr}(V_\mu\tilde{\phi}\overleftrightarrow{\partial}_\mu\tilde{\phi})$$
$$+1/m_V{}^2(1-Z^2-2m_V{}^2\zeta f_V{}^2Z^4/g)\,\mathrm{Tr}(F_{\mu\nu}{}^V\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi})],$$

and ζ is given by

$$\zeta = -\tfrac{1}{2}g(1-Z^2)\delta/4f_r{}^2m_V{}^2Z^4.$$

Now we compute the canonical axial-vector current for our Lagrangian. This is given, according to Noether's theorem, by the response of the Lagrangian to a *constant* axial transformation

$$\begin{aligned} l_\mu &\to l_\mu + \delta l_\mu = l_\mu - [l_\mu, E],\\ r_\mu &\to r_\mu + \delta r_\mu = r_\mu + [r_\mu, E],\\ N &\to N + \delta N = N + \gamma_5[E,N],\\ M &\to M + [E,M]_+, \end{aligned} \quad (25)$$

where E is an infinitesimal constant matrix satisfying $E^\dagger = -E$. Equation (25) just corresponds to the difference of (6) and (7). The change in the pseudoscalar field under an axial transformation is more complicated, but, to lowest order,

$$\delta\phi = -(i/f)E + \cdots. \quad (26)$$

The pseudovector current $\mathcal{P}_\mu$ is then to be determined from the formula

$$\mathrm{Tr}(\mathcal{P}_\mu E) = -i\,\mathrm{Tr}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu N)}\delta N + \frac{\partial\mathcal{L}}{\partial(\partial_\mu l_\nu)}\delta l_\nu + \frac{\partial\mathcal{L}}{\partial(\partial_\mu r_\nu)}\delta r_\nu\right). \quad (27)$$

From (18), (19), and (21), we find

$$\begin{aligned} \mathrm{Tr}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right) &= \frac{iZ}{f}\mathrm{Tr}(\partial_\mu\tilde{\phi}E)\\ &+4\alpha Z^2\,\mathrm{Tr}(\bar{N}\gamma_\mu\gamma_5 EN)\\ &+4\beta Z^2\,\mathrm{Tr}(\bar{N}\gamma_\mu\gamma_5 NE)\\ &+(1-Z^2)\,\mathrm{Tr}(\bar{N}\gamma_\mu\gamma_5[E,N]) + \cdots, \end{aligned} \quad (28)$$

$$\mathrm{Tr}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu N)}\delta N\right) = -\mathrm{Tr}(\bar{N}\gamma_\mu\gamma_5[E,N]).$$

[8] See, e.g., J. Wess and B. Zumino, Ref. 1.

[9] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967); see also J. Wess and B. Zumino, Ref. 1.

Thus, to lowest order (switching for convenience to tensor notation), the axial-vector current is given by

$$\begin{aligned} \mathcal{P}_{a\mu}{}^b &= (f/Z)^{-1}\partial_\mu\tilde{\phi}_a{}^b\\ &-iZ^2(1+4\beta)(\bar{N}_a{}^c\gamma_\mu\gamma_5 N_c{}^b - \tfrac{1}{3}\delta_a{}^b\bar{N}_d{}^c\gamma_\mu\gamma_5 N_c{}^d)\\ &-iZ^2(-1+4\alpha)(\bar{N}_a{}^b\gamma_\mu\gamma_5 N_a{}^c\\ &\qquad -\tfrac{1}{3}\delta_a{}^b\bar{N}_d{}^c\gamma_\mu\gamma_5 N_c{}^d) + \cdots. \end{aligned} \quad (29)$$

From (29) we identify $f_r = f/Z$ as the pion-decay constant and

$$\alpha = \tfrac{1}{4}(1-g_A/Z^2), \quad \beta = \tfrac{1}{4}(-1+(F-D)/Z^2), \quad (30)$$

where $g_A \equiv F+D \simeq 1.2$ and $D/F \simeq 1.7$ [compare with Eq. (17) of I]. Now all the parameters of $\mathcal{L}$ are specified in terms of well-known experimental[10] numbers.

There are many higher-order terms to be added to (29) including the characteristic Yang-Mills contributions of the form $[l_\nu, F_{\mu\nu}{}^l] - [r_\nu, F_{\mu\nu}{}^r]$. Some of these terms will be relevant for K_{l4} decays. The equations of motion can also be written in the usual way but they are long without being immediately illuminating. Perhaps they will be useful in a consideration of the consistency of the quantization procedure for this Lagrangian.

III. PION-NUCLEON SCATTERING

The reasonable (within 20%) agreement with experiment of the CA[11] or chiral Lagrangian predictions[1] for the S-wave scattering lengths is considered one of the main triumphs of this approach. As such, the way in which it comes about would seem be to worth careful study. These results can be obtained in a Lagrangian model of nucleons and pseudoscalar mesons. They can also be obtained from a pure ρ-exchange model. At first glance, therefore, we might expect that in a Lagrangian with vector mesons as well as pseudoscalar mesons, we would get answers twice as large as they should be. What actually saves the situation when we add gauge fields to the theory is that the quadrilinear terms of Eq. (21) change in such a way as to cancel out the ρ-exchange contribution and to leave the formulas for the S-wave scattering lengths unchanged. Wess and Zumino[1] have already pointed out that the particular choice $Z^2 = \frac{1}{2}$, corresponding [see (16)] to the KSRF relation and vector-meson-dominance results in the correct prediction for the scattering lengths. Here we show that the formula for the scattering lengths is in fact independent of Z^2, so that, for example, it is not possible to derive the KSRF relation by equating the experimental and predicted scattering lengths. This does not mean, of course, that the choice $Z^2 = \frac{1}{2}$ is not a good one.

[10] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964); H. Courant *et al.*, Phys. Rev. **136**, B1791 (1964).

