Nuclear Interaction of Oxygen with Oxygen*

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Striking and regular gross structure has been observed in excitation functions for the $O^{16}+O^{16}$ elastic scattering interaction in the oxygen-ion laboratory energy range from 20 to 80 MeV. Excitation functions have been measured simultaneously at five c.m. angles from 50° to 90°; systematic angular-distribution measurements have been carried out at narrow energy intervals spanning the gross-structure peak in the 90° excitation function for $19 \leq E_{c.m.} \leq 22$ MeV and at intervals throughout the remainder of the energy range studied. The gross structure is reproduced surprisingly well by a Woods-Saxon optical model having an unusually shallow real well depth ($V=17$ MeV) and an energy-dependent imaginary well depth ($W=0.4$ MeV $+ 0.1\ E_{c.m.}$). Extensive scans have demonstrated that this is the lowest member of a discrete set of equivalent Woods-Saxon potentials; it is the *only* one of these, however, which provides a reproduction of the experimental data without requiring a marked and complex energy dependence of the real potential-well depth. The effects of adding a repulsive core to this lowest Woods-Saxon potential were studied in some detail; it has been found that the model predictions are quite sensitive to the core—a somewhat surprising result, perhaps reflecting the apparent long mean free path of the O^{16} ions under the present conditions. An ambiguity in the imaginary well depth has been examined in some detail; the present data do not permit resolution of this ambiguity, but throw into question the physical significance of this long mean free path. The addition of a core does not provide significantly improved fits to the experimental data. This does not preclude the the existence of the core, but suggests further study with different well parameters and shapes. A phase-shift analysis of the 28 angular distributions spanning the gross-structure peak provided no evidence for underlying resonances; no compelling explanation for the intermediate (~200-keV) width structure superposed on the gross-structure peak is yet available. Finer-grained excitation-function structure is present and is attributed to statistical fluctuations in the usual way. Extensive theoretical studies stimulated by, and concurrent with, these experimental studies have demonstrated that these heavy-ion data may provide unique information on the nuclear-matter problem in finite systems, including an experimental determination of the effective nuclear compressibility.

I. INTRODUCTION

As the simplest interaction involving complex nuclear projectiles and targets, elastic scattering has been the most extensively studied phenomenon in heavy-ion physics. Since the ratio of the de Broglie wavelength to characteristic dimensions of the nuclear system is much less in the case of heavy ions than in that of the lighter more commonly used nuclear projectiles, the optical analogs to the nuclear situation have considerably greater relevance and validity, as might have been anticipated. In the past, in general, the experimental measurements[1,2] have been carried out in restricted ranges of either or both energy and angle and as such did not permit comprehensive study of the phenomena involved.

Partial exceptions were the elastic scattering work carried out by the Chalk River group[3–6] and the more recent studies[2,7–9] on the $B^{10}+B^{10}$, $B^{10}+O^{16}$, $B^{11}+O^{16}$, $C^{12}+C^{12}$, $N^{14}+N^{14}$, and $N^{14}+O^{16}$ systems at various laboratories. All these investigations, however, were limited to the EN tandem energy range. The early Chalk River work[3–6] was carried out with the first EN tandem, which for the first time, permitted detailed study of the elastic scattering excitation functions.

These measurements on the $C^{12}+C^{12}$, $C^{12}+O^{16}$, and $O^{16}+O^{16}$ systems received considerable interest in that in the $C^{12}+C^{12}$ system, at energies near the Coulomb barrier, evidence[3–5,10] was found for an unusual quasi-molecular interaction mechanism. At energies well above the Coulomb barrier, extensive fine structure

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amenable to statistical fluctuation analyses[11–13] was found in the $C^{12}+C^{12}$ system, to a lesser extent in the $C^{12}+O^{16}$ system, and not at all in the $O^{16}+O^{16}$ system. With the availability of considerably higher heavy-ion energies from the Yale MP Van de Graaff accelerator, extension of these studies to much higher c.m. energies became possible.

To initiate this series of measurements it was decided to concentrate on identical particle scattering systems to take advantage of the fact that the exclusion of all odd partial waves from the problem might be expected to simplify significantly the analyses of the scattering data; moreover, the unique kinematic relationships involved in identical particle scattering greatly simplify the experimental arrangements if kinematic coincidences are used for particle detection and identification.

The $O^{16}+O^{16}$ system, in particular, was selected inasmuch as the earlier measurements[5,14] had demonstrated a freedom from marked fluctuation structure in the excitation functions which would simplify extraction and study of any underlying, nonstatistical structures or mechanisms of intrinsically greater interest. It was anticipated, too, that the doubly magic closed-shell structure of O^{16} might result in inherent simplification of the interaction and thus increase the probability of isolating specific interaction mechanisms.

Detailed elastic scattering excitation functions at five different angles were studied in a laboratory energy range overlapping the earlier Chalk River data[5] and extending to 80 MeV; a surprising and quite unexpected structure was observed at all angles. Gross structure having characteristic widths of $\sim$2 MeV and peak to valley ratios in excess of 20:1 was found. Superimposed on this was intermediate width structure having characteristic widths of $\sim$200 keV and over-all weak and very fine structure presumably of statistical origin.

A number of possible explanations for the gross structure were eliminated by corollary experiments and, on the basis of optical-model studies, it has been concluded that this is a specifically nuclear potential phenomenon.

The optical-model analysis yielded a shallow potential of 17 MeV real well depth, which was found to be the lowest member of a (discrete) set of Woods-Saxon potentials, which all predict the same excitation functions and angular distributions. All but the 17-MeV potential, however, require a rather complicated energy dependence of the real potential strength. These differences in the energy dependence may resolve the ambiguity between the different potentials.

In view of recent attempts[15,16] to derive the heavy-ion–nucleus potential from nuclear-matter calculations it is of particular interest to investigate the possibility of determining a unique potential empirically from the present data. The study of the ambiguities in the heavy-ion potentials has therefore been a central part of this investigation. In particular, we have studied the effect of adding a repulsive core to the real Woods-Saxon potential.

Detailed angular distributions were accumulated over the intermediate width structure on one of the gross-structure peaks and were subjected to a phase-shift analysis analogous to those recently used by Singh *et al.*[17] in their study of α-particle scattering on *sd* shell nuclei, wherein they were successful in extracting evidence for specific resonance phenomena underlying statistical fluctuations. No such resonances were found in the present investigation.

During the course of these studies, and stimulated by their preliminary publication,[18] several theoretical investigations have been undertaken. These have been phenomenological in attempting to relate the observed gross structure to certain characteristics of an empirically derived potential[19]; more fundamental, in attempting to derive the interaction potentials from basic nuclear-matter considerations[15]; and intermediate,[16] in applying some aspects of the nuclear matter problem together with phenomenological input to construct a rather detailed dynamical model of the actual collision process.

The outcome of the phenomenological studies has been the identification of an analogy with familiar optical phenomena; of the fundamental studies, the recognition that elastic scattering studies, such as those reported herein, can indeed be used to examine some of the characteristics of the interaction potential with sensitivity, and through this provide information on the fundamental inputs to the nuclear-matter calculations; of the intermediate studies, a new experimental determination of the compressibility of nuclear matter.

Several other theoretical studies are also in process. In view of this interest and the demonstrated potential of these elastic scattering studies, a systematic program involving other light- and medium-mass identical nuclear systems has been undertaken and is continuing in this laboratory.

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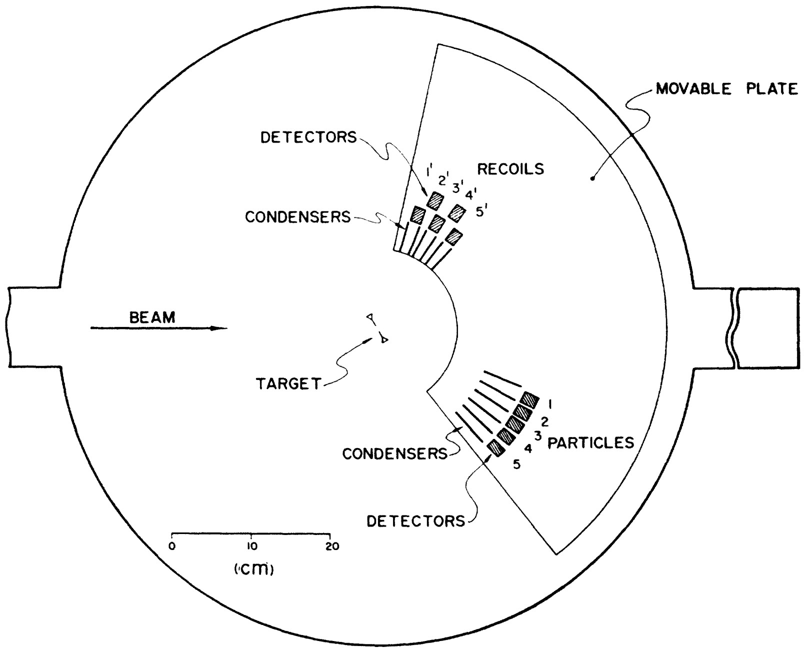

FIG. 1. Diagram of the geometric configuration inside the scattering chamber.

Prior to and during the course of these measurements additional investigations on the $O^{16}+O^{16}$ system have been reported by several authors. Vandenbosch *et al.*,[20] in order to carry out an analysis within the framework of the statistical model, studied the $O^{16}(O^{16}, \alpha)Si^{28}$ reaction, as well as its inverse at $E_\alpha=$ 42 MeV, in the energy range $E_{O^{16}}$ from 35 to 39 MeV. The $O^{16}(O^{16}, \alpha)Si^{28}$ reaction has also been studied by Leachmann and Fessenden[21] in the energy range from 24 to 29 MeV. Feldman and Heikkinen[22] determined an upper limit for the $O^{16}+O^{16}$ radiative capture cross section.

II. EXPERIMENTAL TECHNIQUES

The Yale MP Van de Graaff accelerator was used to accelerate O^{16} beams to laboratory energies in the range from 20 to 80 MeV with a momentum resolution of 2 parts in 10^4. An oxygen-gas stripper was used in the accelerator thus minimizing the problem of beam contamination with ions originating in the stripper gas. Inasmuch as the high specific energy loss characteristic of heavy ions requires high beam line vacuum, thin targets, and thin detector dead layers, the transport lines for the O^{16} beam were held at a pressure of $\sim 10^{-7}$ Torr. The beam impinged on evaporated and self-supporting SiO targets (15–100 $\mu g/cm^2$) positioned in an ORTEC 30-in. scattering chamber. The target thickness was determined through use of standard α-particle ranging techniques and of quartz crystal film thickness monitors.

