Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters *and are not appropriate for regular articles. It includes only the following types of communications: (1) comments on papers previously published in* The Physical Review *or* Physical Review Letters; *(2) addenda to papers previously published in* The Physical Review *or* Physical Review Letters, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Energy Dependence of Pion Asymmetry in the Radiative Decay of Polarized Σ†

L. H. CHAN, A. EBRAHIM, AND R. H. SOCOLOW
Physics Department, Yale University, New Haven, Connecticut 06520
(Received 30 June 1969)

We have calculated the energy dependence of pion asymmetry relative to the direction of Σ polarization in the decay $\Sigma \to n\pi\gamma$, using the inner-bremsstrahlung model. We find that the kinematic enhancement of the pion asymmetry is large only for $\Sigma^- \to n\pi^-\gamma$, and only in the region of low pion energy, which is not yet experimentally accessible.

INTRODUCTION

THE $\Delta I=\frac{1}{2}$ rule and the experimental values of the asymmetry parameters in the decays $\Sigma^\pm \to n\pi^\pm$ and $\Sigma^+ \to p\pi^0$ require that one of the decays $\Sigma^\pm \to n\pi^\pm$ occurs via S wave and the other via P wave. Measurement of neutron polarization now indicates that the decay $\Sigma^- \to n\pi^-$ ($\Sigma^+ \to n\pi^+$) occurs mostly via S wave (P wave).[1]

It has been suggested by Barshay, Nauenberg, and Schultz,[2] on the basis of the inner-bremsstrahlung model, that the branching ratios of the decays $\Sigma^\pm \to n\pi^\pm\gamma$ relative to the nonradiative decays $\Sigma^\pm \to n\pi^\pm$ can be used as an alternative method to determine which of the decays $\Sigma^\pm \to n\pi^\pm$ is dominated by S wave or P wave.[3] The experimental measurement of the branching ratios favors the decay $\Sigma^+ \to n\pi^+$ occurring via P wave.[4]

None of the previous calculations, however, has considered the consequences of the polarization of the decaying hyperon. With the availability of polarized samples of Σ hyperons,[5] it is pertinent to ask whether the polarization of the decaying hyperon can be used to obtain additional information from the radiative Σ decays.

The study of radiative decays from *polarized* Σ can be useful, first of all, because it provides an independent measurement of the two-body asymmetry parameter. The radiative decay mode can also be useful in determining the Σ^- polarization, if a suitable region of enhancement of pion asymmetry is present. In any study of production processes, the Σ^- polarization is a useful parameter. However, the knowledge of this parameter is difficult to obtain experimentally because the most common decay mode $\Sigma^- \to n\pi^-$ is almost isotropic. The radiative decay $\Sigma^- \to n\pi^-\gamma$ is comparable to the leptonic decays and offers an alternative method for determining the Σ^- polarization.

We have searched for a region of pion energy in the radiative decay of polarized Σ where the pion asymmetry is kinematically enhanced relative to the nonradiative decay.

CALCULATION

Our calculation is based on the inner-bremsstrahlung model first proposed by Barshay and Behrends.[6] The gauge-invariant phenomenological Lagrangian[7] for the

† Work (Yale Report No. 2726-553) supported by the U. S. Atomic Energy Commission, under Contract No. AT(30-1)-2726.

[1] D. Berley, S. Hertzbach, R. Kofler, S. Yamamoto, W. Heintzelman, M. Schiff, J. Thompson, and W. Willis, Phys. Rev. Letters **17**, 1071 (1966); D. Berley, S. Hertzbach, R. Kofler, G. Meisner, J. B. Schafer, S. Yamamoto, W. Heintzelman, M. Schiff, J. Thompson, and W. Willis, *ibid.* **19**, 979 (1967).

[2] S. Barshay, U. Nauenberg, and J. Schultz, Phys. Rev. Letters **12**, 76 (1964); **12**, 156(E) (1964).

[3] Subsequent calculations have shown that the magnetic-moment and structure-dependent corrections to the pion spectrum are small. See M.C. Li, Phys. Rev. **141**, 1328 (1966); R. D. Young, M. Sugawara, and T. Sakuma, *ibid.* **145**, 1181 (1966).

[4] M. Bazin, H. Blumenfeld, U. Nauenberg, L. Seidlitz, R. J. Plano, S. Marateck, and P. Schmidt, Phys. Rev. **140**, B1358 (1965).

[5] We have been informed by Professor W. Willis that $\Sigma^\pm$ polarizations of the order of 60–70% are readily available in K^-p scattering near $Y^*(1520)$ resonance. See also R. Bangerter, A. Barbaro-Galtieri, J. P. Berge, J. J. Murray, F. T. Solmitz, M. L. Stevenson, and R. D. Tripp, Phys. Rev. Letters **17**, 495 (1966).

