$\Sigma^- \to n\pi^-\gamma$ with $P<60$ MeV/c, relative to all Σ^- decays. This ratio is unlikely to be wrong by a large factor and is to be compared with the Σ^- leptonic decays, which are about 300 times more probable. To make this point graphically, we display in Fig. 3(b) the pion-energy spectrum, which follows from our calculation (and is identical to that calculated in Ref. 2 because the integration over $\cos\theta$ removes the polarization-dependent term). For simplicity, we show only the limiting cases of pure S wave and pure P wave. In our notation, the pion-energy spectrum, normalized to the two-body decay rate, is

$$\frac{dR}{dP} \equiv \frac{d\Gamma(\Sigma \to n\pi\gamma)}{\Gamma(\Sigma \to n\pi)dP} = \frac{1}{\pi P_0}\frac{e^2}{4\pi}\frac{m}{M}\frac{P^2}{E_0(E_0-E)}T_\pm(P),$$

where the upper (lower) sign refers to pure S wave (P wave).

In conclusion, we note that, since the kinematic enhancement of the pion asymmetry for $\Sigma^- \to n\pi^-\gamma$ is large only in the region of low pion energy, and since this region is at present experimentally inaccessible, the radiative decays offer at this time neither a useful method of identifying the Σ^- polarization nor a useful alternative method for measuring the magnitude of α_0. We emphasize, however, that when the low-pion-energy region becomes experimentally accessible,[11] the kinematic enhancement of the pion asymmetry in the radiative decay can be a useful tool for studying the structure-dependent and magnetic-moment effects, in addition to providing information about Σ^- polarization and the two-body asymmetry parameter.

A complete study of radiative hyperon decays from polarized hyperons is in progress and will be published as a longer article.

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[11] We have been informed by Professor W. Willis that the low-pion-energy region will become experimentally accessible in the not too distant future.

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Equal-Time Commutator and Superconvergence Sum Rule*

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An unambiguous derivation of the superconvergence sum rule from the equal-time commutation relation is presented. This enables us to show that the canonical commutation relations of meson fields lead to the superconvergence sum rules of higher moments.

I. INTRODUCTION

In an earlier work,[1] the superconvergence sum rules for zero-mass-pion–nucleon scattering were derived by applying the infinite-momentum technique to the equal-time commutation relation that involves the pion fields, along with an appropriate subtraction method. Since, however, the method is not unambiguous, especially concerning the dependence of the scattering cross section on the fictitious external mass variables, in this article we present another proof of the above-mentioned assertion that is free of ambiguity. This method shall be applied to the canonical commutation relation of the pion fields. All the notation is the same as in Ref. 1, unless otherwise stated.

II. ALTERNATIVE DERIVATION OF SUPERCONVERGENCE SUM RULE

We start with Eq. (5′) of Ref. 1:

$$\begin{aligned} f^{(+)}(0,0) &= \frac{\mu^4|\mathbf{p}|}{2\pi^2}\int_\mu^\infty \frac{d\nu}{(p_0{}^2+2m\nu)^{1/2}}\frac{1}{(\mu^2-k_0{}^2)^2} \\ &\quad\times\left[\sigma^{(+)}(\nu_L,k_0{}^2)\theta\left(\nu-\mu-\frac{\mu^2}{2m}\right)-\sigma_R{}^{(+)}(\nu_L,k_0{}^2)\right] \\ &\quad+\frac{\mu^4|\mathbf{p}|}{2\pi^2}\int_\mu^\infty \frac{d\nu}{(p_0{}^2+2m\nu)^{1/2}}\frac{\sigma_R{}^{(+)}(\nu_L,k_0{}^2)}{(\mu^2-k_0{}^2)^2} \\ &= I_1+I_2, \end{aligned} \tag{1}$$

where

$$\sigma_R{}^{(+)}(\nu,k_0{}^2)=\sum_{0<\alpha\leq 1}\frac{4\pi\beta_\alpha{}^{(+)}(k_0{}^2)}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha-1}. \tag{2}$$

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[1] Y. Tomozawa, Phys. Rev. 177, 2288 (1969).

Changing the variable in the second integral of Eq. (1), I_2, as follows:

$$k_0 = m\nu_L/p_0 = (p_0^2+2m\nu)^{1/2} - p_0\,,\qquad dk_0 = m\,d\nu/(p_0^2+2m\nu)^{1/2}\,, \tag{3}$$

we have

$$I_2 = \frac{\mu^4|\mathbf{p}|}{2m\pi^2}\sum_{0<\alpha\le 1}\left[\int_0^\infty \frac{4\pi\beta_\alpha^{(+)}(k_0^2)(k_0p_0/m\mu)^{\alpha-1}}{(\mu^2-k_0^2)^2\mu}dk_0 - \int_0^{k_{01}} \frac{4\pi\beta_\alpha^{(+)}(k_0^2)(k_0p_0/m\mu)^{\alpha-1}}{(\mu^2-k_0^2)^2\mu}dk_0\right], \tag{4}$$

where

$$k_{01} = (p_0^2+2m\mu)^{1/2} - p_0\,,$$

which tends to $m\mu/p_0$ as $p_0 \to \infty$. While the second term of Eq. (4) has a definite value in the limit $p_0 \to \infty$, the first term does not have a limit, unless[2]

$$\int_0^\infty \frac{\beta_\alpha^{(+)}(k_0^2)k_0^{\alpha-1}}{(\mu^2-k_0^2)^2}dk_0 = 0\,,\quad 0<\alpha\le 1\,. \tag{5}$$

