approximate value of $\text{Re}F_0(q,E)$ has to be determined by interpolation. An example for the conversion of exact cross sections into real parts of the form factors is given in Fig. 4 for $_{38}\text{Sr}^{88}$. The result is compared with the corresponding MIA curve obtained from Eq. (57) and with the Born-approximation form factor.

VII. SUMMARY

A model-independent analysis of inelastic electron scattering from nuclei can only be performed if the analytical behavior of the Coulomb corrections is known. This behavior has been discussed earlier[1] in the case of low momentum transfer and for monopole as well as quadrupole transitions. These results have been obtained in a second Born approximation with the additional assumption that the charge distribution in the nuclear ground state can be described by a single parameter, the rms radius. In the case of high momentum transfer, this assumption is much too restrictive.

A new method of deriving model-independent expressions for the Coulomb corrections is discussed in this paper It is shown that this new method, which can incorporate more details of the ground-state charge distribution, leads to results similar to those obtained in Ref. 1 for low momentum transfer. The purpose of this method, however, is to explain the analytical behavior of the form factors at high momentum transfer as well.

The most obvious feature of the plane-wave Born approximation at high momentum transfer is the occurrence of zeros in the cross sections at certain values of the momentum transfer q. It has been observed[3,4] that these zeros are filled up and that their position is shifted towards smaller angles in exact results. This behavior is understood in our present method by observing that the Born approximation form factor has to be replaced by a complex form factor if the distortion of the electron wave functions is taken into account. The real part of this redefined form factor is responsible for the shifting of the zeros, and the imaginary part accounts for the filling up.

The method presented in this paper offers a possibility of determining the Coulomb corrections in a model-independent way, but it can also be used to calculate the corrections for any given nuclear model without extensive numerical procedures. Since this method follows closely the formalism of the exact partial-wave calculations, its results as well as its secondary assumptions can be checked step by step by comparison with exact results. It is for the same reason that this method can be generalized to other multipole orders and to magnetic transitions.

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Hypothesis of Partially Conserved Axial-Vector Current and Meson-Exchange Effects in $p+p\to d+\pi^+$ and $\text{H}^3\to\text{He}^3+e^-+\bar{\nu}_e$

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The hypothesis of partially conserved axial-vector current (PCAC) is applied to relate the threshold production amplitude for the process $\alpha\to\beta+\pi^+$ to the meson-exchange contributions in $\alpha\to\beta+e^-+\bar{\nu}_e$, where α and β are nuclear states. The latter contributions are calculated by assuming that the two-nucleon β-decay process $N_i+N_j\to N_i'+N_j'+e^-+\bar{\nu}_e$ is dominated by the one-pion-exchange contribution. Particular attention is given to the radial dependence of the effective two-body operator which is used to describe both $p+p\to d+\pi^+$ and the meson-exchange effects in $\text{H}^3\to\text{He}^3+e^-+\bar{\nu}_e$. It is shown that terms as singular as $x^{-2}e^{-x}$ and $x^{-3}e^{-x}$ can arise naturally in a dynamical (as opposed to a phenomenological) treatment of meson-exchange effects, and that these terms contribute significantly to $p+p\to d+\pi^+$ but not at all to $\text{H}^3\to\text{He}^3+e^-+\bar{\nu}_e$. Some discussion of the present experimental status of these two processes is also given.

I. INTRODUCTION

It is well known that, within the usual impulse approximation, the $V-A$ theory of weak interactions has been extensively tested with impressive success in nuclear β-decay and muon-capture processes.[1,2] To achieve a greater degree of quantitative understanding of these phenomena, it is then natural to begin to study the contributions from meson-exchange effects. Undoubtedly this study will provide an oppor-

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tunity for investigating in more detail questions concerning the validity of the conserved vector-current (CVC) hypothesis[3] and the partially conserved axial-vector-current (PCAC) hypothesis[4] in a realm where they have thus far not been tested extensively. In this paper, we address ourselves to a discussion of meson-exchange effects in nuclear β decay. Recently, one of the authors (WKC) proposed a theory of meson-exchange effects in nuclear β decay based on CVC and PCAC.[5] In this theory some insight was obtained into (1) the expression of meson-exchange mechanisms in terms of experimentally well-established baryon and meson-exchange mechanisms in terms of experimentally well-established baryon and meson resonances, and (2) the detailed structure of the most general phenomenological two-body operator in β decay. When applied to triton β decay, $H^3 \to He^3 + e^- + \bar{\nu}_e$, this theory gave an *enhancement* of approximately 10% (due to meson-exchange effects) to the Gamow-Teller matrix element $\langle He^3 | \boldsymbol{\sigma} | H^3 \rangle$ which is in good agreement with the experimentally determined value of $\sim$8%. In the triton β decay the following consequences of the theory, as will be explained below, are important for our discussion. (1) The radial dependence of the exchange operator is given by a Yukawa form ($\sim \Gamma x^{-1} e^{-x}$, where Γ is a phenomenological strength parameter, $x = \mu r$, μ = pion mass, and $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is the internucleon distance) in contrast to the phenomenological theories previously suggested[6] in which this Yukawa-type radial dependence is a basic *assumption*. (2) The meson-exchange contribution arising from the $N^*(\frac{3}{2}^+, \frac{3}{2})$ π-N resonance rescattering in the intermediate states is identically zero.

Recently, Blin-Stoyle and Tint[7] (BT) have proposed a PCAC theory of low-energy strong pion production in which a relation is obtained between the two-body pion-production operator and the phenomenological meson-exchange operator in β decay. Using this relation, they calculated the meson-exchange effects in $H^3 \to He^3 + e^- + \bar{\nu}_e$ by use of the (near threshold) $p + p \to d + \pi^+$ experimental data and obtained a *reduction* of $\sim$5–25% to $\langle He^3 | \boldsymbol{\sigma} | H^3 \rangle$. The finding of a reduction (and not the expected enhancement) might lead one to express some doubts about the validity of PCAC as applied to the problem of meson-exchange effects in nuclear β decay. However, apart from the fact that the $p + p \to d + \pi^+$ data used by BT differ significantly from the results of a more recent remeasurement,[8] BT's work involves some other (theoretical) difficulties which we wish to discuss. We have previously mentioned that BT calculated the phenomenological meson-exchange strength parameter Γ in $H^3 \to He^3 + e^- + \bar{\nu}_e$ by using the $p + p \to d + \pi^+$ experimental data. However, we shall show in Sec. II B that the intermediate state $N^*(\frac{3}{2}^+, \frac{3}{2})$ rescattering contributes dominantly to the two-body production amplitude in $p + p \to d + \pi^+$, whereas its contribution to the meson-exchange effects in $H^3 \to He^3 + e^- + \bar{\nu}_e$ is identically zero, as we have already noted. In view of this, one is led to doubt the validity of BT's use of the $p + p \to d + \pi^+$ data to deduce the magnitude of the meson-exchange effects in triton β decay. It must be emphasized that we are *not* questioning the validity of the relation [Eq. (2.11)] which connects the two processes $\alpha \to \beta + \pi^+$ and $\alpha \to \beta + e^- + \bar{\nu}_e$ (α and β are nuclear states) but merely the manner in which this relation was applied. Second, the expressions for the radial functions $\alpha_{\rm I}(x)$ and $\alpha_{\rm II}(x)$ taken by BT were very restrictive. We note that, consistent with the idea of a one-pion-exchange (OPE) interaction between two nucleons, the most general forms for $\alpha_{\rm I}(x)$ and $\alpha_{\rm II}(x)$ are

$$\alpha_{\rm I}(x) = (e^{-x}/x)(A_{\rm I} + B_{\rm I}/x + C_{\rm I}/x^2),$$
$$\alpha_{\rm II}(x) = (e^{-x}/x)(A_{\rm II} + B_{\rm II}/x + C_{\rm II}/x^2), \quad (1.1)$$

where $A_{\rm I}, \ldots, C_{\rm II}$ are constant parameters. We may point out that in Ref. 5 these six parameters were shown to be expressible in terms of various strong-interaction coupling constants and hadron masses (see Sec. II B) and that, moreover, these parameters were such that

$$\alpha_{\rm I}(x) + \tfrac{1}{3}\alpha_{\rm II}(x) = (A_{\rm I} + \tfrac{1}{3}A_{\rm II})(e^{-x}/x). \quad (1.2)$$