A kinematic recoil technique,[23] as noted above, was selected for particle detection and identification; i.e., two detectors located to detect the scattered and recoil O^{16} ions simultaneously provide sufficient kinematic information to define the scattering interaction uniquely. Since deep minima in the $O^{16}+O^{16}$ elastic excitation function have been observed in this experiment, it was of particular importance to eliminate all such nonelastic contaminations.

Since the differential cross section changes rapidly with angle in the present study, angular resolution was of crucial importance. (For identical particle scattering, $\theta_{lab}=\frac{1}{2}\theta_{c.m.}$ and $E_{lab}=2E_{c.m.}$.) Apertures of $\pm\frac{1}{2}°$ (lab) were used to provide the highest possible count rate consistent with acceptable angular definition. Because of the thin targets needed for heavy-ion work, and since target deterioration was a problem for beam fluxes greater than 150 nA, counting rates were severely limited. To reduce this problem, rectangular (10×50 mm) surface barrier detectors with annular section apertures were used to maximize detector active areas (larger azimuthal acceptance) without destroying the angular resolution. The dead layer on these detectors was 40 $\mu g/cm^2$ of gold.

To increase the attainable data rate further, an array of ten large area detectors was used to form five parallel and independent coincidence systems. These detectors were rigidly mounted on a base plate which positioned the detectors precisely in order to satisfy the kinematic requirements. [For identical

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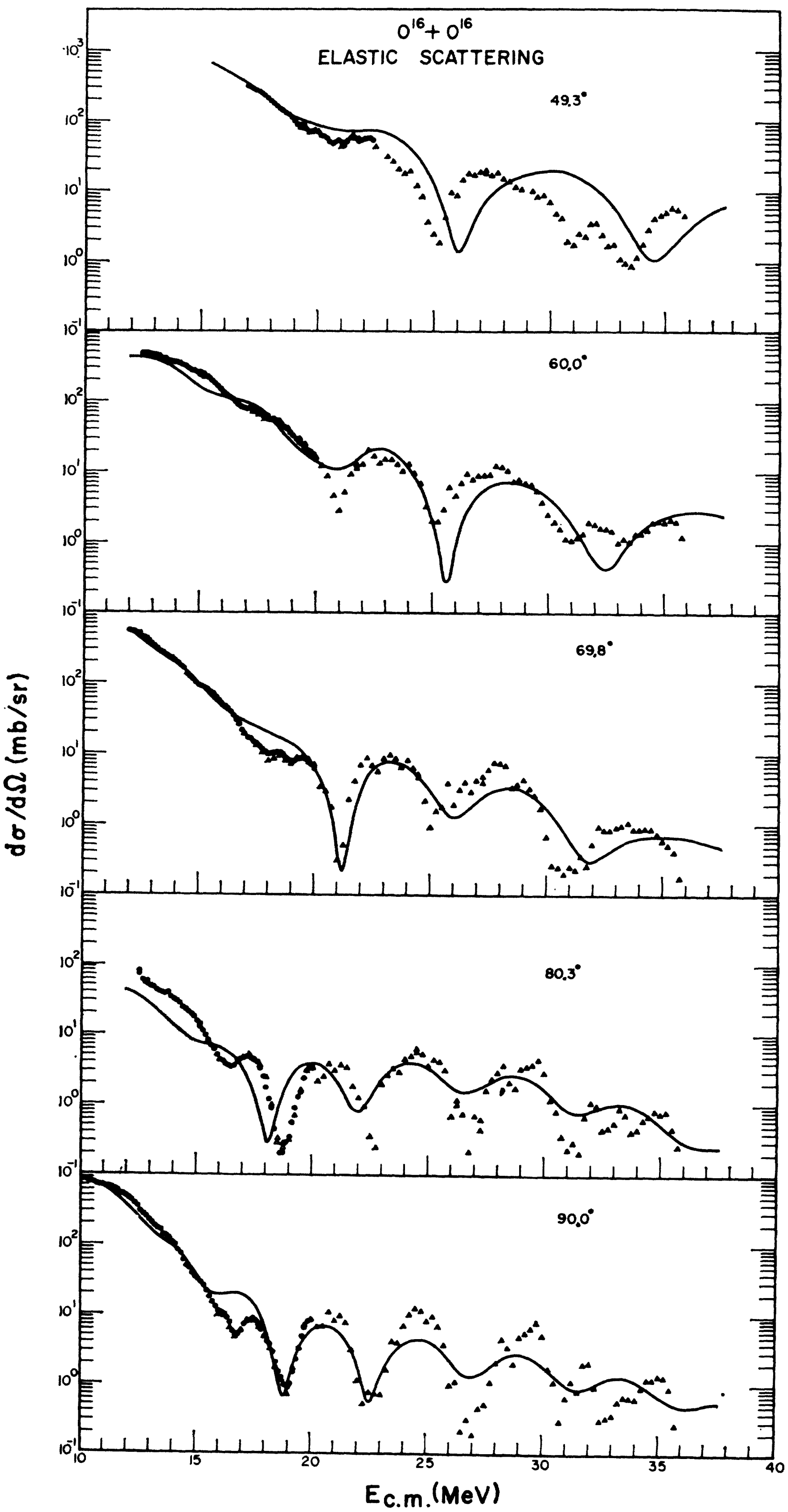

FIG. 2. $O^{16}+O^{16}$ elastic scattering excitation functions. The solid lines represent the cross-section predictions of the optical potential described in Sec. IV B.

particles the two detectors in a given pair must remain 90° apart (lab) independent of energy and of scattering angle.] The plate shown in Fig. 1 was rotated to measure angular-distribution segments or positioned at a fixed angle for measurement of five simultaneous excitation functions.

Secondary electron production by a heavy-ion beam is a more severe problem than is the case with lighter projectile beams. Since magnetic deflection of such electrons from detector faces was not practical for the large area detectors in use in this work, the capacitors shown in the figure were used to sweep away electrons electrostatically before they could reach the detector faces.

The signals from each of the detector pairs were passed through a standard fast coincidence system; simultaneously the analog signal from the forward angle detector of the pairs was sent into a parallel amplifier and linear gate (strobed by the fast coincidence output). This analog signal, following transmission through the gate, was mixed with signals from the parallel coincidence systems and fed into an IBM 1024-channel analog-to-digital converter (ADC). The output of the ADC was identified as to angle by a pulse from the appropriate fast coincidence system and processed and stored by an IBM 360/44 computer by means of an interface and programming system developed for use in experimental nuclear physics in a Yale-IBM joint study.[24] The computer light pen facility could then be used to sum the elastic peaks in the individual spectra, and the spectra themselves were written on magnetic tape and simultaneously printed.

The elastic scattering spectra accumulated in the above fashion each contained only the elastic peak. In no case was any evidence observed for the presence of beam contamination from lower-energy O^{16} ions originating from the stripper gas.

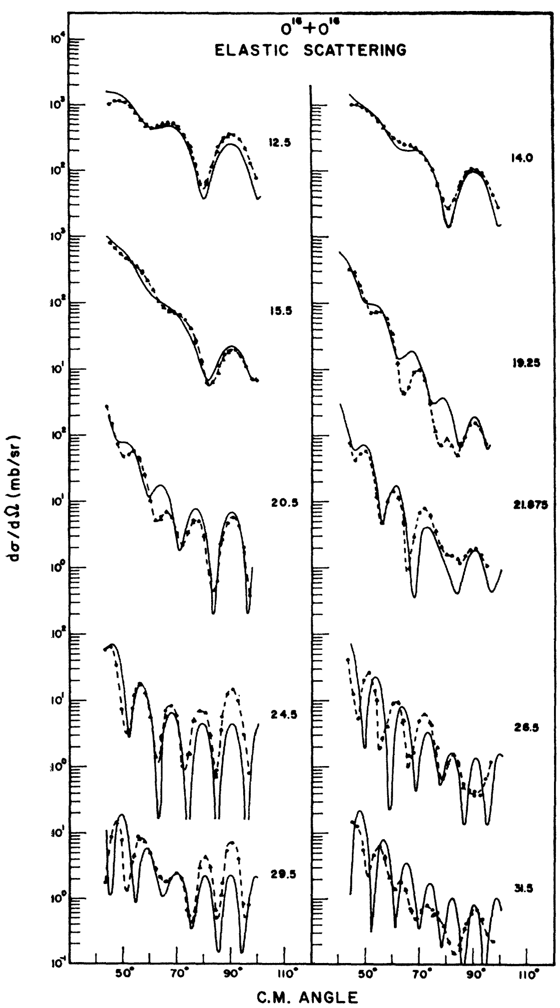

FIG. 3. $O^{16}+O^{16}$ elastic scattering angular distributions. The solid lines represent the cross-section predictions of the optical potential described in Sec. IV B. The dashed curves are hand drawn through the data points.

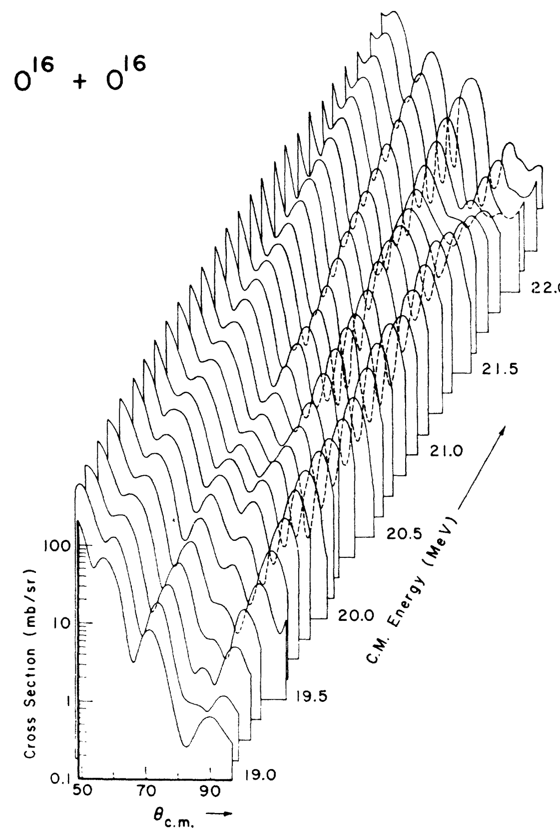

FIG. 4. $O^{16}+O^{16}$ elastic scattering angular distributions. The lines shown in the figure have been hand drawn through the data points.

