where

$$T_{mn}{}^{(\alpha)}=\alpha^2\,\mathrm{Tr}\{V_{mn}{}^{(\alpha)}V_{mn}{}^{(-\alpha)}\}\int_0^\infty dt\,\tilde{\Gamma}_{mnmn}{}^{(\alpha)}(t)e^{i\alpha\omega_0 t}.$$

In the high-field approximation, this gives

$$\Delta\simeq 4\left(\frac{H_l}{H_0}\right)^2\Big[\sum_{s,s'\neq 0}\sum_{ijkl}c_{ij}{}^{(s)}(1-c_{kl}{}^{(s')})T_{ij}{}^{(s)}T_{kl}{}^{(s')}\Big/\sum_{s,s'\neq 0}\sum_{ijkl}c_{ij}{}^{(s)}c_{kl}{}^{(s')}T_{ij}{}^{(s)}T_{kl}{}^{(s')}\Big]. \tag{D11}$$

Therefore,

$$z_+\approx 4\lambda^2\sum_{s\neq 0}\sum_{ij}\left(1+c_{ij}{}^{(s)}\frac{H_0{}^2}{H_l{}^2}\right)T_{ij}{}^{(s)}+O(\lambda^4),$$

$$z_-\approx O(\lambda^4). \tag{D12}$$

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Description of a Many-Atom System in Terms of Coherent Boson States

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The correlated motion of an N-spin system has been described in terms of the coherent states of a harmonic oscillator. We have shown that this description of the system provides us with the closest analog to the classical limit. The macroscopic dipole operator in the coherent-state representation is shown to obey the Bloch equation in which each of the three components is simultaneously well defined.

I. INTRODUCTION

Many interesting aspects of atomic physics are associated with atomic coherence, for example, Dicke's superradiance,[1] optical pumping,[2] photon echo,[3] self-induced transparency,[4] etc. The coherence properties of a quantized radiation field have been recently described in terms of the so-called coherent states[5] of a harmonic oscillator. These states have provided a natural basis for description of coherence phenomenon in the theory of laser[6] and Josephson[7] oscillators as well as certain aspects of superfluidity.[8] In this paper we demonstrate how the correlated or superradiant states of an N-atom or an N-spin system can be described in a convenient and (hopefully) pedagogical fashion in terms of coherent states of a boson field. After briefly outlining the arguments leading to correlations in an atomic system and reviewing the coherent states of a boson field, we show in Sec. II how these states may be used in describing an N-atom system and apply the formalism to a specific problem in Sec. III.

A single atom having two relevant levels $|+\rangle$ and $|-\rangle$ is said to be in a coherent superposition of states

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when its wave function is represented by

$$\Psi_1(t)=a_+(t)e^{-i(E_+/\hbar)t}+a_-(t)e^{-i(E_-/\hbar)t}, \quad (1)$$

where $a_+(t)$ and $a_-(t)$ are the probability amplitudes, and E_+ and E_- are the energy of the two states, respectively. The atom then has an oscillating dipole moment given by

$$\langle P(t)\rangle = e\langle + | r | -\rangle (a_+^* a_- e^{i\omega t}+\text{c.c.}),\ \omega=(E_+-E_-)/\hbar \quad (2)$$

and serves as a source term in Maxwell's equation. The field generated by the atom when viewed at a distant point $\mathbf{R}$ is given by

$$\mathbf{E}(\mathbf{R},t)=(1/cR^2)\omega^2\hat{n}\times\hat{n}\times\langle\mathbf{P}[t-(R/c)]\rangle, \quad (3)$$

where $\hat{n}=\mathbf{R}/R$ is the unit vector from the source and c is the velocity of light.

Next consider two atoms both in the $|+\rangle$ state contained in a resonant traveling-wave cavity. When we shine light on this system, the atom-field coupling is described by the interaction Hamiltonian V, which, expressed in terms of Pauli spin-flip operators for the ith atom σ_i, $\sigma_i^\dagger$ and photon creation and annihilation operators $a^\dagger$, a, is

$$V=g(\sum_{i=1}^{2}\sigma_i)a^\dagger+g^*(\sum_{i=1}^{2}\sigma_i^\dagger)a, \quad (4)$$

where g is a c-number coupling constant.

