Eikonal Model for Current-Nucleon Scattering: Electron-Proton Deep-Inelastic Scattering*

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A high-energy, small-angle diffraction model in the eikonal approximation is adapted to current-nucleon scattering. Within the framework of certain assumptions, both the longitudinal and transverse electroproduction cross sections fall off as $1/|q^2|$ for high-energy ν and large momentum transfer $|q^2|$, with the ratio $\omega=|q^2|/\nu$ fixed in the asymptotic region. Furthermore, expressions are given for the functional dependence on ω of the electroproduction structure functions W_1 and $W_2(\nu,q^2)$.

INTRODUCTION

IN this paper we adapt a high-energy, small-angle diffraction model in the eikonal approximation to asymptotic current-nucleon (hadron) scattering. Certain assumptions are made, the principal ones being (i) a scaling property in terms of the variable[1] $\omega=-q^2/\nu$ for ν, $|q^2|$ large and (ii) a particular phenomenological parametrization for the eikonal. The former property is motivated for the dimensionless eikonal function if the current-nucleon elastic scattering is confined to small impact parameters, while the latter is suggested by high-energy, strong-interaction fits. It is then found that *both* the longitudinal and transverse electroproduction cross sections decrease like $1/|q^2|$ at fixed ω and a physical picture (mathematically expressed) is suggested for this circumstance. Expressions are given for the functional dependence on ω of the structure functions $W_{1,2}(\nu,q^2)$ for inelastic electron-proton scattering. $W_{1,2}$ are defined by[2]

$$\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)W_1(\nu,q^2)+\frac{1}{M^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right)$$
$$\times W_2(\nu,q^2)=(\tfrac{1}{2}\sum_s)\int\frac{d^4x}{(2\pi)}e^{iq\cdot x}$$
$$\times\langle p,s|[J_\mu(x),J_\nu(0)]|p,s\rangle\equiv t_{\mu\nu}(p,q),\quad(1)$$

where J_μ is the electromagnetic current for hadrons, $\nu=p\cdot q$ is the energy variable, and $q^2<0$ (spacelike) is the virtual-photon momentum transfer with p and q the nucleon and photon four-momenta, respectively.

EIKONAL MODEL REPRESENTATION FOR CURRENT-NUCLEON SCATTERING

In the c.m. system of the s channel, we consider the elastic scattering of a spin-averaged proton and virtual photon (coupled to a lepton pair). At large momentum q_s and small angle θ_s, the transverse dimensions of the scattering region may be measured in terms of an impact parameter $b=(J+\frac{1}{2})/q_s$, where $J=$the s-channel angular momentum. Then for a dominant, s-channel helicity-nonflip amplitude[3] $F_{hh}{}^s$ in the normalized current-proton scattering, the usual partial-wave sum

$$F_{hh}{}^s=\sum_{J=h}^{\infty}(2J+1)d_{hh}{}^J(\cos\theta_s)f_h(J;s,q^2)$$

becomes

$$=(2q_s{}^2)\int_{b_0(h)}^{\infty}bdb\,J_0(b\sqrt{-t})f_h(b;s,q^2).\quad(2)$$

Here, the lower limit $b_0(h)\equiv(h+\frac{1}{2})/q_s$ has not been set equal to zero since the q^2 variation of f_h has not yet been specified (and the virtual mass $|q^2|$ can be very large). Note that for large ν, $|q^2|$ and ω fixed within the kinematical region[4] of the inelastic ep experiment,[5] $q_s{}^2\to\nu/(2-\omega+M^2/\nu)$ is always large so that the angle defined by $\cos\theta_s-1=t/2q_s{}^2\ll1$ is indeed small. Since the internal, high-energy dynamics of the current-hadron process cannot possibly depend upon the particular value (e) for the electromagnetic coupling, the currents are normalized [$J_\mu(x)$, not $eJ_\mu(x)$] for the representation (2) above.

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[1] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969). Also R. P. Feynman (unpublished). By "scaling property" is meant a dimensional argument for the variable dependence of the relevant amplitudes when ν, $|q^2|\gg$ masses in the problem.

[2] S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) **28**, 18 (1964); J. D. Bjorken, in Proceedings of the International School of Physics Enrico Fermi, Varenna, 1967 (unpublished), p. 55.

