PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Λ-Σ Conversion in Low-Energy Λd Scattering

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(Received 7 May 1969)

Nonlocal separable potentials are used to represent the $^3S_1$ np and the $^3S_1$ isospin-$\frac{1}{2}$ YN ($Y=\Lambda,\Sigma$) potentials. For different sets of YN potential parameters, both ΛN triplet cross sections far below the ΣN threshold and Λd quartet cross sections far below the ΣNN threshold are calculated. The Λd cross sections are much more sensitive to the coupling of the ΛN and ΣN channels as well as to the presence of a ΛN resonance.

I. INTRODUCTION

THE purpose of this work is to describe the utility of low-energy Λd scattering as a tool for investigation of the contribution of Λ-Σ conversion (i.e., $\Lambda N \leftrightarrow \Sigma N$) to the ΛN interaction. At low energies (Λ laboratory momentum $\lesssim 300$ MeV/c), Λd scattering shares the simplicity of low-energy ΛN scattering that does not exist for ΛN scattering near or above the ΣN threshold. The hyperon-nucleon interaction should be dominated by its S-wave component, and final states with a Σ present are not physically realizable. It is shown here that Λd scattering cross sections far below the threshold for ΣNN production are much more sensitive to the coupling of the ΛN system to the isospin $-\frac{1}{2}$ ΣN system than is the ΛN cross section far below, but relatively closer to, the threshold for ΣN production. Recent developments in both experiment[1–4] and fundamental theory[5] of the YN ($Y=\Lambda$, Σ) interaction make a phenomenological investigation of Λd scattering of more than passing interest.

Ultimately what is desired is at least one fundamental model of the YN interaction [e.g., the one- and two-pion-exchange model[6] (PEM) or the one-boson-exchange model[5] (BEM)] from which a potential-energy operator valid at low energies and parametrized by the masses of the particles exchanged by the baryons and their coupling constants to the baryons may be obtained. An appeal to some experimentally determined parameters (e.g., scattering lengths and effective ranges determined from the binding energies of light hypernuclei) would then be made to fix any unknown values of these parameters. The predictions of this potential for other experimentally determinable quantities (e.g., the binding energy of the Λ in nuclear matter) would then be used as a test of the fundamental theory. Such a potential operator would be a matrix with rows and columns characterized by the particles present (e.g., Λn, Λp, $\Sigma^- n$, $\Sigma^- p$, etc.) and most of the matrix elements would contain noncentral potentials and repulsive-core potentials.

In the simplified calculations reported here, tensor potentials and repulsive-core potentials were not included. Therefore, no direct connection was made between the potential parameters of this work and the coupling constants and exchanged masses of a fundamental theory. Rather, calculations were carried out for a number of different sets of potential parameters as described below.

The n and p were taken to be identical isospin-$\frac{1}{2}$ nucleons and the mass splittings of the isospin-1 Σ multiplet were also ignored. The YN potential operator became a 2×2 matrix whose diagonal elements generate the processes $\Lambda N \leftrightarrow \Lambda N$ and $\Sigma N \leftrightarrow \Sigma N$ and whose off-diagonal elements generate Λ-Σ conversion. Both the YN and NN potentials were taken to be S-wave potentials. Further, only (spin) quartet Λd scattering cross sections were calculated so that only ^3S_1YN and NN potentials were needed. To facilitate the three-body calculations, the NN potential and each of the matrix elements of the YN potential matrix were taken to be nonlocal separable potentials of the Yamaguchi[7] type. Nonrelativistic kinematics were used throughout.

The kernels of the nonlocal potentials used here are those previously described by Toepfer and Schick.[8,9] The kernel of the np potential in a relative-momentum space representation has the form

$$V_N(p,p')=\lambda_N v_N(p)v_N(p'), \qquad (1)$$

with

$$v_N(p)=1/(p^2+\beta_N^2). \qquad (2)$$

The parameters λ_N and β_N were chosen by fitting the deuteron binding energy $\epsilon=2.225$ MeV and the $^3S_1$ scattering length $a_N=5.39$ F. It has been shown recently[10] that for low-energy Λd scattering, this gives an adequate representation of the $^3S_1$ np potential. The AB element (A, $B=\Lambda$ or Σ) of the kernel of the YN potential matrix is

