Surface-Impedance Anomalies at Perpendicular-Field Cyclotron Resonance in the Anomalous-Skin-Effect Regime

G. A. BARAFF
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
and
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 24 April 1969)

A model of a rough surface is proposed in which the physical effects are simulated mathematically in a Boltzmann-equation formalism by allowing the scattering time for electrons to depend upon their depth beneath the surface. The nonlocal conductivity in this model is evaluated by solving the linearized Boltzmann equation, and the field which that conductivity supports is then determined variationally. This model accounts for one of the two types of anomalies in the surface resistance observed at perpendicular-field cyclotron resonance in the anomalous-skin-effect regime. The possibility of constrained electrons producing the other type of anomaly is briefly considered.

I. INTRODUCTION

RECENT measurements of the surface impedance of potassium in a magnetic field normal to the surface of the metal[1] show behavior that cannot be explained on the basis of the classical free-electron model.[2–4] Similar discrepancies have been seen in the transmission of electromagnetic waves across thin sodium slabs under essentially the same conditions.[5] The discrepancy in the surface impedance is illustrated in Fig. 1, where the measured surface resistance is plotted as a solid line and the calculated value is shown as a dashed curve. The experimental curve exhibits a sharp peak at $\omega_c/\omega = -1.025 \pm 0.005$, followed by a decrease over a range of magnetic fields larger (i.e., more negative) than the value at the peak. Not shown on the experimental curve is the leveling off and eventual increase of the surface resistance at still larger fields. The theoretical curve, by contrast, exhibits an uninterrupted rise as the surface resistance increases monotonically with increasing magnetic field. The sharp peak near cyclotron resonance and the decrease in surface resistance, neither of which are predicted by the classical theory, are the anomalies that concern us here. (The apparent replication of these anomalies at $\omega_c/\omega = +1$ is caused by a small admixture of field rotating in the opposite sense from the main component and is of no particular interest.)

Anomalies similar to those that interest us here have been observed in cadmium by Galt, Merritt, and Klauder.[6] The explanations tendered at the time of observation involved the presence of several sets of carriers in that particular metal.[6,7] Such an explanation, whatever its validity for cadmium, cannot possibly apply to potassium, which is known to have a single type of carrier and a spherical Fermi surface. The explanation for the anomalies in potassium must be found elsewhere.

In Ref. 1, we attributed the sharp peak at $\omega_c/\omega = -1.025$ to the excitation of the correlation-produced magnetoplasma mode,[8] and we still believe that that explanation is the correct one. The experimental decrease was attributed[1] to the effects of surface roughness. In the present work, we want to show how the phenomenon of surface roughness may be introduced into a calculation of surface impedance and to present the results of such calculations. We also point out that, although we do not believe in their existence in the potassium experiments, if a very small number of

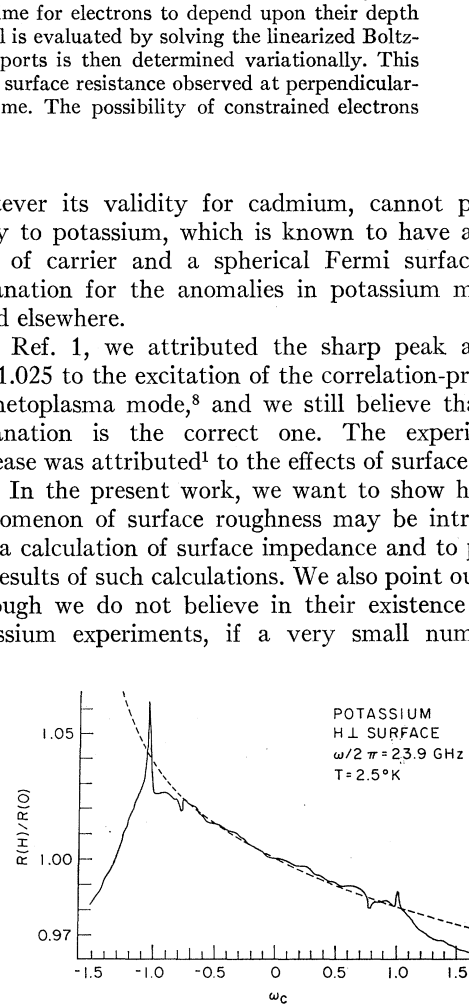

FIG. 1. The dashed curve is the normalized surface resistance as a function of magnetic field predicted by the classical free-electron model for a diffuse surface with mean free path of 1.5 $\times 10^{-2}$ cm and the other parameters as given in Table I. The solid line is the surface resistance of potassium in perpendicular-field geometry measured with predominantly circular polarization. The absolute value of the surface resistance is unkhown and therefore the vertical scale of the experimental curve is arbitrary. For the purposes of display, the scale was chosen to cause the measured curve to parallel the theoretical one.