[3] The helicity-amplitude normalization is defined by

$$(S-1)_{hh}=\frac{i(2\pi)^4\delta(p'+q'-p-q)}{(2\pi)^6(p_0'q_0'p_0q_0)^{1/2}}e^2F_{hh}{}^s.$$

To relate the $W_{1,2}$ to the transverse ($h=+1$) and "longitudinal" ($h=0$) projections of (1), one has

$$\mathrm{Im}F_{hh}{}^s=\epsilon_h{}^{\mu*}(q)t_{\mu\nu}\epsilon_h{}^\nu(q)M\pi/2\,,$$

where $q=(q_0;0,0,q_s)$, $\epsilon_\mu{}^0=(q_s;0,0,q_0)/(q^2)^{1/2}$, and $\epsilon_\mu{}^\pm=(0;\pm1,i,0)/\sqrt{2}$ with $q\cdot\epsilon_h=0,\epsilon_\pm{}^*\cdot\epsilon_\pm=-1$, $\epsilon_0{}^*\cdot\epsilon_0=+1$. From the optical theorem, $e^2\,\mathrm{Im}F_{hh}{}^s(\nu,q^2,0)=\frac{1}{2}Mq_{\mathrm{lab}}\sigma_h$, $Mq_{\mathrm{lab}}=(\nu^2-M^2q^2)^{1/2}$.

[4] In the inelastic ep SLAC-MIT experiment (Ref. 5) for incident electron energy E, ν can vary for fixed $|q^2|=4E(E-\nu/M)\sin^2\frac{1}{2}\theta$ between $\frac{1}{2}|q^2|$ (the nucleon) and $ME(1-|q^2|/4E^2)$, where $|q^2|$ can take values between 0 (Compton scattering) and $2ME/(1+M/2E)$. So $\omega=|q^2|/\nu$ can vary between 0 and 2 (nucleon). The "asymptotic region" as defined here is a large $|q^2|$ region with the larger values of the missing mass $\frac{1}{2}(s-M^2)=\nu-\frac{1}{2}|q^2|$. It does not include the region where distinct-resonance or secondary-trajectory contributions are significant (at least the former may be damped).

[5] For some preliminary results of the SLAC-MIT inelastic experiments, see W. Panofsky, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 23.

We now imagine a bilinear unitarity condition for the *normalized* current amplitude (s large, spacelike q^2):

$$\mathrm{Im}F=\frac{\rho}{4\pi}\int d\Omega\, FF^*+(\text{“inelastic” contribution}).$$

This quasi-two-body unitarity relation is an ansatz for simplifying the multiparticle unitarity relation for a subset of intermediate states which consists of a proton and the continuum of particles having the quantum numbers of the current. The phase function $\rho(\nu,q^2)$ can be taken[6] as $\rho=\rho_0(q_s/2\pi\sqrt{s})$, where ρ_0 is a constant to be determined. Coupling this unitarity with our previous impact-parameter amplitude representation (2) there results[7] the condition $\mathrm{Im}f_h=\rho|f_h|^2+$(“inelastic” contribution). A (complex) eikonal function $\chi(b;s,q^2)$ can now be defined from the relation

$$f_h=\left(\frac{e^{i\chi_h}-1}{2i\rho}\right).$$

In the high-energy, small-angle limit we may then write an impact-parameter representation in the eikonal approximation:

$$F_{hh}{}^s(\nu,q^2,t)=(2q_s{}^2)\int_{b_0(h)}^{\infty} bdb\, J_0(b\sqrt{-t})\left(\frac{e^{i\chi_h}-1}{2i\rho}\right) \qquad (h=0,\,+1). \quad (2')$$

In arriving at (2′), it has been assumed that we need consider only the s-channel “helicity-nonflip” amplitudes $F_{++}{}^s$ and $F_{00}{}^s$ at high energy and small t.[8] In general for virtual-photon ($q^2\neq 0$), spin-averaged-target elastic scattering, there are four independent amplitudes: $f_{00}{}^J$, $\sqrt{2}f_{0+}{}^J$, $f_{++}{}^J+f_{+-}{}^J$ with parity $P=(-1)^J$ and $f_{++}{}^J-f_{+-}{}^J$ with $P=-(-1)^J$ and where $f_{h'h}{}^J=\langle JJ_z;h'|S-1|JJ_z;h\rangle$. An eikonal basis can be conveniently defined if *uncoupled definite* J,P eigenamplitudes result; e.g., $f_{0+}{}^J$, $f_{+-}{}^J\approx 0$ at high energy as for our representation.