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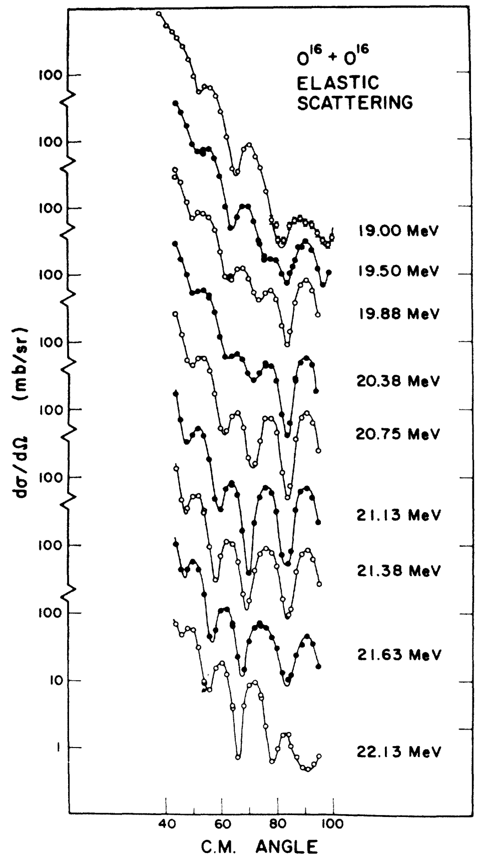

FIG. 5. Typical angular distributions selected from among those shown in Fig. 4. The lines are hand drawn and connect the data points.

III. RESULTS

For the determination of the absolute cross section from the present data two steps were involved. First, the 90° excitation function was extended down to energies below the Coulomb barrier, where the cross section is assumed to be pure Coulomb in nature, thus giving an accurate measurement of the absolute cross section at this energy. In extending this normalization to higher energies, however, correction must be made for energy variation in the mean charge state of the O^{16} ions collected in the Faraday cup of the beam current integrator in converting from beam charge to particle flux. Accurate data on this variation are not available and in consequence an $O^{16}+Au$ elastic scattering excitation function was measured simultaneously from a very thin gold flash overlaying the SiO target in the energy range of this experiment and below the $O^{16}+Au$ Coulomb barrier at approximately 67 MeV in the laboratory frame. Since these latter data would be expected to show the usual Coulomb $1/E^2$ behavior, deviations from such behavior can be ascribed to the variation of the mean beam charge state; appropriate corrections were made to the $O^{16}+O^{16}$ data.

Figure 2 shows the $O^{16}+O^{16}$ elastic scattering excitation functions at five angles, while Figs. 3–5 present angular distributions measured at selected energies throughout the range of this study. The excitation function data at forward angles cannot be extended readily below the Coulomb barrier with the instrumentation described above because multiply scattered recoiling nuclei will fall outside the collimation limits of the backward-angle detector and thus fail to fulfill the coincidence requirements. Because of this possible deviation from 100% coincidence efficiency, empirical detector-to-detector normalization factors, determined from angular-distribution measurements, have been used to fix the absolute values of the forward-angle cross sections. These normalization factors are understood and contribute only ~5% to our error estimate.

Figure 6 shows the 90° excitation function on a linear scale. This figure perhaps best illustrates the three types of structure of interest in these data. Three pronounced broad peaks whose width is in the neighborhood of 2 MeV are evident; in addition, a fourth gross-structure peak (included in Fig. 2 but not in Fig. 6) appears partially submerged in the rapid rise of the cross section as the energy decreases toward the Coulomb barrier. These, and the similar structure at the four other angles, will be discussed later primarily as potential interaction phenomena. Superimposed on this gross structure are peaks of intermediate width in the 200-keV range. It was specifically to provide the input data for a systematic phase-shift analysis of this structure that the 28 angular distributions of Fig. 4 have been measured. Finally, to investigate the possibility of still finer structure, excitation functions over restricted energy ranges were measured with very thin targets; the results are shown in Fig. 7. The left-hand side of the figure shows data taken with a target approximately 25 keV (c.m.) thick, while the target used for the measurements on the right-hand side was approximately twice as thick. The narrow, weak structure evident under this higher-energy resolution is assumed to originate from statistical compound-nuclear fluctuations.

Figure 8 shows the measured 90° c.m. excitation function for the inelastic scattering of O^{16} by O^{16}, leaving one of the two O^{16} nuclei in either of the 6.05 (0^+) and 6.13 (3^-) states. Experimental resolution did not permit the separation of these close lying states. Above 26.5 MeV, the kinematic requirements for inelastic scattering to the 6.92-MeV state allow this channel to enter the experimental resolution limits for the lower doublet; consequently the excitation function was discontinued at that energy.

With the exception of the thin target data of Fig. 7 and the inelastic scattering data of Fig. 8 (in both cases statistical errors are shown in the figures), the data accumulation in this experiment was planned to achieve ~3% (i.e., 10^3 events) statistical error in the

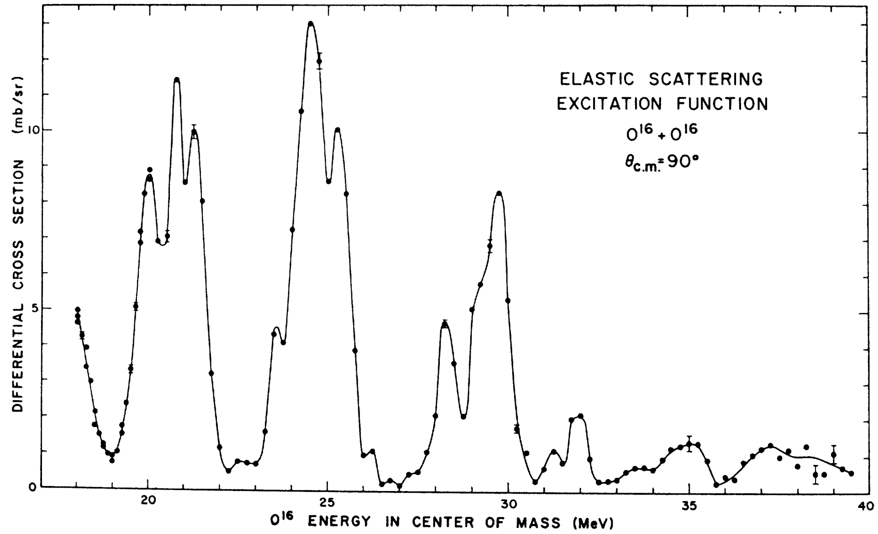

FIG. 6. 90° elastic scattering excitation function for $O^{16}+O^{16}$ system. The solid curve is hand drawn and has no theoretical significance.

detector pair with the lowest count rate. An rms error estimate (which includes not only statistical errors but also estimates of the possible errors in charge state corrections 2%, empirical normalization factors 5%, and the Mott scattering measurement 1%) gives a value of 6% for the relative cross-section determination and of 13% for the absolute values of the cross section. Tables of these cross sections are available on request.

IV. DATA ANALYSIS

A. General

Three very simple approaches to explaining the 90° excitation function structure suggested themselves early in this work and have since been rejected. First, the three gross-structure peaks in the 90° excitation function might be considered as high angular momentum members of a rotational band in S^{32}. It would, of course, be most improbable that such a band could be based on the S^{32} ground state inasmuch as there is no evidence for well-developed rotational structure or well-defined static deformation of the ground state from the low-energy S^{32} level spectrum. Moreover, within the framework of a closed *sd* shell the required angular momenta are not available. Imposing a $J(J+1)$ energy dependence on these three peaks nonetheless requires them to represent states of spin 20, 22, and 24 in the compound system. These angular momenta would already require participation of much higher shell-model configurations. Much more important, optical-model partial-wave amplitudes, to be presented later in this paper, are negligibly small for these partial waves in the relevant energy regions.

Second, Davis[25] has shown that the gross structure of the 90° excitation function can be roughly reproduced with a cutoff model which involves hard-sphere phase shifts. The Davis model does not fit the 90° data as well as does the optical potential to be presented later, and it fails to reproduce the structure at the other angles.

Additionally, an extensive set of parameter scans with a five-parameter cutoff model has been included in this study. The parameters were R_1, S_1, Φ, R_2, and S_2, where

$$d\sigma/d\Omega = |f(\Theta)+f(\pi-\Theta)|^2, \tag{1}$$

$$f(\Theta)=f_{\text{coul}}(\Theta)+(1/2ik)\sum_l (2l+1) \times \exp[2i(\sigma_l-\sigma_0)][1-\eta_l\exp(2i\delta_l)]P_l(\cos\Theta), \tag{2}$$

$$\eta_l = 1/\{1+\exp[(L_1-l)/S_1]\} \tag{3}$$

$$\delta_l = \Phi/\{1+\exp[(l-L_2)/S_2]\} \tag{4}$$

and the pairs L_1, R_1 and L_2, R_2 are related by

$$E = Z^2e^2/R_i + (\hbar^2/2mR_i^2)L_i(L_i+1). \tag{5}$$

Fine mesh scans of the above parameters failed to fit more than one of the five excitation functions in any case. Furthermore, no parameter set reproduced any excitation function as successfully as the optical potential presented in this paper. Thus we have not been able to reproduce the $O^{16}+O^{16}$ gross structure with a cutoff model with energy-independent parameters.

This is only one of several demonstrations, which have appeared in this work, of the importance of forcing *simultaneous* fitting to excitation-function data obtained at *several* angles; it has been found that a variety of approaches, including the simple strong-absorption models, can indeed fit isolated subsets of the data while failing completely to reproduce the entire set.