[1] G. A. Baraff, C. C. Grimes, and P. M. Platzman, Phys. Rev. Letters **22**, 590 (1969).
[2] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **A195**, 336 (1948).
[3] R. B. Dingle, Physica **19**, 311 (1953).
[4] R. G. Chambers, Phil. Mag. **1**, 459 (1956).
[5] S. Schultz (private communication).
[6] J. K. Galt, F. R. Merritt, and J. R. Klauder, Phys. Rev. **139**, A823 (1965).
[7] A similar explanation for anomalies in zinc was suggested by M. H. Cohen, in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), p. 178.
[8] Y. C. Cheng, J. S. Clarke, and N. D. Mermin, Phys. Rev. Letters **20**, 1486 (1968).

electrons—of the order of a twentieth of a monolayer—is for some reason constrained to move parallel to and closely beneath the surface, it will give rise to a peak in the surface resistance which corresponds in shape, amplitude, and position with what has been observed.

II. SURFACE ROUGHNESS

The conventional treatment of anomalous skin effect,[2–4] with or without a magnetic field normal to the surface, proposes that the metal surface be treated as a plane at which electrons suffer either specular or diffuse reflection, or, perhaps, some combination of the two. Calculations performed with this model yield surface impedances whose dependence on magnetic field is roughly independent of whether diffuse or specular reflection is assumed. The dashed curve of Fig. 1 is typical of what is predicted in the diffuse case in the neighborhood of cyclotron resonance. There is a change in the slope of the surface resistance at $\omega_c=\omega$, but no striking structure.

Since the time of Pippard's introduction of the ineffectiveness concept,[9] it has been generally understood that the electrons traveling parallel to the surface within the skin depth interact most strongly with the field, and hence play the major role in determining surface impedance. One might infer from this that if the surface were rough, i.e., had indentations which could intercept the flights of these effective electrons and thereby drastically shorten their mean free time, there would be a profound effect on the surface impedance. Actually, this is not the case because the surface impedance, as calculated by using the ineffectiveness concept, is independent of the mean free time. Thus, the physical condition of the surface, i.e., whether or not it is accurately plane to within a tolerance comparable to a skin depth, and whether or not there are dislocations which enhance the scattering rate of surface electrons, does not usually enter the physics. For this reason, a model wherein the metal is bounded by a simple plane surface has proved adequate in the past for the calculation of surface impedance.

Galt, Merrit, and Klauder[6] were the first to realize that the observed changes in surface resistance were small enough to make it necessary to consider the next higher-order term in the conventional treatment.[2–4] This term is needed for the dashed curve in Fig. 1 to depend on the magnetic field at all. The dashed curve, and the physical model which it represents, still provide a zeroth-order approximation to what is seen. That it is no longer adequate for the effects of interest here is evident from Fig. 1.

Suppose now that the metal were bounded by a rough surface. By rough surface we mean one that is not accurately plane to a tolerance comparable to a skin depth over distances comparable to the electron-cyclotron radius or to the electron bulk mean free path, whichever is the shorter length that normally limits the extent of the electron's lateral excursion. Such a surface will intercept a large fraction of the electrons traveling parallel to the surface with $V_z=0$, provided that they are located close enough to the surface.

[9] A. B. Pippard, Proc. Roy. Soc. (London) **A191**, 385 (1947).

The physical effect of a rough surface can be simulated mathematically in a Boltzmann-equation description by introducing a mean free time $\tau(z)$ which depends on the distance from the surface of the metal. By letting τ have the bulk value τ_b when z is greater than some distance δ from the surface (δ will be the vertical extent of the deviation from flatness) and some much smaller value τ_s, when z is closer to the surface than δ, we have a situation in which electrons with small V_z have their path sharply curtailed if they are closer to the surface than δ, but are undisturbed when they travel at greater depths. Even when the surface is rough, electrons that start their flights at the surface and have large V_z will not be intercepted again by the surface. In the model, these large V_z electrons are, by virtue of their rapid motion through the region δ, out of the small τ region in a time short compared with τ_s, i.e., before the fictitious scattering can act, if $\tau_s \gg \delta/V_F$. Those which start their flight at the surface with small V_z and which, physically, would be intercepted again by a rough surface because they stay in the disturbed region δ, will experience the enhanced scattering rate τ_s^{-1} which can approximate collisions with the surface. Thus, a simple $\tau(z)$ of this sort, used in a Boltzmann equation, can simulate the effects of a rough surface.