Finally, we give the relations[3] between our helicity amplitudes $F_{hh}{}^s(\nu,q^2,\,t=0)$ and the usual structure functions $W_{1,2}$:

$$\tfrac{1}{2}\pi MW_1(\nu,q^2)=\mathrm{Im}F_{++}{}^s(\nu,q^2,0)\,, \quad (3)$$

$$\tfrac{1}{2}\pi MW_2(\nu,q^2)=\left(\frac{q^2}{q^2-\nu^2/M^2}\right)\times\mathrm{Im}[F_{++}{}^s(\nu,q^2,0)+F_{00}{}^s(\nu,q^2,0)]. \quad (4)$$

A SCATTERING PICTURE AND A PARAMETRIZATION

In Eq. (2′), $\chi(b;\,q^2,q_s)$ is given an explicit dependence upon q^2, but not upon any energy variable. We assume an energy-independent,[9] purely absorptive eikonal (and, in “trajectory” parlance, a flat Pomeranchon in the region around $t=0$). Guided by strong-interaction parametrizations[10] for the form of χ, we specialize to the usual Gaussian expression in the impact-parameter variable, but with a *q^2-dependent interaction radius* $R(q^2)$:

$$i\chi_h(b,q^2)=-c_he^{-b^2/2R^2(q^2)}\,. \quad (5)$$

We now assume a scaling property[1] for the dimensionless dynamical amplitude $(e^{i\chi}-1)$ in terms of the variable ω (or, alternatively, $q^2/q_s{}^2$) as ν and $|q^2|$ become large in the asymptotic region. Then, since $b=(J+\frac{1}{2})/q_s$ (and $\Delta b=1/q_s$, the distance between one angular momentum value J and the next one, $J+1$) we have the behavior

$$R^2(q^2)\to\lambda/(-q^2),\quad -q^2\to\text{very large} \qquad (\lambda>0\text{ and dimensionless}). \quad (6)$$

More generally, if $\chi=\chi(b,q_s{}^2,q^2)$ and the important impact parameters are given by $b\propto\Delta b$, then $\chi\equiv\chi(q^2/q_s{}^2)$. Writing Eq. (5) implies Eq. (6). Note that a scattering region $b\propto\Delta b$, can, in fact, be argued in a formal way. The causality condition for the current commutator $t_{\mu\nu}$ in (1) implies contributions when $(x_0-x_L)(x_0+x_L)\geqslant x_T{}^2$, where $x_L=\mathbf{x}\cdot\hat{q}$ and $x_T=\mathbf{x}\cdot\hat{b}$ in the nucleon rest frame, say ($x_L{}^{\text{c.m.}}=\gamma^{-1}x_L{}^{\text{lab}}$, $\gamma^{-1}=$Lorentz contraction factor). When $1/\omega$ becomes large in the Fourier transform (assuming no violent variation in q^2, i.e., when $Mx_L{}^{\text{lab}}\approx\nu/M^2$), it follows that

$$O\left(\frac{M}{\nu}\frac{2}{M}\right)\geqslant x_T{}^2$$

and so $b\sim O(1/q_s)$. As we shall see, the behavior (6) allows both

$$\sigma_{L,T}\propto R^2(q^2)\times(\text{functions of }q^2/q_s{}^2)$$

[6] For normalization, at low q^2 one may use the unitarity relation to order e^2 for $\gamma(q^2)p\to\gamma(q^2)p$ and note the inclusion of the “elastic” intermediate state V_0p (V_0 is a vector meson with current quantum numbers) which gives $\rho=(2\gamma_V)^2(k_s/2\pi\sqrt{s})$, where $k_s=q_s$ with q^2 replaced by $M_V{}^2$ and $\langle 0|J_\mu|V^0\rangle=(M_V{}^2/2\gamma_V)\epsilon_\mu{}^{V^0}$. For purposes of the ω variation of (2′), $(q_s{}^2/\rho)=2\pi(Mq_{\text{lab}})/\rho_0\to 2\pi\nu/\rho_0$ as ν, $|q^2|$ become large (for fixed ω).

[7] For optical models in hadron scattering, see R. Blankenbecler and M. Goldberger, Phys. Rev. **126**, 766 (1962); R. C. Arnold, *ibid.* **153**, 1523 (1967).

[8] It may be motivated by strong-amplitude, phenomenological fits with dominant s-channel helicity diagonalization in the eikonal model. For example, in pp scattering refer to C. B. Chiu and J. Finkelstein, Nuovo Cimento **49A**, 92 (1969); and A. Capella *et al.*, *ibid.* **63A**, 141 (1969).