A third approach involved explanation of the $O^{16}+$

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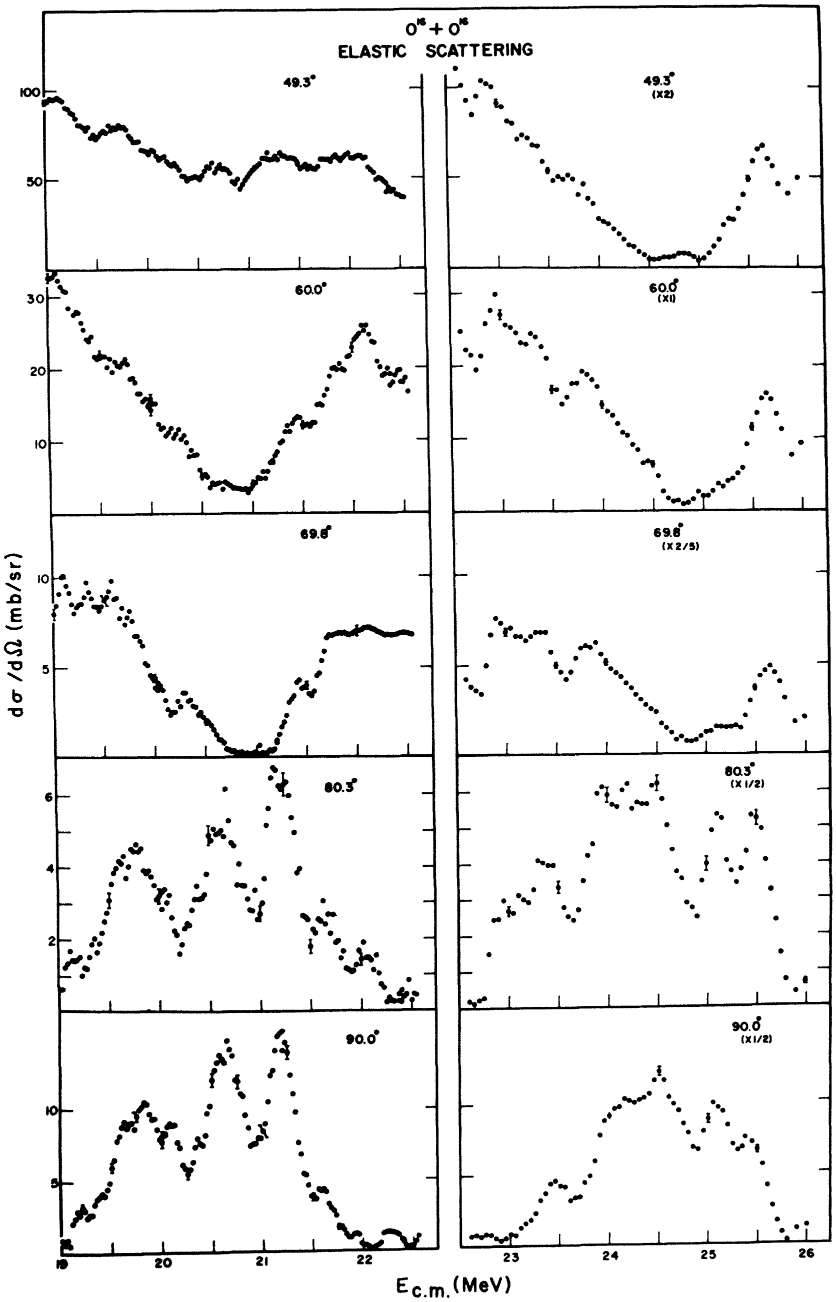

FIG. 7. $O^{16}+O^{16}$ elastic scattering excitation functions. The data on the left-hand side have been measured at 25-keV (c.m.) intervals, while those on the right-hand side were measured at intervals of 50 keV.

O^{16} excitation-function gross structure in terms of a resonant transfer mechanism first discovered in ion-atom studies by Everhart and co-workers[26] and suggested as of possible interest in nuclear physics by Temmer.[27] Evidence for a somewhat similar process has been presented by van Oertzen *et al.*[28] and by Gobbi[8] in independent studies of nonidentical particle scattering. For example, at back angles van Oertzen *et al.* have reported an enhancement of the $O^{16}(C^{12}, C^{12})O^{16}$ scattering cross section and has attempted to explain this result in terms of an addition of contributions from elastic scattering and the transfer reaction $O^{12}(C^{12}, O^{16})C^{12}$.

In the present study it would be anticipated, on binding energy grounds, that the α particle would perhaps play a role equivalent to the electron in the work of Everhart and co-workers. The $O^{16}(O^{16}, C^{12})Ne^{20}$

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reaction, conjugate to the elastic channel in a resonant α transfer, has been investigated in this context using appropriate kinematic detection techniques and has been found to show no structure indicative of such a resonant process.

B. Optical-Model Analyses

The earliest optical-model analysis of heavy-ion nucleus elastic scattering was that of Porter.[29] Since then the optical model has been successfully applied to heavy-ion scattering by many authors, the most extensive investigation probably being that of Kuehner and Almqvist.[6] A table of all heavy-ion optical-model parameters used until 1967 may be found in Ref. 2. Following Porter, real well depths of approximately 50 MeV have been used in most of these studies.

For the present analysis the optical-model code ABACUS II, developed by Auerbach,[30] was modified for the scattering of identical heavy ions and has been used exclusively in this study. Also a modified integration routine (with improved accuracy for heavy-ion large-KR calculations) was supplied by Auerbach and was incorporated. A routine was also added which calculates the Coulomb potential appropriate for two equally charged spheres of uniform charge density. This Coulomb potential, however, introduced very slight changes when compared with results predicted by using the potential appropriate to a point charge impinging on a uniformly charged sphere.

Although the applicability of ABACUS has already been tested for some cases of heavy-ion scattering,[31] studies were made to test the accuracy of the code for the present study. These tests included large variations in the integration mesh, changes in the integration routine (second-order difference equations replaced the first-order equations), checks on the Coulomb functions, and comparison with the results of other codes. In no case did any variation in the predicted cross section exceed 5%. This is well within the accuracy of the results to be presented in this experiment.

The relatively narrow ($\sim$200 keV) intermediate width structure observed in all five excitation functions was found to affect individual angular distributions strongly. Thus angular-distribution fits could not yield consistent parameter sets and were abandoned in favor of fitting energy averaged excitation functions at all five angles. Further, standard search code attempts to minimize χ^2 were found to converge on local minima which merely corresponded to small adjustments of the average level of the theoretical cross

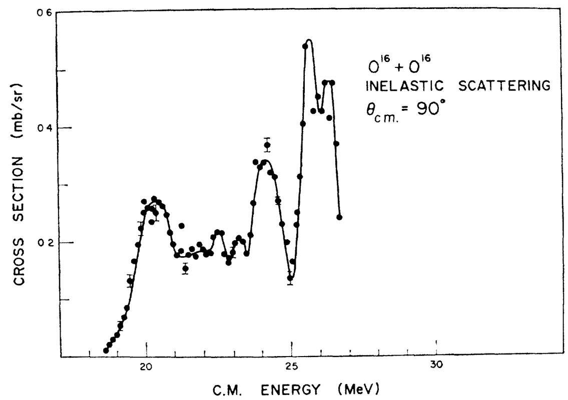

FIG. 8. 90° $O^{16}+O^{16}$ inelastic scattering, leaving either O^{16} nucleus in either the 6.05- or 6.12-MeV state.

section. These, furthermore, frequently distorted the correspondence of peaks and valleys in the predicted and experimental data. Parameter scans have thus been emphasized in this work.

In the initial analysis, a shallow potential of 17 MeV real well depth was found, which is very reminiscent of the molecular-type potentials suggested by various authors[5,32] for the heavy-ion-nucleus interaction. In spite of the known ambiguities associated with the heavy-ion optical model, rather extensive parameter scans failed to give other energy-independent potentials which would likewise reproduce the excitation functions at all angles. On the other hand, it was immediately noticed that any one angular distribution could be easily fit with more than one potential. From this it is evident that one can construct other potentials which will equally well fit the excitation function if only one allows for an energy dependence of the parameters. Indeed we find, as will be discussed below, that the 17-MeV potential is the shallowest member of a family of Woods-Saxon potentials with the same geometrical parameters but different real and imaginary well depths. Whereas the real well depth V of the 17-MeV potential is constant with energy, all other potentials have a rather complicated and steep energy dependence of V. Although all these potentials must presently be considered equivalent, future calculations, based on nuclear models, may predict the correct energy dependence for such a potential with sufficient accuracy to restrict, or possibly remove, the ambiguity.

The parameters of the 17-MeV potential are

$$V=17 \text{ MeV}, \qquad W=0.4 \text{ MeV}+0.1E_{\text{c.m.}},$$
$$R=6.8 \text{ F}, \qquad a=0.49 \text{ F}.$$

The potential is of the standard Woods-Saxon form

[29] C. E. Porter, Phys. Rev. **112**, 1722 (1958).
[30] E. H. Auerbach (unpublished).
[31] E. H. Auerbach and C. B. Porter, in *Proceedings of the Third Conference on Reactions between Complex Nuclei, Asilomar, Pacific Grove, California*, edited by A. Ghioiso. R. M. Diamond, and H. E. Congett (University of California Press, Berkeley, 1963), p. 19.
[32] E. Almqvist, D. A. Bromley, J. A. Kuehner, and B. Whalen, Phys. Rev. **130**, 1140 (1963); A. S. Kompaneets, Zh. Eksperim. i Teor. Fiz. **39**, 1713 (1960) [English transl.: Soviet Phys.—JETP **12**, 1196 (1961)].

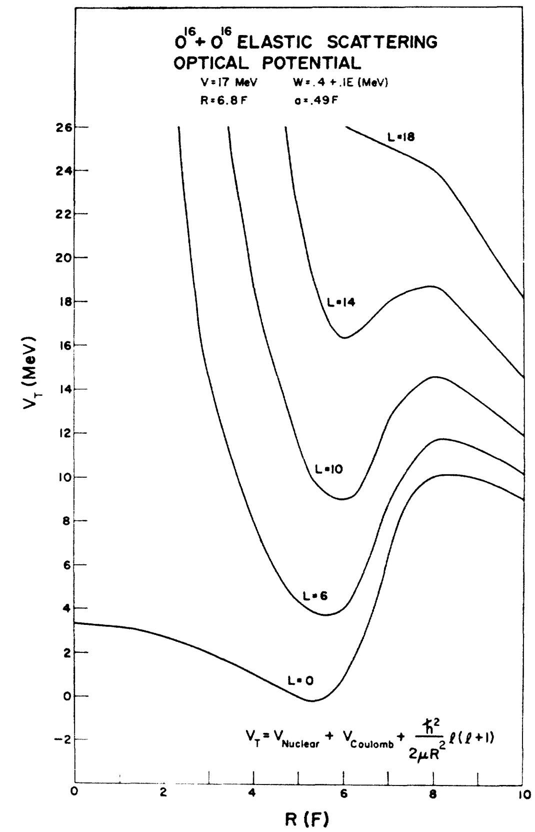

FIG. 9. Radial dependence of the real part of the optical potential for several partial waves.

with identical form factors for the real and the imaginary part. The radial dependence of the effective potentials for several partial waves is shown in Fig. 9, where the above potential and the Coulomb potential have been added to the appropriate barrier in each case. It is interesting to note that all but the S-wave potential are essentially repulsive. It should be noted that the above potential does not have bound states. It is not obvious, however, that the potential representing the interaction of two O^{16} nuclei should have eigenvalues corresponding to certain bound states of the S^{32} system.

The shallowness of the imaginary potential W, in particular at low energies, is somewhat surprising as it formally implies a long mean free path

$$\lambda=\frac{4.6}{-W}\left(\frac{E-V}{M}\right)^{1/2}$$

of the interpenetrating ions ($\lambda=4.85$ F at 15 MeV, 3.27 F at 30 MeV). As discussed below, the exact physical significance of this long mean free path remains open to question. Wildermuth and Carovillano[33] were the first to point out that because of the Pauli principle, the mean free path of complex particles in nuclear matter may be greatly extended. In the present investigation, the shallow imaginary potential is required, if the same geometry is used for the real and the imaginary potentials, to reproduce the excitation-function structure. A similarly shallow imaginary potential was obtained by Kuehner and Almqvist[6] in their analysis of the $C^{12}+O^{16}$ elastic scattering, but not for oxygen beams on N^{14} or Be^9 targets. The linear energy dependence of W found is not surprising, as one might expect the absorption to increase with increasing bombarding energy, as more degrees of freedom for the excitation of the compound system become available. Indeed Bassel *et al.*[34] find a much deeper imaginary well for the $C^{12}+O^{16}$ system at 168 MeV than did Kuehner and Almqvist at 10-MeV incident energy.