[11] Y. Tomozawa, Nuovo Cimento **46**, 803 (1967); A. P. Balachandran, G. M. Gundzik, and F. Nicodemi, *ibid.* **44A**, 1257 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

We follow the Chew-Goldberger-Low-Nambu[12] notation for pion-nucleon scattering and write the amplitude as

$$T^I=-A^I+i\gamma\cdot\tfrac{1}{2}(q_1+q_2)B^I, \tag{31}$$

where the superscript I denotes the isotopic spin of the particular channel. The amplitude is computed in perturbation theory, including only diagrams without closed loops ("tree" approximation). The interaction Hamiltonian is taken to be the negative of the relevant interaction terms of $\mathcal{L}$. The ρ-exchange contribution comes from (17)–(19) and (24):

$$\begin{aligned}&A_\rho{}^{1/2}=A_\rho{}^{3/2}=0,\\&B_\rho{}^{1/2}=(m_\rho{}^2-t)^{-1}(g^2+bgt), \quad B_\rho{}^{3/2}=-\tfrac{1}{2}B_\rho{}^{1/2},\end{aligned} \tag{32}$$

where s, t, and u are the usual Mandelstam variables, and

$$b=4\zeta f_r{}^2Z^4-\tfrac{1}{2}g(1-Z^2)/m_\rho{}^2.$$

Actually, the term involving b does not contribute to the S-wave scattering lengths. The contributions from the nucleon-exchange type of diagrams and from the quadrilinear-interaction contact-type diagrams are computed using (19)–(21). They turn out to be

$$\begin{aligned}&A_\pi{}^{1/2}=A_\pi{}^{3/2}=2mf_r{}^2g_A{}^2,\\&B_\pi{}^{1/2}=m^2f_r{}^2g_A{}^2[6/(m^2-s)+2/(m^2-u)]\\&\qquad\qquad-2f_r{}^2(g_A{}^2+1-2Z^2), \qquad (33)\\&B_\pi{}^{3/2}=[4m^2f_r{}^2g_A{}^2/(u-m^2)]+f_r{}^2(1+g_A{}^2-2Z^2).\end{aligned}$$

In our approximation, the S-wave scattering lengths are found from the sum of (32) and (33) to be

$$\begin{aligned}a_1&=\frac{2m\mu f_r{}^2}{4\pi(m+\mu)}\left(2Z^2-1+\frac{g^2}{2m_\rho{}^2f_r{}^2}-\frac{\mu(m-\mu)}{4m^2-\mu^2}g_A{}^2\right),\\a_3&=\frac{-\mu f_r{}^2m}{4\pi(m+\mu)}\left(2Z^2-1+\frac{g^2}{2m_\rho{}^2f_r{}^2}+\frac{\mu g_A{}^2}{2m-\mu}\right).\end{aligned} \tag{34}$$

Noting (16), we see that the combination

$$2Z^2-1+g^2/2m_\rho{}^2f_r{}^2$$

appearing in each of Eqs. (34) can be replaced by 1. Thus the Z^2 dependence has dropped out and we are left with the old results in I, corresponding to the case when there were no vector and axial-vector mesons in the Lagrangian. In the CA limit where $\mu/m\to 0$, Eqs. (34) become

$$a_1\to 2m\mu f_r{}^2/4\pi(m+\mu), \quad a_3\to -\tfrac{1}{2}a_1,$$

as expected. [Note that the factor $\mu/(m+\mu)$ comes from kinematics so that it does not suffer any change on going to the limit.]

After considering the threshold values (S-wave scattering lengths) of the pion-nucleon amplitudes, the natural question is what are the predictions for these amplitudes away from threshold. A start on this problem can be made by computing the P-wave scattering lengths that involve the first derivatives of the amplitudes evaluated at the threshold point. It has been found[13] that, in a chiral Lagrangian of nucleons and pseudoscalar mesons, all the P-wave scattering lengths, except for the one in the "3,3" N^* resonance channel, are in reasonable agreement with experiment. Here the situation is essentially unchanged.

The P-wave phase shifts in this gauge-field model, which has no anomalous terms, are still given by Eqs. (21) of I, except that the very small term

$$\frac{1}{12\pi}\frac{m\mu}{m+\mu}\frac{g^2}{m_\rho{}^4}(1+Z^2)$$

should be added to a_{11} and a_{13}, while minus half that should be added to a_{31} and a_{33}. This would tend to make one believe that good over-all results could be achieved if the Lagrangian we have written is in some sense a fundamental one. In that event, we expect that the scattering lengths for the nonresonant channels would be given quite well by the Born-approximation type of procedure we have used, but that the calculation for the resonant channel would require higher iterations of the Lagrangian and would also predict the N^* itself. Since this is hard to carry out in a belivable way, we will no longer pursue this line of approach.

An alternative procedure is, as mentioned in Sec. II, to regard the Lagrangian as a convenient mnemonic for keeping track of the particles and symmetries of the theory. In this approach, we hand place, in a chiral-invariant way, a $\frac{3}{2}^+$ decuplet into the Lagrangian. In order to compute pion-nucleon scattering, the only relevant interaction is of the $(N^*N\pi)$ type. This can be easily written[14] in a locally chiral-invariant form by using the matrix M of (3). This gives rise to additional terms, which are of no concern at present but which would be interesting in the computation of N^* production. There is an over-all free parameter that can be related to the N^* decay width. For the effective interaction Hamiltonian density we thus find

$$\mathcal{H}_{\mathrm{I}}=h[\epsilon_{ebd}\bar{\mathfrak{D}}_\mu{}^{abc}N_c{}^d\partial_\mu\tilde{\phi}_a{}^e+\epsilon^{ebd}\bar{N}_d{}^c\mathfrak{D}_{\mu abc}\partial_\mu\tilde{\phi}_e{}^a]+\cdots,$$

where $\mathfrak{D}_{\mu abc}$ is the decuplet field normalized so that $N_{++}{}^*=(1/\sqrt{6})\mathfrak{D}_{111}$ and h is related to the decay width for $N_{++}{}^*\to p\pi^+$ by

$$\Gamma=\frac{h^2}{4\pi}\frac{|\mathbf{p}|^3}{m^{*2}}[(m+m^*)^2-\mu^2],$$

giving $h^2/4\pi\simeq 0.062\mu^{-2}$. Using this interaction, we compute the N^*-exchange contributions to the S-wave

[12] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

[13] J. A. Cronin, Ref. 1.

[14] For discussion of the chiral transformation properties of the N^* see, e.g., J. Schechter and Y. Ueda, Phys. Rev. **177**, 2300 (1969).

scattering lengths to be

$$a_1(N^*) = -\frac{4}{3}\frac{m}{m+\mu}\frac{h^2\mu^2}{\pi m^{*2}}(m+m^*-\mu) \simeq -0.053\mu^{-1},$$
$$a_3(N^*) = -\frac{2}{3}\frac{m}{m+\mu}\frac{h^2\mu^2}{\pi m^{*2}}[2(m+m^*)+\mu] \simeq -0.059\mu^{-1}. \quad (35)$$

First, we notice that these contributions are of order μ/m compared to the expressions in (34), so that they do not appear in the CA limit where $\mu \to 0$. However, adding these contributions to (34) gives predictions of $a_1 \simeq 0.08\mu^{-1}$ and $a_3 \simeq -0.137\mu^{-1}$, which no longer compare reasonably well with the experimental values $a_1 = 0.171\mu^{-1}$ and $a_3 = -0.088\mu^{-1}$. For the P-wave scattering lengths the expressions are long, so we just give the numbers: $a_{33}(N^*) \simeq 0.100\mu^{-3}$, $a_{13}(N^*) \simeq 0.017\mu^{-3}$, $a_{11}(N^*) \simeq 0.076\mu^{-3}$, and $a_{31}(N^*) \simeq 0.017\mu^{-3}$. These are not especially bad and, not surprisingly, bring the 3,3 scattering-length prediction into good shape (see I, for example). In order to solve the serious difficulty of bad predictions for a_1 and a_3, Mani *et al.*[1] and Peccei[15] have proposed that a non-chiral-invariant quadrilinear ($NN\pi\pi$) term be added to the Lagrangian. Some justification based on the requirement of good asymptotic behavior has been given for this term but, of course, in the present context it would be nice to relate such a term more directly to the chiral scheme. We may note that an anomalous (magnetic type) coupling of vector mesons to nucleons does not contribute to the S-wave scattering lengths, so that this would not help.