Therefore, in connection with the approach of BT, one may demand purely phenomenologically that Eq. (1.2) hold together with

$$B_{\rm I} + \tfrac{1}{3}B_{\rm II} = 0 \quad \text{and} \quad C_{\rm I} + \tfrac{1}{3}C_{\rm II} = 0. \quad (1.3)$$

We remark that in imposing Eqs. (1.2) and (1.3) the number of parameters is reduced from six to four in the meson-exchange calculation of $p + p \to d + \pi^+$, while at the same time the $x^{-2}e^{-x}$ and $x^{-3}e^{-x}$ terms in $\alpha_{\rm I}(x)$ and $\alpha_{\rm II}(x)$ become entirely irrelevant to the meson-exchange calculation in $H^3 \to He^3 + e^- + \bar{\nu}_e$. It is now clear that, with a certain latitude in suitably choosing the remaining four phenomenological parameters, it is possible to fit both the two-body production amplitudes in $p + p \to d + \pi^+$ and an *enhancement* of 5–20% because of meson-exchange effects in $H^3 \to He^3 + e^- + \bar{\nu}_e$. In this way, we can reconcile the apparent discrepancy discussed in BT between PCAC and an enhancing meson-exchange contribution to the Gamow-Teller matrix element $\langle He^3 | \boldsymbol{\sigma} | H^3 \rangle$.

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For the sake of completeness, we will review, in Sec. II, the theory of BT as well as the theory given in Ref. 5. We will also comment on the basic difference between our approach and that of BT. In Sec. III, we present our calculation of the two-body production amplitudes for $p+p\to d+\pi^+$ near threshold using the results of Sec. II. Finally, in Sec. IV, a discussion is given of some of the questions considered in this paper.

II. THEORY

A. Blin-Stoyle and Tint

In this section, we summarize the method BT[7] for relating the production amplitude for the process $p+p\to d+\pi^+$ to an effective Lagrangian describing meson-exchange effects. We begin by assuming PCAC in the form

$$\partial_\lambda A_\lambda{}^a(x)=C\phi^a(x)=(\mu^3 a_\pi/\sqrt{2})\phi^a(x),\qquad a=1,2,3 \quad (2.1)$$

where $\phi^a(x)$ is the field operator which annihilates a pion with isospin a, and $a_\pi=0.94$. Using Eq. (2.1), a relation may be derived[9] between $\langle\beta\,|\,\partial_\lambda A_\lambda{}^a(x)\,|\,\alpha\rangle$ and the transition amplitude $T(\pi^+ +\alpha\to\beta)$, where α and β are nuclear states. We have

$$\begin{aligned}\langle\beta\,|\,\partial_\lambda A_\lambda{}^a(x)\,|\,\alpha\rangle&=\langle\beta\,|\,C\phi^a(x)\,|\,\alpha\rangle\\&=[C/(q^2+\mu^2)]\langle\beta\,|\,(-\Box+\mu^2)\phi^a(x)\,|\,\alpha\rangle\\&=[C/(q^2+\mu^2)]\langle\beta\,|\,j_\pi{}^a(x)\,|\,\alpha\rangle,\end{aligned}\quad (2.2)$$

where $j_\pi{}^a(x)\equiv(-\Box+\mu^2)\phi^a(x)$, and $q=p_\beta-p_\alpha$ is the pion four-momentum. $\langle\beta\,|\,j_\pi{}^a(0)\,|\,\alpha\rangle$ may be related to $T(\pi^a+\alpha\to\beta)$ by noting that

$$\begin{aligned}&\langle\beta\,|\,S(\pi^a+\alpha\to\beta)-1\,|\,\alpha\pi^a\rangle\\&\equiv i(2\pi)^4\delta^4(p_\alpha+q-p_\beta)N_\alpha N_\beta(2q_0V)^{-1/2}T(\pi^a+\alpha\to\beta)\\&=i(2q_0V)^{-1/2}\int d^4x\,e^{iq\cdot x}\langle\beta\,|\,j_\pi{}^a(x)\,|\,\alpha\rangle\\&=i(2q_0V)^{-1/2}(2\pi)^4\delta^4(p_\alpha+q-p_\beta)\langle\beta\,|\,j_\pi{}^a(0)\,|\,\alpha\rangle.\end{aligned}\quad (2.3)$$

In Eq. (2.3), N_α and N_β denote the usual normalization factors $(m_i/E_iV)^{1/2}$ for the ith fermion and $(2E_jV)^{-1/2}$ for the jth boson. In going to the second line of Eq. (2.3), we have performed a Lehmann-Symanzik-Zimmermann (LSZ) reduction on the incoming pion and have denoted $|\,\alpha\rangle_{in}$ by $|\,\alpha\rangle$, etc. Combining Eqs. (2.2) and (2.3), we then have

$$\langle\beta\,|\,\partial_\lambda A_\lambda{}^a(0)\,|\,\alpha\rangle=[C/(q^2+\mu^2)]N_\alpha N_\beta T(\pi^a+\alpha\to\beta),\quad (2.4)$$

which is just Adler's[9] result. We have in mind that $|\,\alpha\rangle$ and $|\,\beta\rangle$ are nuclear states, and we are interested in evaluating the left-hand side of Eq. (2.4) by relating $T(\pi^a+\alpha\to\beta)$ to an effective P-wave pion-production Lagrangian which induces the process $\pi^a+\alpha\to\beta$. BT assume that $\mathcal{L}_\pi$ has the form

$$-\mathcal{L}_\pi=\int d^3r\,S_\lambda{}^a(\mathbf{r})\partial_\lambda\phi^a(\mathbf{r}),\quad (2.5)$$

where $S_\lambda{}^a(\mathbf{r})$ [whose detailed structure is given Eq. (2.12)] is a function of the momenta, spins, and isospins of the nucleons in $|\,\alpha\rangle$ and $|\,\beta\rangle$. $T(\pi^a+\alpha\to\beta)$ and $\mathcal{L}_\pi$ are related as follows:

$$\begin{aligned}&\langle\beta\,|\,-\mathcal{L}_\pi\,|\,\pi^a\alpha\rangle\\&=i(2\pi)^4\delta^4(p_\alpha+q-p_\beta)N_\alpha N_\beta(2q_0V)^{-1/2}T(\pi^a+\alpha\to\beta),\end{aligned}\quad (2.6)$$

and hence, from Eqs. (2.4) and (2.5),

$$\begin{aligned}&\frac{(2q_0V)^{1/2}C}{q^2+\mu^2}\langle\beta\,|\,-\mathcal{L}_\pi\,|\,\pi^a\alpha\rangle\\&=\frac{(2q_0V)^{1/2}C}{q^2+\mu^2}\langle\beta\,|\int d^3r'\,S_\lambda{}^a(\mathbf{r}')\partial_\lambda\phi^a(\mathbf{r}')\,|\,\pi^a\alpha\rangle\\&=i(2\pi)^4\delta^4(p_\alpha+q-p_\beta)\langle\beta\,|\,\partial_\lambda A_\lambda{}^a(0)\,|\,\alpha\rangle.\end{aligned}\quad (2.7)$$

If we confine our attention to allowed β decays, then $|\,\alpha\rangle$ and $|\,\beta\rangle$ will be states having the same parity, and therefore we need only consider $\langle\beta\,|\,\mathbf{S}^a(\mathbf{r})\,|\,\alpha\rangle$ and $\langle\beta\,|\,\mathbf{A}^a(\mathbf{r})\,|\,\alpha\rangle$. Furthermore, since the momentum transfer q can be taken as zero, we can write

$$\begin{aligned}&(2\pi)^3\delta^3(\mathbf{p}_\alpha-\mathbf{p}_\beta)\langle\beta\,|\,\boldsymbol{\nabla}\cdot\mathbf{A}^a(\mathbf{r})\,|\,\alpha\rangle\\&=-(C/\mu^2)(2\pi)^3\delta^3(\mathbf{p}_\alpha-\mathbf{p}_\beta)\langle\beta\,|\,\boldsymbol{\nabla}\cdot\mathbf{S}^a(\mathbf{r})\,|\,\alpha\rangle\end{aligned}\quad (2.8)$$

after a partial integration. Multiplying Eq. (2.8) by $\mathbf{l}\cdot\mathbf{r}$, where $\mathbf{l}=i\bar{u}(p_e)\boldsymbol{\gamma}(1+\gamma_5)v(p_\nu)$ is the lepton current in momentum space, and integrating over all space we find

$$\begin{aligned}&(2\pi)^3\delta^3(\mathbf{p}_\alpha-\mathbf{p}_\beta)\langle\beta\,|\int d^3r\,\mathbf{A}^a(\mathbf{r})\cdot\mathbf{l}\,|\,\alpha\rangle\\&=-(2\pi)^3\delta^3(\mathbf{p}_\alpha-\mathbf{p}_\beta)(C/\mu^2)\langle\beta\,|\int d^3r\,\mathbf{S}^a(\mathbf{r})\cdot\mathbf{l}\,|\,\alpha\rangle.\end{aligned}\quad (2.9)$$