One conveniently describes the temporal dependence of the atomic state $|\Psi(t)\rangle$ by an iteration in the interaction picture:

$$|\Psi(t)\rangle=\sum_{n=0}^{\infty}\left(\frac{-i}{\hbar}\right)^n\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}}dt_n V_I(t_1) \times V_I(t_2)\cdots V_I(t_n)|+_1+_2\rangle. \quad (5)$$

When we confine our attention to the resonant case in which the atomic frequency is equal to the radiation frequency, the interaction Hamiltonian $V_I(t)$ becomes independent of time. Thus if one turns off the light after some time T, $|\Psi(t)\rangle$ up to the second order in g is

$$|\Psi(t)\rangle=(1-g^2T^2)|+_1+_2\rangle +\sqrt{2}gT[(1/\sqrt{2})(|+_1-_2\rangle+|-_1+_2\rangle)] +\sqrt{2}g^2T^2|-_1-_2\rangle, \quad (6a)$$

where we have taken the initial photon number to be zero. From Eq. (6a), we note that the interaction term V couples the initial state $|+_1+_2\rangle$ to the two other symmetric stationary states

$$(1/\sqrt{2})(|+_1-_2\rangle+|-_1+_2\rangle) \quad (6b)$$

and

$$|-_1-_2\rangle. \quad (6c)$$

However, in order to have a complete set of states for a two-atom system, we clearly need four eigenstates, three of which are already seen to be connected by V. The remaining one is the antisymmetric combination

$$(1/\sqrt{2})(|+_1-_2\rangle-|-_1+_2\rangle), \quad (7)$$

which does not radiate. This antisymmetric state is not coupled to the other states by the interaction Hamiltonian (4); hence, the two-atom system may be locked into the state (7) (if it ever gets there in the first place). As an example of how the state (7) could be prepared, consider two atoms placed in the cavity, the first in the $|+\rangle$ state and the second in $|-\rangle$. This state may be resolved in terms of (6b) and (7) as

$$|+_1-_2\rangle=(1/\sqrt{2})[(1/\sqrt{2})(|+_1-_2\rangle+|-_1+_2\rangle) +(1/\sqrt{2})(|+_1-_2\rangle-|-_1+_2\rangle)]. \quad (8)$$

As time evolves, the symmetric part of the wave function decays to $|-_1-_2\rangle$, leaving the system in the antisymmetric state, which means that even after a long time there is still a chance of finding an excited atom. Interesting results such as this provide a stimulus for considering the cooperative aspects of a many-atom system.

Next consider a system of N atoms, n_+ in the upper level and n_- in the lower level. For simplicity let us take the initial-state vector of this system in a symmetric combination,

$$|n_+,n_-\rangle=(n_+!n_-!/N!)^{1/2} \times\sum_p|+_1+_2\cdots+_{n_+}-_{n_++1}\cdots-_N\rangle, \quad (9)$$

where $\sum_p$ denotes the sum over all possible permutations among the N atoms. The interaction Hamiltonian for the atoms and field is identical to Eq. (4) except that the subscript i extends from 1 to N. The matrix element associated with spontaneous radiation resulting in a transition from state $|n_+,n_-\rangle$ to $|n_+-1, n_-+1\rangle$ is then equal to $g[(n_-+1)n_+]^{1/2}$. Thus, if $n_+=n_-=\frac{1}{2}N$, the emission rate is proportional to N^2 rather than the usual population factor N. Dicke[1] first investigated these collective aspects of the spontaneous radiation problem and called these states superradiant.