Thus we see that the alternative procedure is not without problems. Possibly it is necessary to introduce a fairly large set of additional particles in the second approach to get good agreement again for the S-wave scattering lengths.

IV. K^+-NUCLEON SCATTERING

We shall regard the S-wave K^+-nucleon scattering lengths as a test for the proper way of introducing $SU(3)$-symmetry breaking into the Lagrangian. The K^--nucleon reaction, on the other hand, is exothermic, so that it is unreasonable to expect good agreement in a treatment that does not take account of unitarity; it will not be considered here.

The $SU(3)$ breaking for the baryons was investigated in I for a theory without gauge fields. In that case, the optimum situation was where all the symmetry breaking resided in the mass terms, so that the coupling constants retained their $SU(3)$-symmetric values. In the present theory, we must investigate, in addition, $SU(3)$ breaking in the vector- and axial-vector-meson systems. There are two aspects to this. The first concerns the symmetry breaking among the octet members of the vector and axial-vector families. Depending on the particular scheme chosen, this has different consequences for the Cabibbo theory of weak semileptonic decays, as well as for the S-wave scattering lengths. This will be discussed in Sec. V. The second aspect concerns the introduction of vector- and axial-vector-meson unitary singlets and their mixing with the corresponding octet isosinglets. The method chosen manifests itself in different couplings of the physical ζ meson to the nucleons. (Experimentally, it seems to decouple.)

When introducing the vector and axial-vector singlets, we have a choice as to whether or not they should be considered as gauge fields. If they are taken as gauge fields, the natural procedure is to enlarge the initial symmetry group from $SU(3)\times SU(3)$ to $U(3)\times U(3)$. The additional additive conservation laws may be taken as "triplet number" (or, equivalently, baryon number) and its axial analog. However, since $U(3)$ is not a *simple* group, the singlet couplings and the octet couplings are not related to each other[16] in a gauge theory. Thus we do not gain any additional information by treating the singlets as gauge particles. The use of a higher symmetry [e.g., $SU(4)\times SU(4)$, which ultimately breaks down to Okubo's nonet symmetry] would give us the usual nonet relations, but that is beyond the context of the present work.

In this section we shall introduce the $SU(3)$ breaking for the spin-1 mesons only in the relevant mass terms. The axial-vector unitary singlet will not be considered here because we do not need it for present applications. The terms that are to be *added* to the previous Lagrangian are

$$-\tfrac{1}{4}\varphi_{\mu\nu}\varphi_{\mu\nu} - \tfrac{1}{2}C_1\varphi_\mu\varphi_\mu - [m^2(K^*) - m_V^2]\,\mathrm{Tr}(V_\mu V_\mu S)$$
$$- C_2\varphi_\mu\omega_\mu - [m^2(K_A) - m_A^2]\,\mathrm{Tr}(\tilde{A}_\mu\tilde{A}_\mu S), \quad (36)$$

where $\varphi_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu$, φ_μ being the (unmixed) unitary singlet vector field. C_1 and C_2 are constants.

It is important to note that the *physical* axial-vector field is used in (36). This has the consequence that the same renormalization factor is good for the pion and the kaon.

For the vector-meson system, we are presently using the "mass-mixing" model. Physical $\tilde{\omega}_\mu$ and $\tilde{\varphi}_\mu$ fields are defined in the usual way by

$$\omega_\mu = (\sqrt{\tfrac{1}{3}})\tilde{\omega}_\mu + (\sqrt{\tfrac{2}{3}})\tilde{\varphi}_\mu,$$
$$\varphi_\mu = (\sqrt{\tfrac{2}{3}})\tilde{\omega}_\mu - (\sqrt{\tfrac{1}{3}})\tilde{\varphi}_\mu. \quad (37)$$

The above transformation will diagonalize the ω-φ part of (36) when the constants are chosen as follows:

$$C_1 = \tfrac{1}{2}m^2(\tilde{\omega}) - \tfrac{1}{6}m_V^2 + \tfrac{2}{3}m^2(K^*),$$
$$C_2 = (1/\sqrt{2})m^2(\tilde{\omega}) + \tfrac{1}{6}\sqrt{2}m_V^2 - \tfrac{2}{3}\sqrt{2}m^2(K^*),$$

and

$$m^2(\tilde{\varphi}) = 2m^2(K^*) - \tfrac{1}{2}[m_V^2 + m^2(\tilde{\omega})].$$

[15] Our results are essentially the same numerically as those of R. D. Peccei [Phys. Rev. 176, 1812 (1968)] even though a slightly different interaction is used.

[16] See M. Gell-Mann and S. Glashow, Ref. 6.

Before computing the K^+-nucleon scattering, it is necessary to specify the coupling of φ_μ to the baryons. To ensure that, assuming (37) to hold, the physical $\tilde{\varphi}_\mu$ decouples from the neutron and proton, this is chosen as

$$\tfrac{1}{2}\sqrt{3}ig\,\mathrm{Tr}(\bar{N}\gamma_\mu N)\varphi_\mu. \tag{38}$$

The scattering lengths are calculated in about the same way as the pion-nucleon case. Besides the ρ_0 and $\tilde{\omega}$ exchange diagrams, we must include the Λ and Σ exchanges as well as the quadrilinear "contact" diagram. For the S-wave scattering lengths we then find

$$\begin{aligned} a_S(K^+p) &= \frac{m}{4\pi(m+\mu_K)}\left[-4\mu_K f_r^2(Z^2-F)-\frac{\mu_K g^2}{m_V^2}\right. \\ &\qquad \left. +4mf_r^2-\sum_{Y=\Lambda,\Sigma}\frac{(f_{pYK}\mu_K-g_{pYK})^2}{m_Y+m-\mu_K}\right] \\ &= \frac{m}{4\pi(m+\mu_K)}\left[-4\mu_K f_r^2(1-F)\right. \\ &\qquad \left. +4mf_r^2-\sum_{Y=\Lambda,\Sigma}\frac{(f_{pYK}\mu_K-g_{pYK})^2}{m_Y+m-\mu_K}\right], \end{aligned} \tag{39a}$$

$$\begin{aligned} a_S(K^+n) &= \frac{m}{4\pi(m+\mu_K)}\left[2\mu_K f_r^2(-1+F-D)\right. \\ &\qquad \left. +2mf_r^2-\frac{(f_{n\Sigma K}\mu_K-g_{n\Sigma K})^2}{m_\Sigma+m-\mu_K}\right], \end{aligned} \tag{39b}$$

where

$$\begin{aligned} g_{p\Lambda K} &= -(\sqrt{6})mf_r, & f_{p\Lambda K} &= (1/\sqrt{6})f_r(-3+D+3F), \\ g_{n\Sigma K} &= -2mf_r, & f_{n\Sigma K} &= f_r(-1-D+F), \\ g_{p\Sigma K} &= -\sqrt{2}mf_r, & f_{p\Sigma K} &= (1/\sqrt{2})f_r(-1-D+F). \end{aligned}$$