[9] This is an assumption which may be exact when the missing mass $s\to\infty$, in the spirit of the diffraction nature of the model. Since we are at highest energies and assuming there is damping in q^2, Regge trajectories and their associated cuts are excluded from discussion.

[10] For a comprehensive review with references see, C. Hong-Mo, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 391.

to drop off mildly like the inverse power $1/q^2$ for fixed ω, thereby allowing considerable inelastic production[5] at high $|q^2|$.

In the same kinematic region we make the simplest assumption and take the dimensionless quantity c_h to be a constant, although, for example, it could be a polynomial in $b^2/R^2(q^2)$. [A polynomial would not disturb the qualitative behavior for $\sigma_{L,T}$ already cited; in fact, it gives rise to the presence in these quantities of additional (convergent) polynomials in ω which, in the large-missing-mass limit $s\to\infty$ and $\omega\to 0$, essentially reduces to the $c_h=$const case.[11]]

Since, restated, our scaling assumption says that the scattering takes place at small impact parameters $b\propto\Delta b=1/q_s$, a possible intuitive picture for the behavior (5) and (6) can be given. Not only is the current projectile slamming into a "pancakelike" hadron target as $\nu\to\infty$, but also (in contradistinction to a purely *strong*, absorptive process) it is probing deeper and deeper into the target structure as $q^2\to-\infty$ so that the effective radius of interaction presented to the projectile decreases with increasing $|q^2|$. At some fixed impact parameter b, according to (6), the nth absorptive iteration $(i\chi)^n$ decreases for increasing $|q^2|$ and n and the scattering amplitude, $[e^{i\chi(b)}-1]$, vanishes as the target presents a diminishing radius off which to scatter.

As an example, we may write χ_h as a product form in the impact-parameter variable b^2 or a convolution in the variable t[12]

$$i\chi_h(b,q^2)=-c_h\exp(-b^2/2R_H^2)\exp(-b^2/2R_\gamma^2(q^2)),\quad(7)$$

so that

$$1/R^2(q^2)=1/R_H^2+1/R_\gamma^2(q^2).\quad(8)$$

Here, R_H is the hadronic target radius (dimensionally, $R_H\sim 1/M$). R_γ is a photon radius which, assuming there are no masses to determine *its* scale, behaves as $R_\gamma^2(q^2)\to\lambda/-q^2$, $q^2\to-\infty$. Equation (8) then illustrates the behavior $R^2(q^2)\to\lambda/-q^2$, just as stated by Eq. (6). In this example, the condition upon q^2 is given by $|q^2|\gg\lambda/R_H^2$. If $\lambda\approx\lambda_V\equiv M_V^2R_V^2$ [see discussion in the next section, preceding Eq. (17)], the necessary condition for the size of $|q^2|$ in the model is roughly estimated to be $|q^2|\gg 2M_V^2$, where the vector-meson squared radius $R_V^2\simeq 2R_H^2$ and M_V is a vector-meson mass.

Using representation (2′) at $t=0$ together with (5), we get

$$\frac{1}{(2\pi i\nu)}F_{hh}{}^s(\nu,q^2)=\frac{1}{2\rho_0}\int_{b_0^2(h)}^{\infty}db^2[1-e^{i\chi_h}]$$
$$=R^2(q^2)\sum_{n=1}^{\infty}-\frac{(-c_h)^n/\rho_0}{n(n!)}\exp\left(\frac{-nb_0^2(h)}{2R^2}\right)$$
$$\equiv R^2(q^2)f\left[c_h\exp\left(-\frac{(h+\frac{1}{2})^2}{2q_s^2}\frac{1}{R^2}\right)\right].\quad(9)$$

Here,

$$\rho_0 f(a)\equiv\int_0^a\frac{1-e^{-x}}{x}dx$$

is an elementary integral[13] which has been numerically tabulated.[14] Thus in our asymptotic region,

$$\sigma_h(\nu,q^2)=16\pi^2\alpha R^2(q^2)f(c_he^{-b_0^2(h)/2R^2})$$
$$=\frac{16\pi^2\alpha\lambda}{|q^2|}f\left[c_h\exp\left(\frac{(h+\frac{1}{2})^2}{2\lambda}\frac{q^2}{q_s^2}\right)\right],\quad(10)$$
$$q^2/q_s^2=-\omega(2-\omega).$$