In his original paper, Dicke made use of the close analogy existing between N two-level atoms and a system of N spin-$\frac{1}{2}$ particles, and introduced two quantum numbers r and m in specifying the 2^N eigenstates of the atomic system. In this notation the state (9) would be

$$|n_+,n_-\rangle=|r,m\rangle,$$

where

$$r=\tfrac{1}{2}(n_++n_-) \quad\text{and}\quad m=\tfrac{1}{2}(n_+-n_-).$$

The states of the two-atom system considered earlier expressed in terms of $|r,m\rangle$ are

$$|+_1+_2\rangle=|1,1\rangle,$$
$$(1/\sqrt{2})(|+_1-_2\rangle+|-_1+_2\rangle)=|1,0\rangle,$$
$$|-_1-_2\rangle=|1,-1\rangle,$$
$$(1/\sqrt{2})(|+_1-_2\rangle-|-_1+_2\rangle)=|0,0\rangle.$$

In concluding this introduction let us briefly review the coherent states[5] of a quantized radiation field. In terms of the photon number states $|n\rangle$, these classical limit states are

$$|\alpha\rangle=\sum_n (\alpha^n/\sqrt{n!})e^{-|\alpha|^2/2}|n\rangle\,, \tag{10}$$

and have the property of being an eigenstate of the photon annihilation operator a with the complex number α as an eigenvalue:

$$a|\alpha\rangle=\alpha|\alpha\rangle\,. \tag{11}$$

Furthermore, these states are complete in the sense that

$$1=\frac{1}{\pi}\int d^2\alpha|\alpha\rangle\langle\alpha|\,, \tag{12}$$

and we may therefore expand an arbitrary state of the field $|\Psi\rangle$ as

$$|\Psi\rangle=\frac{1}{\pi}\int d^2\alpha\langle\alpha|\Psi\rangle|\alpha\rangle\,. \tag{13}$$

These $|\alpha\rangle$ states constitute a convenient basis for investigating the properties of an almost classical field. In the next section we demonstrate how these coherent states may be used in describing the state of a many-atom system.

II. N-ATOM SYSTEM AND $|\alpha\rangle$ STATES

Denoting the upper state of the ith atom by $|+_i\rangle$ and the lower state by $|-_i\rangle$, an arbitrary state of the N-atom system can be expressed in terms of basis vectors such as

$$|+_1+_2-_3\cdots+_N\rangle\,, \tag{14}$$

or in terms of the basis vectors $|r,m\rangle$. In general,

$$m=\tfrac{1}{2}(n_+-n_-) \tag{15}$$

is the difference in population in two levels and is formally analogous to the magnetic quantum number m; r was termed by Dicke as the cooperation number and is given by

$$|m|\leq r\leq\tfrac{1}{2}(n_++n_-)=\tfrac{1}{2}N\,. \tag{16}$$

As a specific example, let us consider the case where all the spins are initially pointing up, i.e., all atoms are excited into their upper level. Then the state would evolve in time via the spontaneous emission of radiation into a linear combination of $|n_+n_-\rangle$,

$$|\Psi\rangle=\sum_{n+n-} a_{n_+,n_-}|n_+,n_-\rangle\,, \tag{17}$$

where the symmetric basis vectors $|n_+,n_-\rangle$ are given in Eq. (9) and a_{n_+,n_-} is the corresponding probability amplitude. When we consider a dipole interaction of this system with the radiation field, the state $|n_+,n_-\rangle$ is coupled to other states by the operator

$$\Sigma=\sum_{i=1}^{N}(\sigma_i+\sigma_i^\dagger) \tag{18}$$

as discussed in Sec. I. The explicit matrix elements of interest are easily found to be

$$\begin{aligned}\langle n_+\mp1,\, n_-\pm1|\Sigma|n_+,\, n_-\rangle&=[n_+(n_-+1)]^{1/2}\\&=[n_-(n_++1)]^{1/2}.\end{aligned} \tag{19}$$

