Problem of Neutron-Proton Mass Difference

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The problem of the n-p mass difference is carefully analyzed in the framework of dispersion theory. The author has studied the models recently used to obtain interesting properties of the electroproduction structure functions, in order to calculate an important piece of the mass shift coming from the subtraction constant that enters into the dispersion relation for the spin-nonflip forward Compton amplitude of the nucleon. It was hoped that in some simple model one might enhance this piece in the right direction and thereby obtain at least the required sign reversal. It is shown that this is not so. Thus, unless the contributions of the higher resonance states (in addition to the nucleon pole) or those of the fixed poles to the Compton amplitude are important, the n-p mass shift cannot be understood from the viewpoint of dispersion theory.

THE problem of calculating the electromagnetic mass differences of hadrons within an isomultiplet has turned out to be very complicated. It is well known that the n-p or K^+-K^0 mass shifts of order α with only the lowest states retained in the self-energy diagrams yield the wrong sign,[1] whereas the same procedure gives the correct magnitude and sign for the π^+-π^0 mass difference.[2]

In a rather convincing argument based on dispersion theory, Harari[3] has shown that if the high-energy behavior of the forward Compton amplitude is controlled by the crossed-channel Regge exchanges, then the spin-nonflip amplitude $[t_1(\nu,q^2)]$ in the Cottingham formula must satisfy a once-subtracted dispersion relation in the energy variable for all $\Delta I=1$ mass differences such as the n-p mass splitting. On the other hand, the "no subtraction hypothesis" may hold for all $\Delta I=2$ mass shifts such as the π^+-π^0 problem with the standard assumption, in view of the absence of low-lying $I=2$ mesons, that the $I=2$ Regge intercept $\alpha_{I=2}(t=0)<0$. The unknown subtraction constants present in $\Delta I=1$ mass shifts may be identified with the "tadpole" terms proposed by Coleman and Glashow.[1]

Strong objections have been raised against all the n-p mass splitting calculations reported in the literature.[4] In this paper we show that it is not possible to understand the n-p mass difference of order α in the framework of dispersion theory in a natural way with the customary approximation of neglecting the fixed $I=1$, $J=0$ poles in the complex angular momentum plane and the higher $I=\frac{1}{2}$ nucleon resonances and multiparticle intermediate states in the dispersion integral.

Recently Gross and Pagels[5] (GP) systematically analyzed the problem of electromagnetic mass differences and attempted to estimate the subtraction-constant contributions. Let us briefly review the central part of their work.

To order α, the electromagnetic self-energy of a hadron is

$$\delta m=\frac{i}{8\pi^2}\int_{-\infty}^{\infty} d^4q\frac{\delta_{\mu\nu}}{q^2-i\epsilon}T_{\mu\nu}(q^2,\nu)\,,$$

where $T_{\mu\nu}(q^2,\nu)$ is the forward Compton amplitude of a photon of mass q^2 and lab energy $\nu=-p\cdot q/M$ scattered from a hadron of mass M and four-momentum p. We are free to choose the Feynman gauge for the photon propagator, since $T_{\mu\nu}$ is gauge-invariant. Lorentz and gauge invariance of $T_{\mu\nu}$ (after summing over the hadron spins) allows us to express it in terms of only two invariant amplitudes[6]:

$$T_{\mu\nu}(q^2,\nu)=t_1(q^2,\gamma)[q^2\delta_{\mu\nu}-q_\mu q_\nu]+t_2(q^2,\nu)\times[\nu^2\delta_{\mu\nu}+(q^2/M^2)p_\mu p_\nu+(\nu/M)(p_\mu q_\nu+p_\nu q_\mu)]\,. \quad (1)$$

By rotating the integration contour in the q_0 plane in a counterclockwise direction through $\frac{1}{2}\pi(\nu\to i\nu)$ and carrying out the angular integrations, Cottingham[1] obtained an elegant formula for the n-p mass shift:

$$\Delta M=\delta m_p-\delta m_n=-\frac{1}{4\pi}\int_0^{\infty}\frac{dq^2}{q^2}\int_{-q}^{+q}d\nu(q^2-\nu^2)^{1/2}\times[3q^2t_1^{(1)}(q^2,i\nu)-(q^2+2\nu^2)t_2^{(1)}(q^2,i\nu)]\,. \quad (2)$$

The time-ordered product of two electromagnetic currents in the Compton amplitudes contains pieces of isospin 0, 1, and 2. The $I=0$ part cancels out in mass-difference problems, the $I=2$ part cannot contribute to mass shifts of $I=\frac{1}{2}$ objects, and therefore the n-p mass difference transforms like a pure $I=1$ object.