$$V_{AB}(p_A,p_B')=\lambda_{AB}v_A(p_A)v_B(p_B'), \qquad (3)$$

[1] D. Cline, R. Laumann, and J. Mapp, Phys. Rev. Letters **20**, 1452 (1968).
[2] G. Alexander, B. H. Hall, N. Jew, G. Kalmus, and A. Kernan, Phys. Rev. Letters **22**, 483 (1969).
[3] R. Laumann, D. Cline, and J. Mapp, Bull. Am. Phys. Soc. **14**, 520 (1969).
[4] J. A. Kady, G. H. Trilling, G. Alexander, and P. J. Gaposchkin, Bull. Am. Phys. Soc. **14**, 591 (1969).
[5] J. T. Brown, B. W. Downs, and C. K. Iddings, Bull. Am. Phys. Soc. **14**, 520 (1969).
[6] J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).
[7] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[8] L. H. Schick and A. J. Toepfer, Phys. Rev. **170**, 946 (1968).
[9] A. J. Toepfer and L. H. Schick, Phys. Rev. **175**, 1253 (1968).
[10] L. H. Shick, Phys. Rev. **182**, 1106 (1969).

TABLE I. ΛN $S=1$ potential characteristics. All potentials have $a=-1.95$ F and $r_0=3.50$ F.

Potential label	β_Λ^{-1} (F)	$\beta_\Lambda/\beta_\Sigma$	$-\lambda_\Lambda$ [MeV^2/(20π)^3]	$\lambda_\Sigma/\lambda_\Lambda$	λ_x [MeV^2/(20π)^3]	$\sigma_{\Lambda N}$ (mb) at ΛN c.m. energy (MeV) 25.1	39.2	56.5	a_Σ (F)
A	0.766$^-$		0.95241	0.0	0.0	31.17	14.78	7.26	$0.0\ -i0.0$
$C1$	0.796	1.0	0.59593	1.0	1.2139	33.08	17.01	10.03	$-0.446-i0.809$
$E1$	0.828	1.0	0.27404	1.0	1.6535	34.93	19.14	12.64	$+0.432-i0.802$
$G1$	0.8600$^+$	1.0	0.00000	1.0	1.9345	36.60	21.01	14.83	$+0.908-i0.701$
$C2$	0.793	2.0	0.66835	0.1	0.5857	33.05	17.04	10.26	$+0.262-i1.780$
$E2$	0.822	2.0	0.41525	0.1	0.7846	34.96	19.35	13.39	$+1.735-i1.557$
$G2$	0.848	2.0	0.22434	0.1	0.8927	36.58	21.29	15.98	$+2.373-i1.288$
$C3$	0.784	2.0	0.80132	1.0	0.3103	33.05	17.70	20.05	$+4.137-i0.041$
$E3$	0.807	2.0	0.61531	1.0	0.4914	34.92	20.15	20.16	$+4.557-i0.208$
$G3$	0.834	2.0	0.40117	1.0	0.6706	36.60	21.98	19.90	$+5.187-i1.208$

where $v_A(p)=v_N(p)$, with $\beta_N\to\beta_A$. V_{AB} denotes the potential that describes the process $A+N\to B+N$. For convenience the notation $\lambda_\Lambda\equiv\lambda_{\Lambda\Lambda}$, $\lambda_\Sigma\equiv\lambda_{\Sigma\Sigma}$, and $\lambda_x\equiv\lambda_{\Sigma\Lambda}=\lambda_{\Lambda\Sigma}$ is used throughout the rest of the work. The momenta in Eq. (3) are given by $p_A=(2\mu_AE_A)^{1/2}$, where E_A is the c.m. energy in the A channel relative to its threshold value, $\mu_A=M_AM_N/(M_A+M_N)$, and $E_\Sigma=E_\Lambda+M_\Lambda-M_\Sigma$. The values (in MeV) $M_N=938.9$, $M_\Lambda=1115.4$, and $M_\Sigma=1193$ were used for the particle masses. With these values the threshold for the Σ channel occurs at a ΛN c.m. energy of 77.6 MeV, i.e. at a c.m. Λ momentum of 281.3 MeV/c.