[6] S. Barshay and R. E. Behrends, Phys. Rev. **114**, 931 (1959).

[7] The first term in the Lagrangian used in Ref. 6 is of the vector-pseudovector type, whereas the corresponding term in our Lagrangian is of the scalar-pseudoscalar type. We note, however, that in the absence of magnetic-moment terms, the two types of Lagrangians are completely equivalent. Furthermore, for the scalar-pseudoscalar type of Lagrangian, the four-field coupling $\bar{\psi}_n\gamma^\mu\psi_\Sigma A_\mu\phi_\pi$, and hence the Feynman diagram with a four-point vertex, is absent.

decay $\Sigma \to n\pi\gamma$ is taken as

$$\mathcal{L}_{\mathrm{int}} = -\bar{\psi}_n(\lambda_+ A_S + \lambda_- A_P\gamma^5)\psi_\Sigma\phi_\pi - e\bar{\psi}_\Sigma\gamma^\mu\psi_\Sigma A_\mu$$

$$-ie\phi_\pi^\dagger \overleftrightarrow{\partial^\mu}\phi_\pi A_\mu - \sum_{i=n,\Sigma}\frac{e\mu_i}{2M_i}\bar{\psi}_i\sigma^{\mu\nu}\psi_i F_{\mu\nu}\,,$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $\lambda_\pm=\{4mM/[(M\pm m)^2-\mu^2]\}^{1/2}$; M, m, and μ are the hyperon, nucleon, and pion masses, respectively; μ_i is the anomalous magnetic moment of the ith baryon; and A_S, A_P are dimensionless, real parameters.[8] The case $A_S=0$ ($A_P=0$) corresponds to pure P-wave (S-wave) coupling.

As a first approximation, we neglect the magnetic-moment terms and calculate the decay rate for the Feynman diagrams shown in Fig. 1. The gauge-invariant decay amplitude for these diagrams is found to be

$$m(\Sigma\to n\pi\gamma) = e\bar{u}_n(\lambda_+A_S+\lambda_-A_P\gamma^5)\left(\frac{\gamma\cdot k\gamma\cdot\epsilon}{2Q\cdot k}-\frac{Q\cdot\epsilon}{Q\cdot k}+\frac{P\cdot\epsilon}{P\cdot k}\right)u_\Sigma\,.$$

The differential decay rate in the Σ rest frame as a function of pion momentum and angle θ with respect to Σ polarization can be written in the form

$$\frac{d\Gamma(\Sigma\to n\pi\gamma)}{dPd(\cos\theta)}=\frac{1}{4\pi^2}\frac{e^2}{4\pi}\left(\frac{m}{M}\right)^2\frac{P^2}{E_0(E_0-E)}$$

$$\times[A_S^2T_+(P)+A_P^2T_-(P)][1+\alpha(P)\mathcal{P}\cos\theta]\,,$$

where

$$\alpha(P)=\frac{2A_SA_PT_0(P)}{A_S^2T_+(P)+A_P^2T_-(P)}\frac{P}{P_0}$$

is the pion asymmetry parameter for the radiative decay, and $\mathcal{P}$ is the magnitude of the Σ polarization. In these expressions, $E_0=(M^2+\mu^2-m^2)/2M$ and $P_0=2m/\lambda_+\lambda_-$ are the *fixed* pion energy and momentum in the two-body nonradiative decay $\Sigma\to n\pi$, and, therefore, also the maximum pion energy and momentum in the three-body radiative decay $\Sigma\to n\pi\gamma$; E and P are the *variable* pion energy and momentum in the three-body radiative decay.

In writing the differential decay rate, we have separated out the factor $P^2/E_0(E_0-E)$, which represents the dominant energy dependence of the expression. We then find that

$$T_\pm(P)=\frac{ME_0}{mE}\left\{\lambda_\pm^2\frac{(E_0-E)^2}{4mP}\ln\left(\frac{M-E+P}{M-E-P}\right)+\left[\frac{E}{P}\ln\left(\frac{E+P}{E-P}\right)-2\right]\right\}$$

and

$$T_0(P)=\frac{ME_0}{mE}\left\{\frac{(E_0-E)^2}{2P^2}\left[\frac{M-E}{P}\ln\left(\frac{M-E+P}{M-E-P}\right)-2\right]+\frac{EE_0-\mu^2}{P^2}\left[\frac{E}{P}\ln\left(\frac{E+P}{E-P}\right)-2\right]\right\}$$

are smooth functions of P, as shown in Fig. 2. The expressions for $T_\pm(P)$ and $T_0(P)$ are written such that the second term in each case dominates in the infrared limit as $P\to P_0$, while the first term in $T_\pm(P)$ dominates as $P\to 0$.