An energy dependence of the imaginary potential similar to that in the present investigation was recently found by Okuma[2] in the scattering of O^{16} from the boron isotopes.

Excitation functions and angular distributions calculated with the 17-MeV potential are compared with the data in Figs. 2 and 3.[35] The over-all agreement between the calculated and the measured cross sections is good, and, in particular, the agreement in shape between some of the calculated and the measured angular distributions is rather striking. The gross features of the excitation functions are seen to be well reproduced over the *entire* energy range from the Coulomb barrier up to 35 MeV (c.m.). A partial exception is the 80° excitation function at low bombarding energies. However, at these energies 80° corresponds to a diffraction minimum, and therefore the theoretical prediction for this angle may be less accurate.

The most marked deficiency of the 17-MeV potential is reflected in the fact that the calculated gross structure at 50° and 80° is out of phase with the data. We consider this shortcoming as significant, in that might well indicate that a potential other than the ordinary Woods-Saxon potential may be required, i.e., a cored or an l-dependent potential similar to the one used in α-He^4 scattering.[36] All attempts, however, to find such alternative potentials empirically failed. Introducing an energy dependence of the real well strength and uncoupling the geometrical parameters of the real, imaginary, and Coulomb potentials also yielded no improvement.

From Fig. 2, it is also apparent that the predicted 90° cross sections do not reproduce the experimental

[33] K. Wildermuth and R. L. Corovillano, Nucl. Phys. **28**, 636 (1961).

[34] R. H. Bassel, G. R. Satchler, and R. M. Drisko, Nucl. Phys **89**, 419 (1966).

[35] The scale of the optical-model cross sections in Fig. 1 of Ref. 18 was incorrect. Here the scale is presented correctly.

[36] S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. **41**, 247 (1969).

gross structure in magnitude beyond the first gross-structure peak. This is a direct consequence of the linear energy dependence of W, which was chosen to damp the 90° oscillations sufficiently beyond the third gross-structure peak. It is obvious that keeping W more or less constant between 15 and 39 MeV and increasing it then rather sharply with energy would produce the observed effects. Without further theoretical justification such attempts are in essence numerology and have not been further pursued in this study.

C. Ambiguities of the Potentials

Since the early work by Igo[37] on α-particle scattering, ambiguities in the optical potentials of complex particles have received considerable interest, the most recent investigation being that by Jackson and Morgan.[38] With one exception all these studies have concentrated on the ambiguities encountered in the analysis of data taken at one energy. As will be shown in this section, the energy dependences of the ambiguous potentials often differ from each other and thus may lead to a resolution of the ambiguities.

There are two types of ambiguities which are usually referred to as the continuous and the discrete ambiguities. It is today generally believed that the continuous ambiguities are due to insufficient data, as for instance, structureless angular distributions at incident energies close to the Coulomb barrier. Kuehner and Almqvist[6] report such ambiguities in their optical-model analysis of heavy-ion scattering, where for some cases their results appear to be totally insensitive to the interior of the potential well and only sensitive to the extreme surface potential shape. Igo found a similar result for α-particle scattering and has formulated a prescription for finding alternative potentials in two ways: First, the diffuse surface of the nuclear potential well may be replaced by an exponential function and, since the interior well depth seems unimportant, the appropriate exponential should duplicate the prediction of the original well. Second, this prescription reduces, for constant diffuseness a, to the requirement that given a potential characterized by parameters V_0 and R_0, any other potential will be equivalent if it has parameters V_1 and R_1 such that

$$V_1 \exp(R_1/a) = V_0 \exp(R_0/a). \qquad (6)$$

This has been tested in this study both by fitting exponential functions to the surface shape of the 17-MeV-deep potential (and truncating the corresponding effective wells at various depths) and by scanning R with small mesh for $V=40$ MeV and $a=0.49$ F. The latter approach is not obviously redundant since in some cases of the scattering of lighter projectiles it has been reported[39] that VR^2 is the quantity to be conserved rather than $Ve^{R/a}$. By scanning the parameter R from 4.5 to 6.84 in 0.1-F steps all such possibilities have been explored. No continuous ambiguity of the type reported by Igo or Kuehner has been found in this study. Additionally, no ambiguities have been found, discrete or continuous, excepting those predicted by the prescription of Drisko *et al.*,[40] which will be discussed below.

As discussed in the previous section, it is possible to construct alternative potentials to fit the excitation functions, if the parameters are allowed to vary with energy. To arrive at such alternative potentials the following procedure was used. Rather than searching on the experimental data the 17-MeV potential and the angular distributions calculated with this potential have been used as a starting point.

On the basis of Austern's work,[41] Drisko, Satchler, and Bassel[40] have formulated a prescription for finding alternative parameter sets to any particular potential for strongly absorbed particles. They calculate the WKB phase for the most sensitive partial wave and demand that all alternative potentials yield a phase different from this by an integral multiple of π. This is equivalent to demanding that the alternative potentials hold one or more additional half wavelengths. Thus, the S-matrix element

$$N_l = \exp[2iS_l(r_c)], \qquad (7)$$

where

$$S_l(r) = C_l + \int_{r_c}^{\infty} K_l(r)\,dr \qquad (8)$$

is the WKB phase and

$$K_l^2(r) = (2M/\hbar^2) \times [E_{\text{c.m.}} - V(r) - V_c(r) - iW(r) - (\hbar^2/2mR^2)l(l+1)] \qquad (9)$$

is the wave number, C_l is a constant independent of well depth, and r_c is the classical turning point of the Lth partial wave.

Drisko *et al.* contend that the interference between the waves reflected at the nuclear surface with those reflected at the centrifugal barrier is important even for cases such as those which Igo[37] or Kuehner[42] present where surface scattering seems to be the dominant effect and that therefore there are no continuous

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[37] G. Igo, Phys. Rev. Letters **1**, 72 (1958); Phys. Rev. **115**, 1665 (1959).

[38] D. F. Jackson and C. G. Morgan, Phys. Rev. (to be published).

[39] J. S. Nodvik, in *Proceedings of the International Conference on the Nuclear Optical Model, Tallahassee, 1959* (Florida State University Press, Tallahassee, Fla. 1959), p. 16.

[40] R. M. Drisko. G. R. Satchler, and R. H. Bassel, Phys. Letters **5**, 347 (1963).

[41] N. Austern, Ann. Phys. **15**, 299 (1961).

[42] J. A. Kuehner and E. Almqvist, in *Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, Pacific Grove, California*, edited by A. Ghioriso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 11.

TABLE I. Ambiguities of predictions of the optical potential; $V=17$, $W=0.4+0.1E$, $R=6.8$, $a=0.49$.

$E_{c.m.}$ (MeV)	$V+iW$ (MeV)	$V+iW$ (MeV)	$V+iW$ (MeV)	$V+iW$ (MeV)	$V+iW$ (MeV)
12.5	$22.25+i1.50$	$28.95+i1.24$	$36.58+i1.107$	$48.05+i7.40$	$54.55+0.93$
15	$23.61+i2.98$			$47.91+i3.89$	$58.22+i3.81$
175	$24.74+i3.12$	$33.20+i3.72$	$42.63+i4.06$		$64.31+i4.66$
20	$25.59+i3.02$	$35.07+i3.32$	$45.45+i3.72$	$50.11+i3.33$[a]	$68.71+i4.75$
22.5	$26.39+i2.88$	$36.71+i3.41$	$47.72+i4.09$	$59.53+i4.77$	$72.08+i5.37$
25	$27.50+i3.01$	$38.93+i3.92$[a]	$50.04+i5.07$[a]		$75.45+i5.79$[a]
27.5	$28.28+i3.19$	$40.60+4.19$[a]	$53.05+i5.32$[a]	$65.52+i5.79$[a]	$78.65+i5.79$[a]
30	$29.51+3.83$[a]	$41.18+i5.19$[a]	$53.68+i5.61$[a]		
32.5	$30.49+i4.58$[a]	$43.86+i6.27$[a]	$59.96+i6.61$[a]		$85.40+i6.51$[a]
35	$32.16+i6.01$	$44.17+i7.39$[a]	$57.78+i6.62$[a]		$88.47+i7.34$[a]

[a] Cases wherein no adequate reproduction of the synthetic data corresponding to the potential described in Sec. IV B could be obtained with fixed geometric parameters.

ambiguities. Rather, there are discrete ambiguities which may be found by holding the surface constant and deepening the well to satisfy the requirement that

$$\Delta S=S_l^n(r_c)-S_l^0(r_c)=N\pi. \tag{10}$$

Here S^n corresponds to the nth ambiguity in the well depth and S^0 to the original parameter set.

Using this WKB prescription, a set of well depths may be predicted at any given incident beam energy which, for the same potential radius and diffuseness, will reproduce the same WKB phase shift for any given partial wave. Within this framework, and selecting the most sensitive partial waves at four representative energies (from the transmission coefficients given by the 17-MeV potential), a series of alternative potential well depths has been calculated at each incident energy. These alternative well depths are plotted in Fig. 10. The index n in the figure labels each discrete ambiguity and indicates the integral multiple of π by which the most sensitive phase shift has been increased. (This sensitivity has been defined in terms of the energy rate of change of the transmission coefficient calculated for the individual partial waves.) The most striking observation from Fig. 10 is the close spacing in well depth of the ambiguous potentials. This readily explains why in a four-parameter search on angular distributions at a given energy almost any real potential well depth can be obtained with reasonable geometrical parameters.

The alternative potential strengths derived with the WKB approximation can only be considered as approximate values. Using these as the initial guess, we have searched on angular distributions which have been computed with the 17-MeV potential. (The search was done on synthetic cross sections rather than on the data in order to limit us to a study of the ambiguities associated with a given potential. Clearly the ambiguities connected with the fact that none of the predicted differential cross sections fit the data perfectly are not included in this investigation.) In these calculations, the search was restricted to varying V and W, the geometrical parameters R and a being identical to those of the 17-MeV potential. Excellent fits were obtained to the computed angular distributions except at the highest bombarding energies. The parameters obtained for the first five ambiguities are listed in Table I. Except for the 17-MeV potential, all other potentials have a rather complicated and steep energy dependence of the real well strength, with V increasing with increasing incident energy.

