Theory of Partially Conserved Axial-Vector Current and Mesonic-Exchange Effects in Nuclear Beta Decay

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The size of mesonic-exchange effects in nuclear β decay are compared with the value given by the theory of partially conserved axial-vector current in the light of new experimental data. It is concluded that, allowing for the experimental and theoretical uncertainties, there is no violent disagreement.

INTRODUCTION

In an earlier paper,[1] by assuming partially conserved axial-vector current (PCAC) theory, a relationship was obtained between the phenomenological meson-exchange operator in β decay and the two-body pion-production operator. However, an analysis of the process $p+p\to d+\pi^+$ in terms of this operator was found to lead to exchange effects inconsistent with the experimental data for the β decay of ^{3}H. Since the publication of the paper[1] new experimental data have become available on the two-body pion-production process[2] and on the lifetime of the neutron,[3] both of which influence the conclusions reached in Ref. 1.

NEW EXPERIMENTAL DATA

In terms of the amplitudes $a(^1S_0)$ and $a(^1D_2)$ for p-wave pion production near threshold from initial $^1S_0$ and $^1D_2$ diproton states, the result of Ref. 2 can be written[4]

$$|a(^1S_0)|^2+|a(^1D_2)|^2=2.02\pm0.80 \text{ mb}. \quad (1)$$

This is to be compared with the relation

$$|a(^1S_0)|^2+|a(^1D_2)|^2=4.1\pm0.6 \text{ mb} \quad (2)$$

used in Ref. 1.

The new measurement of the lifetime of the neutron[3] leads to $ft=1120\pm21$ sec, after including electromagnetic radiative corrections as given by Berman and Sirlin.[5] This is to be compared with the previous value[6] $ft=1213\pm35$ sec. The corresponding figure for the ^{3}H decay is $ft=1173\pm20$ sec. Comparing these two ft values then gives for the square of the axial-vector matrix element for the ^{3}H decay (see Blin-Stoyle[7] for an earlier analysis)

$$|M_A|^2_{\text{expt}}=2.83\pm0.14. \quad (3)$$

Theoretically, we can write

$$|M_A|^2_{\text{theor}}=|M_A{}^0|^2(1+\delta_{\text{ex}})^2, \quad (4)$$

where $M_A{}^0$ is the axial-vector matrix element calculated using the usual single-particle β-decay operator $\sum\sigma\tau_i{}^\dagger$ and whose value is well determined in terms of the percentage of D state $[p(D)]$ and mixed-symmetry S' state $[p(S')]$ in the three-body wave function. δ_{ex} represents the enhancement of M_A due to exchange effects.

In Table I, values of $|M_A{}^0|^2$ are given on the basis of various assumptions about $p(D)$ and $p(S')$ together with the value of δ_{ex} necessary to reconcile theory and experiment.

The minimum percentage of D state is probably around 6% (e.g., Blatt and Delves[8]), and recent calculations by Delves *et al.*[9] suggest that $p(D)\approx9\%$ and $p(S')\approx2\%$. Thus, although δ_{ex} may be zero or even just negative, it could be as high as $+10\%$.

INTERPRETATION OF NEW DATA

In order to estimate the magnitude of exchange effects from the new data of Rose,[2] it is necessary to combine it with data from polarization measurements near threshold in order to determine a_S and a_D separately. The only experimental polarization measurements near threshold are those of Crawford and Stevenson,[10] and taking their result at face value and combining them with the Rose[2] data gives

$$|a_S|=(0.28\pm0.13) \text{ mb}^{1/2},$$
$$|a_D|=(1.55\pm0.9) \text{ mb}^{1/2},$$

to be compared with

$$|a_S|=(0.60\pm0.20) \text{ mb}^{1/2},$$
$$|a_D|=(1.93\pm0.10) \text{ mb}^{1/2},$$

used in Ref. 1.

[1] R. J. Blin-Stoyle and Myo Tint, Phys. Rev. **160**, 803 (1967).
[2] C. M. Rose, Jr., Phys. Rev. **154**, 1305 (1967).
[3] C. J. Christensen, A. Nielsen, A. Bahnsen, W. K. Brown, and B. M. Rustad, Phys. Letters, **26B**, 11 (1967).
[4] We take the result obtained by fitting the low-energy data since our concern here is with π production near threshold.
[5] S. M. Berman and A. Sirlin, Ann. Phys. (N. Y.) **20**, 20 (1962).
[6] C. P. Bhalla, Phys. Letters **19**, 691 (1966).
[7] R. J. Blin-Stoyle, Phys. Rev. Letters **13**, 55 (1964).
[8] J. M. Blatt and L. M. Delves, Phys. Rev. Letters **12**, 544 (1964).
[9] L. M. Delves, J. M. Blatt, C. Pask, and B. Davies, Phys. Letters **28B**, 472 (1969).
[10] F. S. Crawford, Jr., and M. L. Stevenson, Phys. Rev. **97**, 1305 (1955).