[In Eq. (2.9) it is now understood that we are specializing to the charged currents corresponding to $a=1\pm i2$.] Hence, if we define an effective axial-vector β-decay operator $\mathcal{L}_\beta$ by

$$\mathcal{L}_\beta=(G\cos\theta/\sqrt{2})\mathbf{A}_\beta\cdot\mathbf{l},\quad (2.10)$$

where $G\cong 10^{-5}/m^2$ ($m=$ nucleon mass) and θ is the Cabibbo angle, then

$$\mathbf{A}_\beta=-\frac{C}{\mu^2}\int d^3r\,\mathbf{S}(\mathbf{r}).\quad (2.11)$$

Equation (2.11) establishes BT's connection between the axial-vector matrix element describing the process $\alpha\to\beta+e^-+\bar{\nu}_e$ and the source term $\mathbf{S}(\mathbf{r})$ describing the process $\alpha\to\beta+\pi^+$. The two-body contributions to $\mathbf{A}_\beta$

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[9] S. L. Adler, Phys. Rev. **137**, B1022 (1965).

which arise from meson-exchange effects may thus be calculated from Eq. (2.11), if $\mathbf{S}(\mathbf{r})$ is known once the previously mentioned difficulties have been straightened out. (Recall that by CVC the polar-vector matrix elements are unaffected by meson-exchange contributions, as we shall discuss in more detail in Sec. II.)

On general symmetry grounds $\mathbf{S}(\mathbf{r})$ can be taken as

$$\begin{aligned}\mathbf{S}(\mathbf{r})=&\sum_i (g_{\pi NN}/m)K_{\pi NN}(0)\tau_i^{(+)}\boldsymbol{\sigma}_i\delta^3(\mathbf{r}-\mathbf{r}_i)\\&+\sum_{i<j}\{(S_{ij}^{\sigma}T_{ij}^{\tau}+T_{ij}^{\sigma}S_{ij}^{\tau})[\alpha_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\\&+\alpha_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^+-\tau_j^+)\\&+(T_{ij}^{\sigma}T_{ij}^{\tau}+S_{ij}^{\sigma}S_{ij}^{\tau})[\beta_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\\&+\beta_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^+-\tau_j^+)\\&+[\gamma_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)+\gamma_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^++\tau_j^+)\}\\&\times\tfrac{1}{2}[\delta^3(\mathbf{r}-\mathbf{r}_i)+\delta^3(\mathbf{r}-\mathbf{r}_j)],\\ \tau^{\pm}=&\tfrac{1}{2}(\tau_x\pm i\tau_y),\end{aligned}\tag{2.12}$$

where $g_{\pi NN}^2/4\pi=14.4$ and $K_{\pi NN}(q^2)$ is the pionic for factor of the nucleon normalized to unity at $q^2=-\mu^2$. S_{ij} and T_{ij} are the singlet and triplet projection operators

$$S_{ij}^{\sigma}=\tfrac{1}{4}(1-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j),\qquad S_{ij}^{\tau}=\tfrac{1}{4}(1-\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),$$
$$T_{ij}^{\sigma}=\tfrac{1}{4}(3+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j),\qquad T_{ij}^{\tau}=\tfrac{1}{4}(3+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j).\tag{2.13}$$

$\alpha_i(r)$, $\beta_i(r)$, and $\gamma_i(r)$ ($i=$I or II) are as yet unspecified functions of $r=|\mathbf{r}_i-\mathbf{r}_j|$. From Eqs. (2.11) and (2.12), we then have

$$\begin{aligned}\mathbf{A}_\beta=&\mathbf{A}_\beta^{(1)}+\mathbf{A}_\beta^{(2)},\\ \mathbf{A}_\beta^{(1)}=&-(C/\mu^2)\sum_i\frac{g_{\pi NN}}{m}K_{\pi NN}(0)\tau_i^+\boldsymbol{\sigma}_i,\\ \mathbf{A}_\beta^{(2)}=&-(C/\mu^2)\sum_{i<j}\{(S_{ij}^{\sigma}T_{ij}^{\tau}+T_{ij}^{\sigma}S_{ij}^{\tau})[\alpha_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\\&+\alpha_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^+-\tau_j^+)\\&+(T_{ij}^{\sigma}T_{ij}^{\tau}+S_{ij}^{\sigma}S_{ij}^{\tau})[\beta_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\\&+\beta_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^+-\tau_j^+)\\&+[\gamma_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)+\gamma_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i^++\tau_j^+)\}.\end{aligned}\tag{2.14}$$

By use of the Goldberger-Treiman[10] relation

$$G_A=Cg_{\pi NN}K_{\pi NN}(0)/m\mu^2,\tag{2.15}$$

where $G_A=1.23$ is the usual axial-vector form factor,[11] the one-body term $\mathbf{A}_\beta^{(1)}$ in Eq. (2.14) is seen to reproduce the conventional Gamow-Teller operator, while the two-body term $\mathbf{A}_\beta^{(2)}$ gives the most general phenomenological form of the meson-exchange operator (see Sec. II C). We note that the detailed form of the functions $\alpha_i(r)$, $\beta_i(r)$, and $\gamma_i(r)$ can be predicted only on the basis of a detailed dynamical model. In the Sec. II, we summarize the model of Ref. 5 and present its main conclusions.

B. Cheng

To discuss the meson-exchange contributions to the effective hadron weak-current operator, consider first the following fundamental two-nucleon nuclear β-decay process:

$$\begin{aligned}N_i^{(i)}(p_i^{(i)},m)+N_j^{(i)}(p_j^{(i)},m)\rightarrow&N_i^{(f)}(p_i^{(f)},m)\\&+N_j^{(f)}(p_j^{(f)},m)+e^-(p_e,m_e)+\bar{\nu}_e(p_{\bar{\nu}},0),\end{aligned}\tag{2.16}$$

where the subscripts i and j designate the ith and jth nucleons, and the superscripts (i) and (f) denote (initial) and (final) states, respectively. The p's are the momenta of the respective particles and the m's their masses. We assume the simultaneous presence of strong and weak interactions but neglect entirely the electromagnetic interaction. Just as in any actual nuclear β-decay, the momentum transfer q to the lepton pair $(e^-\bar{\nu}_e)$,

$$q^2\equiv(p_e+p_{\bar{\nu}})^2=[(p_i^{(i)}+p_j^{(i)})-(p_i^{(f)}+p_j^{(f)})]^2\tag{2.17}$$

is small and can be taken as zero. As noted previously, CVC implies that, in the $q=0$ limit, the meson-exchange contribution to the effective polar vector current V_μ is zero. This is so because the value of the hadron→hadron matrix element of V_μ depends, in this limit, only on the spin (J), parity (P), and isospin (I) quantum numbers of the two hadrons involved. Thus we need only calculate the meson-exchange contribution to the effective axial-vector hadron weak current A_μ. We further assume that

$$\{N_i^{(i)}+N_j^{(i)}\rightarrow N_i^{(f)}+N_j^{(f)}+e^-+\bar{\nu}_e\}\cong\begin{Bmatrix}N_i^{(i)}\rightarrow N_i^{(f)}+e^-+\bar{\nu}_e+\pi; & \pi+N_j^{(i)}\rightarrow N_j^{(f)}\\ N_j^{(i)}\rightarrow N_j^{(f)}+e^-+\bar{\nu}_e+\pi; & \pi+N_i^{(i)}\rightarrow N_i^{(f)}\end{Bmatrix}\tag{2.18}$$

and

$$\begin{aligned}\{N^{(i)}\rightarrow N^{(f)}+e^-+\bar{\nu}_e+\pi\}\cong&\begin{Bmatrix}N^{(i)}\rightarrow B+e^-+\bar{\nu}_e; & B\rightarrow N^{(f)}+\pi\\ N^{(i)}\rightarrow B+\pi; & B\rightarrow N^{(f)}+e^-+\bar{\nu}_e\end{Bmatrix}\quad\text{(Class I processes)}\\&+\{N^{(i)}\rightarrow N^{(f)}+M;\quad M\rightarrow\pi+e^-+\bar{\nu}_e\}\quad\text{(Class II processes)}.\end{aligned}\tag{2.19}$$

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[11] C. J. Christensen, A. Nielsen, A. Bahnsen, W. K. Brown, and B. M. Rustad, Phys. Letters **26B**, 11 (1967).