In presence of fixed $J=0$ poles in the complex J plane, the high-energy behavior of the two invariant Compton

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[1] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959); N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963); J. H. Wojtaszek, R. E. Marshak, and Riazuddin, Phys. Rev. **136**, B1053 (1964); S. Coleman and S. Glashow, *ibid.* **134**, B671 (1964).

[2] Riazuddin, Phys. Rev. **114**, 1184 (1959); V. Barger and E. Kazes, Nuovo Cimento **28**, 385 (1963).

[3] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

[4] For example, R. Dashen's calculation [Phys. Rev. **135**, B1195 (1964)] has been criticized by G. Barton [*ibid.* **146**, 1149 (1966)] and Y. S. Kim [*ibid.* **142**, 1150 (1966)]. Objections against Srivastava's calculation (Ref. 15) have been raised by Gross and Pagels (Ref. 5).

[5] D. J. Gross and H. Pagels, Phys. Rev. **172**, 1381 (1968). There is a small printing error in Eq. (2.15) of this paper.

[6] For details, see Cottingham's paper in Ref. 1. We are using the Pauli metric.

amplitudes is given by

$$t_1^{(1)}(q^2,\nu) \underset{\nu\to\infty}{\longrightarrow} R_1^{(1)}(q^2)+\beta_1^{(1)}(q^2)\nu^{\alpha_{A2}(0)},$$
$$t_2^{(1)}(q^2,\nu) \underset{\nu\to\infty}{\longrightarrow} R_2^{(1)}(q^2)\nu^{-2}+\beta_2^{(1)}(q^2)\nu^{\alpha_{A2}(0)-2}. \quad (3)$$

The superscripts refer to the isospin exchanged in the t channel. For the present, we confine our attention to $\Delta I=1$ mass differences only, especially the neutron-proton problem. The $R_{1,2}^{(1)}(q^2)$ are related to the residues of the $I=1$, $J=0$ fixed pole; $\beta_{1,2}^{(1)}(q^2)$ are the residues of the leading charge conjugation even natural-parity $I=1$ Regge pole, and the $t=0$ intercept of this trajectory is $\alpha_{A_2}(0)\simeq 0.4$.

GP[5] introduce the function

$$H^{(1)}(q^2,\nu)=t_1^{(1)}(q^2,\nu)-[\beta_1^{(1)}(q^2)/\beta_2^{(1)}]\nu^2 t_2^{(1)}(q^2,\nu). \quad (4)$$

From Eq. (3) it is clear that the A_2^0 trajectory contribution cancels out in Eq. (4), and consequently $H_1^{(1)}(q^2,\nu)$ has improved the high-energy behavior at the expense of introducing the ratio of Regge residues.

The fixed-q^2 dispersion relations in ν for the Compton amplitudes, after separating out the pole terms, have been written down by several authors[7] and are given by

$$t_1^{(1)}(q^2,\nu)=t_1^{(1)}(q^2,0)+\frac{16M^3\nu^2 f_1^{(1)}(q^2)}{q^2(q^4-4M^2\nu^2)}+\frac{\nu^2}{\pi}\int_{\nu_t^2}^{\infty}\frac{\operatorname{Im}t_1^{(1)}(q^2,\nu')}{\nu'^2(\nu'^2-\nu^2)}d\nu'^2$$

and (5)

$$t_2^{(1)}(q^2,\nu)=\frac{4Mq^2 f_2^{(1)}(q^2)}{q^2-4M^2\nu^2}+\frac{1}{\pi}\int_{\nu_t^2}^{\infty}\frac{\operatorname{Im}t_2^{(1)}(q^2,\nu')}{\nu'^2-\nu^2}d\nu'^2.$$

We have used the crossing symmetry for both the amplitudes and have subtracted the $t_1^{(1)}(\nu,q^2)$ dispersion relation at $\nu=0$; $f_{1,2}^{(1)}(q^2)$ are certain combinations of the electric and magnetic form factors of the nucleon, M is the nucleon mass, and ν_t is the inelastic threshold. So, the dispersion relation for $H^{(1)}(q^2,\nu)$ is[5]

$$H^{(1)}(q^2,\nu)=R_1^1(q^2)-\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}R_2^{(1)}(q^2)+\frac{4Mq^2 f_1^{(1)}(q^2)}{q^4-4M^2\nu^2}-\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}\frac{4Mq^6 f_2^{(1)}(q^2)}{4M(q^4-4M^2\nu^2)}+\frac{1}{\pi}\int_{\nu_t^2}^{\infty}\frac{\operatorname{Im}H^{(1)}(q^2,\nu')}{\nu'^2-\nu^2}d\nu'^2. \quad (6)$$

