Superconvergent Sum Rules for a $u=0$ Elastic Process

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We have constructed and tested sum rules for a $u=0$ elastic process $\bar{K}+\Xi\rightarrow\bar{K}+\Xi$ by assuming superconvergent behavior based on a Regge hypothesis and saturation by a finite number of low-lying intermediate states.

RECENT work on superconvergent sum rules for backward ($u=0$) elastic pion-nucleon scattering rests on the doubtful assumption that sum rules may be obtained from the amplitudes that are not superconvergent.[1–3] These sum rules are used to investigate the applicability of the following assumptions: (1) that the Reggeized u-channel trajectories determine the asymptotic behavior of the amplitudes for $u=0$, and (2) that the resulting sum rules are well approximated by the contributions of a finite number of low-lying intermediate states. Since these are important assumptions for all such calculations, it would be worthwhile to test their applicability under conditions where the superconvergence of the amplitudes is not merely plausible, but definitely established.

We should consider an elastic process for which the u-channel Regge trajectory is not N but, say, Λ or Σ. We know that[4,5] $\alpha_{(I=0,1)}(u=0)<-\frac{1}{2}$ which will make both $A^I\rightarrow s^{\alpha(u)-1/2}$ and $B^I\rightarrow s^{\alpha(u)-1/2}$ separately superconvergent.

We consider the elastic process

$$\bar{K}+\Xi\rightarrow\bar{K}+\Xi. \tag{1}$$

The u-channel Regge trajectories are Λ and Σ. Therefore, we can write the following superconvergence relations:

$$\int_{-\infty}^{+\infty} A^{(I=0,1)}(s,u=0)ds=0, \tag{2}$$

$$\int_{-\infty}^{+\infty} B^{(I=0,1)}(s,u=0)ds=0. \tag{3}$$

These sum rules can be rewritten as

$$\sum_i \Delta_{us}C_{us}\int_0^\infty \mathrm{Im}A_i^{(I=0,1)}(s,u=0)ds+\sum_j \Delta_{ut}C_{ut}\int_0^\infty \mathrm{Im}A_j^{(I=0,1)}(s,u=0)ds=0 \tag{2'}$$

and

$$\sum_i \Delta_{us}C_{us}\int_0^\infty \mathrm{Im}B_i^{(I=0,1)}(s,u=0)ds+\sum_j \Delta_{ut}C_{ut}\int_0^\infty \mathrm{Im}B_j^{(I=0,1)}(s,u=0)ds=0, \tag{3'}$$

where the suffix i stands for s channel and j for t channel. C_{us} and C_{ut} are the crossing matrices given by

$$C_{us}:\begin{pmatrix}-\frac{1}{2} & \frac{3}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix},\quad C_{ut}:\begin{pmatrix}-\frac{1}{2} & \frac{3}{2}\\ -\frac{1}{2} & -\frac{1}{2}\end{pmatrix},$$

and Δ_{us} and Δ_{ut} denote the crossing-symmetry phase factors. Δ_{us} for the A_i^I (B_i^I) amplitude is $+1$ (-1) and Δ_{ut} for the A_i^I (B_i^I) amplitude is $(-1)^{I_t}$ $((-1)^{I_t+1})$, where I_t is the total isotopic spin in the t channel.

In evaluating these sum rules, we will take contributions from Ω in the s channel and ρ, ω, ϕ, and f in the t channel in the narrow-width approximation. On projecting the u-channel isospin contributions, we find the following sum rules evaluated at $u=0$:

$$A^{(0)}:\ -g_{\Omega\Xi K}{}^2A_\Omega-3g_{\rho KK}A_\rho-g_{\omega KK}A_\omega-g_{\phi KK}A_\phi-\tfrac{1}{2}A_f=0, \tag{4}$$

$$A^{(1)}:\ g_{\Omega\Xi K}{}^2A_\Omega+g_{\rho KK}A_\rho-g_{\omega KK}A_\omega-g_{\phi KK}A_\phi-\tfrac{1}{2}A_f=0, \tag{5}$$

$$B^{(0)}:\ -g_{\Omega\Xi K}{}^2B_\Omega-3g_{\rho KK}B_\rho-g_{\omega KK}B_\omega-g_{\phi KK}B_\phi-\tfrac{1}{2}B_f=0, \tag{6}$$