For missing mass $s\to\infty$ $(\omega\to 0)$,

$$\sigma_h(\nu,q^2)=16\pi^2\alpha\lambda f(c_h)/|q^2|.\quad(11)$$

Both cross sections σ_L and σ_T fall off as $(q^2)^{-1}$ for fixed ω, and their ratio is given by (constant c_h case)

$$\frac{\sigma_T(\nu,q^2)}{\sigma_L(\nu,q^2)}=f\left[c_1\exp\left(-\frac{9}{8\lambda}\frac{|q^2|}{q_s^2}\right)\right]\Big/ f\left[c_0\exp\left(-\frac{1}{8\lambda}\frac{|q^2|}{q_s^2}\right)\right].\quad(12a)$$

For example, if we could neglect multiple iterations in this ratio,

$$\frac{\sigma_T}{\sigma_L}\simeq\frac{c_1}{c_0}\exp\left(-\frac{1}{\lambda}\frac{|q^2|}{q_s^2}\right).\quad(12b)$$

This behavior differs from that in which (i) asymptotic T products are related to (model-dependent) canonical commutators[15] so that $q^2\sigma_L\to 0$ (quarks) and $q^2\sigma_T\to 0$ (spinless bosons, vector mesons) and (ii) a vector-dominance model[16] as extrapolated to $-q^2$ large (large

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[11] Polynomials in R^2/b^2 (singular near $b=0$) give rise to growing powers of q_s^2/q^2, but still with the over-all $R^2\sim 1/q^2$ factor for fixed ω in $\sigma_{L,T}$.

[12] That is, (7) can be written as a Fourier transform, $\int d^2k\,e^{i\mathbf{k}\cdot\mathbf{b}}$, of the convolution $\int d^2k'\,[R_H^2\exp(-\frac{1}{2}\mathbf{k}'^2R_H^2)]\times\{R_\gamma^2\exp[-\frac{1}{2}(\mathbf{k}'-\mathbf{k})^2R_\gamma^2]\}$, where $\mathbf{b}=(b_x,b_y)$, $\mathbf{k}=(k_x,k_y)$, and $\mathbf{k}^2=-t$. On the other hand, one may also consider χ to be determined linearly in terms of a "Born" amplitude, $\exp(\frac{1}{2}tR_H^2)\times\exp(\frac{1}{2}tR_\gamma^2)$, $R^2(q^2)=R_H^2+R_\gamma(q^2)$, so that the Pomeranchon is expressed as another kind of exchange (a convolution in b), like an ordinary Regge trajectory (Ref. 7). However, this consideration (a) describes asymptotic scattering over a large region of the impact parameter up to $b\sim O(1/M)$, (b) is probably restricted to small values of $|q^2|$ (like a strong scattering), and (c) in its most consistent form would give rise to rather large production since $\sigma_{L,T}\propto R_H^2$.

[13] Some elementary properties of $\rho_0 f(a)$ are as follows: (i) It possesses the series expansion $-\sum_{n=1}^{\infty}[(-a)^n/n(n!)]$ which is especially useful when a is small, i.e., $\rho_0 f(a)=a-\frac{1}{4}a^2+\cdots$; (ii) when $a_1>a_2>0$, $f(a_1)>f(a_2)>0$.

[14] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. Stegun (Dover Publications, Inc., New York) Chap. 5, p. 227. In this reference $\rho_0 f(a)$ is denoted by $\mathrm{Ein}(a)=E_1(a)+\ln a+\gamma$ (Euler's constant).

[15] C. G. Callan and D. J. Gross, Phys. Rev. Letters **22**, 156 (1969).

[16] J. J. Sakurai, Phys. Rev. Letters **22**, 981 (1969).

missing mass) so that σ_T/σ_L decreases as M_ρ^2/q^2. However, recently[17] the breakdown has been studied of the relation between the asymptotic behavior of T products in perturbation theory and the canonical commutators.

From the equations for W_1 and W_2,

$$MW_1(\nu,q^2)=\left(\frac{-\nu}{q^2}\right)4\lambda f\left[c_1\exp\left(-\frac{9}{8\lambda}\frac{|q^2|}{q_s^2}\right)\right]$$

$$\frac{\nu}{M}W_2(\nu,q^2)=4\lambda\left\{f\left[c_1\exp\left(-\frac{9}{8\lambda}\frac{|q^2|}{q_s^2}\right)\right]+f\left[c_0\exp\left(-\frac{1}{8\lambda}\frac{|q^2|}{q_s^2}\right)\right]\right\}, \quad (13)$$

we see that for highest missing masses, $\omega\to 0$, the behavior $\nu W_2\propto$ const and $W_1\propto\nu/q^2$ results as would follow from (flat) Pomeranchon asymptotics and the scale property.