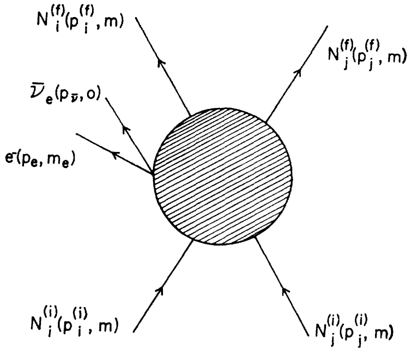

FIG. 1. General Feynman diagram for the two-nucleon nuclear β-decay process [$N_i^{(i)}+N_j^{(i)}\to N_i^{(f)}+N_j^{(f)}+e^-+\bar{\nu}_e$]. The relevant kinematic variables are also displayed.

In Eq. (2.19), B and M represent, respectively, various intermediate baryons and mesons with appropriate J, P, and I quantum numbers. The scheme described in Eqs. (2.18) and (2.19) is diagrammatically represented in Figs. 1–4. Consider next the β-decay processes $B\to N+e^-+\bar{\nu}_e$ and $M\to\pi+e^-+\bar{\nu}_e$. We note that only allowed and $\Delta J=|J(\text{initial})-J(\text{final})|=0$, 1 first-forbidden β-decay transitions have nonvanishing axial-vector-current matrix elements in the $q=0$ limit. Hence, the entire complex of OPE diagrams can be reduced to only four diagrams with B being identified as the pion-nucleon resonances $N^*(J^P, I)=N^*(\frac{3}{2}^+, \frac{3}{2})$, $N^*(\frac{3}{2}^-, \frac{1}{2})$, $N^*(\frac{1}{2}^+, \frac{1}{2})$, and with M taken as the ρ meson.[12] To calculate the weak axial-vector $B\to N$ or $M\to\pi$ matrix elements of A_μ in the $q=0$ limit, we use PCAC and derive in the usual manner a Goldberger-Treiman type relation. This permits the weak axial-vector form factor $F_A(N^*\to N; q^2=0)$ or $F_A(\rho\to\pi; q^2=0)$ to be expressed in terms of the corresponding $N^*N\pi$ or $\rho\pi\pi$ strong coupling constant and the neutron β-decay axial-vector form factor $G_A\equiv F_A(n\to p;\ q^2=0)$. The strong-interaction vertices are described by explicitly specifying effective Lagrangians in which strong-interaction-stable "particles" and strong-interaction-unstable "resonances" are treated in essentially the same way. All relevant coupling constants in these effective Lagrangians are calculated using the experimentally determined decay widths or scattering cross sections. After all of this has been done, what remains is then some lengthy calculations involving the following steps: (a) Expressing in momentum space the Lorentz-covariant S-matrix elements corresponding to the OPE diagrams shown in Figs. 1–4, (b) executing a nonrelativistic reduction of the nucleon Dirac spinors, retaining terms up to order $|\mathbf{k}|/m$ ($|\mathbf{k}|=|p^{(f)}-p^{(i)}|$), and finally, (c) Fourier-transforming the results into coordinate space. To summarize the final results, we define an effective Lagrangian $\mathcal{L}_\beta{}^{\text{exch}}$ for β decay by

$$\mathcal{L}_\beta{}^{\text{exch}}=(G\cos\theta/\sqrt{2})\mathbf{A}_\beta{}^{\text{exch}}(\pi)\cdot\mathbf{l}, \qquad (2.20)$$

where $\mathbf{l}=i\bar{\psi}_e\boldsymbol{\gamma}(1+\gamma_5)\psi_{\bar{\nu}}$ is the lepton current in co-

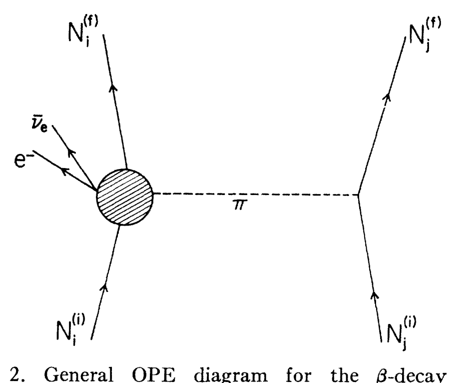

FIG. 2. General OPE diagram for the β-decay process [$N_i^{(i)}\to N_i^{(f)}+e^-+\bar{\nu}_e+\pi+\pi$; $N_j^{(i)}\to N_j^{(f)}$].

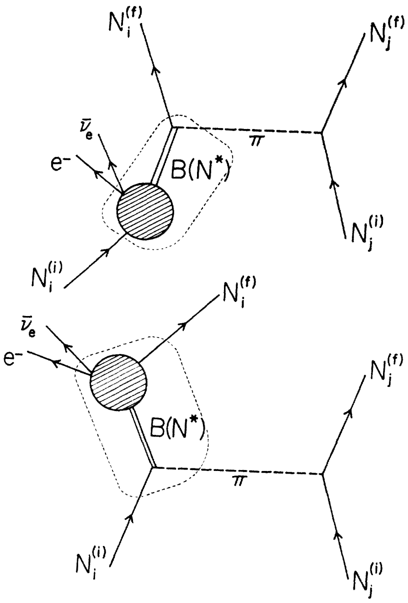

FIG. 3. Class-I pion-exchange diagrams for the β-decay processes [$N_i^{(i)}\to N^*+e^-+\bar{\nu}_e\to N_i^{(f)}+\pi+e^-+\bar{\nu}_e$; $\pi+N_j^{(i)}\to N_j^{(f)}$] (upper diagram) and [$N_i^{(i)}\to N^*+\pi\to N_i^{(f)}+e^-+\bar{\nu}_e+\pi$; $\pi+N_j^{(i)}\to N_j^{(f)}$] (lower diagram).

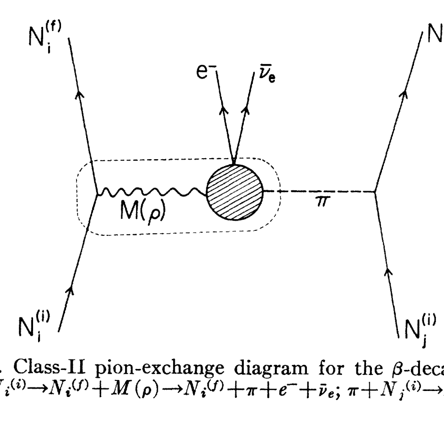

FIG. 4. Class-II pion-exchange diagram for the β-decay process [$N_i^{(i)}\to N_i^{(f)}+M(\rho)\to N_i^{(f)}+\pi+e^-+\bar{\nu}_e$; $\pi+N_j^{(i)}\to N_j^{(f)}$].