II. ΛN CALCULATIONS

Table I contains the potential parameters, ΛN cross sections (at energies corresponding to Λ c.m. momenta of 160, 200, and 240 MeV/c, respectively), and ΣN scattering lengths for each of the four YN potential models used. Each of the models 0, 1, 2, and 3 contains in effect an assumption about the masses of the exchanged particles reflected in the ratio $\beta_\Lambda/\beta_\Sigma$ and an assumption about the symmetry character of the interaction reflected in the ratio $\lambda_\Sigma/\lambda_\Lambda$. Each YN potential has been adjusted to give the charge-symmetric version of the ΛN $^3S_1$ scattering length and effective range as determined from the Herndon-Tang[11] analysis of the light hypernuclei. No doubt the values for these scattering parameters could have been chosen to be those obtained from low-energy Λp scattering[12] without changing the basic conclusions of this work.

The "zero" model consists of the single potential A of Table I, for which $\lambda_x=\lambda_\Sigma=0$; i.e., the Σ channel is turned off. The 1 model includes potentials $C1$, $E1$, and $G1$. In these the ranges in each channel were assumed equal (i.e., $1/\beta_\Lambda=1/\beta_\Sigma$), as were the strengths λ_Λ and λ_Σ. This would be the parametrization obtained from complete $SU(3)$ symmetry.[13] With the purely attractive potentials in the Λ and Σ channels used in this work, complete $SU(3)$ symmetry gives too small an effective range for the correct ΛN scattering length. Thus, instead of fixing β_Λ at its $SU(3)$ value, β_Λ was allowed to vary such as to take λ_Λ over the range from its most negative value at $\lambda_x=0$ all the way up to zero. The potentials $C1$, $E1$, and $G1$ cover this range, as may be seen in Table I.

For the remaining two models the range in the Λ channel was taken to be one-half the range in the Σ channel, as might be the case for a PEM. In model 2 (potentials $C2$, $E2$, and $G2$), λ_Σ was changed to $0.1\lambda_\Lambda$, whereas in model 3 (potentials $C3$, $E3$, and $G3$) the relation $\lambda_\Lambda=\lambda_\Sigma$ was retained. The determination of the parameters in these two models was completed by requiring that $C2$ and $C3$ give the same value as given by $C1$ for $\sigma_{\Lambda N}$ at 25.1 MeV, and similarly for the E and G types of potential.

Within each of the models 1–3 the variation of the ranges is relatively small so that potentials C, G, and E may be thought of as representing different amounts of coupling of the Λ to the Σ channel (i.e., different strengths for Λ-Σ conversion) for that model.

From Table I it is clear that for each of the given models $\sigma_{\Lambda N}$ at energies $\lesssim 40$ MeV is an insensitive function of the amount of Λ-Σ conversion present. But for higher energies, ΛN P-wave scattering is no longer negligible[11]; the model used here breaks down, and the analysis of ΛN scattering loses a great deal of its simplicity.

Another point of importance is the singularity structure of the ΛN scattering amplitude. This amplitude for each of the four models used has the Λ-channel unitarity cut in the ΛN c.m. energy plane beginning at the threshold energy for this channel. The amplitude for model zero has no Σ channel and hence no cut for this channel. The ΛN amplitudes for the other models all have a cut beginning at the Σ channel threshold. In the Λ c.m. momentum plane all four models generate an amplitude which has one pole very close to $p_\Lambda=-i64.7$ MeV/c and another pole on the negative imaginary p_Λ axis that falls in the range $-i370$ to $-i450$ MeV/c. In addition, each potential of model three produces a ΛN resonance. This resonance occurs at p_Λ (in MeV/c)$=251-i2.34$, $263-i5.19$, and $276-i6.49$ for potentials $C3$, $E3$, and $G3$, respectively.

[11] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967); **159**, 853 (1967); **165**, 1093 (1968).

[12] G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, and W. Lughofer, Phys. Rev. **173**, 1452 (1968).

[13] A. Deloff and H. W. Wyld, Jr., Phys. Letters **12**, 245 (1964).

The $G3$ resonance has the properties (i.e., it lies about 2.5 MeV below the ΣN threshold and has a width <10 MeV) such as to give agreement with the results of Ref. 1.