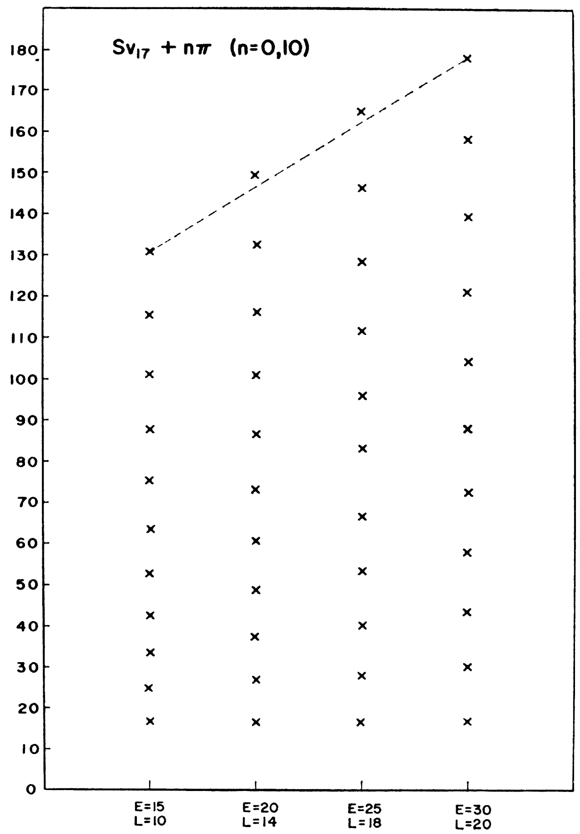

FIG. 10. Positions of the real well depths of ten potentials predicted to be equivalent to the 17-MeV potential [by the criterion of Drisko *et al.* (Ref. 40)] are indicated at each of four energies. The dotted line emphasizes the nonlinear energy dependence predicted. In practice, these predictions served only to indicate the approximate positions of alternative potentials. The best values of the alternative potentials found in this study are listed in Table I.

Another discrete ambiguity, closely related to the V, W ambiguity discussed above, has been observed by Kuehner and Almqvist[6,42] in the geometrical parameters R and a. A similar ambiguity was found in the initial phase of this study when it was observed that a radius of approximately 6.1 F gave equally good fits to the 90° excitation function. Extensive parameter scans over the two-dimensional region 6.0 F$\leq R \leq$8.0 F and 0.4 F$\leq a \leq$0.6 F have not yielded an alternative parameter set which fits all five excitation functions as well as does the R=6.8-F, a=0.49-F potential. It is possible, however, to find a variety of parameter sets which fit some combination of four excitation functions. Thus, it appears that the existence of five excitation functions limits the availability of this sort of discrete ambiguity drastically if not completely.

Very recently, Krubasik *et al.*[43] found in the analysis of the $B^{10}+O^{16}$ elastic scattering a continuous ambiguity in the imaginary potential. Readjusting the radius and diffuseness of the imaginary well, they were able to fit their data equally well with the imaginary potential varying anywhere from 2 to 282 MeV in depth. They therefore conclude that the shallow imaginary potentials obtained in many heavy-ion elastic scattering analyses are accidental and therefore not physically significant.

The results of Krubasik *et al.*[43] are attractive in that they could eliminate the apparent long mean free path of the heavy ions implied by the shallow imaginary potentials, which has been the source of some concern for a long time. Since Krubasik *et al.*[43] obtained their results from the analysis of rather structureless angular distributions, it was not obvious that they would be applicable for the present case, in which the angular distributions and excitation functions show pronounced diffraction patterns.

Searching on the synthetic cross sections obtained with the 17-MeV potential, however, we indeed find[44] the same ambiguity in W for the $O^{16}+O^{16}$ potential. It is also found that this ambiguity, in principle, may be resolved by appeal to the energy dependence of the calculated cross sections, although in the present instance the differences are too small to allow this distinction to be drawn with confidence. It must therefore be concluded that, for data which can be fitted by a Woods-Saxon potential, the well depth and geometry of the imaginary part of the potential are highly ambiguous and any conclusion depending upon the interior form of the radial wave functions of the lower partial waves must be considered ambiguous.

Recently, it has been proposed by Eck, LaSalle, and Robson[45] that the ambiguities in the heavy-ion potentials may be resolved by measuring and fitting excitation functions close to the Coulomb barrier, since at these energies the differential cross sections sensitively depend on the interference between the Coulomb and the nuclear amplitude. In view of the present investigation, it seems highly questionable that this method can resolve the discrete ambiguities discussed above, since the energy range covered by Eck *et al.* in their experiment on the $Mg^{24}+O^{16}$ scattering appears to be too limited for the energy dependence of the potentials to play a role.

D. Nuclear-Matter Considerations

Brueckner *et al.*[15] have calculated an $O^{16}+O^{16}$ interaction potential using the Brueckner theory of finite nuclear matter and have found both a shallow attractive well of 20 MeV depth and a roughly Gaussian repulsive core some 60 MeV in height and 1 F in half-width. They have performed a real phase-shift analysis of angular-distribution data in the vicinity of the Coulomb barrier employed in an attempt to demonstrate that only real phase shifts, and consequently only a real potential, are needed to reproduce the data. This implies neglect of all inelastic channels. Further, they argue that a potential with a repulsive core is superior to one with only a Woods-Saxon shape in terms of fitting a low-energy $O^{16}+O^{16}$ angular distribution and the $O^{16}+O^{16}$ 90° (c.m.) excitation function at energies below 14 MeV (c.m.).

It has been found, in this study, that the number of ambiguities in the potential increases sharply if only one excitation function is considered, particularly the flat 90° data below 14 MeV. Additionally, the reaction channel data for the $O^{16}+O^{16}$ system at low energies[4] indicates that absorption from the elastic channel is important at all energies above the Coulomb barrier; hence it is not obvious that the success of the Brueckner phase-shift analysis vitiates the need or usefulness of a complex potential. Indeed Chatwin *et al.*[46] have recently shown that the calculations by Brueckner *et al.* with a purely real potential are erroneous, and that good fits to the data at energies around the Coulomb barrier can be obtained with essentially the 17-MeV potential with slightly adjusted parameters.

Scheid, Ligensa, and Greiner[16] also apply the nuclear-matter theory to calculate an $O^{16}+O^{16}$ potential in many ways similar to that of Brueckner *et al.* They empirically fix the parameters of the nuclear-matter theory to give the correct binding energies and radii of S^{32} and O^{16} and then adjust the compressibility of nuclear matter to give the complex potential which

[43] E. Krubasik, H. Voit, E. Blatt, H. D. Helb, and G. Tschenko, Z. Physik **219**, 185 (1969).

[44] J. V. Maher, R. H. Seimssen, M. W. Sachs, A. Weidinger, and D. A. Bromley, in Proceedings of the Heidelberg International Conference on Heavy Ion Reactions, Heidelberg, 1969 (to be published).

[45] J. S. Eck, R. A. LaSalle, and D. Robson, Phys. Letters **27B**, 420 (1968).

[46] R. A. Chatwin, J. S. Eck, R. Richter, and D. Robson, Phys. Rev. (to be published).

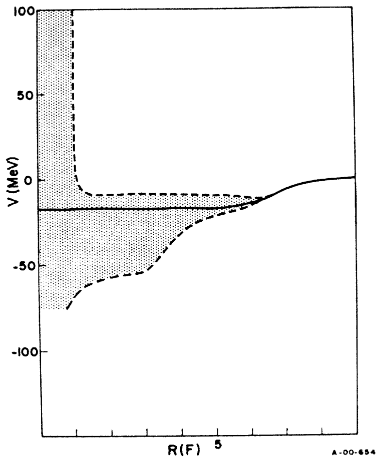

FIG. 11. Radial dependence of the 17-MeV Woods-Saxon real potential discussed in Sec. IV B is shown as a solid curve. The shaded area indicates the region wherein the Woods-Saxon well can be deformed without appreciably affecting the cross-section predictions of the potential.

best fits the $O^{16}+O^{16}$ 90° (c.m.) excitation functions obtained here in the energy range below 30 MeV (c.m.). (This result is heavily dependent upon the imaginary part of the complex potential, however, and the determination of the imaginary potential is only approximate.) Scheid *et al.* obtain a nuclear compressibility of approximately 200 MeV, a value which is in good agreement with the results obtained from previous theoretical investigations,[47] which give compressibilities between 100 and 300 MeV. (See, however, Migdal,[48] who finds that a value of the compressibility of ~900 MeV is required in his calculations on finite Fermi systems.) Since none of the previous indirect determinations of this important parameter are considered very accurate, this independent method by Scheid *et al.* of extracting the nuclear compressibility from heavy-ion scattering data may prove very valuable.

The potential derived in the investigation by Scheid *et al.*[16] must again be considered as possibly ambiguous since only one excitation function has been considered. The amount of structure involved and reproduced, is, however, considerably greater than in the low-energy study of Brueckner *et al.*

Block and Malik[19] carried out a simple one-dimensional calculation on the $O^{16}+O^{16}$ results for purposes of orientation and a more extensive calculation utilizing an empirically derived interaction potential incorporating a repulsive-core region to reflect Pauli principle considerations in the overlap of the interacting O^{16} nuclei. Here again the actual data fitting has been restricted to sections of the 90° excitation function reported herein, but their conclusion is also that a quite shallow real potential well is required by the data.

All of the above arguments agree in indicating that the $O^{16}+O^{16}$ potential should be shallow or repulsive depending upon the individual partial wave involved. At first sight this would appear in direct conflict with the now well-understood optical potentials for light projectiles where the real potential depths are given approximately by $50N$ MeV, where N is the number of nucleons in the projectile. In view of the saturation of nuclear forces, it would not be anticipated that this simple dependence could be directly extrapolated to heavier projectiles yet it is surprising that such a shallow well as 17 MeV seems preferred.

Some relevant information is obtainable from the relatively detailed optical-model studies on α-particle scattering. In that case, shallow potentials have also been found, however; they have customarily been rejected in favor of the deeper wells (150–250 MeV). There are three factors, however, which suggest that this rejection is not unambiguous.

First, one major reason for favoring deep α-particle potentials stems from the need for a cutoff radius in DWBA calculations using shallow wells. Although it is true that deep potentials do not generally need such a cutoff, increased oscillation of the optical wave function in the interior of the well, induced by the deeper well, may simply introduce an effective cutoff without these interior wave functions having a greater physical significance. Second, McFadden and Satchler[49] had difficulty in finding satisfactory deep wells for α-particle scattering from light nuclei. And finally, it has long been held (although with some controversy) that the scattering of α particles by He^4 requires a very shallow and l-dependent potential with a repulsive core.[36]

E. Sensitivity of $O^{16}+O^{16}$ Potentials to Inclusion of a Core

Since the theoretical approaches described above all predict a repulsive core for the $O^{16}+O^{16}$ interaction potentials, it is of interest to study the sensitivity to such a core of the differential cross sections, *calculated* with the 17-MeV potential $[V+iW=17\text{ MeV}+i\ (0.4\text{ MeV}+0.1E)]$. As discussed above, nothing can be inferred about the interior of the nucleus if the data are fit by a complex Woods-Saxon potential because of the ambiguity in W.[50] Only if there are effects which cannot be described by the Woods-Saxon potential can it be hoped to deduce anything about the interior.