At this point, let us again emphasize that our application of the representation (2′) should be more valid for the higher values of the missing mass $\frac{1}{2}(s-M^2)=\nu-\frac{1}{2}|q^2|$. However, it will be experimentally interesting to consider the utility of these types of formulas (say in plots versus $1/\omega\gtrsim 1$) over a broad, but always asymptotic, region[4] with $|q^2|\gg 2M_V^2$. Also note that if one wants to speak of a well-defined projectile "trajectory," then the condition that the actual momentum q_s is much greater than the wave-packet momentum spread Δp is given by $q_s\gg\Delta p$ or $q_s^2\gg|q^2|/\lambda$ and $\lambda\gg\omega(2-\omega)$ —from the uncertainty principle,

$$\Delta p\sim 1/\Delta x\approx 1/R=(|q^2|/\lambda)^{1/2}.$$

It is clear that the considerations above may be applicable as well to asymptotic, neutrino-induced inelastic scattering off hadrons.

MODEL CONSTANTS AND APPROXIMATIONS

In an attempt to obtain the constants in the above equations, we propose the following method only as an illustration of the situation. Since the diffractive representation also applies to small q^2, it is supposed (i) that the essential q^2 variation of $\chi(b,q^2)$ for $q^2\leqslant 0$ is confined to variation of the interaction radius $R^2(q^2)$ and (ii) that explicit q^2 threshold factors are only inserted where kinematically demanded. For example, $R^2(0)$ is taken to be comparable to some vector meson, proton radius (call it R_V^2) and, consistent with current conservation, $F_{00}{}^s\to O(q^2)$ as $q^2\to 0$ implies $i\chi_0\to O(q^2)$. We then assume the forms $i\chi_1=-c_1e^{-b^2/2R^2(q^2)}$ and $i\chi_0=-(c_0/\lambda)\times R^2(q^2)|q^2|e^{-b^2/2R^2(q^2)}$ for the entire spacelike range $q^2\leqslant 0$. We immediately get from (9), for example, an equation for $f(c_1)$ involving the Compton cross section $\sigma_C(\nu)=\sigma_T(\nu,0)$:

$$f(c_1)=\sigma_C(\infty)/16\pi^2\alpha R^2(0). \quad (14)$$

To estimate c_1, we make use of the unitarity basis which underlies our method:

$$\sigma_h{}^{\text{elastic}}(\nu,q^2)/\sigma_h{}^{\text{total}}(\nu,q^2)$$
$$=\sigma(\gamma_h,q^2p\to\mathcal{V}^0p)/\sigma(\gamma_h,q^2p\to\text{hadrons})$$
$$=\int bdb\left|\frac{e^{i\chi_h}-1}{2i}\right|^2\Big/\int bdb\,\text{Im}\left(\frac{e^{i\chi_h}-1}{2i}\right), \quad (15)$$

where $\mathcal{V}^0$ represents the *spectrum* of all produced states which have current quantum numbers. Near $q^2=0$, if we take the state ρ^0 to represent the set $\mathcal{V}^0$, then[18]

$$\left[1-\frac{1}{2}\frac{f(2c_1)}{f(c_1)}\right]\simeq\sigma(\gamma p\to\rho^0p)/\sigma_c\simeq 14/111,$$

and so $c_1\simeq 4/9$. Further, using[19] $R^2(0)=R_V^2=8.5$ $(\text{GeV}/c)^{-2}$ in (14), we can obtain from $f(c_1)\simeq 0.03$ a value for the constant ρ_0 which appears in $\rho=\rho_0(q_s/2\pi\sqrt{s})$; i.e., $\rho_0\simeq 14$. From (15), the asymptotic relation then follows:

$$\sigma_T{}^{\text{elastic}}(\nu,q^2)/\sigma_T(\nu,q^2)=\tfrac{1}{4}c_1\exp\left[\frac{-9}{8\lambda}\omega(2-\omega)\right]+\cdots$$
$$\simeq\tfrac{1}{9}\exp\left[\frac{-9}{8\lambda}\omega(2-\omega)\right].$$

If in addition, $c_0\approx c_1$, then one can roughly estimate the electroproduction, large $|q^2|$-integrated, cross section ratio for very large incident electron energies: $\sigma(ep\to e'\mathcal{V}^0p)/\sigma(ep\to e'+\text{hadrons})\approx\frac{1}{8}$.

