dispersion relation in ν. Thus,

$$\delta m_1^{\text{Harari}} = -\tfrac{3}{8}\int_0^\infty dq^2 q^2 t_1(q^2,0).$$

Harari proves that

$$\lim_{q^2\to 0} q^2 t_1(q^2,0) = \frac{\alpha}{\pi}\frac{1}{m}(\mu_p{}^2-\mu_n{}^2-1)>0$$

(which is satisfied by the present model of course) and then conjectures that $h(q^2)$, defined by

$$q^2 t_1(q^2,0) = \frac{\alpha}{\pi}\frac{1}{m}(\mu_p{}^2-\mu_n{}^2-1)h(q^2),$$

is a positive and rapidly decreasing function. In our specific model,

$$q^2 t_1(q^2,0) = \frac{2m}{\mu^2}\Gamma(-\alpha)q^2 f_1(q^2)\frac{\Gamma(q^2/2\mu^2)}{\Gamma(q^2/2\mu^2-\alpha)}$$

and crosses over quite soon to negative values at $q^2=2\alpha\mu^2$, before the damping from the form factor has set in. Finally, we need not emphasize again that the calculation presented here is specifically model-dependent and serves only as an illustration.

ACKNOWLEDGMENTS

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Duality and the Pomeranchuk Singularity*

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The conjecture on the role played by the Pomeranchuk singularity in finite-energy sum-rule (FESR) calculations and within the duality framework is reviewed and subjected to various experimental tests. It is assumed that in the FESR sense the Pomeranchon is built from nonresonating background contributions, while all other trajectories are constructed from s-channel resonances. Previous results based on this conjecture are reviewed first. A detailed model for πN elastic scattering is then compared with experiment. All $I=1$ t-channel amplitudes for πN scattering are entirely accounted for by the N^*-resonance contributions, while the $I=0$ t-channel amplitudes require significant nonresonating background. This background is predominantly imaginary, and is presumably associated with the Pomeranchon-exchange term. The residue functions of the P and P' trajectories are calculated, using FESR and assuming our conjecture. The calculated functions are then used to predict high-energy differential cross sections and polarizations for πN scattering, in reasonable agreement with experiment. The P' trajectory seems to favor the Gell-Mann ghost-eliminating mechanism both in πN and in KN elastic scattering. Inelastic processes such as $K^+n\to K^0p$, $KN\to K\Delta$, and $KN\to K^*N$ are predicted to have purely real amplitudes at large s and small t. Various phenomenological models are shown to be consistent with this prediction. The paper concludes with a few remarks concerning various properties of the Pomeranchon, the connection of the model with multiparticle production and to photon initiated reactions, and the (only) failure of the model in baryon-antibaryon scattering.

I. INTRODUCTION

IN the absence of a better understanding of strong-interaction dynamics, it is customarily assumed that, in many cases, direct-channel resonances dominate the strong-scattering amplitudes over a wide energy range. This working hypothesis has gained new popularity in the last year or two. Empirically, the ever-increasing number of experimentally identified hadronic states[1] seems to support this idea. Theoretically, it is now believed that the duality between direct-channel resonances and crossed-channel exchanges[2] allows the resonance-dominance assumption to coexist with the usual exchange mechanisms or even to replace them in some cases. In practice, many model calculations actually

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[1] See, e.g., the rapporteur talks of B. French, A. Donnachie, R. D. Tripp, and H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).

assume that most t-channel Regge trajectories are entirely built from s-channel resonances either in the local sense or in the average sense of finite-energy sum rules (FESR).

The Pomeranchuk singularity appears to be an exception. Its crossed-channel contribution is presumably built from some nonresonating background in the direct channel.[3] The conjecture that the Pomeranchon is associated with the nonresonating part of the amplitude, as well as the resonance-dominance assumption for all other t-channel trajectories, has led to many interesting results in the past year. Our main purpose in this paper is to review some of these results and to present a detailed analysis of various new experimental tests of the conjecture. Our attention is particularly focused on two questions:

(i) To what extent can we really assume that amplitudes to which the Pomeranchon does not contribute can be completely described in terms of direct-channel resonances?

(ii) How well does the "Pomeranchon + resonances" model work, and what can we learn from it about the nature of the Pomeranchuk singularity?