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[47] E. Feenberg, Phys. Rev. **149**, 593 (1941); D. S. Falk and L. Wilets, *ibid.* **124**, 1887 (1961); K. A. Brueckner, Rev. Mod. Phys. **30**, 561 (1968); L. R. B. Elton and A. Swift, Proc. Phys. Soc. (London) **84**, 125 (1964); R. A. Uher and R. A. Sorenson, Nucl. Phys. **86**, 1 (1966); T. Dahlblom as referenced in H. A. Bethe, Phys. Rev. **167**, 879 (1968).

[48] A. Migdal, *The Theory of Finite Fermi Systems and the Properties of Nuclei* (Wiley-Interscience, Inc. New York, 1968).

[49] L. McFadden and G. R. Satchler, Nucl. Phys. **84**, 177 (1966).

[50] J. A. McIntyre, S. D. Baker, and T. L. Watts, Phys. Rev. **116**, 1212 (1959).

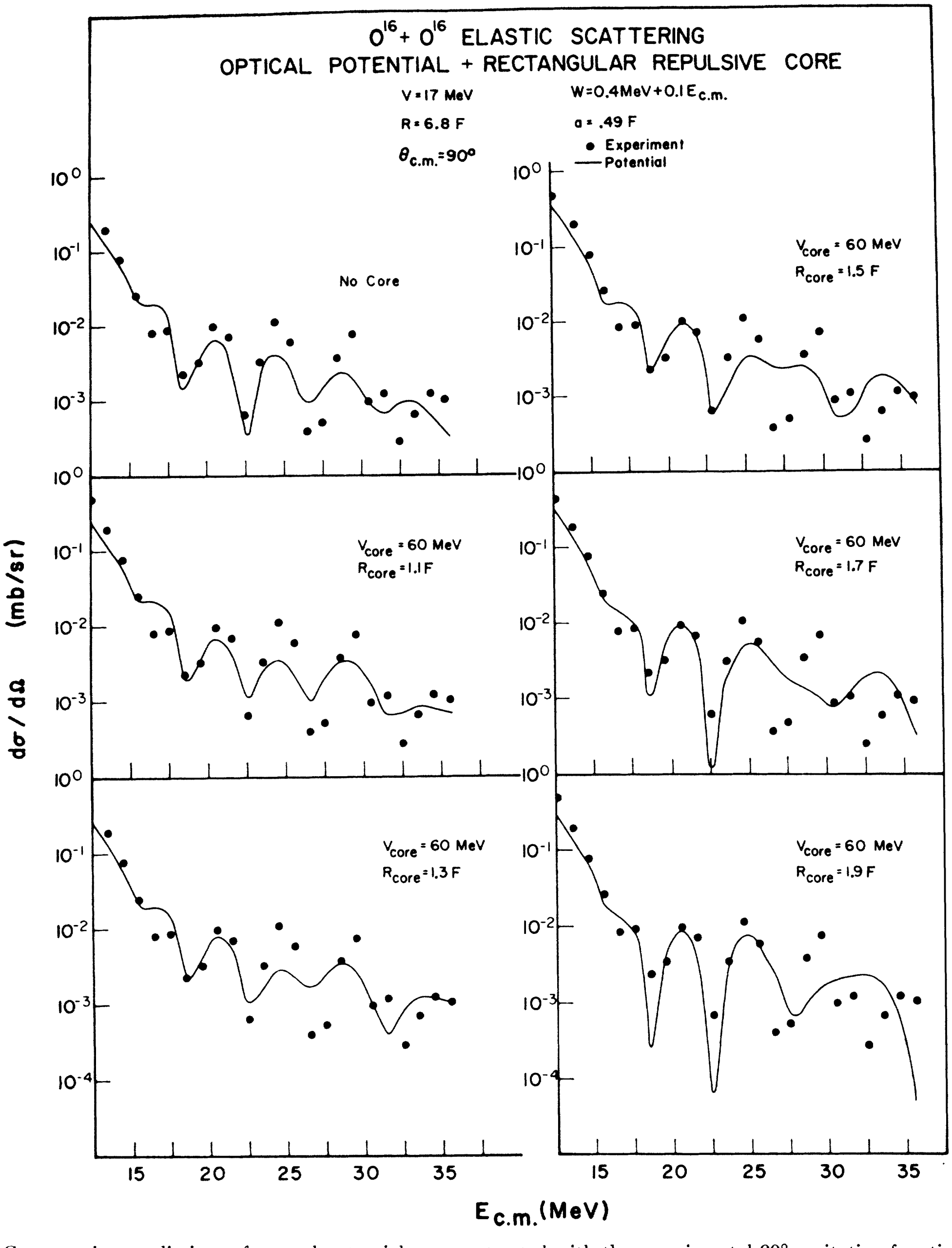

FIG. 12. Cross-section predictions of several potentials are contrasted with the experimental 90° excitation function. In each case, the 17-MeV Woods-Saxon real potential is used but 60-MeV repulsive barriers of various radii have been added as indicated. The criterion discussed in Sec. IV D judges the 1.1-F core to be not significantly different from the 17-MeV Woods-Saxon potential, although small differences can be seen. The dividing line shown in Fig. 11 falls at 1.2 F between the 1.1-F and the 1.3-F cases shown above.

The 17-MeV potential with the shallow imaginary well has been chosen for the investigation to be discussed below, since it was felt that this potential is most sensitive to the addition of a repulsive core.

This sensitivity test was performed by gradually distorting the 17-MeV real potential (i.e., adding rectangular, trapezoidal, or exponential wells or cores to the Woods-Saxon well of larger radius). The calculated excitation functions were compared visually with the predictions of the pure Woods-Saxon well, and subjective decisions were made to classify the distorted potential as equivalent or significantly different from the Woods-Saxon well.

In general, an appreciable effect was found only when cores or additional wells were added which placed part of the interior of the potential outside the shaded

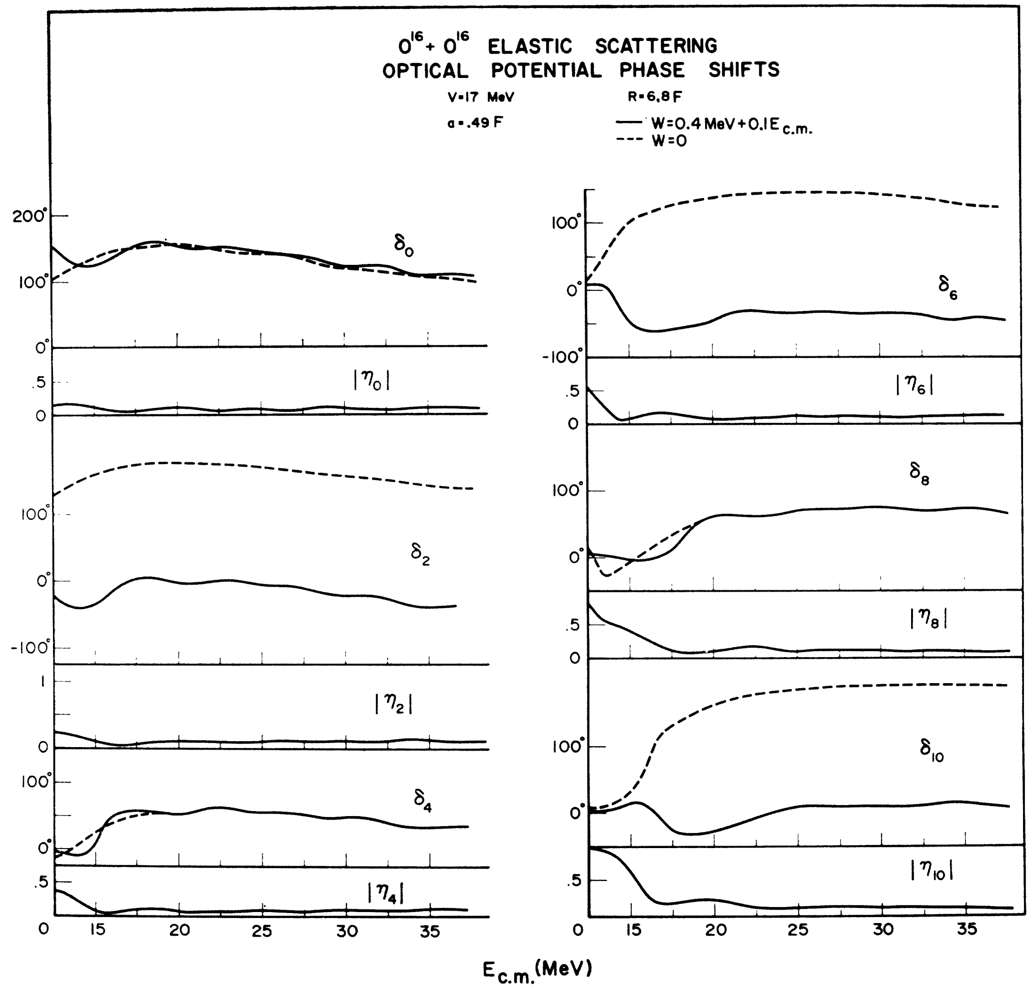

FIG. 13. Optical-model phase shifts and partial-wave amplitudes are shown as a function of energy for even partial waves from L=0, 10. The solid curves represent the predictions of the complex potential discussed in Sec. IV B. The dotted curves are predicted by the same real potential with the imaginary potential set to zero.

area shown in Fig. 11. Thus a repulsive core must have a radius greater than ~1.3 F in order to be detectable in $O^{16}+O^{16}$ scattering data.

The effects obtained by adding a repulsive core to the 17-MeV potential are illustrated in Fig. 12. Here excitation functions have been calculated with a rectangular repulsive core of 60 MeV, with several different radii, and the calculated excitation functions have been compared with the data. The only partial waves affected by a 60-MeV barrier of radius 1.5 F are the $L=0$, 2, 4, and 6 waves.

Altogether we find that the 17-MeV O^{16} potential is surprisingly sensitive to the inclusion of a repulsive core. The present investigation does not determine whether or not there are effects produced by the repulsive core which cannot equally well be obtained by an ordinary Woods-Saxon potential with a reasonable energy dependence, i.e., whether or not there are effects which are characteristic of the repulsive core only. Any attempt, however, to obtain significantly improved fits to our data with a cored potential failed.

F. Phase-Shift Analysis

As discussed in the Introduction, it was one aim of this investigation to perform a phase-shift analysis on the $O^{16}+O^{16}$ elastic scattering data similar to that done by Singh *et al.*[17] on α-particle scattering in order to search for possible underlying resonances. To this end 28 angular distributions were measured in the c.m. energy range from 19 to 22.5 MeV.