One could easily generalize by choosing a shape for $\tau(z)$ which was related to the probability that a surface depression extends into the metal at least as far as depth z, and then if need be, take an ensemble average (over probable roughness distributions) of the resulting conductivity. This would yield an effective conductivity which could probably reproduce any smooth curve of the same general shape, as will be exhibited below. It is not, however, our purpose to fit curves here. We choose the simplest model, namely, an added layer of additional scattering, to show the sort of effect to be expected.

III. IMPEDANCE OF ROUGH SURFACE

The Boltzmann equation governing the electron distribution is written for a transverse wave propagating in the z direction:

$$\left(\frac{\partial}{\partial t}+V_z\frac{\partial}{\partial z}\right)f(z,\mathbf{p},t)+q(\mathbf{e}+\mathbf{V}\times\mathbf{B})\cdot\nabla_p f=\frac{-(f-f_0)}{\tau(z)}, \quad (3.1)$$

where f_0 is the zero-temperature Fermi distribution, $\mathbf{V}=\nabla_p\epsilon$ is the velocity, $\epsilon(|p|)$ is the energy giving rise to a single spherical Fermi surface, and $\mathbf{e}(z)$ is a circularly polarized transverse electric field varying in the z direction. The steady uniform magnetic field, B_0 points along z.

This equation is linearized by putting

$$f(z,\mathbf{p},t)=f_0(p)+f_1(z,p)e^{-i\omega t},$$
$$\mathbf{B}(z,t)=\mathbf{B}_0+\mathbf{b}_1(z)e^{-i\omega t},$$
$$\mathbf{e}(z,t)=\mathbf{e}_1(z)e^{-i\omega t},$$

and extracting from the resulting equation the terms containing the factor $e^{-i\omega t}$. We can solve the linearized equation exactly, and, for a given field $\mathbf{e}_1(z)$, we can determine f_1, the current resulting, and the conductivity $\sigma(z,z')$ which this current implies. That is, we calculate

$$J_\pm(z)\equiv J_x(z)\pm iJ_y(z)=q\int d^3p(V_x\pm iV_y)f_1(z,\mathbf{p})$$
$$=\int_0^\infty \sigma_\pm(z,z')e_\pm(z')dz', \quad (3.2)$$

where

$$e_\pm(z)=e_{1x}(z)\pm ie_{1y}(z).$$

This procedure yields a $\sigma(z,z')$ virtually identical with that of the usual specular (s) or diffuse (d) surface calculations when specular or diffuse boundary conditions are adjoined to (3.1). The sole difference is that here the quantity

$$l(z,z')\equiv V_F^{-1}\int_{z'}^{z} dz''[\tau^{-1}(z'')-i(\omega-\omega_c)]$$

replaces the quantity

$$(z-z')[\tau_b^{-1}-i(\omega-\omega_c)]/V_F$$

on which the usual conductivity depends. In particular,

$$\sigma_d(z,z')=\frac{3nq^2}{4p_F}\int_0^{\pi/2}\frac{\sin^3\theta}{\cos\theta}e^{-l(z,z')/\cos\theta}d\theta \quad (z>z') \quad (3.3a)$$
$$=\sigma_d(z',z) \quad (z'>z) \quad (3.3b)$$

and

$$\sigma_s(z,z')=\sigma_d(z,z')+\sigma_d(z,-z').$$

(In evaluating the second term of σ_s, one must calculate as though τ were an even function of z.)

The field in the metal satisfies the nonlocal wave equation

$$\left(\frac{d^2}{dz^2}+k_0^2\right)e(z)=-i\omega\mu_0\int_0^\infty \sigma(z,z')e(z')dz', \quad (3.4a)$$
$$e(z)\to 0 \quad \text{as} \quad z\to\infty \quad (3.4b)$$

plus a boundary condition at $z=0$. If we choose the boundary condition $e(z=0)=e_0$ a fixed number, then because of (3.3b) and (3.4), the functional

$$K[e]\equiv\int_0^\infty dz\left[\left(\frac{de}{dz}\right)^2-k_0^2e^2(z)\right]$$
$$-2i\omega\mu_0\int_0^\infty dz\int_0^z dz'\, e(z)\sigma(z,z')e(z') \quad (3.5)$$

is stationary with respect to variations of e,

$$\delta K/\delta e(z)=0, \quad \delta e_0=0, \quad (3.6)$$

and, moreover, its stationary value is such that the dimensionless surface impedance

$$Z\equiv\left(\frac{ik_0e(z)}{de/dz}\right)_{z=0} \quad (3.7)$$

is given by

$$Z=-ik_0e_0^2/K[e]. \quad (3.8)$$

Equations (3.5)–(3.8) provide a variational principle for the surface impedance quite similar to that used by Jones and Sondheimer.[10] What we are doing, then, is using an exact solution of the linearized Boltzmann equation to calculate the conductivity, and then using a variational principle to determine the field which that conductivity will support.

