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Charged-Vortex-Ring Creation by Ions in Superfluid Helium

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The critical-threshold electric fields and the critical velocities for the formation of charged vortex rings have been measured at temperatures $0.35 < T < 1°K$. Below $T = 0.85°K$, two different thresholds are shown to exist, connected with two different processes of formation. These two processes are found to be detectable only when a stability condition is satisfied.

1. INTRODUCTION

The existence of quantized vortex rings in liquid helium was hypothesized to explain the critical velocities.[1] The most direct evidence for the existence of vortex rings was given by experiments utilizing He ions.[2] An ion in a sufficiently intense electric field generates a vortex ring and then is bound to the vortex, forming a stable complex. The dynamics of a charged vortex ring with radius greater than the ion radius is essentially the same as that of the uncharged one, the motion of which can be described by classical hydrodynamics.[3]

The existing literature gives little information about the electric-field thresholds for formation and stability of charged vortex rings and the critical velocities at temperatures below 0.9°K. Some previous work was done in our laboratory on the threshold fields of positive charges,[4] and partial data were already published for negative ions.[5] Measurements at some temperatures of the electric field above which bare ions do not exist because all ions have formed charged vortex rings were recently published.[6] The velocities reached by the last fraction of bare ions at the threshold fields were, respectively, 34 m/sec for negative ions and 40 m/sec for positive ions.

In our experiment, we have observed at $T < 0.85°K$ two different threshold fields E_{c1} and E_{c2}, the corresponding critical velocities V_{c1} and V_{c2}, and their temperature dependence. We believe that the lower threshold is connected to the formation of charged vortex rings from vortex germs already present in the helium bath. We have also shown that the lower threshold is related to a stability condition, and the data have been compared to the behavior given by the Huang-Olinto (HO) model.[7] The higher threshold is regarded as the onset of the process of formation of charged vortex rings because of the direct interaction between bare ions and liquid helium. Previous results at $T > 0.9°K$ by Careri *et al.*[2b] are in agreement with our measurements.

A property peculair to charged vortex rings close to the liquid-vapor interface was recently revealed in our laboratory.[8] When the vortex ring approaches the liquid surface, the charge is ejected into the vapor. This phenomenon was used to determine the threshold fields and to check the effect of friction at higher temperatures.

2. EXPERIMENTAL PROCEDURE

The temperatures below 0.9°K were reached by using a single-shot helium-3 refrigerator. A schematic view of the cell is shown in Fig. 1. The charge sign of the ion, produced by a Po^{210} source, was chosen by the sign of the electric field e_{s1} applied between the source and grid 1.

The threshold fields for negative ions were determined by applying the electric field e_{s1}, with zero electric field e_{12} between grid 1 and grid 2 and collecting the current I_{2c} on the collector and grid 2 as a function of e_{s1}. In this way a characteristic like the one shown in Fig. 2 is obtained. Such a characteristic exhibits two threshold fields, E_{c1} and E_{c2}. E_{c1} and E_{c2} can also be obtained by applying a constant field $e_{s1} < E_{c1}$, varying e_{12} and recording the current I_c, at the collector with zero electric field e_{2c} between grid 2 and collector. By comparing the characteristics obtained in the two conditions, we observed that f, (the ratio of the plateau current above E_{c1} to that above E_{c2}) depends partially on the presence of the source and also on the preexisting vortex germs produced by the vorticity decay. No essential difference was noticed when using a tritium β source; in this case f was lower than for the α source. By varying the distance between the electrodes it was checked that the two threshold fields at fixed temperature, were independent of this distance.

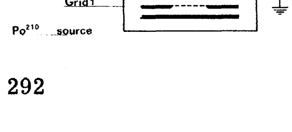

FIG. 1. Schematic diagram of an experimental cell.