From the Cottingham formula [Eq. (2)], the contribution of the subtraction constant to the n-p mass shift is

$$\Delta M_{\text{sub}}=-\frac{1}{4\pi}\int_0^{\infty}\frac{dq^2}{q^2}\int_{-q}^{+q}d\nu(q^2-\nu^2)^{1/2}[3q^2t_1^1(q^2,0)]$$
$$=-\frac{3}{8}\int_0^{\infty}dq^2q^2t_1^{(1)}(q^2,0)$$
$$=-\frac{3}{8}\int_0^{\infty}dq^2q^2H^{(1)}(q^2,0). \quad (7)$$

Equation (6) leads to

$$\Delta M_{\text{sub}}=-\frac{3}{8}\int_0^{\infty}\left\{q^2R_1^{(1)}(q^2)+4Mf_1^{(1)}(q^2)-\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}q^2\left[R_2^{(1)}(q^2)+\frac{1}{M}q^2f_2^{(1)}(q^2)\right]+\frac{q^2}{\pi}\int_{\nu_t^2}^{\infty}d\nu'^2\frac{\operatorname{Im}H^{(1)}(q^2,\nu')}{\nu'^2}\right\}dq^2$$
$$=\Delta M'_{\text{el}}+\Delta M'_{\text{inel}}+\Delta M_{\text{Regge}}+\Delta M_{\text{fixed pole}}, \quad (8)$$

where

$$\Delta M_{\text{fixed pole}}=-\frac{3}{8}\int_0^{\infty}\left[q^2R_1^1(q^2)-\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}q^2R_2^{(1)}(q^2)\right]dq^2, \quad (9)$$

$$\Delta M_{\text{Regge}}=\frac{3}{8M}\int_0^{\infty}dq^2\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}q^4f_2^{(1)}(q^2), \quad (10)$$

and $\Delta M'_{\text{el}}$ and $\Delta M'_{\text{inel}}$ are the nucleon pole and continuum contribution, respectively.

The net expression for the mass shift is

$$m_p-m_n=\Delta M=\Delta M_{\text{sub}}+\Delta M_{\text{el}}^0+\Delta M_{\text{inel}}^0$$
$$=\Delta M_{\text{el}}+\Delta M_{\text{inel}}+\Delta M_{\text{fixed pole}}+\Delta M_{\text{Regge}}, \quad (11)$$

where $\Delta M_{\text{el}}=\Delta M'_{\text{el}}+\Delta M_{\text{el}}^0$, and similarly for ΔM_{inel}; ΔM_{el}^0 and ΔM_{inel}^0 are the contributions of terms not involving the subtraction term in Eq. (5).

The elastic piece coming from the nucleon pole contribution has been calculated by several authors[1]:

$$\Delta M_{\text{el}}\cong +0.8\text{ MeV}. \quad (12)$$

The well-known 1236-MeV nucleon resonance does not contribute because of isospin. The somewhat higher $I=\frac{1}{2}$ nucleon resonances and even the multiparticle states can certainly contribute, but it is almost impossible to make a reliable quantitative estimate of their contributions. For the moment, we ignore ΔM_{inel} and shall make further comments later on in this connection.

Next, we study the ΔM_{Regge} piece coming from the subtraction-term contribution. For this, we require information regarding the ratio of Regge residues. GP[5] consider crossing relations between s and t channels and

[7] See Refs. 3 and 5, for example.

the virtual-photon helicity decompositions to obtain

$$t_1(q^2,\nu)=\frac{F_{++}{}^s(s,0)+(\nu^2/q^2)F_{00}{}^s(s,0)}{\nu^2+q^2}=R_1{}^{(1)}(q^2)+\beta_1{}^{(1)}(q^2)\nu^{\alpha_{A2}(0)}$$

and (13)

$$\nu^2 t_2(q^2,\nu)=\frac{\nu^2F_{++}{}^s(s,0)-\nu^2F_{00}{}^s(s,0)}{\nu^2+q^2}=R_2{}^{(1)}(q^2)+\beta_2{}^{(1)}(q^2)\nu^{\alpha_{A2}(0)}.$$

The subscripts $+$, $-$, and 0 refer to virtual-photon helicities. The (ν^2+q^2) in the denominator gets rid of kinematic singularities. Thus, in the limit of $\nu\to\infty$,

$$\frac{\beta_1{}^{(1)}(q^2)}{\beta_2{}^{(1)}(q^2)}=\frac{1}{q^2}\frac{\gamma_L{}^{(1)}(q^2)}{\gamma_T{}^{(1)}(q^2)-\gamma_L{}^{(1)}(q^2)},\tag{14}$$

where $\gamma_{T,L}{}^{(1)}(q^2)$ refer to the coupling of the $A_2{}^0$ trajectory to transverse and longitudinal photons, respectively.