The paper is organized as follows. In Sec. II we review previous work related to the resonance-dominance assumption for ordinary trajectories and to the conjecture on the special role played by the Pomeranchon. In Sec. III we outline the main qualitative features of an analysis of the πN elastic scattering amplitude. We show that, below 2 GeV, the $I=1$ t-channel πN amplitude is fully accounted for by s-channel resonances, while the $I=0$ t-channel amplitude requires significant background contributions. Section IV includes a detailed quantitative study of a model for the πN amplitude in which the extrapolated Pomeranchon contribution is added to the s-channel resonances. The properties of the P and P' trajectories are investigated. Section V deals with KN scattering, using a similar point of view. In Sec. VI we discuss the status of various processes which are predicted by our model to possess purely real amplitudes. Finally, in Sec. VII we make a few remarks on the nature of the Pomeranchuk singularity and mention several additional possible applications of the model.

The reader who is not particularly interested in the details of our analysis is advised to focus his attention on Sec. III and Figs. 1–3, in which our most important new qualitative results are presented.

II. REVIEW OF PREVIOUS RESULTS

A. Assumptions

We consider strong-interaction scattering amplitudes with two particles in the final state. We assume that, in general, such an amplitude consists of two parts[3]:

(i) The contribution of the crossed-channel Pomeranchuk singularity. We propose that this term is not directly related to resonances in any channel and that from the direct-channel point of view it can be considered as an isospin-independent nonresonating background.

(ii) The contributions of all the "ordinary" trajectories in the crossed channel. These can be alternatively described as the contributions of all resonances in the direct channel.

The immediate consequences of this assumption are the following:

(a) Amplitudes to which the Pomeranchon cannot contribute (charge or strangeness exchange, etc.) are entirely accounted for by direct-channel resonances.

(b) Channels in which no resonances are observed (K^+p, $\pi^+\pi^+$, $K^+\pi^+$, etc.) are completely described by the Pomeranchon-exchange part of the amplitude.

(c) In general, the Pomeranchon contribution together with the direct-channel resonances should give the entire amplitude.

The description of a given scattering amplitude in a given channel in terms of resonances only, or in terms of the Pomeranchon alone, is supposed to hold locally only for the imaginary part of the amplitude. The main effect of a resonant state on the real part of the amplitude is not felt at the resonance energy. On the contrary, the real part of the amplitude at a given positive s value may have important contributions from distant resonances such as those having negative s (or positive u). Thus, for example, the absence of resonances in elastic K^+p scattering leads us to believe that the Pomeranchon controls the imaginary part of the amplitude, while the real part of the K^+p amplitude does not necessarily vanish and may have contributions from distant K^-p resonances.

B. Processes in which P Exchange Is Forbidden

Inelastic processes such as $\pi\pi \to \pi\omega$, $\pi\eta \to \pi\rho$, and $\pi\pi \to \pi A_2$ do not allow Pomeranchon-exchange contributions. They should therefore be completely dominated by s-channel resonances. An extensive analysis of such reactions was carried out using FESR and assuming resonance dominance in the s channel and the exchange of a few "ordinary" Regge trajectories in the t channel. Many successful results were obtained from this bootstrap-type analysis without any indications that background contributions were needed.[4]

The Veneziano formula,[5] first proposed for $\pi\pi \to \pi\omega$, demonstrates the consistency between the resonance-dominance assumptions in all channels.

[2] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968); C. Schmid, Phys. Rev. Letters **20**, 689 (1968).

[3] H. Harari, Phys. Rev. Letters **20**, 1395 (1968).

[4] M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. Virasoro, Phys. Rev. Letters **19**, 1402 (1967); Phys. Letters **27B**, 99 (1968); Phys. Rev. **176**, 1904 (1968).

[5] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

The original Dolen-Horn-Schmid analysis[2] of the πN charge-exchange FESR indicates that the sum of s-channel resonance contributions is a good approximation of the full charge-exchange amplitude. We will return to this process in Secs. III and IV.

C. Elastic Processes: Pomeranchon Contributes but Is Ignored in Calculation

The $\pi\pi \to \pi\pi$ FESR-bootstrap calculations of Schmid[6] and of Freund,[7] as well as the $SU(3)$ calculation of Schmid and Yellin,[8] indicate clearly that the resonance approximation in the direct channel accounts successfully for the leading ordinary trajectories, while the Pomeranchon's contribution cannot be reproduced without adding extra terms.

Recent applications of the Veneziano formula to $\pi\pi \to \pi\pi$ [9] and $\pi N \to \pi N$ [10] lead to consistent descriptions of elastic processes in terms of s-channel resonances or ordinary t-channel trajectories *only* when the Pomeranchuk singularity is ignored. It is not possible to construct Veneziano-type combinations of s-channel resonances which will reproduce the t-channel properties of the Pomeranchon without introducing exotic mesons.