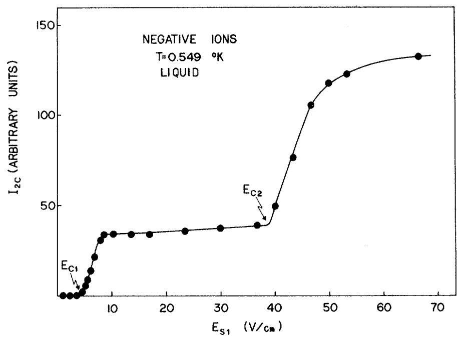

FIG. 2. Characteristic of the collected current versus the electric field. The two different threshold fields E_{c1} and E_{c2} are shown.

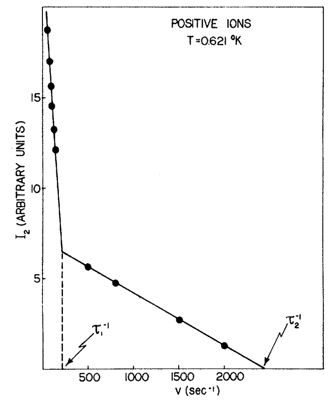

FIG. 3. Collected current versus the frequency of the square wave multiplied by a factor of 2, with amplitude $E = 8$ V/cm.

A different method of detecting the threshold fields is to condense liquid helium up to a level between grid 1 and grid 2. By applying $e_{s1} = e_{12}$ and $e_{2c} = 0$, one collects the current I_c as a function of e_{s1}. When $e_{s1} < E_{c1}$, the current I_c is zero, since thermalized charges do not cross the liquid surface at $T < 1\,°\mathrm{K}$,[9] but if $e_{s1} > E_{c1}$, charges can cross the liquid surfaces.[8] This procedure was used to minimize the effect of friction. This was strictly necessary at $T > 0.65$ °K, though the method was applied also at lower temperature. However, with $T < 0.6$ °K, consistent results were found also without applying any field at the interface ($e_{12} = 0$) and by simply varying e_{s1}.

The positive interface current is at most one order of magnitude smaller than that in bulk liquid (the currents in the liquid were usually of the order of 10^{-11}–10^{-12} A for both ions); for this reason the determination of E_{c1} is usually difficult. Hence, E_{c1} was determined by a time-of-flight method.[10] In fact, when the square-wave electric field E is greater than E_{c1}, a fraction of the ions moves at a lower velocity; this is clear from Fig. 3. This behavior suggests the existence of two components: one consisting of bare ions with time of flight τ_2 and the other of charged vortex rings with time of flight τ_1. We assume that the amplitude of the square-wave at which two different times of flight τ_1 and τ_2 appear corresponds to E_{c1}. When the threshold field E_{c2} is reached, the high velocity component strongly decreases. The bare-ion velocity V_{c2} corresponding to e_{c2} is so determined.

To verify that charges traveling in the field-free space above E_{c1} or E_{c2} are in both cases bound to vortices of quantum of circulation 1, we developed a technique which permits the measurement of the distance Δx traveled by the ion-vortex complex in a time Δt under the action of the electric field. The experimental apparatus for this measurement is shown schematically in Fig. 4. The charges are extracted by an electric field $e_{s1} > E_{c1}$, and then between grid 1 and grid 2 they experience a single pulse of amplitude $e_{12} > E_{c1}$. The energy gained because of the pulse between grid 1 and grid 2 was measured by the change of the retarding potential U_{2c} at which the current I_c is zero. The distance Δx traveled by the charged vortex ring during the time of the pulse is the change of U_{2c} times the distance between grid 1 and grid 2 divided by the amplitude of the pulse. Naturally, this is true when friction is negligible. When Δt is greater than the time of flight Θ, the energy gained by the ion-vortex complex because of the pulse no longer increases and the distance Δx, traveled during the time the pulse was applied, is just the whole distance between grid 1 and grid 2.

A different technique, to study the dynamics of the charged vortex rings, was based on the determination of the time of flight in field-free space. Charged vortices were gated in the space between grid 1 and grid 2 by high-amplitude square pulses clamped to zero and, by varying the pulse length, the time of flight could be determined.

