PHYSICAL REVIEW VOLUME 188, NUMBER 3 15 DECEMBER 1969

Green's Function for an Electron in a Lattice*

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(Received 14 August 1969)

The dynamics of an electron in a solid (with or without uniform external electromagnetic fields) is studied by confining attention to a single lattice cell. The Green's function is presented and computed in the limit of the periodic potential going to zero.

1. INTRODUCTION

The dynamics of an electron in a solid is studied by confining attention to a single lattice cell. The periodicity of the lattice is built into the Green's function so that periodic potentials, although extending throughout all space, may (depending on the potential) be thought of as small perturbations.

The feasibility of this restriction is to be expected because of the physical equivalence of all lattice cells. However, in the presence of uniform electromagnetic fields, although physical equivalence is retained, the potentials of the field destroy the mathematical equivalence. In this paper, essentially by doing something similar to a gauge transformation, we present a Green's function for a single cell even in the presence of fields.

There is some similarity of our method to Zak's[1] *kq* representation. He too is able to deal with a single cell. As in his work, an "extra" three-dimensional continuum of parameters (**k**) therefore appears. In his representation they are additional variables, to us they are boundary conditions (just as they are for Bloch wave functions). Related to this work and a precursor to Zak's is the paper of Pradduade,[2] who likewise found it useful to treat **k** as a variable for the purpose of retaining formal periodicity in **r**.

In the limit of the periodic potential going to zero, but retaining the periodicity of the lattice (by imposing boundary conditions in the cell), we obtain the Green's function in closed form (with or without electromagnetic fields) as a multidimensional Jacobi theta function. Some tentative applications of this form are presented.

2. GREEN'S FUNCTION FOR A SINGLE CELL

We wish to confine attention to a single cell W of the lattice and to express physical results for the entire lattice in terms of functions on W.

In the absence of magnetic or electric fields all cells are both mathematically and physically equivalent. Under these circumstances we can appeal to a general theory[3] for the construction of Green's functions on W. There is a periodic potential V which is defined through all space $\mathbb{R}^3$ and which, in effect, defines W. Similarly defined are the Bravais lattice vectors

$$\boldsymbol{\varrho}_j = j_1\mathbf{R}_1 + j_2\mathbf{R}_2 + j_3\mathbf{R}_3, \qquad (2.1)$$

where $\mathbf{R}_i$ are the primitive translation vectors. A Green's function G_c can be defined for all of $\mathbb{R}^3$ (which is the covering space for W, cf. Ref. 3) with the potential V. This is now used to define Green's functions G_W on W. Let $\mathbf{x}_1, \mathbf{x}_2 \in W$. For each (arbitrary) $\mathbf{k}$ we can define

$$G_W(\mathbf{k}; \mathbf{x}_2,t_2; \mathbf{x}_1,t_1) = \sum_j \exp(-i\boldsymbol{\varrho}_j\cdot\mathbf{k})\, G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j, t_2; \mathbf{x}_1,t_1), \qquad (2.2)$$

where $j=(j_1,j_2,j_3)$ runs through triples of integers. Such a sum for $V=0$ and in one dimension was carried out in Ref. 3; it may be quite singular but can be given meaning (as the boundary value of an analytic function) by adding a small positive imaginary part to the mass parameter that appears in G_c. Obviously,

$$G_W(\mathbf{k}; \mathbf{x}_2+\mathbf{R}_i, t_2; \mathbf{x}_1,t_1) = \exp(i\mathbf{R}_i\cdot\mathbf{k})\, G_W(\mathbf{k}; \mathbf{x}_2,t_2; \mathbf{x}_1,t_1). \qquad (2.3)$$

Now for each $\mathbf{k}$, a Bloch function $\Psi_{\mathbf{k}}(\mathbf{x})$ can be considered to be defined on W alone. It is thus a solution of Schrödinger's equation (in W but of the same form as in $\mathbb{R}^3$) with the boundary conditions

$$\Psi_{\mathbf{k}}(\boldsymbol{\xi}+\mathbf{R}, t) = \exp(i\mathbf{k}\cdot\mathbf{R})\Psi_{\mathbf{k}}(\boldsymbol{\xi},t), \qquad (2.4)$$

where $\mathbf{R}$ is any of the $\mathbf{R}_i$, and $\boldsymbol{\xi}$ is on the "left" boundary of W, i.e., let ∂W = boundary surfaces of W, then the left boundary $\equiv \partial_L W = \partial W - \partial_R W$; $\partial_R W = \{\boldsymbol{\xi}\in W \mid \boldsymbol{\xi} - \mathbf{R}_i \in W \text{ for some } \mathbf{R}_i\}$.[4] Of course Ψ may be trivially taken to be of the form

$$\Psi_{\mathbf{k}}(\mathbf{x},t) = \exp(i\mathbf{k}\cdot\mathbf{x})u_{\mathbf{k}}(\mathbf{x},t), \qquad (2.5)$$

with u periodic, but for later purposes it suits us only to impose the condition (2.4).