III. Λd CALCULATIONS

In turn each of the potentials described in Table I was used to calculate[14] the Λd quartet elastic and total cross sections at Λ laboratory momenta of 200 and 300 MeV/c, but only the lower-momentum results are reported here. The calculations used a multiple-scattering formalism of the Faddeev[15] type, which has been amply described previously.[16] Elastic angular distributions were also calculated, but in view of the present experimental situation these were not seriously analyzed further. The Λ laboratory momenta used yield a maximum ΛN c.m. energy of 9.02 and 23.1 MeV, respectively. Values of Λ laboratory momenta in the range 200–300 MeV/c constitute the high end of the momentum range for which a purely S-wave representation of the ΛN potential should be quite realistic and the low end of the range for which Λd scattering is experimentally feasible.

The results for Λd elastic and total scattering cross sections at 200 MeV/c are shown in Table II. Clearly Λ-Σ conversion can influence Λd scattering far below the threshold for ΣNN production.

For a given model the marked sensitivity of the Λd cross sections to the amount of Λ-Σ conversion is manifest. For the ΛN amplitudes without a resonance this sensitivity may be understood if it is remembered that although the position of the pole closest to the physical region is for all practical purposes fixed, the variation in λ_Λ and λ_x from potential to potential causes the residues of the $\Lambda N \to \Lambda N$, $\Sigma N \to \Sigma N$, and $\Lambda N \leftrightarrow \Sigma N$ amplitudes at this pole to vary, and in fact to vary in quite different ways. Thus at low energies the $\Lambda N \to \Lambda N$ amplitude is insensitive to the exact value of λ_x, whereas the amplitude for Λ-Σ conversion, which appears explicitly in the multiple-scattering terms of the Λd scattering amplitude, is proportional to λ_x. For the model-3 ΛN amplitudes the change in the position of the resonance and the change in the value of the residues at the resonance also play a role.

Fitting a theoretical model to the Λd cross sections at a Λ lab momentum of 200 MeV/c is clearly a much tougher test of the model than fitting it to the ΛN cross sections at ΛN c.m. energies that are less than half of the Λ-Σ mass difference, yet each involves only the ΛN S-wave interaction. By the time the effects of the contributions of the other spin channel present in each of these scatterings (i.e., the singlet ΛN channel and the doublet Λd channel) are added into the cross sections, the relatively much greater sensitivity of the Λd cross sections may be reduced somewhat[17] but it will still be significant.

TABLE II. Λd quartet cross section at Λ lab momentum of 200 MeV/c.

Potential	σ_{el} (mb)	σ_{tot} (mb)
A	193	244
$C1$	157	195
$E1$	115	143
$G1$	80	103
$C2$	155	195
$E2$	110	139
$G2$	81	106
$C3$	177	223
$E3$	150	187
$G3$	105	139

The other important result exhibited in Table II is that, given two different models for which the ΛN cross sections are the same up to energies several times greater than the maximum ΛN energy encountered in a given Λd scattering, the Λd cross sections will agree (to within 5% for potentials $C1$ and $C2$, $E1$ and $E2$, and $G1$ and $G2$) if the singularity structure of the two models is the same. Results (not shown here) obtained from Λd calculations carried out at 200 and 300 MeV/c with model-2 potentials that fit $\sigma_{\Lambda N}$ at 39.2 MeV for potentials $C1$, $E1$, and $G1$ reinforce the results shown in Table II. The Λd cross sections will differ markedly (e.g., $G1$ and $G2$ versus $G3$) if one model has a ΛN resonance and the other does not.

In combination, these results describe the limits of the information on Λ-Σ conversion that can be drawn from low-energy Λd scattering cross sections. Given two models (say, the PEM and the BEM) of the ΛN interaction which fit the ΛN scattering length and effective range, if both models do, or do not, predict a ΛN resonance, then low-energy Λd scattering experiments might very well distinguish between them where low-energy Λp experiments could not. On the other hand, as with low-energy ΛN scattering, low-energy Λd scattering gives the same cross sections for a resonant ΛN interaction as it does for some other nonresonant interaction with a smaller coupling of the Λ and Σ channels. ΛN scattering near the threshold for Σ production is still required to answer the question of the existence of the ΛN resonance.

[14] All computations were performed at the USC Computer Sciences Laboratory.

[15] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory* (Daniel Davey and Co., Inc., New York, 1965).

[16] L. H. Shick, Nuovo Cimento Letters **1**, 313 (1969).

[17] According to the PEM of Ref. 6, as a result of tensor forces, Λ-Σ conversion effects should be larger in the YN spin-1 system than in the spin-0 system.