Recently, there has been considerable interest in the study of electroproduction structure functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ which are the absorptive parts of the invariant Compton amplitudes $t_1(\nu,q^2)$ and $t_2(\nu,q^2)$.[8] Let us first consider the vector-dominance model of the inelastic electron scattering suggested by Sakurai.[9] Since fixed poles are absent from the absorptive parts,[5]

$$\frac{\gamma_L{}^{(1)}(q^2)}{\gamma_T{}^{(1)}(q^2)}=\lim_{\nu\to\infty}\frac{\sigma_L{}^{(1)}(q^2,\nu)}{\sigma_T{}^{(1)}(q^2,\nu)},\tag{15}$$

where $\sigma_{L,T}{}^{(1)}(q^2,\nu)$ are the difference between the total cross sections of proton and neutron scattered from longitudinal or transverse photons, respectively.

In Sakurai's model[9]

$$\sigma_T{}^{(1)}(q^2,\nu)=\left(\frac{e}{f_\rho}\right)^2\left(\frac{m_\rho{}^2}{q^2+m_\rho{}^2}\right)^2[\sigma_{\rho Tp}(K)-\sigma_{\rho Tn}(K)],$$

$$\sigma_L{}^{(1)}(q^2,\nu)=\left(\frac{e}{f_\rho}\right)^2\left(\frac{m_\rho{}^2}{q^2+m_\rho{}^2}\right)^2\frac{q^2}{m_\rho{}^2}\left(\frac{K}{\nu}\right)^2\times[\sigma_{\rho Lp}(K)-\sigma_{\rho Ln}(K)],$$

$$K=\nu-q^2/2M;$$

f_ρ is related to the coupling of a ρ meson to a photon.

$$\frac{\sigma_L{}^{(1)}(q^2,\nu)}{\sigma_T{}^{(1)}(q^2,\nu)}=\xi(K)\frac{q^2}{m_\rho{}^2}\left(1-\frac{q^2}{2M\nu}\right)^2,\tag{16}$$

with

$$\xi(K)=\frac{\sigma_{\rho Tp}(K)-\sigma_{\rho Tn}(K)}{\sigma_{\rho Lp}(K)-\sigma_{\rho Ln}(K)}.$$

Equations (15) and (16) tell us

$$\frac{\gamma_L{}^{(1)}(q^2)}{\gamma_T{}^{(1)}(q^2)}=\xi(\infty)\frac{q^2}{m_\rho{}^2}.\tag{17}$$

When one considers the virtual Compton scattering of the proton and not the difference between the proton and neutron scattering, it is possible to fix $\xi(\infty)$ by relating the total photoabsorption cross section for the proton to the constant approached by $\nu W_2(\nu,q^2)$ for large ν/q^2.[9]

In the present calculation, we shall treat $\xi(\infty)$ as a free parameter and study the $\xi(\infty)$ dependence of the mass shift:

$$\Delta M_{\text{Regge}}=-\frac{3}{8M}\int_0^\infty dq^2\frac{\gamma_L{}^{(1)}(q^2)}{\gamma_L{}^{(1)}(q^2)-\gamma_T{}^{(1)}(q^2)}q^2f_2{}^{(1)}(q^2)=-\frac{3}{8M}\int_0^\infty dq^2\frac{q^2}{q^2-m_\rho{}^2/\xi(\infty)}q^2f_2{}^{(1)}(q^2),\tag{18}$$

$$f_2{}^{(1)}(q^2)=\frac{\alpha}{\pi}\frac{q^2[G_{M,p}{}^2(q^2)-G_{M,n}{}^2(q^2)]+4M^2[G_{E,p}{}^2(q^2)-G_{E,n}{}^2(q^2)]}{q^2(q^2+4M^2)}=\frac{\alpha}{\pi}\frac{q^2(\mu_p{}^2-\mu_n{}^2)+4M^2}{[1+q^2/0.71\text{ BeV}^2]^4q^2(q^2+4M^2)}.\tag{19}$$

We have used the standard dipole fit to the nucleon electromagnetic form factors with the usual normalization

[8] H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters **22**, 500 (1969); R. Brandt, *ibid.* **22**, 1149 (1969); H. Harari, *ibid.* **22**, 1078 (1969); S. D. Drell, D. J. Levy, and T. M. Yan, *ibid.* **22**, 744 (1969); also see J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[9] J. J. Sakurai, Phys. Rev. Letters **22**, 981 (1969).