D. Elastic Scattering When No s-Channel Resonances Are Observed

The imaginary part of the amplitude of elastic processes in which no s-channel resonances are observed is predicted to be entirely given by the Pomeranchon contribution. The total cross sections for such channels should remain constant in energy.[3] This is successfully demonstrated in the cases of K^+p, K^+n, pp, and pn scattering, in contrast with the behavior of the K^-p, K^-n, $\bar{p}p$, $\bar{p}n$, π^+p, and π^-p total cross sections in which many s-channel resonances are observed and the energy dependence at high energy is not flat. Our model also predicts that energy-dependent total cross sections should always decrease towards their asymptotic values,[3] in accord with the experimental situation.

The predicted constancy[3] of $\pi^+\pi^+$ and π^+K^+ cross sections leads to relations among the couplings of the vector and tensor meson trajectories. It also predicts the degeneracy among the ρ, ω, A_2, and P' trajectories.

E. Processes in which No s-Channel Resonances Are Observed and P Exchange Is Forbidden

In such processes the imaginary part of the amplitude should vanish.[3] This should be the case in $K^+n \to K^0p$, $pn \to np$, $KN \to K\Delta$, $KN \to K^*N$, $KN \to K^*\Delta$, $pp \to p\Delta$, etc. Various methods of comparing these predictions with experiment are discussed in Sec. VI. Here we only state that no disagreement was found.

F. Self-Consistency of Sets of Reactions

Assuming that except for the Pomeranchon's contribution, all amplitudes are dominated by $SU(3)$ singlets and octets in all mesonic channels and by singlets, octets, and decuplets in all baryonic channels, self-consistent solutions have been obtained for the meson-meson, meson-baryon, and baryon-baryon scattering amplitudes.[3,11,12] Among the most interesting features of these solutions we find the prediction of the usual ω-ϕ and f^0-f^* $SU(3)$ mixing angles.[11,12] The various aspects of this type of approach have recently been thoroughly discussed by various authors and we refer the reader to these papers.[13,14]

Rosner has shown[12] that in baryon-antibaryon scattering our model leads to an inconsistency. He proposed that exotic mesons exist, which couple to $B\bar{B}$ but not to two mesons. This could be one way of removing the discrepancy. Another alternative would be to consider $B\bar{B}$ annihilation into two mesons on the same footing as s-channel resonances.[14] In our opinion both possibilities are not very attractive and we consider this problem to be a serious difficulty of the model.

Schwimmer has shown[15] that if one assumes that all nonstrange, natural-parity mesons have $C=P$ (as predicted, for example, by the quark model), our model leads to a determination of the usual $SU(3)$ mixing angle for ω-ϕ and f^0-f^*, to a degeneracy between the ρ, ω, A_2, and f^0 trajectories, and to the decoupling of the f^* from the $\pi\pi$ system. He did *not* assume the absence of exotic states.

III. QUALITATIVE ANALYSIS OF πN ELASTIC SCATTERING

The existence of a detailed partial-wave analysis for πN elastic scattering below 2 BeV led us to select this process as an appropriate test case for our conjecture. A quantitative analysis of this amplitude in terms of our model must depend on details which are outside the framework of the model itself. These details include the specific choice of shape of the resonance curves, the precise values of the parameters of the known πN resonances, the parametrization of the Pomeranchon amplitude, and the nature of the Pomeranchuk singularity itself. We therefore propose to discuss in this section the qualitative aspects of our analysis which do not depend on these details, and to postpone to Sec. IV a detailed numerical comparison of our model with the experimental πN amplitude.

[6] C. Schmid, Phys. Rev. Letters **20**, 628 (1968).
[7] P. G. O. Freund, Phys. Rev. Letters **20**, 235 (1968).
[8] C. Schmid and J. Yellin, Phys. Letters **27B**, 19 (1968).
[9] C. Lovelace, Phys. Letters **28B**, 264 (1968); J. Shapiro and J. Yellin (unpublished); J. A. Shapiro, Phys. Rev. **179**, 1345 (1969).
[10] M. Virasoro, Phys. Rev. **184**, 1621 (1969); K. Igi, Phys. Letters **28B**, 330 (1968).
[11] C. B. Chiu and J. Finkelstein, Phys. Letters **27B**, 510 (1968).
[12] J. L. Rosner, Phys. Rev. Letters **21**, 950 (1968).
[13] M. Kugler, Phys. Rev. **180**, 1538 (1969).
[14] H. J. Lipkin, Nucl. Phys. **B9**, 349 (1969).
[15] A. Schwimmer, Phys. Rev. **184**, 1508 (1969).