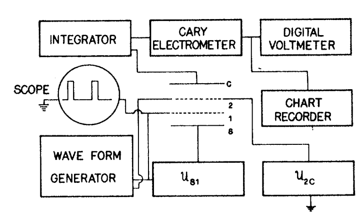

FIG. 4. Schematic representation of the method used for determining the displacement Δx as function of time Δt.

3. EXPERIMENTAL RESULTS

The current-voltage characteristic represented in Fig. 2 and the results of ac measurements (Fig. 3) show that there are two different threshold electric fields E_{c1} and E_{c2}. The temperature dependence of such thresholds is shown in Figs. 5 and 6. The higher threshold field E_{c2}, for both negative and positive ions, exhibits a dependence upon temperature $E_{c2} \propto T^4$, like the density of phonons.

For negative ions the lower threshold E_{c1} has a nearly exponential dependence $E_{c1} \propto e^{\Delta/T}$ down to 0.65° K ($\Delta = 8.5$° K is the energy gap for rotons). At 0.85 °K one finds that $E_{c1} = E_{c2}$ and above 0.9° K the single threshold has an exponential temperature dependence.[2b]

The solid line shown in Fig. 5 represents an extension of the stability condition given by the HO model[7] to negative ions [by assuming the parameter $\xi = \frac{1}{5}$ and taking the friction coefficient[2] $\alpha(T)$ and the zero-field mobility[11] from existing data] and fits our results rather well. Some determinations of the threshold fields, where friction plays a rather important role, were made by using the fact that negatively charged vortex rings annihilate at the liquid surface.[8] At temperature $T < 0.53$°K, E_{c1} exhibits a cubic temperature dependence like the number of phonons.

For positive ions (Fig. 6) the lower threshold E_{c1} is in good agreement with the stability condition given by the HO model ($\xi = \frac{1}{3}$).

The bare-ion critical velocities are shown in Figs. 7 and 8 for positive and negative ions, respectively. For positive ions the critical velocity V_{c2} slowly increases by decreasing the temperature, and from about 30 m/sec at $T = 1$°K reaches the value of about 42 m/sec at $T = 0.4$°K. For negative ions V_{c2} at $T = 0.4$°K reaches instead 34 m/sec. The lower critical velocity V_{c1} for positive ions is in good agreement with that calculated by the stability condition of the HO model down to $T = 0.5$°K. It must be stressed, however, that though E_{c1} seems to indicate that the HO model can be extrapolated at $T < 0.5$°K, V_{c1} instead disagrees below such temperature from the values obtainable by a reasonable extrapolation of the stability condition. Again there is agreement between the values calculated by the stability condition and V_{c1}, E_{c1} for negative ions down to $T = 0.53$°K. For low temperatures neither V_{c1} nor E_{c1} agrees with the stability condition provided by the HO model. The critical velocity V_{c1} reaches a constant value $V_{c1} = V_0 = 4.8$ m/sec.

A typical example of measurements, showing that the dynamics above both critical fields are that of a single quantized vortex ring, is shown in Figs. 9 and 10. The effect of friction was con-

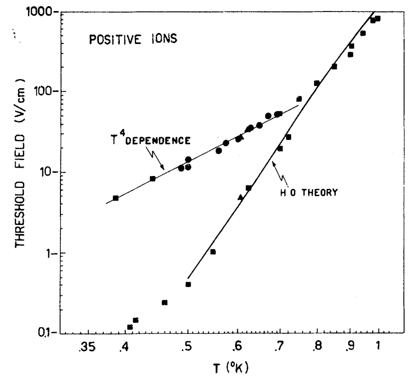

FIG. 6. Threshold fields E_{c1} and E_{c2} versus temperature. Squares refer to ac measurements, circles to dc measurements and the others to dc measurements with the liquid level between grid 1 and grid 2.