These coupling constants are numerically the same as those in I. In deriving (39) we have set $m_V = m(\tilde{\omega})$.

Again in this case the effects of vector-meson exchange have been canceled out by part of the contact term, leaving us with a result independent of Z. This is indicated explicitly in (39a), where the use of (16) is shown. The result is identical to that obtained in I, where only baryons and pseudoscalar mesons were included in the theory. Therefore, the discussion given in I still applies; briefly, (39a) and (39b) are in good agreement with experiment and, furthermore, in the CA limit, where $\mu_K/m \to 0$ and $m_Y = m$, they reduce to the relatively poor CA results.

To review, the assumptions involved in deriving (39) are that (a) $SU(3)$ breaking exists in the mass terms of the physical particles, not in the coupling constants; (b) the mass-mixing scheme should be used for ω-ϕ mixing; and (c) the $\tilde{\varphi}$ particle decouples from the nucleons.

If these assumptions are relaxed, there are many possibilities. Some of the most apparently reasonable ones will be discussed in the next section. However, they do not seem to give quite such good results for the S-wave scattering lengths. Nevertheless, as mentioned in the discussion of the pion-nucleon case, it may be necessary to add additional particles to the Lagrangian or to make a more complicated symmetry-breaking assumption so that these other cases cannot be clearly ruled out. Furthermore, they have additional consequences for the weak interaction that give us more points to check.

If assumptions (a) and (b) are made, then (c) will, of course, automatically follow, if our results (39) are equated to the experimental scattering lengths and if the over-all factor in (38) is considered arbitrary. This proof of the decoupling of the $\tilde{\varphi}$ from the nucleons was essentially given in I.

It is interesting to note that Eqs. (39) are the ones that would be obtained in a theory where there were just eight vector mesons, so that the ω-φ mixing problem would not exist.

V. SEMILEPTONIC DECAYS AND ALTERNATIVE $SU(3)$-BREAKING POSSIBILITIES

The $SU(3)$ breaking for the spin-1 mesons represented by (36) does not lead to any renormalization of the weak Cabibbo matrix elements for processes like neutron decay, hyperon β decay, K_{l2} decay, and K_{l3} decay. This is not necessarily inconsistent with present data, since our experimental knowledge of the strangeness-changing semileptonic decays including the K_{l3} modes is far from precise. Nevertheless, there is some indication that a small amount of renormalization is required and it would be interesting to accommodate this in our Lagrangian. One simple way to do this is to modify (36) slightly. Then the renormalizations for all semileptonic decays in the Cabibbo theory can be expressed in terms of one parameter. This parameter is explicitly given in our theory but it may be that the correlations between the different modes, allowing this parameter to be arbitrary, are even more general.

We consider the "modified mass-mixing" model to be one where the term

$$-[m^2(K_A)-m_A^2]\,\mathrm{Tr}(\tilde{A}_\mu\tilde{A}_\mu S)$$

in (36) is replaced by

$$-[m^2(K_A)-m_A^2]\,\mathrm{Tr}(A_\mu A_\mu S). \tag{40}$$

Since the axial-vector fields appearing in (40) are the *mixed* unphysical ones and (40) explicitly violates $SU(3)$, the *separate* diagonalizations of the π-A_1 and K-K_A systems give rise to different renormalizations for strangeness-changing and strangeness-conserving objects.

We note that if the $SU(3)$-symmetry breaking given by the modified (36) is required to transform as

(8,1)+(1,8), we would have the relation

$$m^2(K_A)+m_V{}^2=m^2(K^*)+m^2(A_1)\,,$$

which is *roughly* satisfied.

Now, proceeding analogously to (12), we define physical (tilde) fields that reduce the bilinear terms in our new Lagrangian to diagonal form:

$$(A_1)_\mu=(\tilde{A}_1)_\mu+(g/2fZm_A{}^2)\partial_\mu\tilde{\pi}\,, \tag{41a}$$

$$(K_A)_\mu=(\tilde{K}_A)_\mu+[g/2fZ_Km^2(K_A)]\partial_\mu\tilde{K}\,, \tag{41b}$$

$$\pi=(1/Z)\tilde{\pi}\,, \tag{42a}$$

$$K=(1/Z_K)\tilde{K}\,. \tag{42b}$$

The renormalization constants Z and Z_K are given by

$$Z^2=1-g^2/4f^2m_A{}^2=(m_V/m_A)^2\,, \tag{43a}$$

$$Z_K{}^2=1-g^2/4f^2m^2(K_A)\,. \tag{43b}$$

The quantity Z is exactly the old one we had in the $SU(3)$ limit. [We still have $m_V=m(\rho)$ and $m_A=m(A_1)$.] We could also define Z_η for the η field. It is apparent that, if the axial-vector unitary singlet is neglected, we would have a kind of Gell-Mann–Okubo relation:

$$\frac{3}{1-Z_\eta{}^2}+\frac{1}{1-Z^2}=\frac{4}{1-Z_K{}^2}\,.$$

The quantity which appears in the semileptonic matrix elements is

$$x=\frac{Z}{Z_K}=\frac{m(K_A)}{m_A}\left(\frac{m_V{}^2}{m_V{}^2+[m^2(K_A)-m_A{}^2]}\right)^{1/2}. \tag{44}$$

Numerically, $x\simeq 0.87$ if K_A is identified as $K_A(1320)$, and $x\simeq 0.90$ if K_A is taken to be $K_A(1240)$.