In the r, m notation[9] this reads

$$\langle r,\, m\mp1|\Sigma|r,m\rangle=[(r\pm m)(n\mp m+1)]^{1/2}. \tag{20}$$

One readily observes that the matrix elements of Eqs. (19) and (20) have formally the same connotation as those which connect the number states of two boson field operators a_+, a_- in the presence of interaction of the type $a_+^\dagger a_-$ or $a_-^\dagger a_+$, i.e.,

$$\begin{aligned}\langle n_+\mp1,\, n_-\pm1|a_+^\dagger a_-+a_-^\dagger a_+|n_+,n_-\rangle&\\=[n_+(n_-+1)]^{1/2}&\\=[n_-(n_++1)]^{1/2},&\end{aligned} \tag{21}$$

where $|n_+,n_-\rangle$ is defined by

$$|n_+,n_-\rangle=\frac{(a_+^\dagger)^{n+}(a_-^\dagger)^{n-}}{(n_+!n_-!)^{1/2}}|0\rangle\,. \tag{22}$$

In fact, it has been shown by Schwinger[10] that a general angular momentum can be profitably described in terms of the second-quantized boson operators a_+, a_- obeying the commutation relations

$$[a_\zeta,a_{\zeta'}^\dagger]=\delta_{\zeta\zeta'}\,,\quad \zeta,\zeta'=+,\,-\,. \tag{23}$$

The total number of spins and the angular momentum J are given by

$$N=\sum_\zeta a_\zeta^\dagger a_\zeta=n_++n_-\,, \tag{24}$$

$$J=\sum_{\zeta\zeta'} a_\zeta^\dagger\langle\zeta|\tfrac{1}{2}\sigma|\zeta'\rangle a_{\zeta'}\,, \tag{25}$$

i.e.,

$$J_+=J_1+iJ_2=a_+^\dagger a_-\,, \tag{26a}$$

$$J_-=J_1-iJ_2=a_-^\dagger a_+\,, \tag{26b}$$

$$J_3=\tfrac{1}{2}(a_+^\dagger a_+-a_-^\dagger a_-)\,. \tag{26c}$$

We may now transcribe $|n_+n_-\rangle$ or $|r,m\rangle$ in terms of these operators as

$$|n_+,n_-\rangle=\frac{(a_+^\dagger)^{n+}(a_-^\dagger)^{n-}}{(n_+!n_-!)^{1/2}}|0\rangle\,, \tag{27}$$

$$|r,m\rangle=\frac{(a_+^\dagger)^{r+m}(a_-^\dagger)^{r-m}}{[(r+m)!]^{1/2}[(r-m)!]^{1/2}}|0\rangle\,. \tag{28}$$

[9] In this paper J, J_3 will represent operators of the angular momentum, and r, m the associated eigenvalues.

[10] J. Schwinger, in *Quantum Theory of Angular Momentum; Perspectives in Physics*, edited by L. C. Biedenharn and H. VanDam (Academic Press Inc., New York, 1965).

	$r=3/2$	$r=1/2$, $P_{\rm I}$	$r=1/2$, $P_{\rm II}$
$m=3/2$	$\|+++\rangle$		
$m=1/2$	$(1/3)^{1/2}(\|++-\rangle+\|+-+\rangle+\|-++\rangle)$	$(1/6)^{1/2}(\|++-\rangle+\|+-+\rangle-2\|-++\rangle)$	$(1/2)^{1/2}(\|++-\rangle-\|+-+\rangle)$
$m=-1/2$	$(1/3)^{1/2}(\|--+\rangle+\|-+-\rangle+\|+--\rangle)$	$(1/6)^{1/2}(\|--+\rangle+\|-+-\rangle-2\|+--\rangle)$	$(1/2)^{1/2}(\|--+\rangle-\|-+-\rangle)$
$m=-3/2$	$\|---\rangle$		