The phase-shift fitting was performed by using the variable metric minimization routine DAVIDON[51] to minimize the sum of squares of deviations between the data of a given angular distribution and the differential cross-section predictions of the phase-shift expansion [Eqs. (1) and (2)].

It is well known that phase-shift parametrizations are highly ambiguous in general. This problem is accentuated in the case of $O^{16}+O^{16}$ scattering, since

[51] Davidon, Argonne National Laboratory Report No. ANL 5990, 1959 (unpublished).

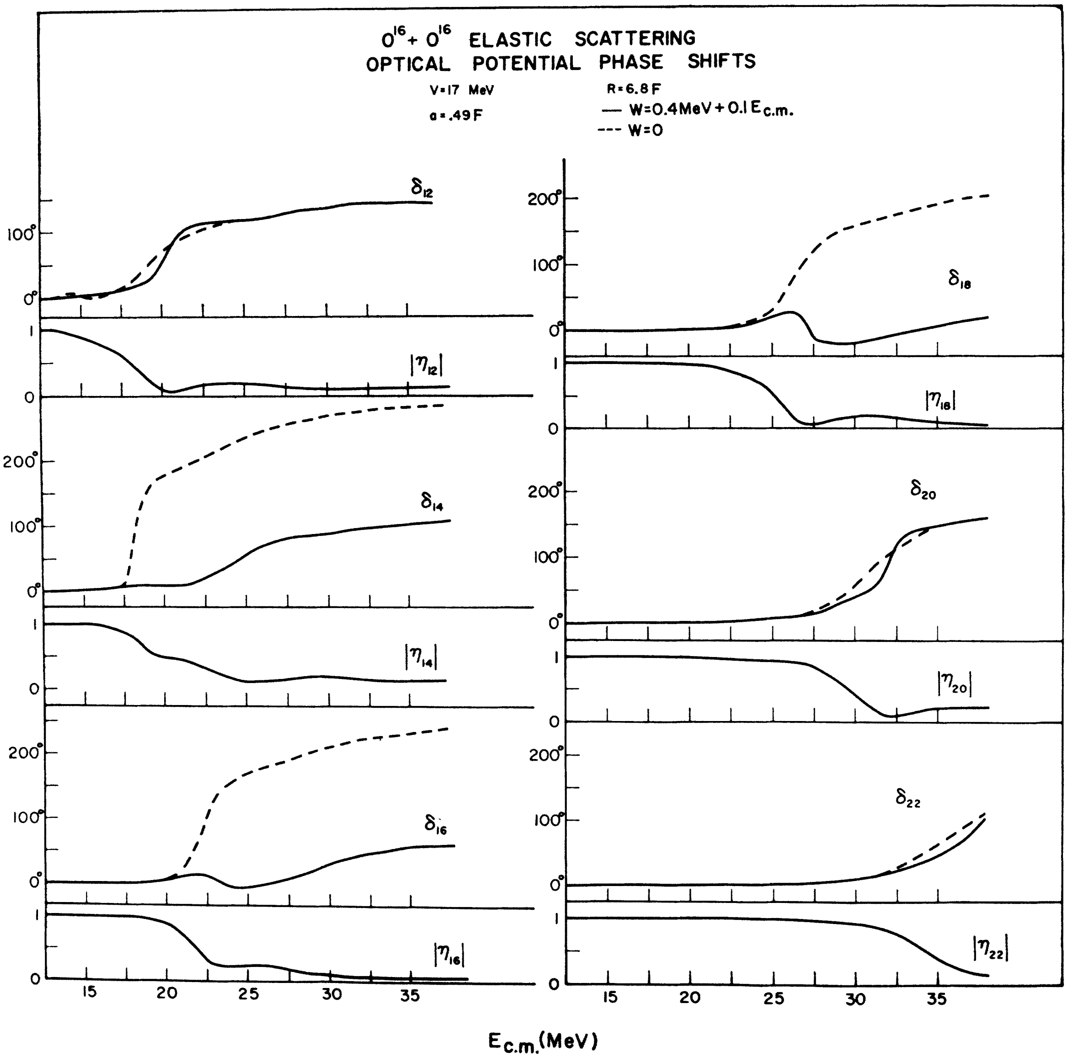

FIG. 14. Optical-model phase shifts and partial-wave amplitudes are shown as a function of energy for even partial waves from $L=12$, 22. The solid curves represent the predictions of the complex potential discussed in Sec. IV B. The dotted curves are predicted by the same real potential with the imaginary potential set to zero.

in the energy range involved in this part of the study (19–22 MeV c.m.) partial waves up to $l=28$ produce non-negligible contributions to the cross section. Since fitting with one partial wave involves two parameters ($S_l=\eta_l e^{2i\delta l}$) this allows 30 possible parameters (only the even partial waves contribute), and ambiguities are possible in any phase-shift set with so many parameters. For this reason, physical intuition must be utilized to limit the number of possibilities involved.

There are several considerations involved in choosing the approaches taken in this work. First, careful attention must be given to the choice of a background set of parameters, i.e., the fixed values of the relatively insensitive partial waves. Second, insight from the potential scattering considerations can be useful in choosing the most sensitive partial waves.

Two sets of background parameters were used. The phases δ_l and amplitudes η_l corresponding to the selected optical potential discussed above [$V+iW=17+i(0.4+0.1E)$] provided the most useful background parameters. An energy dependence was incorporated in this set, since the optical potential obviously predicts slight changes in most parameters with changing energy and large changes in only a few parameters.

An alternative set of background parameters was obtained by searching on all parameters simultaneously starting at a c.m. energy of 19.0 MeV with the optical-model parameter set appropriate to that energy. The parameter set resulting from that search was then used as a starting set for a search on all parameters at 19.125 MeV and so on up to 22.375 MeV. It was found that a smooth set of parameters could be obtained in this way and that all ~200 keV width structure in the angular distributions shown in Fig. 4 can be described in terms of small variations of a large number of parameters about this set. No systematic energy dependence is involved in this set and the set has not proved any significant improvement over the optical parameters as a set of background parameters.

Figures 13 and 14 show the phase shifts and amplitudes of the diagonal S-matrix elements predicted as a function of energy by the 17-MeV optical potential described above. It is evident from these plots that only the $l=10$ and $l=14$ waves change appreciably in the energy range of this investigation, $l=14$ resonating at an energy slightly above the range of this study, and $l=10$ damped by the imaginary part of the complex potential but resonating at $E\cong 16$ MeV for the same real potential with $W=0$. (As Figs. 13 and 14 indicate, the phase shifts in several partial waves do, at least formally, resonate. None of these resonances, however, appears at an energy where its corresponding amplitude η_l is sufficiently large to warrant attributing any of the excitation-function gross structure solely to the resonance. Rather, it is felt that the gross structure arises from the effects of a number of partial waves, thus being a diffraction phenomenon.) The foregoing indicates that $l=10$ and 14 are sensitive parameters in this energy region and they have been emphasized in this phase-shift analysis. Additionally, from the partial-wave amplitudes shown in Figs. 13 and 14 it is clear that only the $l=10$, 12, 14, and 16 waves have transmission coefficients significantly different from 0 or 1. The higher partial waves have not yet felt the nuclear potential at these energies and so are not promising candidates for analysis. The low partial waves have been almost totally absorbed, and although the study of sensitivity to a central core discussed above has revealed that variations in the low partial waves can affect the differential cross section, preliminary studies have indicated that typically fitting on these parameters resulted in very small changes with no obvious systematic trend. For this reason, the major emphasis in this study has been restricted to searching primarily on the $l=10$, 12, 14, and 16 waves and, to a much lesser extent, on the $l=8$ and 18 waves.

Several sets of background phases and several combinations of search parameters have been used in extensive phase-shift calculations. *In no case has any partial wave resonated.* There are also several sets of final parameter values which give equally good results.

G. Statistical Considerations

With the failure of the phase-shift analysis, the ~200-keV width structure discussed above remains as possibly a consequence of statistical fluctuations. There is additional structure which is also presumed to arise from statistical compound-nuclear effects; this is evident in the data shown in Fig. 7. Autocorrelation functions were calculated for this data for a variety of averaging intervals. The coherence width was found to be between 50 and 100 keV, while the compound-nuclear contribution lies between 1 and 3%. The uncertainty in these results arises from counting statistics and difficulties in averaging the background contributions to the cross sections. The results of this analysis are in agreement with, but less accurate and detailed than, those of Vandenbosch *et al.*[21] in their study of statistical effects in this same $O^{16}+O^{16}$ system.

V. CONCLUSION

The $O^{16}+O^{16}$ elastic scattering excitations functions have been shown to exhibit three types of structure. The broad strong structure has been qualitatively reproduced with a complex potential. This potential fit encompasses an exceptionally large range of angle, energy, and structure and represents one of the most striking successes of the potential model in heavy-ion studies. The narrowest width structure represents a compound statistical contribution to the cross section. The ~200-keV structure has not as yet received an adequate explanation.

The interaction potentials derived phenomenologically from these data have stimulated extensive theoretical study of heavy-ion scattering mechanisms from a more fundamental viewpoint than has previously been the case. These studies indicate that these, and similar heavy-ion studies may provide unique information on the nuclear-matter problem; in particular, Scheid *et al.*[16] have reported extraction, for the first time, of an experimental value of the nuclear compressibility, appropriate to the O^{16} system, from these data. This determination is entirely independent of previous methods of determining this important constant of the nuclear-matter theory.

A continuing and systematic study of identical-particle elastic scattering is underway in this laboratory with the goal of exploiting the new insight which has been gained in this work. In particular, current work on the $C^{12}+C^{12}$ and $N^{14}+N^{14}$ systems will examine the effects of nuclear static deformations and of possible loose cluster structure on the scattering interaction; attempts will be made to examine the variation of the nuclear compressibility in moving away from the closed-shell structure of O^{16}.

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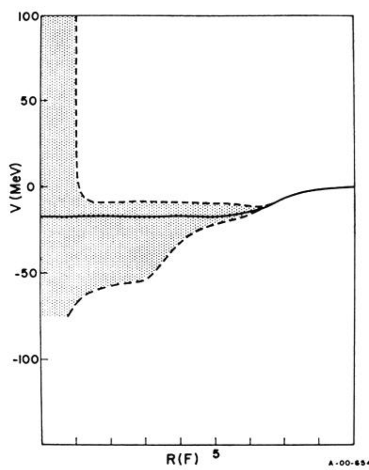

FIG. 11. Radial dependence of the 17-MeV Woods-Saxon real potential discussed in Sec. IV B is shown as a solid curve. The shaded area indicates the region wherein the Woods-Saxon well can be deformed without appreciably affecting the cross-section predictions of the potential.