These new values for $|a_S|$ and $|a_D|$ make little difference to the size and magnitude of δ_{ex}, which is still negative with a magnitude of at least 5% and, therefore, still in disagreement with experiment, although not so violently.

In reconsidering this maintained discrepancy, two points must be stressed. First, the theoretical result for δ_{ex} depends critically on the results of the polarization experiment of Crawford and Stevenson.[10] The measurements are difficult and were carried out 14 years ago. Obviously, they should be repeated. Second, the theoretical value for δ_{ex} rests heavily on the values taken for the contributions to a_S and a_D from the one-body pion-production operator. It will be recalled that one can write

$$a(^1S_0)=[b(^1S_0)+c(^1S_0)]e^{i\tau_0}\eta^{3/2}\ \mathrm{mb}^{1/2},$$

$$a(^1D_2)=[b(^1D_2)+c(^1D_2)]\eta^{3/2}\ \mathrm{mb}^{1/2},$$

where b and c refer to one- and two-body pion production, respectively, η is the center-of-mass pion momentum, and τ_0 is a phase factor. In Ref. 1, $b(^1S_0)$ and $b(^1D_2)$ were taken from the work of Woodruff[11] to have the values

$$b(^1S_0)=0.24\ \mathrm{mb}^{1/2},\qquad b(^1D_2)=0.86\ \mathrm{mb}^{1/2}.$$

These values were obtained by using the Gammel-Thaler potential[12] to obtain the diproton wave function and the Gartenhaus potential[13] to obtain the deuteron wave function. Clearly, considerable uncertainties exist, for example, as far as the percentage of D state in the deuteron is concerned and particularly in respect to high-momentum components in the deuteron wave function which give the greatest contribution to the one-body matrix element. In this connection, it is interesting to note that in an earlier calculation which used the Gartenhaus potential[13] for both the diproton and the deuteron, Lichtenberg[14] obtained

$$b(^1S_0)\approx 0,\qquad b(^1D_2)\approx 1.55\ \mathrm{mb}^{1/2}.$$

These values lead to $\delta_{ex}\approx +10\%$.

[11] A. E. Woodruff, Phys. Rev. **117**, 1113 (1960).
[12] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).
[13] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[14] D. B. Lichtenberg, Phys. Rev. **100**, 303 (1955).

TABLE I. Values of $|M_A{}^0|^2$ for given values of $p(D)$ and $p(S')$, with corresponding values of δ_{ex}.

$p(D)$ (%)	$p(S')$ (%)	$\|M_A{}^0\|^2$	δ_{ex} (%)
4	0	2.84	$+0.2\pm2.5$
6	0	2.76	$+1.2\pm2.5$
9	2	2.47	$+7.0\pm2.7$

In summary then, the uncertainties in the experimental polarization data and in the detailed nature of the internucleon potential mean that the discrepancy between the experimental value for δ_{ex} as set out in Table I and between δ_{ex} determined from the Woodruff values of $b(^1S_0)$ and $b(^1D_2)$ should not be regarded as serious. Further, recent work by Cheng and Fischbach[15] shows that the assumption of a simple Yukawa form for the radial dependence of the meson-exchange operator and the pion-production operator as in Ref. 1 is too restrictive. In particular, more singular terms which can contribute to the $p+p\to d+\pi^+$ process may not contribute to the β-decay exchange process, thus enabling agreement between theory and experiment to be achieved. Even so, there are uncertainties still outstanding, and to proceed further it is necessary to (a) remeasure the neutron ft value in order to be sure of the magnitude of δ_{ex},[16] (b) remeasure the polarization phenomena in the process $p+p\to d+\pi^+$ near threshold, and (c) repeat the various calculations when more definite information becomes available about the fine details of the ^{3}H and ^{2}H wave functions.

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[15] W. K. Cheng and E. Fischbach, preceding paper, Phys. Rev. **186**, 1530 (1969).
[16] There no longer seems to be so much doubt about the end-point energy for the ^{3}H β decay which is usually taken to be $E_0=18.61\pm0.1$ keV [F. T. Porter, Phys. Rev. **115**, 450 (1959)]. For example, a recent measurement by Bergkvist [K. E. Bergkvist, in *Proceedings of the Topical Conference on Weak Interactions*, (CERN, Geneva, 1969), and private communication] gives $E_0=18.63\pm0.05$ keV in good agreement with Porter's value.