As the trial function for the field, we use

$$e(z)=ae^{-x\alpha z}+(1-a)e^{-y\alpha z}, \quad \alpha\equiv[\tau_b^{-1}-i(\omega-\omega_c)]/V_F$$

and we fix the three complex parameters a, x, and y by demanding that K be stationary:

$$\partial K/\partial a=0, \quad (3.9a)$$
$$\partial K/\partial x=0, \quad (3.9b)$$
$$\partial K/\partial y=0. \quad (3.9c)$$

Equation (3.9a) can be used to remove the parameter a from (3.9b) and (3.9c) and the result is a pair of simultaneous equations for x and y. This procedure introduces a third-order spurious solution $x=y$ which can be removed by dividing (3.9b) and (3.9c) by $(x-y)^3$. The resulting pair of equations allows an iterative solution whose convergence is quite rapid and, in the cases in which it can be compared with known results, yields a sufficiently accurate value of the surface impedance.

The dimensionless surface resistance, the real part of the Z resulting from this calculation, is shown in Figs. 2–7, which were computed by using diffuse boundary conditions, i.e., using σ_d. The parameters listed in Table I are those appropriate to potassium at 23 GHz, which puts cyclotron resonance at 10 kOe, and were used in all of the calculations.

Figure 2 shows the effect of varying δ, the thickness of the roughened region. The value $\delta=0$ corresponds

[10] M. C. Jones and E. H. Sondheimer, Proc. Roy. Soc. (London) **A278**, 256 (1964).

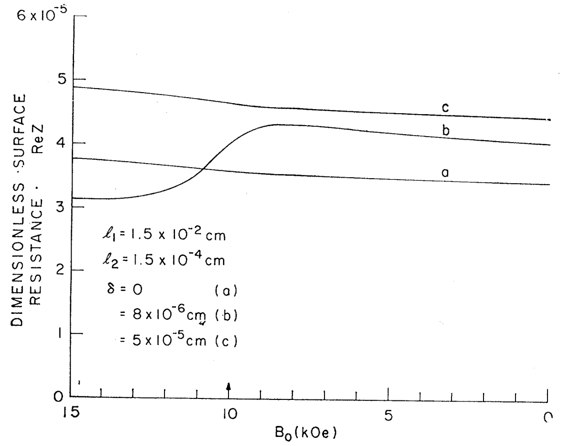

FIG. 2. These curves display the dimensionless surface resistance, the real part of Eq. (3.7), as a function of magnetic field B_0 for various values of the roughness depth δ.

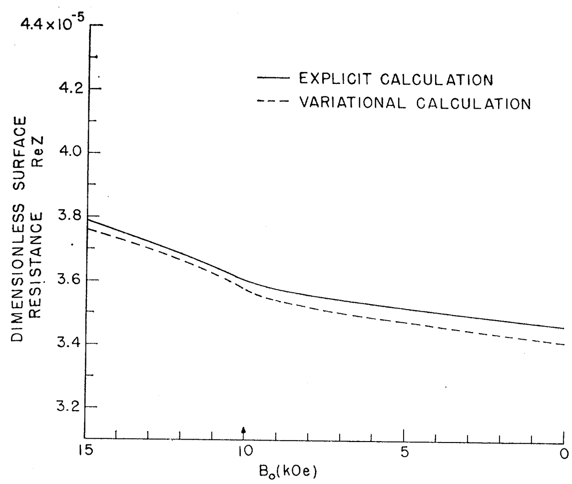

FIG. 3. Comparison between variational and explicit calculation of surface resistance for a diffuse surface with a mean free path $l_1=1.5\times10^{-2}$ cm.