at $q^2=0$. Thus,

$$\Delta M_{\rm Regge}=-\frac{(\mu_p{}^2-\mu_n{}^2)3\alpha}{8\pi M}\int_0^\infty dq^2\frac{q^4}{(q^2+4M^2)[q^2-m_\rho{}^2/\xi(\infty)](1+q^2/0.71)^4}$$
$$-\frac{3\alpha M}{2\pi}\int_0^\infty dq^2\frac{q^2}{(q^2+4M^2)[q^2-m_\rho{}^2/\xi(\infty)](1+q^2/0.71)^4}. \quad (20)$$

With the dimensionless integration variable $y=q^2/m_\rho{}^2$,

$$\Delta M_{\rm Regge}=-\frac{(\mu_p{}^2-\mu_n{}^2)3\alpha}{8\pi M}\frac{m_\rho{}^4}{2M^2}\int_0^\infty\frac{dy\,y^2}{(ym_\rho{}^2/4M^2+1)[y-1/\xi(\infty)](ym_\rho{}^2/0.71+1)^4}$$
$$-\frac{3\alpha M}{2\pi}\frac{m_\rho{}^2}{4M^2}\int_0^\infty\frac{dy\,y}{(ym_\rho{}^2/4M^2+1)[y-1/\xi(\infty)](ym_\rho{}^2/0.71+1)^4}. \quad (21)$$

Because of the presence of squares of the electromagnetic form factors, we have very strong damping in q^2 and therefore the low-q^2 region is the most important in the piece of the mass-shift expression under consideration. The vector-dominance model is known to be valid for low q^2 and hence is expected to work well in the present application.

We shall have to take the principal parts of the integrals. The evaluations are lengthy but straightforward. Because of the Pomeranchuk theorem, $\xi(\infty)$ must go to a constant independent of ν. We substitute various values of $\xi(\infty)$ after carrying out the integrations and the following set of discouraging numbers emerge.

With $\xi(\infty)=1$, $\Delta M_{\rm Regge}\simeq-0.01$ MeV, which is far too small for sign reversal of the over-all mass shift. From its definition, clearly $\xi(\infty)$ may assume negative values too. $\xi(\infty)\rightarrow\infty$ corresponds to $\gamma_L{}^{(1)}(q^2)\gg\gamma_T{}^{(1)}(q^2)$ considered by GP[5] and indeed we recover $\Delta M_{\rm Regge}\simeq-0.34$ MeV from our general expression by setting $\xi(\infty)\rightarrow\infty$. Several cross checks on the evaluation of integrals have been made. We summarize our results in Table I. To obtain the correct sign and magnitude of the n-p mass difference with $\Delta M\simeq\Delta M_{\rm el}+\Delta M_{\rm Regge}$, we need $\Delta M_{\rm Regge}\cong-2.1$ MeV. Thus, the expected enhancement of $\Delta M_{\rm Regge}$ required to achieve the sign reversal does not occur even when $\gamma_T{}^{(1)}(q_0{}^2)\simeq\gamma_L{}^{(1)}(q_0{}^2)$ for some small q^2 in the neighborhood of $m_\rho{}^2$; $\Delta M_{\rm Regge}$ merely oscillates near zero and does not attain large and negative values in the domain of interest.

It is well known that the application of the Bjorken limit at large q^2 applied to the time-ordered product of two electromagnetic current leads to logarithmically divergent mass shifts. This divergence comes from large-q^2 contributions and even in a model with

$$[\partial j_\mu(x)/\partial t, j_\nu(0)]\delta(x_0)=0,$$

which gives finite mass shifts,[10] the q^2 integral is barely convergent if $[\partial^2 j_\mu(x)/\partial t^2, j_\nu(0)]\delta(x_0)\neq0$ and the high-q^2 domain is still important. The general feature of dispersion calculations, on the other hand, is strong damping in q^2 because of the appearance of form factors. If we demand consistency between the two approaches, the sum of the coefficients of terms with large negative powers of q^2 in the dispersion approach must add up to zero as a consistency condition.[11]

Recently Pagels[12] has discussed the question of the convergence of the q^2 integration in the Cottingham mass-shift formula if the Bjorken scaling law for the inelastic-electron-scattering structure functions W_1 and W_2 in the limit of large q^2 and ν holds, and has shown that the mass shift is at least logarithmically divergent unless some remarkable cancellations occur. This conclusion depends on the assumption of analyticity of a partial-wave amplitude in the J plane for $\mathrm{Re}J>-\frac{1}{2}$. Whether such cancellations indeed occur can be answered by performing difficult experiments. It has been pointed out by Pagels that the above conclusion may be avoided if there are fixed poles at $J=0$.