We shall assume here that the Cabibbo currents are the ones computed from the Lagrangian according to the Noether prescription [e.g., (27)]. The Lagrangian to be used is the one where the substitutions indicated by (41) and (42) are made. Then we find that the strangeness-conserving axial-vector current is still given by (29). However, the strangeness-changing axial-vector current now becomes

$$(\mathcal{P}_3{}^1)_\mu=(1/f_r{}^K)\partial_\mu\tilde{\phi}_3{}^1-iZ_K{}^2(1+4\beta)\bar{N}_3{}^c\gamma_\mu\gamma_5N_c{}^1 -iZ_K{}^2(-1+4\alpha)\bar{N}_c{}^1\gamma_\mu\gamma_5N_3{}^c+\cdots\,, \tag{45}$$

where

$$f_r{}^K=f/Z_K\,. \tag{46}$$

If the *true* Cabibbo factor is written as $\sin\theta$, then the effective Cabibbo factor for K_{l2} decay, which we denote as $(\sin\theta_A)^M$, is seen from (45) to be

$$(\sin\theta_A)^M=(1/x)\sin\theta\,. \tag{47}$$

From our model, $1/x\equiv f_r/f_r{}^K=1.15$ or 1.11, which is in the same direction as the numbers usually quoted.[17]

The effective Cabibbo factor for the baryon axial-vector strangeness-changing current is also seen from (45) to be

$$(\sin\theta_A)^B=(1/x^2)\sin\theta\,. \tag{48}$$

It is amusing that the baryon renormalization turns out to be the square of the meson renormalization. The experimental situation is not at all clear for the meson case (K_{l3} is the stumbling block), but $1/x^2\simeq 1.32$ is not too different from the values found, from baryon experiments alone, by Nieh and Nieto.[18] Note that, since the quantities α and β appear in the same combination in $(\mathcal{P}_2{}^1)_\mu$ and in $(\mathcal{P}_3{}^1)_\mu$, the d/f ratio of the axial-vector baryon strangeness-changing currents by themselves is the same as the d/f ratio of the strangeness-conserving currents by themselves. The numerical determination of α and β from (30) should thus be made using either the $\Delta s=0$ or $|\Delta s|=1$ currents.

The vector currents are similarly[19] found to be

$$(\mathcal{V}_2{}^1)_\mu=i(\bar{N}_c{}^1\gamma_\mu N_2{}^c-\bar{N}_2{}^c\gamma_\mu N_c{}^1) +i\tilde{\phi}_2{}^c\overleftrightarrow{\partial}_\mu\tilde{\phi}_c{}^1+\cdots\,, \tag{49a}$$

$$(\mathcal{V}_3{}^1)_\mu=i(\bar{N}_c{}^1\gamma_\mu N_3{}^c-\bar{N}_3{}^c\gamma_\mu N_c{}^1) -i[(1/x)\tilde{\pi}^-\partial_\mu(\tilde{K}^0)^\dagger-x(\tilde{K}^0)^\dagger\partial_\mu\tilde{\pi}^-] +(-i/\sqrt{2})[(1/x)\tilde{\pi}^0\partial_\mu\tilde{K}^--x\tilde{K}^-\partial_\mu\tilde{\pi}^0]+\cdots\,. \tag{49b}$$

It is seen that the baryon part of the strangeness-changing vector current suffers no renormalization due to $SU(3)$ breaking, so that we may write

$$(\sin\theta_V)^B=\sin\theta\,. \tag{50}$$

Equation (49b) shows that the meson part of the strangeness-changing vector current is renormalized; the corresponding K_{l3} matrix element is

$$(4p_0q_0)^{1/2}\langle\pi^0(q)|(\mathcal{V}_3{}^1)_\mu|K^-(p)\rangle =(1/2\sqrt{2})[(1/x+x)(p+q)_\mu+(1/x-x)(p-q)_\mu]. \tag{51}$$

Equation (51) evidently obeys the low-energy theorem $f_++f_-=(1/\sqrt{2})\times 1/x$, where $f_\pm$ are the coefficients of $(p\pm q)_\mu$.

For the effective Cabibbo angle at zero momentum transfer (which is not all that is needed in this case), we have, from (51),

$$(\sin\theta_V)^M=\tfrac{1}{2}(1/x+x)\sin\theta\,. \tag{52}$$

Equations (47), (48), (50), and (52) amount to a definite prescription for modification of the Cabibbo Hamiltonian, which in our notation reads

$$\mathcal{H}_W{}^{SL}=(G/\sqrt{2})J_\mu l_\mu+\text{H.c.}\,, \tag{53a}$$

$$J_\mu=(\mathcal{P}_2{}^1+\mathcal{V}_2{}^1)_\mu\cos\theta+(\mathcal{P}_3{}^1+\mathcal{V}_3{}^1)_\mu\sin\theta\,, \tag{53b}$$

where $G\simeq 10^{-5}m^{-2}$ and l_μ is the leptonic current.

[17] See, e.g., B. W. Lee, Phys. Rev. Letters **20**, 617 (1968).

[18] H. T. Nieh and N. M. Nieto, Phys. Rev. **172**, 1694 (1964). For different results, see N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. **B6**, 255 (1968). For $(1/x)\simeq 1.15$ and assuming $(\sin\theta_A)^M=0.265$, we have $(\sin\theta_V)^M\simeq 0.23$, $(\sin\theta_V)^B\simeq 0.23$, and $(\sin\theta_A)^B\simeq 0.30$.

[19] We have, for a vector transformation, $\delta N=[E,N]$ and $\delta\phi=[E,\phi]$, with $E=-E^\dagger$. The vector currents are then given by (27), where $\mathcal{P}_\mu$ is replaced by $\mathcal{V}_\mu$ on the left-hand side.

In the present treatment no account of the form-factor dependence of the currents has been taken, so that the expressions above are most useful for decays where the hadronic momentum transfer is small. To get momentum-dependent form factors, which we expect to be a better approximation to the real world, the field-current identity assumption can be made. This is indicated for K_{l3} decays in Appendix B, where results similar to well-known ones[16] are derived. Equation (51) is seen to be the limit, where $p_\mu \to q_\mu$, of a more complicated expression.

The above modified mass-mixing model does lead to a change in the K^+-nucleon scattering lengths. In (39a) the quantity $4mf_r^2$ should be replaced by $4m(f_r^K)^2$, in (39b) $2mf_r^2$ should be replaced by $2m(f_r^K)^2$, and the coupling constants should be

$$g_{p\Lambda K} = -(\sqrt{6})mf_r^K,$$
$$f_{p\Lambda K} = (f_r^K/\sqrt{6})[-3+(1/x^2)(3F+D)],$$
$$g_{n\Sigma K} = \sqrt{2}g_{p\Sigma K} = -2mf_r^K,$$
$$f_{n\Sigma K} = \sqrt{2}f_{p\Sigma K} = f_r^K[-1+(1/x^2)(F-D)],$$

where it has been assumed that the values of F and D have been obtained from (say) the *strangeness-conserving* axial-vector matrix elements by themselves. Then (39) yields $a_S(K^+p) \simeq -0.36\mu^{-1}$ and $a_S(K^+n) \simeq -0.19\mu^{-1}$, which are somewhat larger than they should be. This result is evidently a drawback to the modified mass-mixing model, but may not be serious if additional particles are added to the theory. Using the alternative forms for $\mathcal{L}_{\Delta m}$ given in Appendix A does not improve the situation.