Note also that λ can be isolated[20] experimentally for highest[4] missing masses by the relation

$$\frac{W_1(\nu,q^2)}{W_1(\nu,0)}=\frac{\sigma_T(\nu,q^2)}{\sigma_C(\nu)}\simeq\frac{\lambda}{|q^2|R^2(0)}. \quad (16)$$

If in addition we were now to join the $q^2<0$ (spacelike) solution at $q^2=0$ with a purely strong, vector-dominance solution (also in the eikonal approximation) presumed valid for $0\leq q^2\leq M_V^2$ (timelike), and to assume (approximate) spin independence for on-mass-shall vector-meson, proton scattering at very high

[17] R. Jackiw and G. Preparata, Phys. Rev. Letters **22**, 975 (1969); S. Adler and Wu-Ki Tung, *ibid.* **22**, 978 (1969); K. Johnson and F. Low, Progr. Theoret. Phys. (Kyoto) Suppl. Nos. **37–38**, 74 (1966).

[18] R. Morrison (private communication); experimental cross-section values quoted at $E_\gamma\simeq 18$ GeV.

[19] S. Ting, *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 58; W. G. Jones *et al.*, *ibid.* Paper 679.

[20] Provided that $f(c_1e^{-(9/8\lambda)|q^2|/q_s^2})\simeq f(c_1)$ for highest missing masses ($\omega\to$ small), i.e., provided that λ is not anomalously small, $\lambda\ll\lambda_V=M_V^2R_V^2$. The second term in the expansion (16) is independent of λ:

$$-\frac{9}{8q_s^2R^2(0)}\frac{(1-e^{-c_1})}{\rho_0 f(c_1)}.$$

energy, we would get[21] the relation

$$c_0 = (\lambda/\lambda_V)\rho_0 f(c_1), \quad \lambda_V \equiv M_V{}^2 R_V{}^2 \quad (\approx 5 \text{ for } M_V{}^2 = M_\rho{}^2). \tag{17}$$

If $\lambda \lesssim O(\lambda_V)$, then $\rho_0 f(c_0) \simeq c_0$ as well so that the ratio (12) reduces to the especially simple form

$$\frac{\sigma_T}{\sigma_L} \simeq \frac{\lambda_V}{\lambda} \exp\left(-\frac{1}{\lambda}\frac{|q^2|}{q_s{}^2}\right). \tag{18}$$

Similarly, simple expressions are obtained for W_1 and W_2 since

$$F_{hh}{}^s/(i\nu) \simeq \frac{2\pi\lambda}{|q^2|}\rho_0{}^{-1}c_h \exp[-(h+\tfrac{1}{2})^2/2\lambda|q^2|/q_s{}^2];$$

for example,

$$(\nu/M)W_2 \underset{\rightarrow}{\simeq} 4(1+\lambda/\lambda_V)\lambda\rho_0{}^{-1}c_1, \quad 1/\omega \to \infty.$$

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Note added in proof. Recently, the SLAC inelastic e-p scattering data at 10° have been published [E. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); **23**, 935 (1969)]. For example, experimental plots are made of $(\nu/M)W_2$ for the extreme limits $R=0$ and $R=\infty$ of the quantity $R \equiv \sigma_L/\sigma_T$ and where

$$\frac{\nu}{M}W_2 = \frac{\nu}{M}\frac{d^2\sigma/d\Omega dE'}{(d\sigma/d\Omega)_{\text{Mott}}}\left[1+\frac{2\tan^2(\frac{1}{2}\theta)}{1+R}\left(1+\frac{\nu^2}{M^2|q^2|}\right)\right]^{-1}.$$

Note that for sizable $1/\omega = \nu/|q^2|$ data points, such plots are fairly sensitive to the ratio R, since the quantity $2\tan^2(\frac{1}{2}\theta)(1+\nu^2/M^2|q^2|) \approx 1$. So a point plotted for nonzero R rises relative to the same $1/\omega$ point for which $R=0$ is taken.