$G_W(\mathbf{k})$ is now seen to have been defined just so as to propagate solutions of Schrödinger's equation with boundary conditions (2.4).

Since the $\mathbf{k}$ in (2.4) is essentially the same as the k in Zak's kq representation, it is worth remarking how the present approach regards the appearance of three extra degrees of freedom (k_1,k_2,k_3) in the wave functions.

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* Work supported in part by the National Science Foundation and the U. S. Army Research Office (Durham).

[1] J. Zak, Phys. Rev. Letters **19**, 1385 (1967); Phys. Rev. **168**, 686 (1968); **177**, 1151 (1969).

[2] H. C. Pradduade, Phys. Rev. **140**, A1292 (1965).

[3] L. Schulman, Phys. Rev. **176**, 1558 (1968), Sec. 3.

[4] If $\mathbf{x} = \sum \alpha_i \mathbf{R}_i$, then W is defined by $0 \leq \alpha_i \leq 1$, $i=1, 2, 3$; on ∂W at least one of the α_i is 0 or 1; on $\partial_R W$ one or more of the α_i is 1.

They are simply the three-dimensional continuum of boundary conditions that are made possible because the space W is not simply connected; their particular form arises because W has the fundamental homotopy group $\mathbf{Z}^3$.

G_c can be recovered by integration in the first Brillouin zone:

$$G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,\, t_2;\, \mathbf{x}_1,t_1) = (1/V)\int \exp(i\mathbf{k}\cdot\boldsymbol{\varrho}_j)\, G_W(\mathbf{k};\, \mathbf{x}_2,t_2;\, \mathbf{x}_1,t_1)d^3k\,, \quad (2.6)$$

where V is the volume in k space of this zone. [An over-all phase ambiguity can be used to ensure that $G_W(\mathbf{x}_2+\mathbf{R},\, t_2;\, \mathbf{x}_1+\mathbf{R},\, t_1)=G_W(\mathbf{x}_2,t_2;\, \mathbf{x}_1,t_1)$ so that $\mathbf{x}_1$ can also be selected anywhere in $\mathcal{R}^3$.]

Various applications of this formalism come immediately to mind. We can give an integral equation, equivalent to Schrödinger's equation and entirely defined in W, which incorporates both the usual sort of initial conditions and also the periodicity. If

$$H=H_0+H_1 \quad (2.7)$$

then in the usual way it follows that

$$G(t)=G_0(t)-i\int_0^t G_0(t-\tau)H_1(\tau)G(\tau)d\tau\,, \quad (2.8)$$

where G and G_0 have the same $\mathbf{k}$ and are defined by[5]

$$(H-i\partial/\partial t)G=-i\delta(t-t_0)1\,,$$
$$(H_0-i\partial/\partial t)G_0=-i\delta(t-t_0)1\,. \quad (2.9)$$

For example, if $H_0=p^2/2m$, then G_0 can be evaluated in closed form. If g denotes the Green's function for a free particle,

$$g(\mathbf{x}_2,t_2;\, \mathbf{x}_1,t_1)=g(\mathbf{x}_2-\mathbf{x}_1,\, t_2-t_1)\,,$$
$$g(\mathbf{y},t)=(m/2\pi it)^{3/2}\exp(im\mathbf{y}^2/2t)$$

(of course $g=0$ for $t<0$), then letting $\mathbf{y}=\mathbf{x}_2-\mathbf{x}_1$, $t=t_2-t_1$ gives

$$G_{W0}(\mathbf{k};\, \mathbf{x}_2,t_2;\, \mathbf{x}_1,t_1)=(m/2\pi it)^{3/2} \times\sum_j \exp[(im/2t)(\mathbf{y}+\boldsymbol{\varrho}_j)^2-i\mathbf{k}\cdot\boldsymbol{\varrho}_j]\,. \quad (2.10)$$