One popular way[20] of introducing $SU(3)$-symmetry breaking for the spin-1 mesons is the "current-mixing" method. In this case, (36) is to be replaced by

$$-\tfrac{1}{2}[(m_V/m(K^*))^2-1]\,\mathrm{Tr}(F_{\mu\nu}{}^V F_{\mu\nu}{}^V S) -\tfrac{1}{2}[(m_A/m(K_A))^2-1]\,\mathrm{Tr}(F_{\mu\nu}{}^A F_{\mu\nu}{}^A S) -\tfrac{1}{4}\varphi_{\mu\nu}\varphi_{\mu\nu}-\tfrac{1}{2}(C_0)^2\varphi_\mu\varphi_\mu - C_3\varphi_{\mu\nu}\omega_{\mu\nu}, \quad (54)$$

where C_0 and C_3 are constants to be determined. Here $[(8,1)+(1,8)]$-type symmetry breaking implies the relation

$$m(K^*)/m_V = m(K_A)/m_A,$$

which does not seem too bad ($1.15 \simeq 1.14$ or 1.22).

Diagonalization of the π-A_1 system and of the K-K_A system is achieved with the substitutions

$$(A_1)_\mu = (\tilde{A}_1)_\mu + (g/2fZm_A{}^2)\partial_\mu\tilde{\pi}, \quad (55a)$$

$$(K_A)_\mu = [m(K_A)/m_A](\tilde{K}_A)_\mu + (g/2fZm_A{}^2)\partial_\mu\tilde{K}, \quad (55b)$$

$$\pi = (1/Z)\tilde{\pi}, \quad K = (1/Z)\tilde{K}, \quad (55c)$$

$$Z = m_V/m_A. \quad (55d)$$

From (55) we see that the π and K symmetry-breaking renormalizations are the same in this model. Thus the Cabibbo scheme for semileptonic decays is unaffected, as are the non-vector-meson contributions to the K^+-N scattering-length formulas (39a) and (39b).

To diagonalize the ω-ϕ system, we set

$$\omega_\mu = \frac{m(\tilde{\varphi})}{m_V}\cos\theta\,\tilde{\varphi}_\mu - \frac{m(\tilde{\omega})}{m_V}\sin\theta\,\tilde{\omega}_\mu, \quad (56a)$$

$$\varphi_\mu = \frac{m(\tilde{\varphi})}{C_0}\sin\theta\,\tilde{\varphi}_\mu + \frac{m(\tilde{\omega})}{C_0}\cos\theta\,\tilde{\omega}_\mu, \quad (56b)$$

where $\tilde{\varphi}_\mu$ and $\tilde{\omega}_\mu$ are the physical particles. The angle $|\theta|$ can be determined from

$$\frac{\cos^2\theta}{m^2(\tilde{\varphi})} + \frac{\sin^2\theta}{m^2(\tilde{\omega})} = \frac{1}{3}\left(\frac{4}{m^2(K^*)} - \frac{1}{m_V{}^2}\right), \quad (57)$$

which implies $|\theta| \simeq 26°$. (We shall take $\theta = -26°$.) $|C_0|$ is found from

$$\frac{\sin^2\theta}{m^2(\tilde{\varphi})} + \frac{\cos^2\theta}{m^2(\tilde{\omega})} = \frac{1}{(C_0)^2}, \quad (58)$$

while C_3 is determined by

$$C_3 = \tfrac{1}{4}C_0 m_V \tan 2\theta\left[\frac{1}{3}\left(\frac{4}{m^2(K^*)} - \frac{1}{m_V{}^2}\right) - \frac{1}{(C_0)^2}\right]. \quad (59)$$

Now let us replace the coupling of φ to the nucleon given in (38) by

$$id_1\,\mathrm{Tr}(\bar{N}\gamma_\mu N)\varphi_\mu, \quad (60)$$

where d_1 is an arbitrary constant. The requirement that the physical singlet $\tilde{\varphi}_\mu$ decouples from the nucleons leads to the relation

$$d_1 = -(C_0/m_V)(3g/2\sqrt{6})\cot\theta.$$

Then after some computation we find $a_S(K^+p) \simeq -0.5\mu^{-1}$ and $a_S(K^+n) \simeq -0.3\mu^{-1}$, which are rather bad. If we instead choose $d_1 = 0$ ($\tilde{\varphi}_\mu$ would, however, in this case not decouple from the nucleons), the reasonable results $a_S(K^+p) \simeq -0.29\mu^{-1}$ and $a_S(K^+n) \simeq -0.09\mu^{-1}$ emerge. At the present stage, therefore, the current-mixing scheme does not seem to be favored.

To sum up our discussion on $SU(3)$ breaking, we note that, if mass mixing for the ω-φ system is accepted, then the K^+N-scattering-length results come out well if the ratio of the π and K renormalization constants [denoted by x in Eq. (44)] is 1, while the fact that the *effective* Cabibbo angles differ from each other tends to suggest that x should be slightly larger than 1. (However, the exact experimental status of the differences between the effective Cabibbo angles is not clear.) Our modified mass-mixing model predicts $x = 1.15$ or 1.11, but perhaps a slightly different modification would give a somewhat lower value of x, which might lead to reasonable results for the scattering lengths and possible agreement with the data from semileptonic decays. In this connection

[20] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964); N. Kroll, T. D. Lee, and B. Zumino, *ibid.* **157**, 1376 (1967); R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

we mention that $SU(3)$-symmetry breaking can be introduced in various combinations, *both* in the vector-meson mass and kinetic terms. However, this procedure suffers from being inelegant. In any case, there is a fair amount of theoretical flexibility in addition to a certain amount of experimental uncertainty, so that we will not explore any of these other cases now. More information can also be obtained from other meson-baryon scattering processes. Further investigations will be reported elsewhere.