FIG. 1. The 2^3 eigenstates of the three-atom system in terms of r, m. $r=\frac{1}{2}$ states are doubly degenerate. We have used the index P to indicate the orthogonal degenerate subsets. Each column is associated with a pair of boson operators. For example,

$$|r=\tfrac{3}{2},\, m=\tfrac{3}{2}\rangle=[(a_+^\dagger)^3(a_-^\dagger)^0/(3!0!)^{1/2}]|0\rangle,$$
$$|r=\tfrac{1}{2},\, m=\tfrac{1}{2},\, P_{\rm I}\rangle=[(b_+^\dagger)^1(b_-^\dagger)^0/(1!0!)^{1/2}]|0\rangle,$$
$$|r=\tfrac{1}{2},\, m=\tfrac{1}{2},\, P_{\rm II}\rangle=[(c_+^\dagger)^1(c_-^\dagger)^0/(0!1!)^{1/2}]|0\rangle,$$

where

$$a_+^\dagger a_+ + a_-^\dagger a_- = 3,\quad b_+^\dagger b_+ + b_-^\dagger b_- = c_+^\dagger c_+ + c_-^\dagger c_- = 1.$$

A few comments are perhaps in order at this point. The states with a given r value are always coupled to the other states of the same r value (as well as the same value of P)[11] when subject to the dipole interaction with an external field. In order to simplify the discussion let us consider a specific example of three atoms as in Fig. 1. It is clear from Fig. 1 that any state with a given r and m is uniquely specified by an index P that indicates to which of the two degenerate columns having the same r the state actually belongs. Having specified uniquely a set of states corresponding to one column, we may introduce a pair of boson field operators for these states. In general, one should note that n_+ and n_- are not to be thought of as the number of atoms in the upper and lower states, except in the first column from the left having the maximum r value.

We now proceed to expand $|r,m\rangle$ in terms of coherent states $|\alpha_+,\alpha_-\rangle$; using the completeness relation (12), we find

$$\begin{aligned}|r,m\rangle &= \left(\frac{1}{\pi^2}\right)\int d^2\alpha_+\int d^2\alpha_- \langle\alpha_+|(a_+^\dagger)^{r+m}/[(r+m)!]^{1/2}|0\rangle \\ &\quad\times\langle\alpha_-|(a_-^\dagger)^{r-m}/[(r-m)!]^{1/2}|0\rangle \\ &\quad\times\exp[-\tfrac{1}{2}(|\alpha_+|^2+|\alpha_-|^2)]|\alpha_+,\alpha_-\rangle \\ &= \left(\frac{1}{\pi^2}\right)\int d^2\alpha_+\int d^2\alpha_- \frac{(\alpha_+^*)^{r+m}(\alpha_-^*)^{r-m}}{[(r+m)!]^{1/2}[(r-m)!]^{1/2}} \\ &\quad\times\exp[-\tfrac{1}{2}(|\alpha_+|^2+|\alpha_-|^2)]|\alpha_+,\alpha_-\rangle. \end{aligned}\tag{29}$$

For an arbitrary state $|\Psi\rangle$ given by

$$|\Psi\rangle=\sum_{r,m} a_{r,m}|r,m\rangle,$$

we may write

$$|\Psi\rangle=\left(\frac{1}{\pi^2}\right)\int d^2\alpha_+\int d^2\alpha_- f(\alpha_+,\alpha_-)|\alpha_+,\alpha_-\rangle,\tag{30}$$

where

$$f(\alpha_+,\alpha_-)=\sum_{r,m} a_{r,m}\frac{(\alpha_+^*)^{r+m}(\alpha_-^*)^{r-m}}{[(r+m)!]^{1/2}[(r-m)!]^{1/2}} \times\exp[-\tfrac{1}{2}(|\alpha_+|^2+|\alpha_-|^2)].\tag{31a}$$

As an interesting case, consider the state corresponding to the classical limit of an N-spin system, that is,

$$f(\alpha_+,\alpha_-)=\delta(\alpha_+-c_+)\delta(\alpha_--c_-).\tag{31b}$$

The state (30) is then given by

$$|\Psi\rangle=|\alpha_+,\alpha_-\rangle\equiv\sum_{r=0}^{\infty}\sum_{m=-r}^{r}\frac{(\alpha_+^*)^{r+m}(\alpha_-^*)^{r-m}}{[(r+m)!]^{1/2}[(r-m)!]^{1/2}} \times\exp[-\tfrac{1}{2}(|\alpha_+|^2+|\alpha_-|^2)]|r,m\rangle.\tag{30$'$}$$

Note that Eq. (30′) defines the coherent state $|\alpha_+,\alpha_-\rangle$ as a linear superposition of angular momentum eigenstates "for all the possible values of r from zero to infinity." Hence the coherent states are defined in an extended Hilbert space where the total number of atoms is not specified exactly (as is the case in a grand canonical ensemble). However, the quantities of interest such as n_+ and n_-, and hence r, m, and N, are extremely well defined[5] if $|\alpha_+|^2\gg1$ or $|\alpha_-|^2\gg1$.