The Jacobi theta function is defined as

$$\theta_3(z,t)=\sum_{l=-\infty}^{\infty}\exp(i\pi l^2t)\exp(2\pi ilz)\,. \quad (2.11)$$

The N-dimensional generalization of this[6] replaces l^2 by $-\sum_{i,j=1}^N t_{ij}l_il_j$ and zl by $\sum_{i=1}^N z_il_i$. Obviously G_{W0} is such a multidimensional theta function. These functions are the subject of a well-developed theory, references to which can be found in Bellman. For example, they satisfy a generalization of the fundamental theorem for theta functions which is useful for obtaining the energy eigenfunctions from the Green's function[3]:

$$\theta(z,T)\equiv\sum_n \exp[-(n,Tn)+2\pi i(n,z)]$$
$$=\frac{\pi^{N/2}}{(\det T)^{1/2}}\sum_n \exp\{-\pi^2[z+n,\, T^{-1}(z+n)]\}\,, \quad (2.12)$$

where z and n are N-tuples and T is the $N\times N$ matrix (t_{ij}). The components of n run through the integers and $(z,n)=\sum_{i=1}^N z_in_i$.

3. ELECTROMAGNETIC FIELDS

If a uniform electromagnetic field pervades the lattice, then

$$H=(1/2m)(\mathbf{p}-e\mathbf{A})^2-\mathbf{E}\cdot\mathbf{x}+V(\mathbf{x})\,,\quad \mathbf{x}\in\mathcal{R}^3\,, \quad (3.1)$$

with $\mathbf{E}=\text{const}$ and $\mathbf{A}=\frac{1}{2}\mathbf{B}\times\mathbf{x}$, $\mathbf{B}=\text{const}$. V is periodic. Although all cells are still physically equivalent, the appearance of potentials has introduced a mathematical inequivalence. However, we shall find that it is still possible to write a Green's function for W alone.

G_W is supposed to satisfy (2.9) with the Hamiltonian (3.1). However, the expansion (2.2) is no longer possible since $G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,\, t_2;\, \mathbf{x}_1,t_1)$ no longer satisfies (2.9) in W. However, the equation that G_c satisfies [(2.9) in $W+\boldsymbol{\varrho}_j$] is related to (2.9) in W by a gauge transformation. Making use of well-known procedures,[7] we modify $G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,\, t_2;\, \mathbf{x}_1,t_1)$ so that it satisfies Schrödinger's equation in W. Instead of $G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,\, t_2;\, \mathbf{x}_1,t_1)$, we use

$$\exp[-ie\Lambda_j(\mathbf{x}_2-\mathbf{x}_1,\, t_2-t_1)]G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,\, t_2;\, \mathbf{x}_1,t_1)\,, \quad (3.2)$$

with

$$\Lambda_j(\mathbf{x},t)=\boldsymbol{\varrho}_j\cdot(t\mathbf{E}+\tfrac{1}{2}\mathbf{x}\times\mathbf{B})\,. \quad (3.3)$$

Λ_j is the generator of the gauge transformation relating H in W to H in $W+\boldsymbol{\varrho}_j$. G_W can now be expressed as

$$G_W(\mathbf{k};\, \mathbf{x}_2,t_2;\, \mathbf{x}_1,t_1) =\sum_j \exp\{-i\boldsymbol{\varrho}_j\cdot[\mathbf{k}+e\mathbf{E}(t_2-t_1)+\tfrac{1}{2}e(\mathbf{x}_2-\mathbf{x}_1)\times\mathbf{B}]\} \times G_c(\mathbf{x}_2+\boldsymbol{\varrho}_j,t_2;\, \mathbf{x}_1,t_1)\,. \quad (3.4)$$

The considerations of Sec. 2 now go through, and the wave functions propagated by G_W have the property

$$\Psi(\mathbf{x}+\boldsymbol{\varrho}_j,\, t)=\exp[i\mathbf{k}\cdot\boldsymbol{\varrho}_j+ie\Lambda_j(\mathbf{x},t)]\Psi(\mathbf{x},t) \quad (3.5)$$

for $\boldsymbol{\varrho}_j$ one of the lattice vectors. (Note: For $\mathbf{x}\notin W-\partial_R W$ this property does not hold.) In this way boundary conditions enter the problem in a dynamical way.

For perturbation theory in this framework, Eqs.