The differential decay rate for the nonradiative decay can be written as

$$\frac{d\Gamma(\Sigma\to n\pi)}{d(\cos\theta)}=\frac{1}{4\pi}(m/M)P_0(A_S^2+A_P^2)(1+\alpha_0\mathcal{P}\cos\theta)\,,$$

where $\alpha_0=2A_SA_P/(A_S^2+A_P^2)$ is the conventional two-body asymmetry parameter. The best experimental values for α_0 are $+0.018\pm0.039$ for $\Sigma^+\to n\pi^+$, -0.955 ± 0.070 for $\Sigma^+\to p\pi^0$, and -0.06 ± 0.05 for $\Sigma^-\to n\pi^-$.[9]

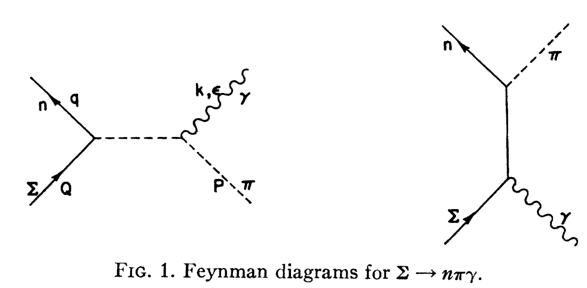

FIG. 1. Feynman diagrams for $\Sigma\to n\pi\gamma$.

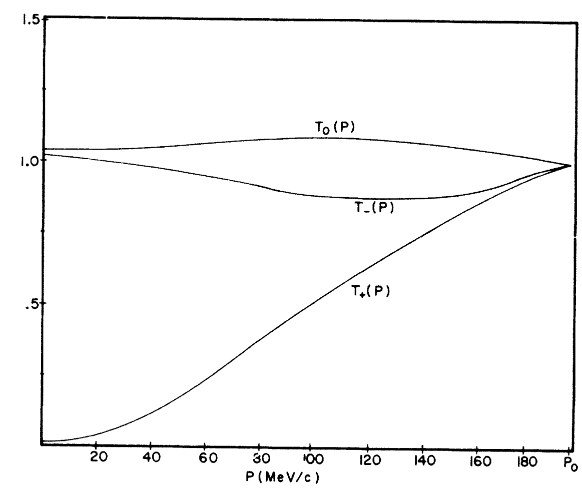

FIG. 2. Energy dependence of $T_\pm(P)$ and $T_0(P)$.

[8] We have chosen our normalization such that

$$\tfrac{1}{2}\textstyle\sum_{\mathrm{spins}}|m(\Sigma\to n\pi)|^2=(A_S^2+A_P^2),$$

where

$$m(\Sigma\to n\pi)=\bar{u}_n(\lambda_+A_S+\lambda_-A_P\gamma^5)u_\Sigma,$$

with the Dirac spinors normalized so that $\bar{u}(P,S)u(P,S)=1$. We also assume time-reversal invariance and neglect final-state interactions throughout.

[9] Particle Data Group. Rev. Mod. Phys. **41**, 109 (1969).

We note that the calculations for Σ^+ and Σ^- are identical provided one uses the appropriate values of A_S, A_P, and M in the final expressions. The result obtained by Barshay, Nauenberg, and Schultz[2] follows from our expressions by integrating over $\cos\theta$.

DISCUSSION AND CONCLUSION

In order to discuss the kinematic enhancement of the pion asymmetry in the radiative decay relative to the nonradiative decay, we study the asymmetry ratio $\alpha(P)/\alpha_0$ as a function of the pion momentum P and the two-body asymmetry parameter α_0. Measurements of the two-body decay rate and asymmetry determine A_S and A_P up to a permutation of the two values. Hence, there is an additional dependence of $\alpha(P)/\alpha_0$ on the nonradiative decay being either predominantly S-wave or predominantly P-wave.

It is convenient to write the asymmetry ratio in the form

$$\frac{\alpha(P)}{\alpha_0}=\frac{2P}{P_0}\left[\left(\frac{T_+(P)}{T_0(P)}+\frac{T_-(P)}{T_0(P)}\right)\pm(1-\alpha_0^2)^{1/2}\left(\frac{T_+(P)}{T_0(P)}-\frac{T_-(P)}{T_0(P)}\right)\right]^{-1},$$

where the plus sign corresponds to the case where the decay $\Sigma\to n\pi$ proceeds predominantly via S wave and hence is appropriate to $\Sigma^-\to n\pi^-$; conversely, the minus sign corresponds to predominantly P-wave decay and is appropriate to $\Sigma^+\to n\pi^+$. We note that the asymmetry ratio is independent of the *sign* of α_0 and also of the absolute two-body decay rate.