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APPENDIX A

The Lagrangian can be presented in many different equivalent forms, since, according to the theorem of Chisholm,[21] the S-matrix elements are unchanged if a *point* transformation on any field ϕ of the type

$$\phi=\phi'+\cdots$$

(where the dots stand for bilinear and higher terms in ϕ') is made. For example, the interaction terms in $\mathcal{L}_1$ [Eq. (20)] can be transformed away if a new baryon field

$$B=\begin{pmatrix}B_L\\ B_R\end{pmatrix},\quad \bar{B}\equiv(\bar{B}_R,\quad \bar{B}_L)\,, \tag{A1}$$

is defined by

$$\begin{aligned}L&=M^{1/2}B_LM^{-1/2}, & \bar{L}&=M^{1/2}\bar{B}_LM^{-1/2},\\ R&=M^{-1/2}B_RM^{1/2}, & \bar{R}&=M^{-1/2}\bar{B}_RM^{1/2},\end{aligned} \tag{A2}$$

where

$$\begin{aligned}M^{1/2}&=1+if\phi-\tfrac{1}{2}f^2\phi^2+\cdots,\\ M^{-1/2}&=1-if\phi-\tfrac{1}{2}f^2\phi^2+\cdots.\end{aligned} \tag{A3}$$

Making the substitutions indicated by (A2) gives

$$\mathcal{L}_1=-m\operatorname{Tr}(\bar{L}MRM^\dagger+\bar{R}M^\dagger LM)=-m\operatorname{Tr}(\bar{B}B)\,, \tag{A4}$$

which is just the usual mass term. In this case the baryon kinetic term picks up a derivative-type Yukawa interaction:

$$\begin{aligned}-\operatorname{Tr}(\bar{N}\gamma_\mu\partial_\mu N)&\\ =-\operatorname{Tr}(\bar{B}\gamma_\mu\partial_\mu B)&-if\operatorname{Tr}(\bar{B}\gamma_\mu\gamma_5[\partial_\mu\phi,B])\\ &-\tfrac{1}{2}f^2\operatorname{Tr}(\bar{B}\gamma_\mu[(\phi\overleftrightarrow{\partial}_\mu\phi),B])+O(\phi^3)+\cdots.\end{aligned} \tag{A5}$$

This is essentially the same as the usual pseudoscalar-pseudovector equivalence theorem. The term $\mathcal{L}_2$ (for simplicity, before the Yang-Mills substitution $\partial_\mu\to D_\mu$ is made) goes to

$$\begin{aligned}\mathcal{L}_2&=2\alpha\operatorname{Tr}(\bar{L}\sigma_\mu\partial_\mu MM^\dagger L+\bar{R}\bar{\sigma}_\mu\partial_\mu M^\dagger MR)+2\beta\operatorname{Tr}(\bar{L}\sigma_\mu L\partial_\mu MM^\dagger+\bar{R}\bar{\sigma}_\mu R\partial_\mu M^\dagger M)\\ &=4i\alpha f\operatorname{Tr}(\bar{B}\gamma_\mu\gamma_5\partial_\mu\phi B)+4i\beta f\operatorname{Tr}(\bar{B}\gamma_\mu\gamma_5B\partial_\mu\phi)+O(\phi^3)+\cdots.\end{aligned} \tag{A6}$$

Note that terms bilinear in the pion fields do not appear in (A6). Additional interaction terms, however, now come from $\mathcal{L}_{\Delta m}$ [Eq. (23)]:

$$\begin{aligned}\mathcal{L}_{\Delta m}=(m-m_\Sigma)&\operatorname{Tr}(\bar{B}B+2if\bar{B}\gamma_5[\phi,B]+4f^2\bar{B}\phi B\phi-2f^2\bar{B}[\phi^2,B]_+-\bar{B}BS+if\bar{B}\gamma_5B[\phi,S]_+\\ &-2if\bar{B}\gamma_5\phi BS-2f^2\bar{B}\phi B[\phi,S]_++\tfrac{1}{2}f^2\bar{B}B[\phi^2,S]_++f^2\bar{B}B\phi S\phi+2f^2\bar{B}\phi^2BS)\\ &+(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{B}SB+if\bar{B}\gamma_5[\phi,S]_+B-2if\bar{B}\gamma_5SB\phi+2f^2\bar{B}[\phi,S]_+B\phi\\ &\qquad-\tfrac{1}{2}f^2\bar{B}[\phi^2,S]_+B-f^2\bar{B}\phi S\phi B-2f^2\bar{B}SB\phi^2)+O(\phi^3)+\cdots.\end{aligned} \tag{A7}$$

If the baryon mass-splitting term is considered to transform as $(3,3^*)+(3^*,3)$, instead of $(8,8)$, we should replace $(\mathcal{L}_1+\mathcal{L}_{\Delta m})$ by $(\mathcal{L}_1+\mathcal{L}_{\Delta m})'$, which is defined and rewritten

$$\begin{aligned}(\mathcal{L}_1+\mathcal{L}_{\Delta m})'&=-m_\Sigma\operatorname{Tr}(\bar{L}MRM^\dagger+\bar{R}M^\dagger LM)+(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{L}SRM^\dagger+\bar{R}SLM)+(m_\Sigma-m)\operatorname{Tr}(\bar{L}MRS+\bar{R}M^\dagger LS)\\ &=-m_\Sigma\operatorname{Tr}(\bar{B}B)+(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{B}SB+if\bar{B}\gamma_5[\phi,S]_+B-f^2\bar{B}\phi S\phi B-\tfrac{1}{2}f^2\bar{B}[\phi^2,S]_+B)\\ &\qquad+(m_\Sigma-m)\operatorname{Tr}(\bar{B}BS-if\bar{B}\gamma_5B[\phi,S]_+-f^2\bar{B}B\phi S\phi-\tfrac{1}{2}f^2\bar{B}B[\phi^2,S]_+)+O(\phi^3).\end{aligned} \tag{A8}$$

If the baryon mass-splitting term goes as $(8,1)+(1,8)$, we would have instead

$$\begin{aligned}(\mathcal{L}_1+\mathcal{L}_{\Delta m})''&=-m_\Sigma\operatorname{Tr}(\bar{L}MRM^\dagger+\bar{R}M^\dagger LM)+\tfrac{1}{2}(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{L}SMRM^\dagger+\bar{R}SM^\dagger LM+\bar{L}MSRM^\dagger+\bar{R}M^\dagger SLM)\\ &\qquad+\tfrac{1}{2}(m_\Sigma-m)\operatorname{Tr}(\bar{L}MRM^\dagger S+\bar{R}M^\dagger LMS+\bar{L}MRSM^\dagger+\bar{R}M^\dagger LSM)\\ &=-m_\Sigma\operatorname{Tr}(\bar{B}B)+(m_\Sigma-m_\Xi)\operatorname{Tr}(\bar{B}SB-\tfrac{1}{2}f^2\bar{B}[\phi^2,S]_+B+f^2\bar{B}\phi S\phi B)\\ &\qquad+(m_\Sigma-m)\operatorname{Tr}(\bar{B}BS-\tfrac{1}{2}f^2\bar{B}B[\phi^2,S]_++f^2\bar{B}B\phi S\phi)+O(\phi^3).\end{aligned} \tag{A9}$$

Note that (A9) gives no contribution to the Yukawa-type interaction.