Then, we obtain

$$\lim_{p_0\to\infty} I_2 = -\lim_{p_0\to\infty}\frac{|\mathbf{p}|}{2m\mu\pi^2}\sum_{0<\alpha\le 1}4\pi\beta_\alpha^{(+)}(0)\left(\frac{p_0}{m\mu}\right)^{\alpha-1}\frac{k_{01}^{\alpha}}{\alpha} = -\frac{2}{\pi}\sum_{0<\alpha\le 1}\frac{\beta_\alpha^{(+)}(0)}{\alpha}\,, \tag{6}$$

with the constraint condition (5) for the Regge residue function $\beta_\alpha^{(+)}(k_0^2)$. Taking the limit $p_0 \to \infty$ in Eq. (1), we may immediately derive the superconvergence sum rule

$$f^{(+)}(0,0) = \frac{1}{2\pi^2}\int_\mu^\infty d\nu\left[\sigma^{(+)}(\nu,0)\theta\left(\nu-\mu-\frac{\mu^2}{2m}\right) - \sum_{0<\alpha\le 1}\frac{4\pi\beta_\alpha^{(+)}(0)(\nu/\mu)^{\alpha-1}}{\mu}\right] - \frac{2}{\pi}\sum_{0<\alpha\le 1}\frac{\beta_\alpha^{(+)}(0)}{\alpha}\,. \tag{7}$$

This is obviously equivalent to Eq. (9) of Ref. 1.

[2] This equation need not be valid for all α's in the region indicated, but must be valid for those values of α corresponding to the relevant poles or cuts, with the appropriate quantum numbers, in the complex angular momentum plane. The integral of this equation is a Cauchy principal-value one. Similar remarks apply to Eqs. (9) and (13).

III. CANONICAL COMMUTATION RELATIONS

A similar method can be applied to the matrix element of the canonical commutation relation

$$\left[\int \varphi^{(-)}(\mathbf{x},x_0)d^3x,\ \int \varphi^{(+)}(\mathbf{y},x_0)d^3y\right] = 0\,, \tag{8}$$

leading to Eq. (27) of Ref. 1, with the constraint

$$\int_0^\infty \frac{\beta_\alpha^{(-)}(k_0^2)k_0^{\alpha}dk_0}{(\mu^2-k_0^2)^2} = 0\,,\quad -1<\alpha<1\,. \tag{9}$$

The other portion of the canonical commutation relation, or a slightly generalized form of it,

$$\left[\int \varphi^{(-)}(\mathbf{x},x_0)d^3x,\ \int \dot{\varphi}^{(+)}(\mathbf{y},x_0)d^3y\right] = C \text{ number}\,, \tag{10}$$

where the dot denotes the time derivative, leads to the relation

$$0 = \int_{\mu+\mu^2/2m} \frac{d\nu}{(p_0^2+2m\nu)^{1/2}}\,\frac{k_0^2\sigma^{(+)}(\nu_L,k_0^2)}{(\mu^2-k_0^2)^2}\,. \tag{11}$$

This can be seen by taking the matrix element of Eq. (10) between the one-proton state and substracting from it the disconnected diagram.[3] Proceeding in a similar way as before, we obtain the relation

$$\int_\mu^\infty d\nu\,\nu^2\left[\sigma^{(+)}(\nu,0)\theta\left(\nu-\mu-\frac{\mu^2}{2m}\right) - \sum_{-2<\alpha\le 1}\frac{4\pi\beta_\alpha^{(+)}(0)(\nu/\mu)^{\alpha-1}}{\mu}\right] = 4\pi\mu^2\sum_{-2<\alpha\le 1}\frac{\beta_\alpha^{(+)}(0)}{2+\alpha}\,, \tag{12}$$

with the constraint

$$\int_0^\infty \frac{\beta_\alpha^{(+)}(k_0^2)k_0^{\alpha+1}}{(\mu^2-k_0^2)^2}dk_0 = 0\,,\quad -2<\alpha\le 1\,. \tag{13}$$

Equation (12) is a superconvergence sum rule with moment ν^2, and can also be derived from a dispersion relation for $f^{(+)}(\nu,0)$.

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[3] N. N. Khuri, Phys. Rev. Letters **16**, 75 (1966); **16**, 601(E) (1966). The effect of subtraction of the disconnected diagram is to change the singularity of Eq. (10) from the pinching type to the Cauchy principal value. For a discussion of the details, see W. I. Weisberger, *ibid.* **14**, 1047 (1965).