Now let us recall that the state $|r,m\rangle$ provides us with the full information concerning the measurement of J and J_3, while the phase information is lost completely, i.e., J_1 and J_2 are indeterminate. But in the classical-limit state (30′) it can be shown that not only J and J_3 but also J_1 and J_2 are known exactly in the limit of large spin number N. To see this we evaluate the expectation values of J_3 and J in the $|\alpha_+,\alpha_-\rangle$ state. By using Eqs. (11) and (26), we obtain

$$\langle(\Delta J_3)^2\rangle=\langle\alpha_+,\alpha_-|(J_3-\langle J_3\rangle)^2|\alpha_+,\alpha_-\rangle =\tfrac{1}{4}(|\alpha_+|^2+|\alpha_-|^2).\tag{32}$$

After performing similar calculations for $\langle(\Delta J_1)^2\rangle$ and $\langle(\Delta J_2)^2\rangle$, we find that the results are exactly the same as Eq. (32). Hence, the relative uncertainty in the three components of J is given in the limit of large N by

$$\langle(\Delta J_i)^2\rangle/J^2\simeq(|\alpha_+|^2+|\alpha_-|^2)^{-1}\propto N^{-1/2},\quad i=1,2,3.\tag{33}$$

It is clear from Eq. (33) that as we increase the number of spins or atoms considered, the three components of J are simultaneously determined almost exactly. We are thus led to the conclusion that $|\alpha_+,\alpha_-\rangle$ is the classical-limit state. We elaborate on this aspect further in the following section.

[11] For example, the states under $P_{\rm I}$ and $P_{\rm II}$ in Fig. 1 are not coupled.

III. MOMENT EQUATIONS AND CLASSICAL LIMIT

We consider in this section a system of N spins coupled to a classical field by a dipole interaction. The Hamiltonian of the system reads

$$H=\tfrac{1}{2}\hbar\omega(a_+^\dagger a_+ - a_-^\dagger a_-)+\epsilon(t)(a_+^\dagger a_- + a_-^\dagger a_+). \quad (34)$$

Here we have taken the external field in the x direction, and the quantity $\epsilon(t)$ is given in terms of the atomic dipole matrix element between two levels, the field frequency ν and the field amplitude E, as

$$\epsilon(t)=ig\hbar\tfrac{1}{2}(e^{i\nu t}-e^{-i\nu t}), \quad g=e\langle +|r|-\rangle E/\hbar. \quad (35)$$

It is convenient to go into an interaction picture. The density operator $\rho(t)$ of this system obeys then the equation

$$\dot{\rho}(t)=(i/\hbar)[V(t),\rho(t)], \quad (36)$$

where, in the rotating-wave approximation,

$$V(t)=i\tfrac{1}{2}ge^{i(\omega-\nu)t}a_+^\dagger a_- - i\tfrac{1}{2}ge^{-i(\omega-\nu)t}a_-^\dagger a_+. \quad (37)$$

We consider the resonant case, viz., $\omega=\nu$, and obtain a set of mean equations of motion for the following combination of operators a_+, a_-:

$$(d/dt)\langle a_+^\dagger a_+ + a_-^\dagger a_-\rangle=0, \quad (38a)$$

$$(d/dt)\langle a_+^\dagger a_+ - a_-^\dagger a_-\rangle=g\langle a_-^\dagger a_+ + a_+^\dagger a_-\rangle, \quad (38b)$$

$$(d/dt)\langle a_-^\dagger a_+ - a_+^\dagger a_-\rangle=0, \quad (38c)$$

$$(d/dt)\langle a_-^\dagger a_+ + a_+^\dagger a_-\rangle=-g\langle a_+^\dagger a_+ - a_-^\dagger a_-\rangle. \quad (38d)$$