In our approach, this divergence (if there is any) is probably hidden in the piece $\Delta M'_{\rm inel}$. Our experimental knowledge of the q^2 dependence of the Regge residue functions and the excitation form factors of resonances is very little, but the study of the electroproduction of several nucleon resonances indicates strongly damped q^2 behavior of the relevant form factors, and finite-energy sum rules (FESR) may be used to infer similar

TABLE I. Calculated results of $\Delta M_{\rm Regge}$ for various values of the parameter $\xi(\infty)$.

$\xi(\infty)$	$\Delta M_{\rm Regge}$ (MeV)	$\xi(\infty)$	$\Delta M_{\rm Regge}$ (MeV)
∞	−0.34	−0.5	+0.03
2	+0.20	−1	+0.47
1	−0.01	−1.33	−0.13
0.5	+0.72	−2	−0.2
0.1	−0.11		

[10] M. B. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967); also see J. D. Bjorken and R. A. Brandt, Phys. Rev. **177**, 2331 (1969).

[11] R. Chanda, R. N. Mohapatra, and S. Okubo, Phys. Rev. **170**, 1344 (1968).

[12] H. Pagels, Phys. Rev. (to be published).

behavior of the A_2 residue functions. If this strong q^2 damping is indeed the case, we do not encounter any divergences.

An alternative approach has been tried by Harari and Elitzur[13] by dropping the scaling assumption for the $I=1$ amplitude and using the duality ideas in the FESR sense to estimate the Regge residue. They also conclude that the sign of the n-p mass difference cannot be explained.

Recently, several authors[8] have studied the joint implications of Bjorken (large q^2) and Regge (large ν) limits on the inelastic-lepton-scattering structure functions. Since W_1 and W_2 are the absorptive parts of Compton amplitudes, they should vanish below the threshold and one can express the asymptotic behavior at fixed q^2 and $\nu\to\infty$ as[14]

$$W_1^{(1)}(q^2,\nu) \xrightarrow[\nu\to\infty]{} \beta_1^{(1)}(q^2,\alpha)(\nu-q^2/2M)^{\alpha},$$
$$W_2^{(1)}(q^2,\nu) \xrightarrow[\nu\to\infty]{} \beta_2^{(1)}(q^2,\alpha)(\nu-q^2/2M)^{\alpha-2}, \tag{22}$$

where $\alpha\equiv\alpha_{A_2}(0)$ is the $t=0$ intercept of the leading $C=+1$, $I=1$ Regge trajectory. Assuming that there exists a nonvanishing Bjorken limit, in the domain of large q^2, one obtains[8]

$$\beta_1^{(1)}(q^2,\alpha)=\lambda_1^{(1)}(\alpha)(1/q^2)^{\alpha}$$
and
$$\beta_2^{(1)}(q^2,\alpha)=\lambda_2^{(1)}(\alpha)(1/q^2)^{\alpha-1}. \tag{23}$$

Therefore,

$$\frac{\beta_1^{(1)}(q^2,\alpha)}{\beta_2^{(1)}(q^2,\alpha)}=\frac{\lambda_1^{(1)}(\alpha)}{\lambda_2^{(1)}(\alpha)}\frac{1}{q^2}. \tag{24}$$

Although we do not expect this result to hold in the low-q^2 domain, let us see if this form of the ratio improves the n-p mass difference in case Eq. (24) holds down to reasonably small values of q^2.

From Eqs. (10) and (24), we obtain

$$\Delta M_{\text{Regge}}=\frac{3}{8M}\int_0^{\infty} dq^2 q^2\frac{\lambda_1^{(1)}(\alpha)}{\lambda_2^{(1)}(\alpha)}f_2^{(1)}(q^2). \tag{25}$$

It is possible to evaluate $\lambda_1^{(1)}(\alpha)/\lambda_2^{(1)}(\alpha)$ from certain sum rules involving the structure functions. The well-known Bjorken inequality for electron scattering, based on the algebra of currents,[15] may be written as

$$\int_{\text{quasi-elastic region}} d\nu[W_{2,p}(q^2,\nu)+W_{2,n}(q^2,\nu)]>\tfrac{1}{2}. \tag{26}$$

The diffractive region should not be included on the left-hand side of this inequality, since the Adler neutrino sum rule,[15] from which this inequality is derived by isospin rotations, has a difference between the νp and $\bar{\nu}p$ cross section and the diffractive contributions cancels out.