We have computed the asymmetry ratio for $\alpha_0=0$, 0.2, and 0.5 for the predominantly S-wave solution; for $\alpha_0=0$ for the predominantly P-wave solution; and for $\alpha_0=1$, which corresponds to equal amounts of S and P waves.[10] The result is shown in Fig. 3(a).

In the limit of vanishing photon momentum ($P\to P_0$), the asymmetry ratio approaches unity as expected classically. Also, the asymmetry ratio is more sensitive to the value of α_0 for the predominantly S-wave solution ($\Sigma^-\to n\pi^-\gamma$) than for the predominantly P-wave solution ($\Sigma^+\to n\pi^+\gamma$); furthermore, $\alpha(P)/\alpha_0\leq 1$ for the predominantly P-wave solution over the entire range of pion momentum. Consequently, the kinematic enhancement of the pion asymmetry in the radiative decay is present only when the nonradiative decay occurs predominantly via S wave ($\Sigma^-\to n\pi^-$).

The magnitude of the kinematic enhancement is largest for nearly pure S-wave two-body decay ($|\alpha_0|\leq 0.2$), but even in that case, the asymmetry ratio rises to only 1.4 at $P=60$ MeV/c. While it is true that the asymmetry ratio continues to rise for pion momenta between 10–60 MeV/c, reaching a maximum value between 2 and 3 depending on the value of α_0, we make the following observations about this kinematic region.

First, the region below 60 MeV/c is where we would expect the structure-dependent and magnetic moment contributions to dominate the inner-bremsstrahlung contribution.[3] As a result, we would consider the details of our calculation of the asymmetry ratio unreliable in this region.

Second, the decays in this kinematic region are likely to be exceedingly rare. We have used our calculation to estimate the decay rate and find a branching ratio of about 5×10^{-6} for the radiative decay of

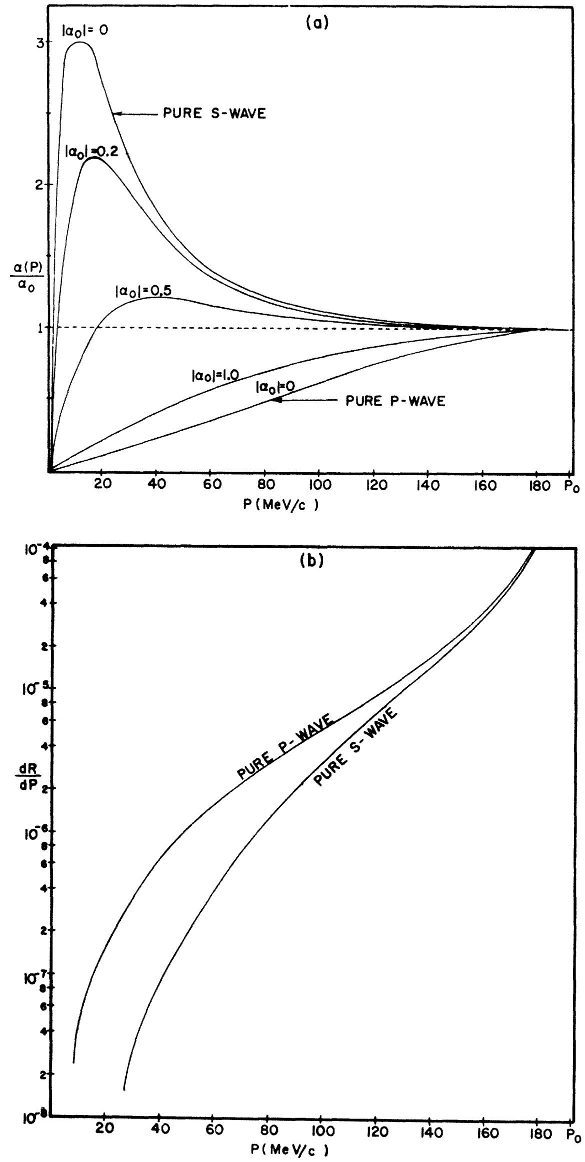

FIG. 3. (a) Energy dependence of asymmetry ratio in $\Sigma\to n\pi\gamma$. (b) Pion-energy spectrum for pure S wave and pure P wave in $\Sigma^-\to n\pi^-\gamma$. dR/dP has the units $(\mathrm{MeV}/c)^{-1}$.