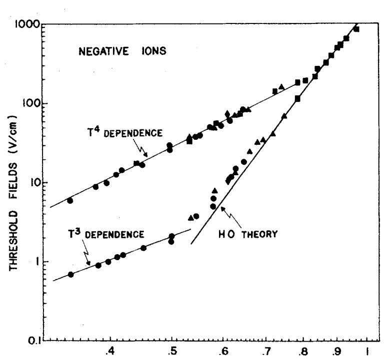

FIG. 5. Threshold fields E_{c1} and E_{c2} versus temperature. Squares refer to ac measurements, circles to dc measurements and the others to dc measurements with the liquid level between grid 1 and grid 2.

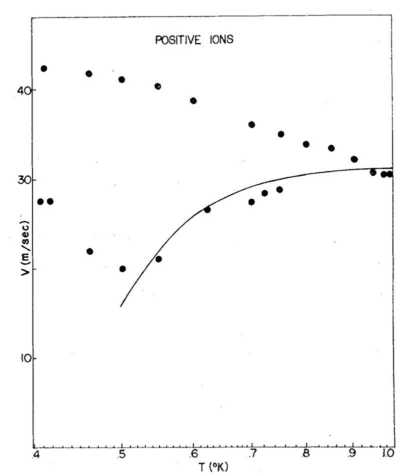

FIG. 7. Critical velocities V_{c1} and V_{c2} versus temperature.

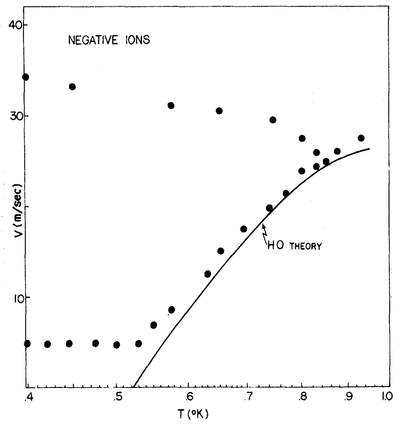

FIG. 8. Critical velocities V_{c1} and V_{c2} versus temperature.

sidered independent of the radius of the vortex, as far as these measurements are concerned. This was demonstrated both for positive and negative charges.

4. DISCUSSION

From the dc and ac measurements we can conclude that above E_{c1} only few ions are bound to vortex rings and at E_{c2} a process begins that causes all the ions to form a ion-vortex ring complex (process A). The temperature dependence $E_{c2} \propto T^4$ for both ions is different from the temperature dependence of the number of rotons and phonons. A simple analysis of the formation process at E_{c2}, in terms of mean free path, by introducing a reduced electric field[6] $E' = \mu(o) \times \frac{2}{3} 10^{-3} T^{-1/2} \times E$, showed that there is a slight dependence of E_{c2} on E' for negative ions and a marked dependence for positive ions. On the other hand, if we consider the velocity as the quantity responsible of the creation process, it

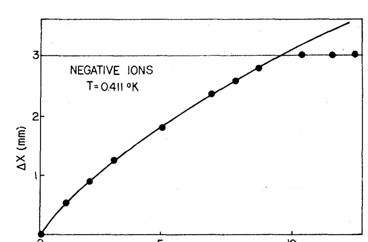

FIG. 9. Example of the determination of the time dependence of the displacement Δx of the charged vortex ring under an electric field $E_{12} = 33$ V/cm $> E_{c2}$. The vortex ring has at $\Delta x = 0$ an energy of 4 eV, and the continuous line was calculated from classical formulas with circulation $K = 1$ and the core of the vortex $a = 1.28$ Å.

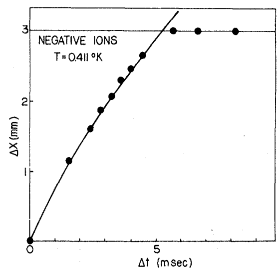

FIG. 10. The plot is analogous to Fig. 9 except for $E_{c1} < e_{12} = 11$ V/cm $< < E_{c2}$ and the energy of vortices at $\Delta x = 0$ of 2.72 eV.

may be assumed that the creation velocity is constant for positive and negative charges. In such a case the temperature dependence of V_{c2} can be explained by considering that the instantaneous ion velocity has a thermal distribution around the drift velocity.