Using the quark model with the naive additivity assumption, it is possible to obtain more detailed sum rules in the $q^2\to\infty$ limit[16]:

$$\int_{\text{quasi-elastic region}} d\nu\, W_{2,p}(q^2,\nu)\cong 1, \tag{27}$$

$$\int_{\text{quasi-elastic region}} d\nu\, W_{2,n}(q^2,\nu)\simeq\tfrac{2}{3}. \tag{28}$$

Equation (27) has also been obtained by Gottfried.[16]

The background scattering inequality of Bjorken, in the case of quark-model space-space component current commutators,[15] is given by

$$|q^2|\int_0^{\infty}\frac{d\nu}{\nu^2}[W_{1,p}(q^2,\nu)+W_{1,n}(q^2,\nu)]>\tfrac{1}{2}. \tag{29}$$

Using the quark model as before, one obtains the following sum rules for large q^2 [16]:

$$|q^2|\int_{\text{quasi-elastic region}}\frac{d\nu}{\nu^2}W_{1,p}(q^2,\nu)\simeq 1 \tag{30}$$

and

$$|q^2|\int_{\text{quasi-elastic region}}\frac{d\nu}{\nu^2}W_{1,n}(q^2,\nu)\simeq\tfrac{2}{3}. \tag{31}$$

Now, it is easy to obtain the $\lambda(\alpha)$ parameters from these sum rules. For example,

$$\int_{\nu_0}^{N} d\nu\, W_{2,p}(q^2,\nu)\simeq 1$$

(ν_0= the quasi-elastic threshold), together with Eqs. (22) and (24), yields

$$\lambda_{2,p}(\alpha)=\frac{(q^2)^{\alpha-1}(\alpha-1)}{(N-q^2/2M)^{\alpha-1}-(\nu_0-q^2/2M)^{\alpha-1}}.$$

Similarly,

$$\lambda_{2,n}(\alpha)=\frac{2}{3}\frac{(q^2)^{\alpha-1}(\alpha-1)}{(N-q^2/2M)^{\alpha-1}-(\nu_0-q^2/2M)^{\alpha-1}}$$

and

$$\lambda_{1,p}=\frac{(q^2)^{\alpha}(\alpha-1)}{[(N-q^2/2M)^{\alpha-1}-(\nu_0-q^2/2M)^{\alpha-1}]|q^2|},$$

$$\lambda_{1,n}=\frac{2}{3}\frac{(q^2)^{\alpha}(\alpha-1)}{[(N-q^2/2M)^{\alpha-1}-(\nu_0-q^2/2M)^{\alpha-1}]|q^2|}.$$

[13] H. Harari and M. Elitzur, Weizman Institute of Science, Israel Report, 1969 (unpublished).

[14] R. Brandt (private communication); however, the ratio $\lambda_1^{(1)}(\alpha)/\lambda_2^{(1)}(\alpha)$ does not change even if we take the asymptotic behavior as $W_1^{(1)}\to_{\nu\to\infty}\beta_1^{(1)}(q^2,\alpha)\nu^{\alpha}$ and $W_2^{(1)}\to_{\nu\to\infty}\beta_2^{(1)}(q^2,\alpha)\nu^{\alpha-2}$.

[15] J. D. Bjorken, Phys. Rev. Letters **16**, 408 (1966); S. L. Adler, Phys. Rev. **143**, 1144 (1966); for "backward" inequality see J. D. Bjorken, *ibid.* **163**, 1767 (1967).

[16] J. D. Bjorken, in *Proceedings of the International School of Physics, "Enrico Fermi," Course IXL,* edited by J. Steinberger (Academic Press Inc., New York, 1968). See also K. Gottfried, Phys. Rev. Letters **18**, 1174 (1967).

In the present case, $\alpha \simeq 0.4$.

$$\frac{\lambda_1^{(1)}(\alpha)}{\lambda_1^{(1)}(\alpha)}=\frac{\lambda_{1,p}(\alpha)-\lambda_{1,n}(\alpha)}{\lambda_{2,p}(\alpha)-\lambda_{2,n}(\alpha)}=\frac{q^2}{|q^2|}=-1. \qquad (32)$$

Since the quark-model sum rules hold for large negative q^2 in our metric, this ratio of parameters leads to

$$\Delta M_{\text{Regge}}=-\frac{3}{8M}\int_0^\infty q^2 f_2^{(1)}(q^2)dq^2=-0.34 \text{ MeV}.$$

It is interesting to note the above set of approximation schemes gives the result obtained by assuming $\gamma_T^{(1)}(q^2)\simeq 0$ or $\xi(\infty)\to\infty$ in the vector-dominance model.