$$B^{(1)}:\ g_{\Omega\Xi K}{}^2B_\Omega+g_{\rho KK}B_\rho-g_{\omega KK}B_\omega-g_{\phi KK}B_\phi-\tfrac{1}{2}B_f=0, \tag{7}$$

where

$$A_\Omega=-[(m_\Omega+m_\Xi)(m_K{}^2-\tfrac{1}{2}m_\Omega{}^2+\tfrac{1}{3}m_\Xi{}^2+\tfrac{2}{3}E^2)+\tfrac{1}{3}(m_\Omega{}^2-m_\Xi{}^2-m_K{}^2)(E+m_\Xi)],$$

$$B_\Omega=\tfrac{1}{3}(1.5m_\Omega{}^2+m_\Xi{}^2-3m_K{}^2-2E^2+2m_\Xi E),$$

$$A_X=-\frac{2m_\Xi{}^2+2m_K{}^2-m_X{}^2}{2m_\Xi}g_{X\Xi\Xi}{}^T,$$

$$B_X=2(g_{X\Xi\Xi}{}^T+g_{X\Xi\Xi}{}^V),$$

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with $E=(m_\Omega^2+m_\Xi^2-m_K^2)/2m_\Omega$ and X denotes ρ, ω, or ϕ. A_f and B_f are the contributions of the f particle to A and B, respectively.

From Eqs. (4)–(7), we get

$$g_{\Omega\Xi K}^2 A_\Omega + 2g_{\rho KK} A_\rho = 0 \tag{8}$$

and

$$g_{\Omega\Xi K}^2 B_\Omega + 2g_{\rho KK} B_\rho = 0 . \tag{9}$$

These two relations immediately yield the following interesting result:

$$g_{\rho\Xi\Xi}^T / g_{\rho\Xi\Xi}^V = -1.98 . \tag{10}$$

We can test this result by noticing that the SU_3 relations[6] are

$$g_{\rho\Xi\Xi}^V = -(1-2\alpha^V) g_{\rho NN}^V$$

and

$$g_{\rho\Xi\Xi}^T = -(1-2\alpha^T) g_{\rho NN}^T .$$

Using[7] $\alpha^V=1$, $\alpha^T=0.25$, we get

$$g_{\rho NN}^T / g_{\rho NN}^V = 3.96 , \tag{11}$$

which is close to the accepted[3,8] value 3.70.

We can also use Eqs. (8) and (10) to estimate $g_{\Omega\Xi K}$ as follows:

$$g_{\Omega\Xi K}^2/4\pi = -(2A_\rho/A_\Omega) g_{\rho KK}/4\pi \simeq 0.033/\mu^2 , \tag{12}$$

where we have used[8] $g_{\rho\pi\pi} g_{\rho NN}^V/4\pi = 1.5$. Following Dass *et al.*,[9] if we put $g_{\rho\pi\pi} g_{\rho NN}^V/4\pi = 1.5\times 1.78$, we get $g_{\Omega\Xi K}^2/4\pi = 0.06/\mu^2$. The closest model-dependent values to that of ours are $0.012/\mu^2$ (Ref. 10), $0.067/\mu^2$, $0.098/\mu^2$ (Ref. 11). If we ignore the f-meson contribution, we find that the sum rules for A^I and B^I cannot be satisfied. Using (4), (6), (8), and (9) to estimate A_f and B_f, we obtain

$$A_f = -2.38 g_{\rho\pi\pi} g_{\rho NN}^T \tag{13}$$

and

$$B_f = 4(g_{\rho NN}^V + g_{\rho NN}^T) g_{\rho\pi\pi} . \tag{14}$$

In obtaining these expressions for A_f and B_f, we have used the SU_3 relations for coupling constants. The values of A_f and B_f are insensitive to the value of α^T. In our case, f is coupled to both A and B. The suppression of f coupling to B noted by earlier workers[1–3] for πN scattering does not occur in our case. The reason may be that when $s=u=0$, we find $t=3.99$ BeV^2 is far away from m_f^2, while for $B^{I_t=0}$ to vanish, $t\rightarrow m_f^2$.

We have thus tested the validity of the two assumptions regarding the superconvergent behavior and finite-pole approximation under more acceptable conditions and have obtained a plausible value for $g_{\rho\Xi\Xi}^T/g_{\rho\Xi\Xi}^V$.

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