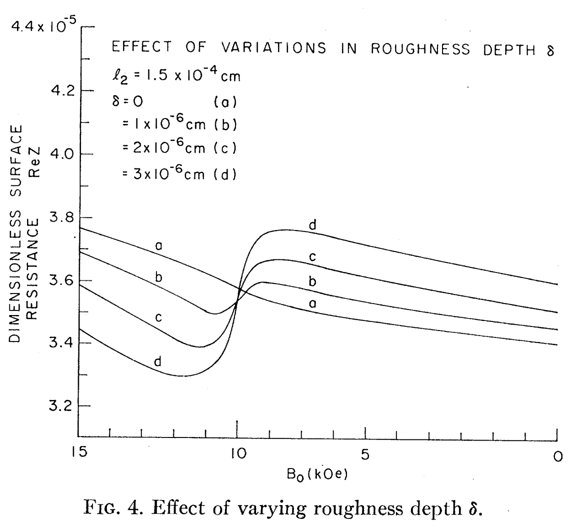

FIG. 4. Effect of varying roughness depth δ.

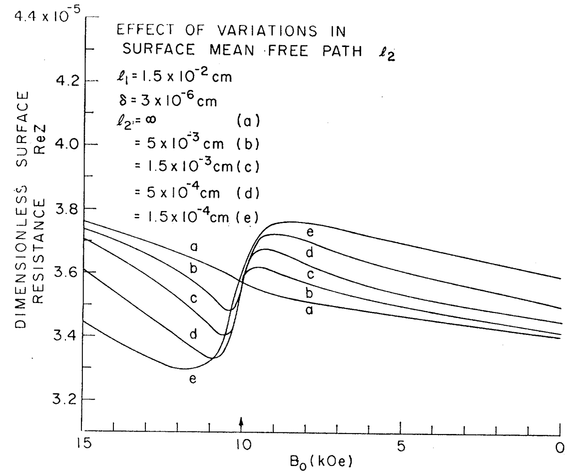

FIG. 5. Effect of varying surface mean free path l_2.

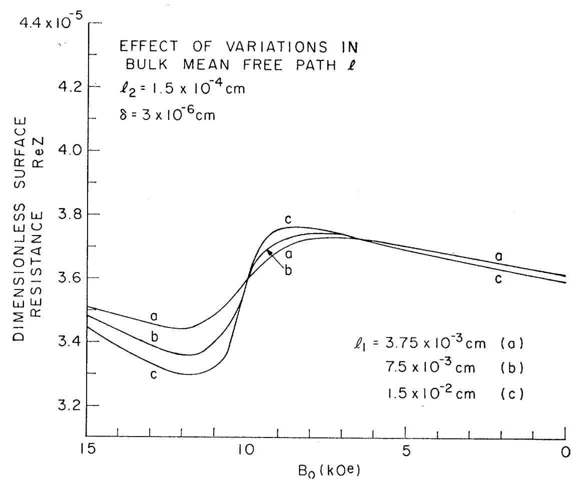

FIG. 6. Effect of varying bulk mean free path l_1.

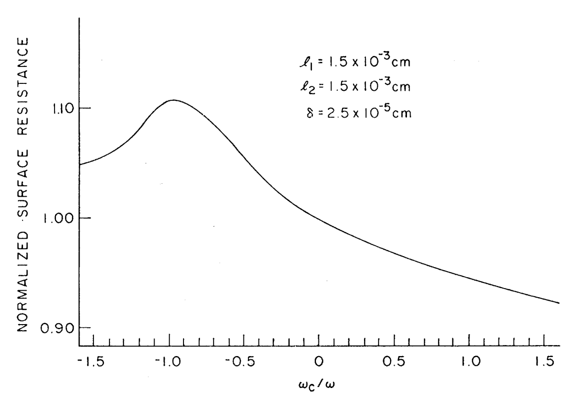

FIG. 7. Normalized surface resistance over a wider range of fields.

physically to the classical diffuse model with $\tau=\tau_b$. Comparison of this variational result with the explicit evaluation of the resistance of a diffuse surface having the same parameters is presented in Fig. 3. The large

TABLE I. Parameters used in the calculations.

ω	Frequency of rf field	1.45×10^{11} rad/sec
m/m_0	Effective mass of electrons	1.21
V_F	Fermi velocity	7.1×10^{7} cm/sec
ω_p	Plasma frequency $=(nq^2/m\epsilon_0)^{1/2}$	6.07×10^{15} rad/sec
δ	Depth of roughness	variable
l_1	Bulk mean free path	$1/\tau_B=V_F/l_1$ variable
l_2	Surface mean free path	$1/\tau_s=V_F/l_1+V_F/l_2$ variable

value of δ exhibited in Fig. 2 is so much larger than an anomalous skin depth that the situation again corresponds to a diffuse surface, one having $\tau=\tau_s$. Agreement with the explicit calculation (although not shown) is as good as in the case of $\delta=0$. The intermediate value of δ used in Fig. 2 is of the order of the anomalous skin depth, a choice that should make the resistance variation the largest.