Finally, we note that the freedom to make point transformations lets us easily demonstrate that the S-matrix elements will be the same if a different form is used for M. First consider the "standard" form

$$\begin{aligned}M_1(\phi)&=\exp(2if\phi)\\ &=1+2if\phi-2f^2\phi^2-\tfrac{4}{3}if^3\phi^3+\cdots.\end{aligned} \tag{A10}$$

Now consider another form $M_2(\psi)$ satisfying

[21] J. S. R. Chisholm, Nucl. Phys. **26**, 469 (1961); S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, *ibid.* **28**, 529 (1961).

$M_2^\dagger(\psi)M_2(\psi)=1$. This can then be written as

$$M_2(\psi)=1+2if\psi-2f^2\psi^2+C\psi^3+\cdots, \tag{A11}$$

where the coefficients of the linear and bilinear terms were fixed by unitarity, but the coefficient of the cubic term and the independent coefficients of higher terms are arbitrary. Let us try to find a point transformation $\phi=\phi(\psi)$ such that

$$M_1(\phi)=M_1(\phi(\psi))=M_2(\psi). \tag{A12}$$

If the proper type of $\phi(\psi)$ can be found, then it is clear, by Chisholm's theorem, that we can use $M_2(\psi)$ instead of $M_1(\phi)$ and still get the same results. From (A10) and (A12) we have

$$\begin{aligned}\phi&=(1/2if)\ln[M_1(\phi)]\\&=(1/2if)\ln\{[M_2(\psi)-1]+1\},\end{aligned} \tag{A13}$$

where the symbol $\ln(A+1)$ means $A-\frac{1}{2}A^2+\frac{1}{3}A^3+\cdots$, and converges for $\mathrm{Tr}(AA^\dagger)<1$. In this case, we expect convergence for sufficiently small $f\psi$, since

$$\mathrm{Tr}[(M_2^\dagger-1)(M_2-1)]=[6-\mathrm{Tr}(M_2^\dagger+M_2)]\to 0$$

as $f\psi\to 0$. Expanding (A13) gives

$$\phi=\psi+(C/2if+\tfrac{2}{3}f^2)\psi^3+\cdots, \tag{A14}$$

which is a point transformation of the required form.

APPENDIX B

According to the field-current identity assumption, the strangeness-changing vector current that appears in the Cabibbo Hamiltonian is to be written as

$$(\mathcal{V}_3{}^1)_\mu=[2m^2(K^*)/g]K_\mu{}^{*-}. \tag{B1}$$

The K_{l3} matrix element is then given by the product of the K^* propagator and the $K^*K\pi$ vertex. The latter receives contributions from (17), (18), and (24) where, in the case of the modified mass-mixing model, we should make the substitutions indicated by (41) and (42) rather than those of (12). Then the $K^{*+}K\pi$ part of the Lagrangian comes out to be

$$\begin{aligned}\mathcal{L}_{K^{*+}K\pi}=\tfrac{1}{2}ig\Bigg\{&\frac{1}{x}\frac{(1-Z^2)}{m_V{}^2}\left[\left(\frac{xm_A}{m(K_A)}\right)^2+\delta\right]\\&\times(\partial_\mu K_\nu{}^{*+}-\partial_\nu K_\mu{}^{*+})\left(\frac{1}{\sqrt{2}}\partial_\mu K^-\partial_\nu\tilde{\pi}^0+\partial_\mu(K^0)^\dagger\partial_\nu\tilde{\pi}^-\right)\\&+K_\mu{}^{*+}\Bigg(-\frac{1}{x}\frac{1}{\sqrt{2}}\partial_\mu\tilde{K}^-\tilde{\pi}^0+\frac{x}{\sqrt{2}}\tilde{K}^-\partial_\mu\tilde{\pi}^0\\&-\frac{1}{x}\partial_\mu(\tilde{K}^0)^\dagger\tilde{\pi}^-+x(\tilde{K}^0)^\dagger\partial_\mu\tilde{\pi}^-\Bigg)\Bigg\}.\end{aligned} \tag{B2}$$

After parametrizing the K_{l3} matrix element in the usual way as

$$\begin{aligned}(4p_0q_0)^{1/2}\langle\pi^0(q)|(\mathcal{V}_3{}^1)_\mu|K^-(p)\rangle\\=f_+(K^2)(p+q)_\mu+f_-(K^2)(p-q)_\mu,\\K_\mu=(p-q)_\mu,\end{aligned} \tag{B3}$$

and using (B2), we find for the K_{l3} form factors the results

$$\begin{aligned}f_+(K^2)=\frac{1}{\sqrt{2}}\frac{m^2(K^*)}{K^2+m^2(K^*)}\Bigg\{&\frac{1}{2}\left(\frac{1}{x}+x\right)\\&+\frac{K^2}{2x}\frac{(1-Z^2)}{m_V{}^2}\left[\left(\frac{xm_A}{m(K_A)}\right)^2+\delta\right]\Bigg\},\end{aligned} \tag{B4}$$

$$\begin{aligned}f_-(K^2)=&\frac{1}{2\sqrt{2}}\left(\frac{1}{x}-x\right)\\&-\frac{(p^2-q^2)}{2\sqrt{2}m_V{}^2}\frac{(1-Z^2)}{x}\left[\left(\frac{xm_A}{m(K_A)}\right)^2+\delta\right]\\&+\frac{p^2-q^2}{m^2(K^*)}f_+(K^2).\end{aligned} \tag{B5}$$

These results are the same as those of Lee,[17] except that his $(1+\delta)$ is replaced by

$$\left(\frac{m(K^*)}{m_V}\right)^2\left[\left(\frac{xm_A}{m(K_A)}\right)^2+\delta\right].$$

Equations (B4) and (B5) coincide with (51) in the limit $p_\mu\to q_\mu$.

For the *ordinary* mass-mixing model or for the current-mixing model the renormalizations are the same for the π and the K fields. Thus the K_{l3} form factors in these cases in the field-current identity model are given by (B4) and (B5), where x is replaced by 1 and the factor $xm_A/m(K_A)$ is also replaced by 1. Explicitly, we have the results

$$f_+(K^2)=\frac{1}{\sqrt{2}}\frac{m^2(K^*)}{K^2+m^2(K^*)}\left(1+K^2\frac{(1-Z^2)}{2m_V{}^2}(1+\delta)\right), \tag{B6}$$

$$\begin{aligned}f_-(K^2)=&\frac{1}{\sqrt{2}}\frac{\mu^2-\mu_K{}^2}{K^2+m^2(K^*)}\\&\times\left[1-\frac{1}{2}\left(\frac{m(K^*)}{m_V}\right)^2(1-Z^2)(1+\delta)\right].\end{aligned} \tag{B7}$$

These equations have been previously obtained[22] by the hard-kaon current-algebra method.

The present experimental data on the K_{l3} form factors are still somewhat ambiguous.

[22] Y. Ueda, Phys. Rev. **174**, 2082 (1968).