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[5] The operator 1 in Eqs. (2.9) is $\sum_j \exp(-i\mathbf{k}\cdot\boldsymbol{\varrho}_j)\,\delta(\mathbf{x}_2+\boldsymbol{\varrho}_j-\mathbf{x}_1)$.

[6] R. Bellman, *A Brief Introduction to Theta Functions* (Holt, Rinehart and Winston, Inc., New York, 1961), Sec. 61.

[7] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1965), p. 422.

(2.7) and (2.8) are again relevant. Taking

$$H_0=(1/2m)(\mathbf{p}-e\mathbf{A})^2\,, \tag{3.6}$$

a result of Glasser[8] can be used to obtain once more a closed-form expression for G_0. Glasser used the path-integral method to obtain the Green's function for a particle (moving in $\mathcal{R}^3$) in a magnetic field. It is[9] ($B=|\mathbf{B}|$):

$$\begin{aligned} g(x_2,t;\,x_1,0)=&\left(\frac{m}{2\pi it}\right)^{3/2}\left(\frac{eBt}{2m}\right)\csc\left(\frac{eBt}{2m}\right)\\ &\times\exp\left\{\frac{im}{2}\left[\frac{eB}{2m}\cot\left(\frac{eBt}{2m}\right)[\hat{B}\times(\mathbf{x}_2-\mathbf{x}_1)]^2\right.\right.\\ &\left.\left.+\frac{1}{t}[\hat{B}\cdot(x_2-x_1)]^2-\frac{e}{m}\mathbf{B}\cdot\mathbf{x}_2\times\mathbf{x}_1\right]\right\}\,. \end{aligned} \tag{3.7}$$

Using g for G_c in (3.4) (with $\mathbf{E}=0$) leads to an expression for G_W as a multidimensional theta function. Before writing this explicitly we introduce some notation. Let

$$\omega=eB/m\,,\quad (\mathbf{R}_i)_{\mathbf{k}}=R_{i\mathbf{k}}\,,$$
$$\mathbf{R}_m\cdot\mathbf{K}_n=2\pi\delta_{mn}\,,\quad (\mathbf{K}_n)_l=K_{nl}\,,$$
$$(\tilde{A})_{ij}=A_{ji}\,,\quad (\boldsymbol{\beta})_{nm}=\hat{B}_n\hat{B}_m\,,\quad \mathbf{y}=\mathbf{x}_2-\mathbf{x}_1\,.$$

Thus $(\boldsymbol{\varrho}_j)_i=j_{\mathbf{k}}R_{\mathbf{k}i}$. If each term in (3.4) is written in the form $A\exp(iS_j)$, then

$$A=(m/2\pi it)^{3/2}(\tfrac{1}{2}\omega t)\csc(\tfrac{1}{2}\omega t)\,,$$

$$\begin{aligned} S_j=&-(\mathbf{y}+\boldsymbol{\varrho}_j)\cdot(\mathbf{k}+\tfrac{1}{2}m\omega\mathbf{x}_2\times\hat{B})\\ &+\tfrac{1}{4}m\omega\cot(\tfrac{1}{2}\omega t)(\mathbf{y}+\boldsymbol{\varrho}_j)\cdot(\mathbf{1}-\boldsymbol{\beta})\cdot(\mathbf{y}+\boldsymbol{\varrho}_j)\\ &+(m/2t)(\mathbf{y}+\boldsymbol{\varrho}_j)\cdot\boldsymbol{\beta}\cdot(\mathbf{y}+\boldsymbol{\varrho}_j)+\mathbf{y}\cdot\mathbf{k}\,. \end{aligned}$$

Since the matrix for the reciprocal-lattice vectors satisfies $R\tilde{K}=2\pi\cdot 1$,

$$(\mathbf{y}+\boldsymbol{\varrho}_j)_i=[(y\tilde{K})_m+2\pi j_m](\tilde{K}^{-1})_{mi}\,.$$