The effect of more modest values of δ is best seen on a scale comparable to that chosen for the experimental results. In Fig. 4, we have shown the surface resistance for several small values of δ. It is clear that a deviation from flatness of the order of 200–300 Å over distances of the order of a micron is sufficient to account for the size of the observed anomalies.

The effect of changing the horizontal scale of surface roughness is shown in Fig. 5. Here the value of δ is held fixed while l_2, a measure of the *added* surface scatter, is varied. The variation of l_2 produces changes qualitatively similar to those produced by changing δ.

The bulk mean free path can vary over rather wide limits with only small effect on the resistance of a diffuse surface. In Fig. 6, however, we see that changes in the bulk mean free path alter the width of the region over which the anomalous resistance dip occurs.

The preceding calculations use a surface mean free path which is much shorter than the bulk mean free path and a very thin region of surface roughness. It is clear physically that the mean free path shortens continuously as one approaches the surface. In Fig. 7, we have approached the physical limit by exhibiting a calculation of the surface resistance in which the roughened region extends deeper but in which the added surface scatter decreases the mean free path by only a factor of 2 compared with the bulk. The bulk mean free path is rather shorter than one could accept as a valid bulk mean free path for potassium. One should regard it as being characteristic of the lower depths which the anomalous-skin-effect fields can sample, rather than of the true bulk. We have chosen a scale and range of fields similar to that used in Fig. 1 and have chosen parameters which facilitate a comparison between the two figures. While the crudeness of the choice of $\tau(z)$ does not motivate detailed comparison between theory and experiment, it does justify the assertion that surface roughness can have a substantial effect on the field dependence of surface resistance, and that it does lead to a decrease in surface resistance in the region just above cyclotron resonance, more or less in accord with what is seen.

IV. EFFECT OF LOCALIZED ELECTRONS

The sharp peak evident in the experiment surface resistance is suggestive of a resonance phenomenon near $\omega_c=\omega$, although the peak is definitely located on the high-field side of the exact cyclotron-resonance point. There is no way of determining the exact line shape to be associated with this peak because we have no way of calculating the exact shape of the background above which the peak rises. One suspects that, with reasonable effort, a variety of smooth background curves could be fitted. If we accept that this may be so and sketch a gentle curve that smoothly continues the data from one side of the peak to the other side, we obtain an estimate of the contribution to the surface resistance which should be ascribed to whatever it is that is causing the peak. That estimate varies, depending on exactly how the background is drawn. For example, one may or may not choose to ascribe the dip between $\omega_c/\omega=-0.9$ and $\omega_c/\omega=-1.0$ to the background rather than to the peak. There is a shoulder between $\omega_c/\omega=-1.1$ and -1.0 which can similarly be assigned to either the peak or the background. The only features on which all are likely to agree are the position of the peak on the high-field side of cyclotron resonance and its asymmetry, and that the positive part extends farther to the high-field side of the peak than to the low-field side. Those who argue that the ineffectiveness concept forbids any structure in the background curve near $\omega=\omega_c$ will choose the gentlest background interpolation, and will assign both the high-field shoulder and the low-field dip to the peak. Others will argue correctly that this assignment is reasonable but not necessary.

It is well to emphasize that the reason the classical calculations of surface impedance in metals show no local cyclotron resonance at $\omega_c=\omega$ is that normally, the large Fermi velocity of the carriers leads to a Doppler shifting which washes out any resonance phenomena at $\omega=\omega_c$.[11] It is interesting to ask, however, what the resonance might be if a very small number of carriers were *constrained* to move at approximately constant z such as they might be if they were in a thin specular film. The Doppler shifting of the cyclotron resonance would be then be absent, and some anomaly should be seen at $\omega=\omega_c$.