[12] That only $M=\rho$ contributes from the mesons of the 0^- and 1^- nonets follows from parity and/or G-parity considerations. The contributions from $B=N$ are effectively accounted for by use of appropriate nuclear wave functions.

ordinate space, and

$$\mathbf{A}_\beta^{\text{exch}}(\pi) = \sum_X \mathbf{A}_\beta^{\text{exch}}(\pi - X), \tag{2.21}$$

$$X = N^*(\tfrac{3}{2}+, \tfrac{3}{2}),\ N^*(\tfrac{3}{2}-, \tfrac{1}{2}),\ N^*(\tfrac{1}{2}+, \tfrac{1}{2}),\ \text{and } \rho.$$

The $\mathbf{A}_\beta^{\text{exch}}(\pi-X)$ are the pion-exchange weak hadron current operators arising from the pion-exchange diagrams of Figs. 1–4. All of the $\mathbf{A}_\beta^{\text{exch}}(\pi-X)$ operators have the general form of $\mathbf{A}_\beta^{(2)}$ in Eq. (2.14) with radial functions $g_{\text{I}}(r), \ldots, j_{\text{II}}(r)$ given by[5,7]

$$\begin{aligned}
-g_{\text{I}}(x) &= \lambda[\alpha_{\text{I}}(x) - \beta_{\text{I}}(x)],\\
-h_{\text{I}}(x) &= \lambda[\alpha_{\text{I}}(x) + \beta_{\text{I}}(x)],\\
-j_{\text{I}}(x) &= 2\lambda\gamma_{\text{I}}(x),\\
-g_{\text{II}}(x) &= \lambda[\alpha_{\text{II}}(x) - \beta_{\text{II}}(x)],\\
-h_{\text{II}}(x) &= \lambda[\alpha_{\text{II}}(x) + \beta_{\text{II}}(x)],\\
-j_{\text{II}}(x) &= 2\lambda\gamma_{\text{II}}(x),\\
\lambda &= mG_A/2g_{\pi NN}K_{\pi NN}(0).
\end{aligned} \tag{2.22}$$

For $N^*(\frac{3}{2}+, \frac{3}{2})$, we have

$$\begin{aligned}
g_{\text{I}}(x) &= |G^{\text{exch}}(N^*(\tfrac{3}{2}+, \tfrac{3}{2}))|(-12)e^{-x}(1+1/x+1/x^2),\\
g_{\text{II}}(x) &= |G^{\text{exch}}(N^*(\tfrac{3}{2}+, \tfrac{3}{2}))|(12)(e^{-x}/x)(1+3/x+3/x^2),\\
h_{\text{I}}(x) &= j_{\text{I}}(x)\\
&= |G^{\text{exch}}(N^*(\tfrac{3}{2}+, \tfrac{3}{2}))|(-24)(e^{-x}/x)(1/x+1/x^2),\\
h_{\text{II}}(x) &= j_{\text{II}}(x) = 2g_{\text{II}}(x),\\
|G^{\text{exch}}(N^*(\tfrac{3}{2}+, \tfrac{3}{2}))| &= |\tfrac{1}{9}(f_{\pi NN^*}{}^2/4\pi)G_A[\mu/(M_{N^*}-m)]|,
\end{aligned} \tag{2.23}$$

with $f_{\pi NN^*}{}^2/4\pi \cong 0.36$.

For $N^*(\frac{3}{2}-, \frac{1}{2})$, we have

$$\begin{aligned}
g_{\text{I}}(x) &= |G^{\text{exch}}(N^*(\tfrac{3}{2}-, \tfrac{1}{2}))|\\
&\quad\times(12)(e^{-x}/x)(1+1/x+1/x^2),\\
g_{\text{II}}(x) &= |G^{\text{exch}}(N^*(\tfrac{3}{2}-, \tfrac{1}{2}))|\\
&\quad\times(-12)(e^{-x}/x)(1+3/x+3/x^2),\\
h_{\text{I}}(x) &= j_{\text{I}}(x) = |G^{\text{exch}}(N^*(\tfrac{3}{2}-, \tfrac{1}{2}))|\\
&\quad\times(-12)(e^{-x}/x)(1/x+1/x^2),\\
h_{\text{II}}(x) &= j_{\text{II}}(x) = |G^{\text{exch}}(N^*(\tfrac{3}{2}-, \tfrac{1}{2}))|\\
&\quad\times(12)(e^{-x}/x)(1+3/x+3/x^2),\\
|G^{\text{exch}}(N^*(\tfrac{3}{2}-, \tfrac{1}{2}))| &= |\tfrac{1}{9}(f_{\pi NN^*}{}^2/4\pi)G_A[\mu/(M_{N^*}+m)]|,
\end{aligned} \tag{2.24}$$

where $f_{\pi NN^*}{}^2/4\pi \cong 0.26$.

For $N^*(\frac{1}{2}+, \frac{1}{2})$, we have

$$\begin{aligned}
g_{\text{I}}(x) &= |G^{\text{exch}}(N^*(\tfrac{1}{2}+, \tfrac{1}{2}))|\\
&\quad\times\eta(12)(e^{-x}/x)(1+1/x+1/x^2),\\
g_{\text{II}}(x) &= |G^{\text{exch}}(N^*(\tfrac{1}{2}+, \tfrac{1}{2}))|\\
&\quad\times\eta(-12)(e^{-x}/x)(1+3/x+3/x^2),\\
h_{\text{I}}(x) &= |G^{\text{exch}}(N^*(\tfrac{1}{2}+, \tfrac{1}{2}))|\\
&\quad\times(6)(e^{-x}/x)(1/x+1/x^2) = j_{\text{I}}(x),\\
h_{\text{II}}(x) &= |G^{\text{exch}}(N^*(\tfrac{1}{2}+, \tfrac{1}{2}))|\\
&\quad\times(-6)(e^{-x}/x)(1+3/x+3/x^2) = j_{\text{II}}(x),\\
\eta &= (M_{N^*}+m)/2m\\
|G^{\text{exch}}(N^*(\tfrac{1}{2}+, \tfrac{1}{2}))| &= |\tfrac{1}{3}(f_{\pi NN^*}{}^2/4\pi)G_A[\mu/(M_{N^*}-m)]|,
\end{aligned} \tag{2.25}$$

where $f_{\pi NN^*}{}^2/4\pi \cong 0.065$.

For the ρ contribution, we have

$$\begin{aligned}
g_{\text{I}}(x) &= |G^{\text{exch}}(\rho)|\tfrac{1}{2}\left[-12\frac{e^{-x}}{x}\left(1+\frac{1}{x}+\frac{1}{x^2}\right) + \left(\frac{m_\rho}{\mu}\right)^3 12\frac{e^{-y}}{y}\left(1+\frac{1}{y}+\frac{1}{y^2}\right)\right],\\
g_{\text{II}}(x) &= |G^{\text{exch}}(\rho)|\tfrac{1}{2}\left[12\frac{e^{-x}}{x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right) - \left(\frac{m_\rho}{\mu}\right)^3 12\frac{e^{-y}}{y}\left(1+\frac{3}{y}+\frac{3}{y^2}\right)\right],\\
h_{\text{I}}(x) &= h_{\text{II}}(x) = j_{\text{I}}(x) = j_{\text{II}}(x) = 0,\\
y &= (m_\rho/\mu)x,\\
|G^{\text{exch}}(\rho)| &= |\tfrac{1}{12}(1+\mu_p-\mu_n)(f_{\rho\pi\pi}{}^2/4\pi)G_A[\mu^3/m(m_\rho{}^2-\mu^2)]|,
\end{aligned} \tag{2.26}$$

where $\mu_p = 1.79$ and $\mu_n = -1.91$ are the anomalous magnetic moments of the proton and neutron, respectively, and $f_{\rho\pi\pi}{}^2/4\pi \cong 2.4$. This completes our discussion of the results of Ref. 5, except for some general remarks which we reserve for Sec. II C.

C. Remarks

(1) We notice that $\mathbf{A}_\beta^{\text{exch}}$ has precisely the same form as the phenomenological meson-exchange Lagrangian obtained by Blin-Stoyle *et al.*[6,13] on general symmetry grounds. Of course, in a phenomenological treatment one cannot determine the numerical value of G^{exch} or the functional dependence of $g_{\text{I}}(x), \ldots, j_{\text{II}}(x)$ on x. We also note that $\mathbf{A}_\beta^{\text{exch}}(\pi)$ has the same space-time and isospace transformation properties as the primitive strangeness conserving hadronic axial-vector current $\mathbf{A}(\mathbf{x})$.

(2) In applying $\mathfrak{L}_\beta^{\text{exch}}$ to the triton β decay, we are led to consider only the combination $g_{\text{I}}(x)+\frac{1}{3}g_{\text{II}}(x)+h_{\text{I}}(x)+\frac{1}{3}h_{\text{II}}(x) = 2\lambda[\alpha_{\text{I}}(x)+\frac{1}{3}\alpha_{\text{II}}(x)]$. It can be easily seen from Eq. (2.23) that the π-$N^*(\frac{3}{2}+, \frac{3}{2})$ pion-exchange contribution to $\langle He^3|\boldsymbol{\sigma}|H^3\rangle$ is identically zero, while the π-$N^*(\frac{3}{2}-, \frac{1}{2})$, π-$N^*(\frac{1}{2}+, \frac{1}{2})$, and π-ρ pion-exchange contributions are each associated with a Yukawa-type radial dependence $\sim \Gamma x^{-1}e^{-x}$ (where $x=\mu r$ or $m_\rho r$) as assumed by Blin-Stoyle and Papageorgiu[6] on purely phenomenological grounds.