[10] The computation was done using the Σ^- mass. The small difference between the Σ^+ and Σ^- masses changes the asymmetry ratio by at most 5% for $P\geq 10$ MeV/c.

$\Sigma^- \to n\pi^-\gamma$ with $P<60$ MeV/c, relative to all Σ^- decays. This ratio is unlikely to be wrong by a large factor and is to be compared with the Σ^- leptonic decays, which are about 300 times more probable. To make this point graphically, we display in Fig. 3(b) the pion-energy spectrum, which follows from our calculation (and is identical to that calculated in Ref. 2 because the integration over $\cos\theta$ removes the polarization-dependent term). For simplicity, we show only the limiting cases of pure S wave and pure P wave. In our notation, the pion-energy spectrum, normalized to the two-body decay rate, is

$$\frac{dR}{dP} \equiv \frac{d\Gamma(\Sigma \to n\pi\gamma)}{\Gamma(\Sigma \to n\pi)dP} = \frac{1}{\pi P_0} \frac{e^2}{4\pi} \frac{m}{M} \frac{P^2}{E_0(E_0 - E)} T_{\pm}(P),$$

where the upper (lower) sign refers to pure S wave (P wave).

In conclusion, we note that, since the kinematic enhancement of the pion asymmetry for $\Sigma^- \to n\pi^-\gamma$ is large only in the region of low pion energy, and since this region is at present experimentally inaccessible, the radiative decays offer at this time neither a useful method of identifying the Σ^- polarization nor a useful alternative method for measuring the magnitude of α_0. We emphasize, however, that when the low-pion-energy region becomes experimentally accessible,[11] the kinematic enhancement of the pion asymmetry in the radiative decay can be a useful tool for studying the structure-dependent and magnetic-moment effects, in addition to providing information about Σ^- polarization and the two-body asymmetry parameter.

A complete study of radiative hyperon decays from polarized hyperons is in progress and will be published as a longer article.

ACKNOWLEDGMENTS

The authors wish to thank Professor W. Willis of Yale University for suggesting this investigation and Dr. P. Yamin of Brookhaven Laboratory for clarifying the current experimental situation.

[11] We have been informed by Professor W. Willis that the low-pion-energy region will become experimentally accessible in the not too distant future.

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Equal-Time Commutator and Superconvergence Sum Rule*

YUKIO TOMOZAWA
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48104
(Received 17 July 1969)

An unambiguous derivation of the superconvergence sum rule from the equal-time commutation relation is presented. This enables us to show that the canonical commutation relations of meson fields lead to the superconvergence sum rules of higher moments.

I. INTRODUCTION

In an earlier work,[1] the superconvergence sum rules for zero-mass-pion–nucleon scattering were derived by applying the infinite-momentum technique to the equal-time commutation relation that involves the pion fields, along with an appropriate subtraction method. Since, however, the method is not unambiguous, especially concerning the dependence of the scattering cross section on the fictitious external mass variables, in this article we present another proof of the above-mentioned assertion that is free of ambiguity. This method shall be applied to the canonical commutation relation of the pion fields. All the notation is the same as in Ref. 1, unless otherwise stated.

II. ALTERNATIVE DERIVATION OF SUPERCONVERGENCE SUM RULE

We start with Eq. (5′) of Ref. 1:

$$\begin{aligned} f^{(+)}(0,0) &= \frac{\mu^4|\mathbf{p}|}{2\pi^2}\int_\mu^\infty \frac{d\nu}{(p_0^2+2m\nu)^{1/2}}\frac{1}{(\mu^2-k_0^2)^2} \\ &\quad\times\left[\sigma^{(+)}(\nu_L,k_0^2)\theta\left(\nu-\mu-\frac{\mu^2}{2m}\right)-\sigma_R^{(+)}(\nu_L,k_0^2)\right] \\ &\quad+\frac{\mu^4|\mathbf{p}|}{2\pi^2}\int_\mu^\infty \frac{d\nu}{(p_0^2+2m\nu)^{1/2}}\frac{\sigma_R^{(+)}(\nu_L,k_0^2)}{(\mu^2-k_0^2)^2} \\ &= I_1+I_2, \end{aligned} \tag{1}$$

where

$$\sigma_R^{(+)}(\nu,k_0^2) = \sum_{0<\alpha\leq 1} \frac{4\pi\beta_\alpha^{(+)}(k_0^2)}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha-1}. \tag{2}$$

* Work supported in part by the U. S. Atomic Energy Commission.

[1] Y. Tomozawa, Phys. Rev. 177, 2288 (1969).