Each electron that is constrained in this way will make a local contribution to the nonlocal conductivity. The contribution per electron, for a transverse circularly

[11] We have reversed the sign convention on field direction here so that local cyclotron resonance occurs at $\omega_c=\omega$.

polarized field will be[12]

$$\sigma_f \delta(z-z') = \frac{q^2\tau/m}{1-i(\omega-\omega_c)\tau}\delta(z-z'). \quad (4.1)$$

If there are N_f electrons per unit area in the film, and the film is localized at the surface, the distribution of these electrons will be $N_f\delta(z)$, and their contribution to the conductivity will be $N_f\sigma_f\delta(z)\delta(z-z')$. This must be added to the nonlocal conductivity to get the total conductivity (call it $\hat{\sigma}$) which governs the field:

$$\hat{\sigma}(z,z') = \sigma(z,z') + N_f\sigma_f\delta(z)\delta(z').$$

This $\hat{\sigma}$ must be used in (3.4a) to describe the field in the metal. Inserting $\hat{\sigma}$ in (3.4) gives

$$\left(\frac{d^2}{dz^2}+k_0^2\right)e(z) = -i\omega\mu_0\int_0^\infty \sigma(z,z')e(z')dz' - i\omega\mu_0 N_f\sigma_f e(0)\delta(z), \quad (4.2a)$$

$$e(z\to\infty) = 0. \quad (4.2b)$$

This equation shows that the field for $z>0$ satisfies exactly Eq. (3.4), i.e., is identical, within a numerical factor, with the field that would be found without the film. The field $e(z)$ is continuous at $z=0$, but suffers a discontinuity in slope there, given by

$$e'(0+) - e'(0-) = -i\omega\mu_0 N_f\sigma_f e(0).$$

It follows then that the local impedance

$$Z(z) \equiv \frac{ik_0 e(z)}{de/dz}$$

changes abruptly at $z=0$:

$$1/Z(0+) - 1/Z(0-) = -i\omega\mu_0 N\sigma_f/ik_0 = -\mu_0 c N_f\sigma_f. \quad (4.3)$$

Since the field at $z>0$ is proportional to that which would have existed without the film, the quantity $Z(0+)$ is the surface impedance of the metal which would have been calculated in the absence of localized electrons. We denote it by Z_m. The quantity $Z(0-)$ is the surface impedance which will be seen experimentally when the conductivity is $\hat{\sigma}$; we denote it by Z_{expt}. From (4.3), one obtains

$$Z_{\text{expt}} = Z_m[1+Z_m\mu_0 c N_f\sigma_f]^{-1}. \quad (4.4)$$

This is a general relationship which is true independent of the details of the theory used to calculate Z_m. In situations in which the second term in square brackets is small, we expand (4.4) to lowest order to get the change in surface impedance caused by the surface electrons:

$$\delta Z = -Z_m^2\mu_0 c N_f\sigma_f = \frac{-Z_m^2\mu_0 c N_f q^2\tau/m}{1-i(\omega-\omega_c)\tau}. \quad (4.5)$$

The real part of δZ is δR, the change in surface resistance caused by the surface electrons. $R\delta$ exhibits a resonance at $\omega_c=\omega$. The line shape depends on the phase of Z_m at $\omega=\omega_c$. If Z_m is pure imaginary, as it will be for a nonabsorbing medium completely cut off, then δR is a positive peak at cyclotron resonance, i.e., the surface electrons absorb energy. If Z_m is pure real, as for a nonabsorbing dielectric in the transmission region, then δR is a negative peak at cyclotron resonance, i.e., the surface electrons establish a resonant shielding current which decreases the energy flow into the dielectric.

In good metals at cyclotron resonance in the extreme anomalous region, we have $Z_m \sim 1-i\sqrt{3}$, and this phase produces a line shape for δR which is a mixture of the typical resistance and reactance line shapes. There is a strong positive peak slightly to the high-field side of cyclotron resonance at

$$(\omega_c-\omega)\tau = 1/\sqrt{3}, \quad (4.6)$$

a slight dip below cyclotron resonance, a sharp rise on the low-field side of the peak, and a gentler falling off on the high-field side. [The τ required by (4.6) to put the peak at the observed value $\omega_c/\omega = 1.025$ is $(1.6\pm0.3)\times10^{-10}$ sec, which is consistent with the value reported in Ref. 1.] This general behavior is apparent in Figs. 8 and 9. In these figures, we have plotted δR as given by (4.5) for perpendicular-field potassium, using the parameters of Fig. 2. In Fig. 8, the Z_m used is that for a diffuse surface (the $\delta=0$ case of Fig. 2) and in Fig. 9, the Z_m is that of a rough surface (the $\delta=8\times10^{-6}$ cm case of Fig. 2). Either curve will account for the high-field shoulder and low-field dip which can be read into the experimental peak, and will also account for

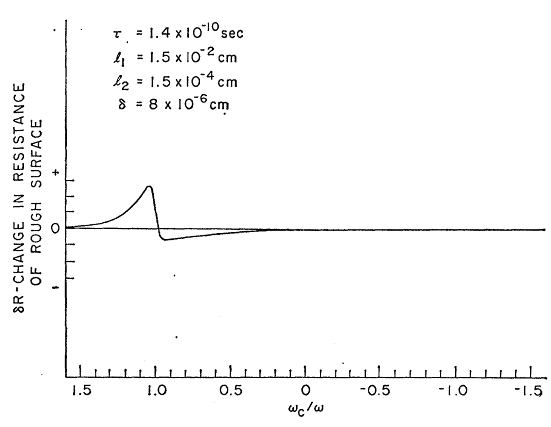

FIG. 8. Added surface resistance caused by a specular film overlying small areas of a diffuse surface.