Absorbing the $\tilde{K}^{-1}$ into the various vectors and dyadics in S_j, it is obvious from (2.12) that S_j is just the argument of the exponent appropriate to the theta function definition. The modular transformation (2.12) can be applied, and a tedious calculation leads to the result

$$\begin{aligned} G_W(\mathbf{k};\,\mathbf{x}_2,t_2;\,\mathbf{x}_1,0)=&(2\pi)^{-3}(\det K)\sec(\tfrac{1}{2}\omega t)\exp(i\mathbf{y}\cdot\mathbf{k})\\ &\times\sum_j\exp[-i\tfrac{1}{4}(\mathbf{q}_j+\mathbf{u})\cdot\mathbf{D}\cdot(\mathbf{q}_j+\mathbf{u})+i\mathbf{q}_j\cdot\mathbf{y}]\,, \end{aligned} \tag{3.8}$$

where $\mathbf{q}_j$ are reciprocal-lattice vectors

$$\mathbf{q}_j=\sum_{n=1}^{3}j_n\mathbf{K}_n\,, \tag{3.9}$$

the dyadic $\mathbf{D}$ is

$$\mathbf{D}=(4/m\omega)\{\mathbf{1}\tan\tfrac{1}{2}\omega t+\boldsymbol{\beta}(\tfrac{1}{2}\omega t-\tan\tfrac{1}{2}\omega t)\} \tag{3.10}$$

and

$$\mathbf{u}=\mathbf{k}+\tfrac{1}{2}m\omega\mathbf{x}_2\times\hat{B}\,. \tag{3.11}$$

Obviously this too can be compactly expressed as a theta function. For $\omega=0$, this form of the Green's function can be used to read off the energy levels of the system by comparison with the general expansion

$$G(\mathbf{k};\,\mathbf{x}_2,t;\,\mathbf{x}_1,0)=\sum_j\Psi\mathbf{k}_j(\mathbf{x}_2)\Psi_{\mathbf{k}j}{}^*(\mathbf{x}_1)\exp(-iE_{\mathbf{k}j}t)\,. \tag{3.12}$$

For $\omega\neq0$ one could, in principle, obtain similar information by Fourier transforming (3.8), but the complicated t dependence in $\mathbf{D}$ makes this difficult.

Each term in the sum over j in (3.8) is recognized as arising from a corresponding one in (3.12), as ω departs from zero. If in a single summand in (3.12) we set $\mathbf{x}_2=\mathbf{x}_1$ and integrate over W, then by the normalization of Ψ, what is left is $\exp(-iE_{\mathbf{k}j}t)$. To lowest order in ω, a similar term can be obtained from the summands in (3.8), and we use this to compute the energy shift in a given band:

$$\Delta E_j=\tfrac{1}{4}\omega\sum_i\mathbf{R}_i\cdot\hat{B}\times(\mathbf{q}_j+\mathbf{k})\,. \tag{3.13}$$

Departures from rigor in this expression are essentially due to band crossing. Another way of saying this is that the approximation $\omega\rightarrow0$ also depends on the size of t. Therefore (3.12) cannot be rigorously identified with the approximated version of (3.8) term by term.

One can obtain the partition function $Z=\mathrm{Tr}(e^{-\beta H})$ from this Green's function with the usual identification $it\rightarrow\beta=1/kT$ and with integration over $\mathbf{x}=\mathbf{x}_1=\mathbf{x}_2$. For small ω and high temperature the integration goes through as above yielding Z with all band-crossing effects exactly accounted for. The integral over k for the computation of Z again yields a theta function.

4. PROSPECTS

An aspect of the foregoing analysis that is amusing is the incorporation of boundary conditions into the dynamics of the problem. This is already apparent in Zak's theory. If the presently proposed method finds practical application, it will probably be because the free (or free plus magnetic) part of the problem can be solved in closed form as a multidimensional theta function. These functions are known for their convenience in computation[6] and also for the illumination they provide when the modular transformation is applied. Furthermore, remaining in a single cell of the lattice (another feature we have in common with Zak) should have the

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[8] M. L. Glasser, Phys. Rev. **133**, B831 (1964). The same formula appears in R. P. Feynman and A. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill Book Co., New York, 1965), p. 64.

[9] This provides yet another example of an *exact* time-dependent Green's function that is equal to $Ae^{iS/\hbar}$, where S is the action along the classical path (i.e., that satisfying the equations of motion), and A is the square root of the Van Vleck determinant $[\det-\partial^2S/\partial(x_2)_i\partial(x_1)_j]$. In Refs. 3 and 8 will be found further remarks on this subject.

advantage that introducing the periodic potential will not have such a traumatic effect on the nature of the problem since the periodicity has already been accounted for and built into the Green's function.

A specific physical situation where the results of Sec. 3 may prove useful is the problem of magnetic breakdown.[10] Under the dominating influence of a strong magnetic field it may be reasonable to consider the lattice potential to be a small perturbation.