The existence of the lower threshold shows that charged vortex rings may be formed at lower velocities ($V_{c1} < V_{c2}$). For positive ions E_{c1} coincides with the field calculated by the stability condition; V_{c1} is explainable at the same way down to $T = 0.5$°K. For $T < 0.5$°K, V_{c1} increases by decreasing the temperature, whereas an extrapolation of the stability-condition curve gives decreasing behavior. As the stability condition is based on the velocity of the ion bound to a vortex ring, while what we determine is the velocity of bare ions, the behavior of V_{c1} for $T < 0.5$°K may mean that the velocity of bare ions at E_{c1} is remarkably greater than the one of charged vortex rings still at E_{c1}.

For the case of negative ions at $T < 0.53$°K both E_{c1} and V_{c1} are much greater than the data obtained by the stability condition and particularly the velocity V_{c1} is constant, $V_{c1} = V_0$. The fact that there are two different thresholds E_{c1} and E_{c2} implies the existence of two different processes (B and A, respectively). The constant velocity V_0 suggests that negative ions form vortex rings according to the process B. The mechanism of the process B may be assumed to be due to the capturing of a preexisting vortex germ.

Two possible ways to originate vortex germs may be either by the radioactivity of the source, or by the decay of vortices previously generated from ions moving in fields $E > E_{c2}$. In fact, the ratio f of the plateau current decreases when the charged vortex rings are formed far from the source. Moreover, the ratio f increases if vortex rings have been previously formed at $E > E_{c2}$ for a certain time, thus showing that the process B is sensitive to the "history" of the bath.

For positive ions the constant velocity corresponding to the process B is not reached because of the high velocity of the bare ions along E_{c1}.

The condition of stability for the electric field and the velocity, allows the two different processes A and B to be revealed only in a limited range of temperatures.

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Electrical Conductivity of a Fully Ionized Plasma in a Strong Magnetic Field

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This paper presents a method of obtaining exact transport coefficients of a plasma in a strong magnetic field. The method is based on Kihara and Aono's unified theory. As an application of the method, the electrical conductivity of a fully ionized plasma in a strong magnetic field has been calculated. The departure from the ordinary theory is expressed by a function χ, which depends on the ratio of the Debye length to the electron gyroradius. Numerical values of χ are given.

1. INTRODUCTION

In the calculation of plasma transport properties, one often takes no account of the influence of the magnetic field on the particle-collision act. Therefore, results of such calculations are restricted to the cases when the external magnetic field is sufficiently weak. For such fields, the average gyroradius of an electron is much longer than the Debye length. In many laboratory plasmas, however, the influence of the magnetic field on the charged particle collisions can not be ignored. Some investigators[1-5] have considered temperature relaxation and transport under conditions of strong magnetic fields.

In simple cases, the rates of relaxation and transport properties of plasmas can be obtained by calculating one or two moments of the change in particle velocity. In general, however, it is necessary to solve a kinetic equation. In a kinetic equation, the charged particle interactions can be represented in one of two alternate ways: One is through impact theory[6] in which binary collisions are taken into consideration by an integral operator with respect to the impact parameter b. The exclusion of collective interactions in this theory leads to logarithmic divergences for b tending to infinity. The order method is based on the wave theory,[7] in which the collective interactions are incorporated in the theory through the dielectric permeability tensor of the plasma, $\overleftrightarrow{\epsilon}(\vec{k}, \omega)$, as a function of the wave vector $\vec{k}$ and frequency ω. Since the orbital curvatures are not included in this theory, the integral operator with respect to k is divergent.

It has been customary to introduce somewhat arbitrary cutoff lengths to deal with the divergence behavior of above integral operators.[8] The results of such calculations are of logarithmic accuracy. When there is no magnetic field present, it is almost true that the shielding distance is of the order of the Debye length around a charged