[12] This result, which is obvious intuitively, can be derived by calculating the nonlocal conductivity for a specular slab using the method of images, and studying the result in the limit that the slab becomes thin enough to neglect the variation of the field across it.

the peak just above resonance and for the asymmetry which the measurement undeniably shows.

There is no vertical scale given in Figs. 8 and 9: The magnitude of δR is directly proportional to N_f, the number of electrons per unit area. If we choose N_f so that δR at the peak is roughly equal to $R_c(\omega_c=\omega)-R(\omega_c=0)$ as the experimental curve suggests, then N_f comes out to be $\sim 3\times 10^{13}$ electrons/cm^2, which, for potassium, is $\sim 1/20$ of a monolayer. That is, if the electrons were constrained to a specular film about 25-Å thick, a film covering about 0.5% of the surface area would yield a peak of the correct magnitude. (The thickness and area just mentioned have no special significance; one can always be traded off for the other, as long as N_f is held constant.)

While there is no basic reason for proposing such a film, it is interesting that so few electrons are needed to give the observed magnitude to the peak, whose general features fit those of experimental without introducing any adjustable parameters. It is, of course, highly unlikely that any *specularly* reflecting thin film could be formed on the surface of a sample, but, because of the extreme sensitivity of the surface resistance to the presence of constrained electrons, it may be worthwhile to verify that they are not masking other effects. The characteristic feature of an electron constrained to move at constant z is that its cyclotron frequency is determined by the z component of magnetic field. Hence, tilting B_0 to an angle θ with the normal would lower the resonance frequency from ω_c to $\omega_c\cos\theta$, or, conversely, would raise the resonant field by a factor $1/\cos\theta$.

V. CONCLUSION

We have shown that surface roughness will introduce structure in the surface impedance at perpendicular-field cyclotron resonance. One is tempted to suggest experiments that might be done with surfaces of varying perfection. Before any of these are carried out, however, one wants to consider the accuracy to which the magnetic field must be aligned normal to the surface, since misalignment would cause a tilt of the orbits which would cause electrons to collide with the surface in the same way as roughness is postulated to do. It would seem that an angle less than δ_a/R (= anomalous skin depth/cyclotron radius) with the normal would be required to allow a smooth surface not to masquerade as a rough one.[13] The smallness of this angle in the anomalous-skin-effect regime is such that the necessary alignment will be difficult to achieve. For this reason, we suggest that the anomalous decrease in surface resistance just above $\omega_c=\omega$ will be the rule, rather than the exception, in perpendicular-field cyclotron resonance measurements in metals. With careful alignment, however, and sample surface preparation, the effects we have described here are capable of experimental verification.

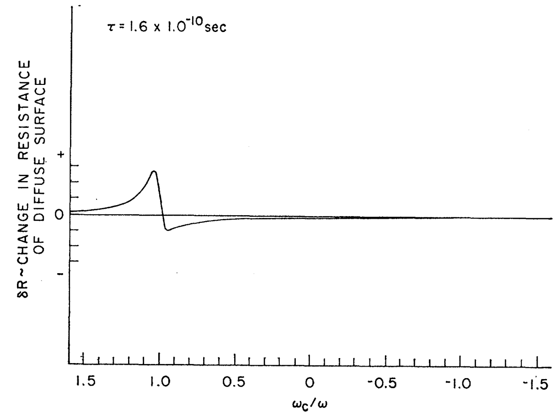

FIG. 9. Added surface resistance caused by a specular film overlying a small area of rough surface.

ACKNOWLEDGMENTS

I should like to express appreciation to C. C. Grimes, J. K. Galt, J. R. Klauder, and D. E. McCumber for a number of useful comments during the course of this work. Part of this work was performed at the Research Laboratory of Electronics, M.I.T. That portion was supported by the National Science Foundation, under Grant No. GK-10472.

[13] J. C. Phillips (see Ref. 7, p. 157) has suggested, for entirely different reasons, that misalignment of the magnetic field by this amount would give structure at cyclotron resonance.