ACKNOWLEDGMENTS

I am indebted to Professor Cecile DeWitt for some interesting discussions on path integration and its applications. I have also benefited from interactions with Professor James Swihart and Professor John Klauder on matters pertaining to this work.

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[10] J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, Cambridge, England, 1964), Sec. 9.8.

PHYSICAL REVIEW VOLUME 188, NUMBER 3 15 DECEMBER 1969

Short-Range Order in Disordered Binary Alloys*

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(Received 28 April 1969; revised manuscript received 23 June 1969)

The Warren short-range-order (SRO) parameter of a binary alloy with arbitrary composition is studied for the case of static, configurational interaction with two-body potentials of arbitrary range. A well-defined straightforward procedure for generating a consistent high-temperature series expansion in powers of a dimensionless parameter related to T_c/T is derived. While the existing theories are correct only to the linear power in T_c/T, our results are worked out exactly to the third power in this ratio. A plausible inversion of the series, which in the linear approximation corresponds to the Clapp-Moss theory, is given. It is shown that, at least up to the second order in T_c/T, such an inversion corresponds to the predictions of a self-consistent decoupling approximation. Using a technique similar to that first introduced by Kramers and Opechowski, the order-disorder transition temperature T_c is computed as a function of the system composition for lattices of cubic symmetry with nearest-neighbor interactions. A remarkable prediction of this study is the suggestion of the existence of a critical concentration below which the system does not order (or separate, as the case may be). For the special case of positive nearest-neighbor interaction in lattices of cubic symmetry, we have also computed the nearest-neighbor SRO parameter for several compositions and temperatures. The results of the present study typically renormalize the corresponding results of the Clapp-Moss theory by several percent. For the stoichiometric composition $m^{(A)}=m^{(B)}$, the results of the present approximation are compared with the very reliable corresponding results of Fisher and Burford (who carried out a careful evaluation of the systematics of the elaborate high-temperature series available for the Ising ferro- and antiferromagnets, i.e., for $m^{(A)}=m^{(B)}$). In general, the quantitative differences between Fisher and Burford's results and those of the present approximation are smaller than the corresponding differences for the linear approximation of Clapp and Moss. An interesting conclusion of the present analysis is that, except for the special case of the nearest-neighbor interactions, the structure of the Fourier transform of the SRO parameter $\alpha(\mathbf{K})$ is different from that conjectured by Clapp and Moss. As such, the originally compelling argument in favor of the Clapp-Moss assumption, that from experimental observations of $\alpha(\mathbf{K})$ the ratios of the strengths of the interparticle potentials for different separations are determined with a higher degree of reliability than the actual magnitudes of the potentials, is found to be somewhat weakened.

I. INTRODUCTION

THE problem of determining the statistical correlations in disordered binary alloys has received much attention in the literature.[1] Clapp and Moss[2] have recently reported a new solution of the correlation function. This solution is applicable to alloys of arbitrary composition with two-body potentials of arbitrary range. However, for arbitrary composition this solution turns out to give only the dominant temperature-dependent term correctly. (For stoichiometric composition, the next-order temperature-dependent term is also correct.) Of course, at sufficiently elevated temperatures the higher-order terms contribute negligibly to the correlations, and as such the Clapp-Moss solution can be expected to be adequate. As the temperature becomes comparable to, or less than, about $2T_c$, where T_c is the transition temperature, these additional temperature-dependent terms become increasingly important.

Fisher and Burford and Moore *et al.*[3] have reported an

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* Supported by the U. S. Atomic Energy Commission.

[1] F. Zernike, Physics **7**, 565 (1940); T. Oguchi, J. Phys. Soc. Japan **6**, 31 (1951); R. J. Elliott and W. Marshall, Rev. Mod. Phys. **30**, 75 (1958); J. M. Cowley, Phys. Rev. **77**, 669 (1950); **120**, 1648 (1960); **138**, A1384 (1965); D. O. Christy and G. L. Hall, *ibid.* **132**, 1959 (1963); R. Brout, *Phase Transitions* (W. A. Benjamin, Inc., New York, 1965).

[2] P. C. Clapp and S. C. Moss, Phys. Rev. **142**, 418 (1966).

[3] M. E. Fisher and R. J. Burford, Phys. Rev. **156**, 583 (1967); M. A. Moore, D. Jasnow, and M. Wortis (to be published).