In the coherent-state representation this set of operator equations (38) leads to the c-number equations in the form

$$(d/dt)\langle|\alpha_+|^2+|\alpha_-|^2\rangle=0, \quad (39a)$$

$$(d/dt)\langle|\alpha_+|^2-|\alpha_-|^2\rangle=g\langle\alpha_-^*\alpha_+ + \alpha_+^*\alpha_-\rangle, \quad (39b)$$

$$(d/dt)\langle\alpha_-^*\alpha_+ - \alpha_+^*\alpha_-\rangle=0, \quad (39c)$$

$$(d/dt)\langle\alpha_-^*\alpha_+ + \alpha_+^*\alpha_-\rangle=-g\langle|\alpha_+|^2-|\alpha_-|^2\rangle. \quad (39d)$$

Let us introduce a vector $\mathbf{\Lambda}$ such that

$$\mathbf{\Lambda}=\hat{x}\langle\alpha_-^*\alpha_+ - \alpha_+^*\alpha_-\rangle+\hat{y}\langle\alpha_-^*\alpha_+ + \alpha_+^*\alpha_-\rangle + \hat{z}\langle|\alpha_+|^2-|\alpha_-|^2\rangle. \quad (40)$$

In this case Eqs. (39b)–(39d) can be compactly written as

$$\partial\mathbf{\Lambda}/\partial t=-g[\mathbf{\Lambda}\times\hat{x}]. \quad (41)$$

This equation is well known as the Bloch equation which describes the precession of a macroscopic dipole in the presence of a classical field. By solving Eq. (41) we can determine with the additional condition of Eq. (39a) the time dependence of the four complex quantities $\alpha_+(t)$, $\alpha_+^*(t)$, $\alpha_-^*(t)$, and $\alpha_-^*(t)$ uniquely. Let us next consider the Heisenberg equation of motion for operators a_ζ and $a_{\zeta'}^\dagger$. Using the Hamiltonian given in Eq. (34), we note that the time rate of change of a_ζ (or $a_{\zeta'}^\dagger$) is functional only of $a_{\zeta'}$ (or $a_\zeta^\dagger$), i.e.,

$$\dot{a}_\zeta=F(a_{\zeta'}:t). \quad (42)$$

For this case we see easily that if a Schrödinger state is initially a coherent state, it remains a coherent state at all times, while its amplitude $\alpha(t)$ evolves in time according to either Eq. (41) or Eq. (42).[12] Hence we conclude that the change in time of the N-spin system $|r,m\rangle$ can be investigated almost classically by expanding the state vector in terms of $|\alpha_+(t)\rangle$, the occupation probability of which is given by a product of two Poisson distributions, viz.,

$$|\langle r,m|\alpha_+(t),\alpha_-(t)\rangle|^2=\frac{[\alpha_+^*(t)\alpha_+(t)]^{r+m}[\alpha_-^*(t)\alpha_-(t)]^{r-m}}{[(r+m)!]^{1/2}[(r-m)!]^{1/2}} \times\exp[-(|\alpha_+|^2+|\alpha_-|^2)]. \quad (43)$$

In conclusion, we have considered in this paper the correlated motion of an N-spin or N-atom system in the presence of a classical driving force, in which case the spins are seen to move together as a macroscopic dipole. Using the fact that angular momentum operators can be represented by boson field operators, and the fact that the boson field can in turn be represented by the coherent states, we have introduced coherent states for N-spin system. We have shown that the dynamics of the system can be conveniently described in terms of coherent states in a way which gives the classical limit immediately. The macroscopic dipole operator of the system was shown to be adequately represented by the ordinary vector whose three components are simultaneously well defined in the limit where N becomes large. Finally, we have shown that the equation of motion of this vector has the same form as a classical Bloch equation.

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