(3) We remark that for any nuclear β-decay process

[13] R. J. Blin-Stoyle, V. Gupta, and H. Primakoff, Nucl. Phys. **11**, 44 (1959); J. S. Bell and R. J. Blin-Stoyle, *ibid.* **6**, 87 (1957).

$[Z, A]\to[Z+1, A]+e^-+\bar{\nu}_e$ the π-$N^*(\frac{3}{2}^+, \frac{3}{2})$ pion-exchange contribution will always be identically zero if both the initial and final nuclear wave functions can be factorized into a spin-isospin part multiplied by a fully symmetric space part.

(4) As was pointed out in the discussion of $\mathbf{A}_\beta{}^{\text{exch}}(\pi)$, the most general phenomenological form of $\alpha_{\text{I}}(x)$ and $\alpha_{\text{II}}(x)$ consistent with the idea of a OPE two-nucleon interaction is[14]

$$\alpha_{\text{I}}(x)=(e^{-x}/x)(A_{\text{I}}+B_{\text{I}}/x+C_{\text{I}}/x^2),$$
$$\alpha_{\text{II}}(x)=(e^{-x}/x)(A_{\text{II}}+B_{\text{II}}/x+C_{\text{II}}/x^2). \quad (2.27)$$

In this sense, a pure Yukawa form

$$\alpha_{\text{I}}(x)=A_{\text{I}}(e^{-x}/x), \qquad \alpha_{\text{II}}(x)=A_{\text{II}}(e^{-x}/x) \quad (2.28)$$

must be regarded as a very restrictive assumption. If one uses $\alpha_{\text{I}}(x)$ and $\alpha_{\text{II}}(x)$ as given in Eqs. (2.22)–(2.26) to calculate the two-body production amplitudes in $p+p\to d+\pi^+$, the results turn out to depend very sensitively on the $x^{-2}e^{-x}$ and $x^{-3}e^{-x}$ terms (see Sec. III). On the other hand, we have already seen that even with the radial functions given in Eqs. (2.22)–(2.26), the *combination of terms* given by $\alpha_{\text{I}}(x)+\frac{1}{3}\alpha_{\text{II}}(x)$ *can* appear as a simple Yukawa form. Thus, it is not unreasonable from a phenomenological point of view to assume in conjunction with Eqs. (2.22)–(2.26) that

$$\alpha_{\text{I}}(x)+\tfrac{1}{3}\alpha_{\text{II}}(x)=\Gamma(e^{-x}/x). \quad (2.29)$$

This corresponds to having

$$A_{\text{I}}+\tfrac{1}{3}A_{\text{II}}=\Gamma, \qquad B_{\text{I}}+\tfrac{1}{3}B_{\text{II}}=0, \qquad C_{\text{I}}+\tfrac{1}{3}C_{\text{II}}=0. \quad (2.30)$$

(5) The quantity δ which characterizes the pion-exchange effects in triton β decay is defined by

$$\delta(\text{H}^3\to\text{He}^3;\pi)$$
$$\equiv\langle\text{He}^3|\mathbf{A}_\beta{}^{\text{exch}}(\pi)|\text{H}^3\rangle/\langle\text{He}^3|G_A\sum_i\boldsymbol{\sigma}_i\tau_i{}^+|\text{H}^3\rangle. \quad (2.31)$$

Our calculation[5] of $\delta(\text{H}^3\to\text{He}^3;\pi)$ with $\mathbf{A}_\beta{}^{\text{exch}}$ as given in Sec. II B yields an enhancement of $\sim$10% to be compared to the best experimentally based estimate of $\sim$8%. Moreover, the value of $\delta(\text{H}^3\to\text{He}^3;\pi)$ is also shown to be insensitive to the various H^3 and He^3 nuclear wave functions which have been used in the literature.[15]

III. APPLICATION TO $p+p\to d+\pi^+$

Given the detailed form of $\mathbf{A}_\beta=\mathbf{A}_\beta{}^{\text{exch}}(\pi)$ in Eq. (2.21) and the connection between $\mathcal{L}_\beta$ and $\mathbf{S}(\mathbf{r})$, we are in a position to calculate the cross section for the *strong* process $p+p\to d+\pi^+$ in terms of the weak-interaction parameters which emerged from the preceding analysis.[16]

The Lagrangian $\mathcal{L}_\pi$ of Eq. (2.5) describes P-wave pion production which can take place from initial $^1S_0$ and $^1D_2$ diproton states. (At low energies, S-wave pion production can take place from an initial $^3P_1$ diproton state, but since S-wave production is not described by $\mathcal{L}_\pi$, we omit a detailed consideration of the $^3P_1$ contribution.) The differential cross section for $p+p\to d+\pi^+$ is given by[17]

$$4\pi(d\sigma/d\Omega)=\tfrac{1}{4}(\{|a(^1S_0)|^2+\tfrac{1}{2}|a(^1D_2)|^2$$
$$+\sqrt{2}\,\text{Re}[a(^1S_0)^*a(^1D_2)]+|a(^3P_1)|^2\}$$
$$+\{\tfrac{1}{2}|a(^1D_2)|^2-\sqrt{2}\,\text{Re}[a(^1S_0)^*a(^1D_2)]\}3\cos^2\theta), \quad (3.1)$$

where $a(^1S_0)$, $a(^3P_1)$, and $a(^1D_2)$ are the amplitudes for pion production from initial diproton states having $l=0$, 1, and 2, respectively. Since, as noted above, $a(^3P_1)$ provides no information about $\mathcal{L}_\pi$, we shall henceforth omit consideration of it. To extract the interesting two-body contribution from $a(^1S_0)$ and $a(^1D_2)$, we write

$$a(^1S_0)=[b(^1S_0)+c(^1S_0)]\exp(i\tau_0)\eta^{3/2}\,\text{mb}^{1/2},$$
$$a(^1D_2)=[b(^1D_2)+c(^1D_2)]\eta^{3/2}\,\text{mb}^{1/2}, \quad (3.2)$$

where the b and c amplitudes derive, respectively, from the one-body and two-body pieces of $\mathbf{S}(\mathbf{r})$, η is the c.m. pion momentum in units of μc, and τ_0 is the relative S-D phase. As noted by BT, the only two-body terms in $\mathbf{S}(\mathbf{r})$ which contribute to $p+p\to d+\pi^+$ are

$$\sum_{i<j}T_{ij}{}^\sigma S_{ij}{}^\tau[\alpha_{\text{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\alpha_{\text{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\hat{r}](\tau_i{}^+-\tau_j{}^+)$$
$$\times\tfrac{1}{2}[\delta^3(\mathbf{r}-\mathbf{r}_i)+\delta^3(\mathbf{r}-\mathbf{r}_j)]. \quad (3.3)$$

The one-body contributions have been evaluated by Woodruff[18] and are given by

$$b(^1S_0)=0.24, \qquad b(^1D_2)=0.86, \qquad \tau_0=2.65. \quad (3.4)$$

(It is understood that all amplitudes are in units of $\text{mb}^{1/2}\cong0.316$ F.) We next turn to a calculation of the two-body contributions $c(^1S_0)$ and $c(^1D_2)$ using Eq. (3.3). We take the deuteron wave function Ψ_d to be

$$\Psi_d=(\mu/4\pi)^{1/2}(1/r)[u(r)\chi_t+w(r)\chi_D]\eta_s, \quad (3.5)$$

where $\chi_D=(1/\sqrt{8})S_{12}\chi_s$, $\chi_s(\chi_t)$ being singlet (triplet) spin functions, $S_{12}=(3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}/r^2-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$, and η_s is a singlet isospin function. $u(r)$ and $w(r)$ are the S- and D-state radial functions for the deuteron which we have taken from Kottler and Kowalski.[19] The initial diproton

[14] The radial dependence of $\alpha_{\text{I,II}}$ in Eq. (2.27) arises from a Fourier transformation of terms of the form $k_lk_m/(k^2+\mu^2)$, where $l, m=1, 2, 3$. Recall that in the nonrelativistic reduction of the OPE diagrams (Figs. 1–4) terms up to $|\mathbf{k}|/m$ were retained for each nucleon leg, so that terms of the form $k_lk_m/(k^2+\mu^2)$ will appear. These terms are similar to those which arise in the derivation of the tensor force in the two-nucleon nuclear potential.