To make a preliminary comparison of our model (applicable to a smooth asymptotic region), we have plotted the 10° SLAC data ($W=\sqrt{s} \gtrsim 2$ GeV, $|q^2| \gtrsim 1$) for the modest, intermediate value $R=\frac{1}{2}$ as well as a theoretical curve (see Fig. 1). The solid line is Eq. (13), $(\nu/M)W_2 = 4\lambda f\{c_1 \exp[-(9/8\lambda)\omega(2-\omega)]\}[1+R(\omega)]$, where the ratio $R(\omega) \equiv \sigma_L/\sigma_T$ is given by Eq. (12) for $R(\omega=0)=\frac{1}{2}$. The quantities $f(c_1)=0.029$, $c_1=0.45$ are, as usual, determined from Eqs. (14) and (15), and $\lambda = \frac{1}{2}\lambda_v$ is taken ($\lambda_v = m_\rho{}^2 R_\rho{}^2 \simeq 5$, $|q^2| \gg m_\rho{}^2$). From Eq. (18), this particular value of λ is just that value which is consistent for $R(\omega=0)=\frac{1}{2}$ with an (approximate) vector-meson-dominance application at $q^2=0$. Then, the over-all normalization of $(\nu/M)W_2$ as illustrated in Fig. 1 is a *prediction*. Let us note that this VMD consideration embodies additional assumptions to our basic model: It is merely intended to estimate the size of λ, while, in general, the model gives $R(\omega=0)=f(c_0)/f(c_1)$ with no restriction upon λ.

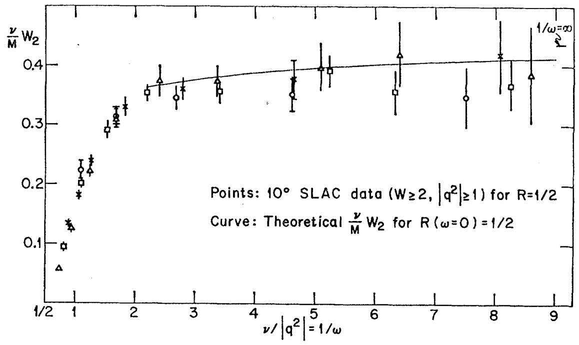

FIG. 1. Plot of 10° SLAC data for $R=\frac{1}{2}$ and theoretical curve for $R(\omega=0)=\frac{1}{2}$.

[21] Applying vector dominance for $\nu \gg M_V{}^2$ (see, for example, Ref. 16), *but for the range* $0 \leq q^2 \leq M_V{}^2$ *only*, we have for $q^2>0$ (timelike),

$$F_{00}{}^s(\nu,q^2) = (q^2/M_V{}^2) f_{00}{}^s(\nu,M_V{}^2) V^2(q^2),$$
$$F_{++}{}^s(\nu,q^2) = f_{++}{}^s(\nu,M_V{}^2) V^2(q^2), \quad V(q^2) \equiv (M_V{}^2/2\gamma_V)/(M_V{}^2-q^2),$$

where $f_{hh}{}^s(\nu,M_V{}^2)$ are the on-mass-shell helicity amplitudes for V^0p scattering. Now writing for $f_{hh}{}^s$ the eikonals, $i\psi_h = -g_h \times \exp(-b^2/2R_V{}^2)$, where the g_h are some constants, we obtain solutions ($\nu \gg M_V{}^2$, $|q^2|$)

$$F_{00}{}^s(\nu,q^2)/2\pi i\nu = (q^2/M_V{}^2)R_V{}^2 f(g_0) V^2(q^2), \quad q^2>0$$
$$= R^2(q^2) f(c_0\lambda^{-1}R^2(q^2)|q^2|), \quad q^2<0$$

and

$$F_{++}{}^s(\nu,q^2)/2\pi i\nu = R_V{}^2 f(g_1) V^2(q^2), \quad q^2>0$$
$$= R^2(q^2) f(c_1), \quad q^2<0.$$

Equating, at $q^2=0$, the coefficients $O(q^2)$ and $O(1)$ for the two solutions of $F_{00}{}^s$ and $F_{++}{}^s$, respectively, and using $R^2(0) \simeq R_V{}^2$ (to equate higher-order coefficients in q^2, one must take into account, among other things, the details of q^2 variation in vector dominance), we obtain $f(g_0) = (2\gamma_V)^2(\lambda_V/\lambda)(\rho_0{}^{-1}c_0)$ ($\lambda_V \equiv R_V{}^2 M_V{}^2$) and $f(g_1) = (2\gamma_V)^2 f(c_1)$. Spin independence ($\nu \gg M_V{}^2$) then says, independent of the value for $\gamma_V{}^2/4\pi$, that $f(g_0)=f(g_1)$; i.e., $c_0 = (\lambda/\lambda_V) \times \rho_0 f(c_1)$.