Srivastava[17] attempted to calculate the n-p mass difference by writing $t_1^{(1)}(q^2,\nu)=t_1^{1,\text{pole}}+\beta_1^{(1)}(q^2)\nu^\alpha$. One of the objections to this calculation has been that one is not allowed to use this expression for $t_1^{(1)}(q^2,\nu)$ directly in the Cottingham formula, since the Regge region does not enter explicitly in that formula.[5] However, in the present approach, we are interested only in the ratio of the Regge residues and, following Srivastava, we can use the FESR for its determination. Let the cutoff in the FESR be N. Then,

$$\int_0^{N^2} d\nu^2\, \text{Im}t_2^{1,\text{pole}}(q^2,\nu^2)=\beta_2(q^2)\frac{(N^2)^{\alpha/2}}{\frac{1}{2}\alpha},$$

$$t_2^{1,\text{pole}}(q^2,\nu^2)=\frac{4Mq^2f_2^{(1)}(q^2)}{q^4-4M^2\nu^2},$$

whence

$$\beta_2^{(1)}(q^2)=\pi(q^2/M)(\tfrac{1}{2}\alpha)f_1^{(1)}(q^2)N^{-\alpha}. \qquad (33)$$

Similarly,

$$\beta_1^{(1)}(q^2)=\pi(q^2/M)(\tfrac{1}{2}\alpha+1)f_2^{(1)}(q^2)N^{-\alpha-2}.$$

Therefore,

$$\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}=\frac{\alpha+2}{\alpha}\frac{f_1^{(1)}(q^2)}{f_2^{(1)}(q^2)}N^{-2}. \qquad (34)$$

Since ν must be larger than q^2 in order that Regge behavior holds, it is reasonable to choose a cutoff dependent on q^2. A choice of $N=q^2/2M$, the threshold value of ν, produces a strong enhancement in ΔM_{Regge}, but it should be noticed that the numerical value of the mass shift varies very strongly, in fact, as the square of N, which is a rather disagreeable feature of this approach. If we choose a cutoff independent of q^2, then $\Delta M_{\text{Regge}}\approx 0$ for all reasonable values of the cutoff.

Thus, we find that it is impossible to understand the n-p mass difference in the framework of dispersion theory in a natural way with the usual assumption that ΔM_{inel} and $\Delta M_{\text{fixed pole}}$ are negligible. It was hoped that in some simple and reasonable models one might be able to enhance the ΔM_{Regge} contribution in the right direction and thereby obtain at least the required sign reversal. We have shown that this is not so.

Let us now present a few conjectures regarding this problem. The failure of the n-p mass difference calculation in this approach may indicate that the ΔM_{inel} piece is in fact quite important. In the past, dispersion sum rules for virtual Compton scattering of pions coupled with low-energy theorems for the process were studied in detail.[18] There were indications that only the lowest-lying states may not be sufficient to satisfy the sum rules. The present investigation may be considered as an indication that the same is true for virtual Compton scattering of nucleons and no accidental cancellation is taking place. Unfortunately, the lack of any experimental data on the radiative decay widths of the $I=\frac{1}{2}$ nucleon resonances makes it impossible to estimate ΔM_{inel}.

There have been some interesting attempts to understand the electromagnetic mass differences using the so-called feedback mechanism by taking the effect of electromagnetic mass splittings themselves on the self-energies due to strong interactions.[19] The "driving term" generated by the electromagnetic field is purely long-range, but the feedback terms may be both of the long- and short-range types. The short-range effects are very difficult to estimate and the results are not very reliable.

Finally, the fixed-pole contributions to the n-p mass shift may be non-negligible. However, it seems to us that no clear statement about fixed poles may be made at the present time. The demonstration that an $I=1$, $J=0$ fixed pole is present in $t_2^{(1)}(q^2,\nu)$ assumes the absence of higher states contributing to the absorptive part of the amplitude.[5] No definite statement can be made regarding $R_1^{(1)}(q^2)$, the residue of $t_1^{(1)}(q^2,\nu)$ at $J=0$ fixed pole. Thus, we do not have sufficient reason to believe that fixed poles alone would account for the puzzling n-p mass shift.

Note added in proof. T. Muta has found the contribution of the Roper resonance to the p-n mass difference to be very small.[20]

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[17] Y. Srivastava, Phys. Rev. Letters **20**, 232 (1968).

[18] See Ref. 10. K. C. Gupta and J. S. Vaisya [Phys. Rev. **176**, 2125 (1968)] argue that that the logarithmic divergence of the pion mass shift indicates that subtraction is required in virtual γ-π scattering dispersion integrals. This is not true, since the divergence in the mass shift comes from the high-q^2 contribution, which has nothing to do with the question of subtraction in dispersion relations in the ν variable; the large-ν behavior is given by Regge theory.

[19] See, for example, G. Barton and D. Dare, Phys. Rev. **150**, 1220 (1966); S. L. Cohen and C. R. Hagen, *ibid.* **157**, 1344 (1967).

[20] T. Muta, Phys. Rev. **171**, 1661 (1968).