[15] The specific wave functions considered were the exponential, Gaussian, Irving, and Irving-Gunn wave functions.

[16] See Ref. 8 for references to earlier theoretical work on $p+p\to d+\pi^+$.

[17] F. Mandl and T. Regge, Phys. Rev. **99**, 1478 (1955).

[18] A. E. Woodruff, Phys. Rev. **117**, 1113 (1960).

[19] H. Kottler and K. L. Kowalski, Nucl. Phys. **53**, 334 (1964).

wave function is assumed to have the form[7]

$$\Psi_{pp}=(2/V)^{1/2}\{R_0(kr)-\tfrac{5}{2}[3(\hat{k}\cdot\hat{r})^2-1]R_2(kr)\}e^{-i\mathbf{K}\cdot\mathbf{R}}\chi_s\eta_t, \tag{3.6}$$

where $\mathbf{k}$ and $\mathbf{K}$ are the relative and total momenta of the two protons. The R's are given by

$$\begin{aligned} R_0(kr)&=0, & r<r_c\\ &=(kr)^{-1}\sin(k(r-r_c)), & r>r_c \end{aligned} \tag{3.7}$$

$$R_2(kr)=j_2(kr)=\left[\frac{3}{(kr)^3}-(kr)^{-1}\right]\sin(kr)-\frac{3}{(kr)^2}\cos(kr),$$

where $k=|\mathbf{k}|$, and $x_c=\mu r_c=0.35$ is the hard-core radius. We assume, in addition, that the pion is described by a plane wave. Collecting the previous results together, we find

$$\begin{aligned} c(^1S_0)&\\ =8\left(\frac{M}{\mu}\right)^{1/4}&\left[\int dx\, u(x)R_0(k'x)j_0(\tfrac{1}{2}q'x)e^{-x}(A_{\rm I}+\tfrac{1}{3}A_{\rm II})\right.\\ &\left.-\sqrt{2}\int dx\, w(x)R_0(k'x)j_0(\tfrac{1}{2}q'x)e^{-x}\left(\tfrac{1}{3}A_{\rm II}-\frac{B_{\rm I}}{x}-\frac{C_{\rm I}}{x^2}\right)\right], \end{aligned} \tag{3.8}$$

$$\begin{aligned} c(^1D_2)&\\ =8\left(\frac{M}{\mu}\right)^{1/4}&\left[\int dx\, w(x)R_2(k'x)j_0(\tfrac{1}{2}q'x)e^{-x}\right.\\ &\times\left(A_{\rm I}+\tfrac{2}{3}A_{\rm II}-\frac{B_{\rm I}}{x}-\frac{C_{\rm I}}{x^2}\right)\\ &\left.-\sqrt{2}\int dx\, u(x)R_2(k'x)j_0(\tfrac{1}{2}q'x)e^{-x}\left(\tfrac{1}{3}A_{\rm II}-\frac{B_{\rm I}}{x}-\frac{C_{\rm I}}{x^2}\right)\right], \end{aligned}$$

where $x=\mu r$, $k'=k/\mu$, and $q'=q/\mu$. For threshold pion-production $q=|\mathbf{q}|\cong 0$, whence $j_0(\frac{1}{2}q'x)\cong 1$. Carrying out the integrations indicated in Eqs. (3.8), we have

$$\begin{aligned} c(^1S_0)&=S_1(k')A_{\rm I}+S_2(k')A_{\rm II}+S_3(k')B_{\rm I}+S_4(k')C_{\rm I}\\ &=1.96A_{\rm I}+0.26A_{\rm II}+1.38B_{\rm I}+1.93C_{\rm I}, \quad k'=1.11\\ &=1.03A_{\rm I}+0.08A_{\rm II}+1.10B_{\rm I}+1.69C_{\rm I}, \quad k'=2.22\\ c(^1D_2)&=D_1(k')A_{\rm I}+D_2(k')A_{\rm II}+D_3(k')B_{\rm I}+D_4(k')C_{\rm I}\\ &=0.15A_{\rm I}-0.14A_{\rm II}+0.32B_{\rm I}-0.04C_{\rm I}, \quad k'=1.11\\ &=0.30A_{\rm I}-0.16A_{\rm II}+0.55B_{\rm I}-0.58C_{\rm I}, \quad k'=2.22. \end{aligned} \tag{3.9}$$

The integrals defining the dimensionless functions $S_i(k')$ and $D_i(k')$ are given explicitly in the Appendix. It is evident from Eqs. (3.9) that the contributions from the singular terms $B_{\rm I}/x$ and $C_{\rm I}/x^2$ (which correspond, respectively, to $B_{\rm I}x^{-1}e^{-x}$ and $C_{\rm I}x^{-3}e^{-x}$ of Sec. II C) are at least as important as those arising from $A_{\rm I}$ and $A_{\rm II}$ if not more so. One may understand the relative magnitude of the contributions from $B_{\rm I}$ and $C_{\rm I}$ (i.e., relative to $A_{\rm I}$ and $A_{\rm II}$) as follows. For small values of $x-x_c$, R_0 is approximately unity and hence the terms proportional to $1/x$ and $1/x^2$ in $c(^1S_0)$ make relatively large contributions for x near x_c. A similar argument for $c(^1D_2)$ serves to make plausible the relative size of the coefficients of $B_{\rm I}$ and $C_{\rm I}$ in Eqs. (3.9). As regards the relative total contributions of $c(^1S_0)$ and $c(^1D_2)$, we note from Eqs. (3.8) that the essential difference between these amplitudes lies in the functions R_0 and R_2. For $x\ll 1$, $R_2(x)\sim x^2$, while $R_0(x)\sim 1$. Hence, R_0 tends to weight the singular $1/x$ and $1/x^2$ terms more heavily than R_2, and these contributions in turn serve to give $c(^1S_0)$ its relatively large magnitude. The preceding argument is obviously not rigorous, but it does at least make plausible the conclusion that $c(^1S_0)$ can be larger than $c(^1D_2)$.

Our results are summarized in Table I. The values of the parameters $A_{\rm I}$, $A_{\rm II}$, $B_{\rm I}$, and $C_{\rm I}$ have been deduced from Ref. 5, as we have already discussed. ρ_a and ρ_b denote the pieces of the effective ρ contribution which are proportional to $e^{-\mu r}$ and $e^{-m_\rho r}$, respectively. Evidently, ρ_b gives rise to a small additional contribution to Eqs. (3.8) in which the factor e^{-x} is replaced by $e^{-m_\rho x/\mu}$ and which must be integrated separately. Hence, the contribution from ρ_b should *not* be calculated using Eqs. (3.9). We note that, as expected, the dominant contribution to both $c(^1S_0)$ and $c(^1D_2)$ comes from intermediate state $N^*(\frac{3}{2}^+,\frac{3}{2})$ rescattering[20] which, however, fails to contribute at all to pion-exchange effects in $H^3\to He^3+e^-+\bar{\nu}_e$ as we have noted previously. In Sec. IV, we discuss our theoretical results as well as the present experimental status of $H^3\to He^3+e^-+\bar{\nu}_e$ and $p+p\to d+\pi^+$.

IV. DISCUSSION

(1) We have not attempted to make a detailed comparison of our results with the experimental data on $p+p\to d+\pi^+$ for several reasons which we proceed to discuss. We first recall that in the analysis of the experimental data, it is customary to reexpress Eq. (3.1) in the form[8]

$$4\pi(d\sigma/d\Omega)|_{P\text{-wave}}=3\beta\eta^3[(X+\cos^2\theta)/(3X+1)], \tag{4.1}$$

where

$$\begin{aligned} \beta&=4\eta^{-3}|a(^1D_2)|^2(1+|\delta_0|^2)=\eta^{-3}\sigma_{\rm tot},\\ X&=\tfrac{1}{3}|1+\sqrt{2}\delta_0|^2(1-2\sqrt{2}\,{\rm Re}\delta_0)^{-1},\\ \delta_0&=a(^1S_0)/a(^1D_2), \end{aligned} \tag{4.2}$$

and where $\sigma_{\rm tot}$ denotes the total P-wave pion-production cross section. Two sets of experimental data for β and

[20] The dominance of the $N^*(\frac{3}{2}^+,\frac{3}{2})$ rescattering contribution has also been noted independently by Woodruff, Ref. 18, in a purely strong-interaction calculation (as contrasted to our weak-interaction calculation).

TABLE I. Summary of $p+p\to d+\pi^+$ amplitudes.[a]

Contribution	$A_{\rm I}(F)$	$A_{\rm II}(F)$	$B_{\rm I}(F)$	$C_{\rm I}(F)$	$k'=1.11$ $c(^1S_0)$ in F	$k'=1.11$ $c(^1D_2)$ in F	$k'=2.22$ $c(^1S_0)$ in F	$k'=2.22$ $c(^1D_2)$ in F
$N^*(\frac{3}{2}^+,\frac{3}{2})$	0.45	−1.35	1.35	1.35	4.98	0.63	4.10	0.31
$N^*(\frac{1}{2}^+,\frac{1}{2})$	−0.17	0.24	−0.24	−0.24	−1.07	−0.13	−0.83	−0.08
$N^*(\frac{3}{2}^-,\frac{1}{2})$	−0.03	0	0	0	−0.08	−0.01	−0.05	−0.01
ρ_a	0.06	−0.06	0.06	0.06	0.29	0.03	0.09	0.02
Total	0.30	−1.16	1.16	1.16	4.12	0.53	3.32	0.24

[a] A small additional contribution from ρ_b has not been included, as explained in the text.

X are available:

(i) Crawford and Stevenson[21]:

$\beta=(1.01\pm0.08)$ mb, $X=0.082\pm0.34$,

(ii) Rose[3]:

$\beta=(0.74\pm0.05)$ mb, $X=$ unpublished.

We remark that apart from a significant difference in the two quoted values for β there appears to be a large uncertainty in the experimental values of X. Quoted values for X range from $X\cong0.05\pm0.05$[22] to $X\cong0.22\pm0.05$.[23] The difficulty in determining β arises in part from the fact that at low energies the differential cross section (when including other partial waves) is relatively insensitive to β.[8] Since $a(^1S_0)$ and $a(^1D_2)$ depend on β and X in a rather complicated way, as may be seen from Eqs. (4.1) and (4.2), it appears unprofitable at the present time to attempt to extract the magnitudes of these amplitudes from experiment in an effort to determine $c(^1S_0)$ and $c(^1D_2)$.

(2) On the basis of the discussion given in previous sections, we conclude that the discrepancy between $\delta(H^3\to He^3;\pi)\cong-(5\text{–}25)\%$ calculated by BT and the experimental value $\delta(H^3\to He^3)\cong8\%$ is specifically a consequence of assuming a Yukawa form for $\alpha_{\rm I}(x)$ and $\alpha_{\rm II}(x)$, and hence should not be taken as a basis for criticizing either PCAC or their effective Lagrangian formalism. At the same time, it should be pointed out that doubts have been expressed concerning the accuracy of the experimental ft value for $H^3\to He^3+e^-+\bar\nu_e$. This ft value is needed along with that of the free neutron in order to determine the experimental value of $\delta(H^3\to He^3)$. It thus appears that to really settle the question (of the magnitude and sign of δ), a remeasurement of the triton ft value is called for, especially in view of the recent remeasurement of the neutron ft value.[11] It is also worth pointing out that with a reliable experimental value of $\langle He^3|\boldsymbol{\sigma}|H^3\rangle$ and more quantitative information about $\delta(H^3\to He^3)$, it would be possible to obtain an accurate estimate of $p(\mathrm{II}^2S)$ and $p(\mathrm{II}^4D)$—the probabilities of the II^2S and II^4D states in the H^3 and He^3 wave functions. These are related to $\langle He^3|\boldsymbol{\sigma}|H^3\rangle$ and $\delta(H^3\to He^3)$ as follows[5]:

$$(|\langle He^3|\boldsymbol{\sigma}|H^3\rangle|^2)_{\rm expt}\cong3[1-(8/3)p(\mathrm{II}^2S)-\tfrac{4}{3}p(\mathrm{II}^4D)+2\delta(H^3\to He^3)]. \tag{4.3}$$

Thus, through Eq. (4.3) the detailed nature of the H^3 and He^3 wave functions may hopefully be elucidated through a knowledge of meson-exchange effects in triton β decay.

Note added in proof. After completing this manuscript, we learned of a paper by Blin-Stoyle [following paper, Phys. Rev. **186,** 1540 (1969)] in which the original calculation of BT is reexamined in the light of new experimental data and existing experimental uncertainties, such as were described in Sec. IV above.

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APPENDIX

In this Appendix, we define the functions $S_1(k'),\ldots,S_4(k')$ and $D_1(k'),\ldots,D_4(k')$.

$$S_1(k')=\frac{\lambda'}{k'}\int_{0.35}^{\infty}dx(d_1e^{-(1+\alpha)x}-d_2e^{-(1+\beta)x})\frac{1}{x}\sin(k'(x-x_c)),$$

$$S_2(k')=\frac{\lambda'}{3k'}\int_{0.35}^{\infty}dx(d_1e^{-(1+\alpha)x}-d_2e^{-(1+\beta)x})\frac{1}{x}\sin(k'(x-x_c))-\frac{\sqrt2\lambda'}{3k'}\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})\frac{1}{x}\sin(k'(x-x_c))$$
$$-\frac{\sqrt2\lambda'}{3k'}\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})\frac{1}{x}\sin(k'(x-x_c)), \tag{A1}$$

[21] F. S. Crawford and M. L. Stevenson, Phys. Rev. **97**, 1305 (1955).
[22] We wish to thank Dr. C. M. Rose, Jr., for communicating to us this unpublished result.
[23] C. Richard-Serre, CERN Report No. 68-40 (unpublished).

$$S_3(k')=\frac{\sqrt{2}\lambda'}{k'}\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})\frac{1}{x^2}\sin(k'(x-x_c))+\frac{\sqrt{2}\lambda'}{k'}\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})\frac{1}{x^2}\sin(k'(x-x_c)),$$

$$S_4(k')=\frac{\sqrt{2}\lambda'}{k'}\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})\frac{1}{x^3}\sin(k'(x-x_c))+\frac{\sqrt{2}\lambda'}{k'}\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})\frac{1}{x^3}\sin(k'(x-x_c)).$$

$$D_1(k')=\lambda'\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})R_2(k'x)+\lambda'\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})R_2(k'x),$$

$$D_2(k')=\frac{2\lambda'}{3}\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})R_2(k'x)+\frac{2\lambda'}{3}\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})R_2(k'x)$$
$$-\frac{\sqrt{2}\lambda'}{3}\int_0^{\infty}dx(d_1e^{-(1+\alpha)x}-d_2e^{-(1+\beta)x})R_2(k'x),$$

$$D_3(k')=-\lambda'\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})\frac{R_2(k'x)}{x}-\lambda'\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})\frac{R_2(k'x)}{x}$$
$$+\sqrt{2}\lambda'\int_0^{\infty}dx(d_1e^{-(1+\alpha)x}-d_2e^{-(1+\beta)x})\frac{R_2(k'x)}{x},\quad \text{(A2)}$$

$$D_4(k')=-\lambda'\int_{0.35}^{3.42}dx(f_1e^{-(1+\alpha_1)x}-f_2e^{-(1+\beta_1)x})\frac{R_2(k'x)}{x^2}-\lambda'\int_{3.42}^{\infty}dx(g_1e^{-(1+\alpha_2)x}+g_2e^{-(1+\beta_2)x})\frac{R_2(k'x)}{x^2}$$
$$+\sqrt{2}\lambda'\int_0^{\infty}dx(d_1e^{-(1+\alpha)x}-d_2e^{-(1+\beta)x})\frac{R_2(k'x)}{x^2},$$

$$\lambda'\equiv 8(M/\mu)^{1/4}.$$

The constants $\alpha, \beta, \ldots, \beta_2$ and $d_1, \ldots, g_2$ are taken from the work of Kottler and Kowalski,[19] and have the following values:

$$\alpha=0.33,\quad \alpha_1=0.66,\quad \alpha_2=0.42,$$
$$\beta=2.90,\quad \beta_1=5.42,\quad \beta_2=1.17,$$
$$d_1=1.05,\quad f_1=0.46,\quad g_1=0.13,$$
$$d_2=2.58,\quad f_2=0.24,\quad g_2=0.86.$$