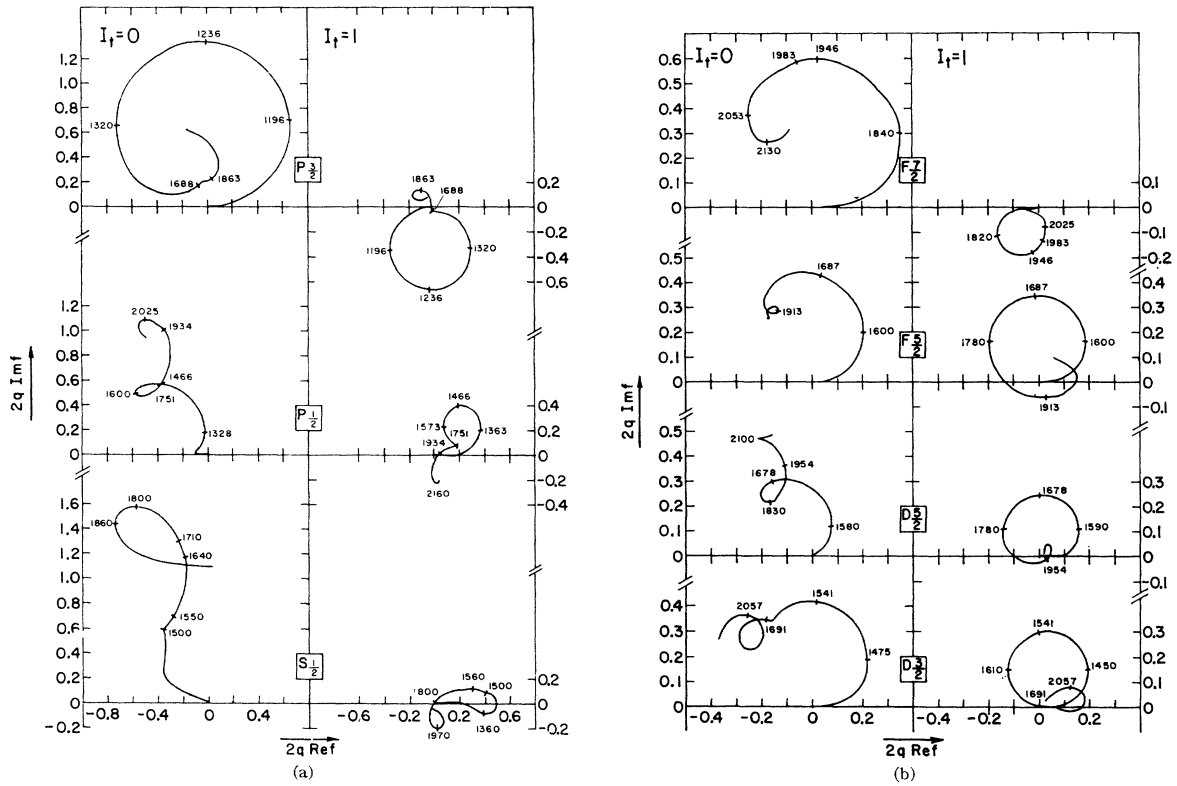

FIG. 1. Argand plots for the s-channel partial-wave amplitudes having definite t-channel isospins. The $I=1$ t-channel combinations are expected to be entirely given by s-channel resonances. Clear closed circles are observed. The $I=0$ amplitudes are predicted to include strong-background contributions which are mostly imaginary. The circles do not close and they are superimposed on the predominantly imaginary background.

The main qualitative predictions of our model for the πN amplitude are the following:

(i) The difference between the elastic $\pi^- p$ and $\pi^+ p$ amplitudes represents a pure $I=1$ amplitude in the t channel. The Pomeranchon does not contribute to it. We therefore predict that this combination of amplitudes is entirely given by s-channel πN resonant states. In other words, if we extract the s-channel partial-wave amplitudes having $I=1$ in the t channel, we should find that a simple sum of resonances accounts for the entire amplitude in every partial wave.

(ii) The $I=0$ t-channel combination of elastic πp amplitudes should not be accounted for by s-channel resonances. What we expect here is a smooth background term, predominantly imaginary, which represents the Pomeranchon term and on which the s-channel resonances are superimposed. This should be the case for every s-channel partial-wave amplitude having $I=0$ in the t channel.

In order to extract the explicit form of the s-channel partial-wave amplitudes having definite t-channel isospins, we have taken the s-channel amplitudes $f_{l\pm}^{1/2}$ and $f_{l\pm}^{3/2}$ obtained from the phase-shift analysis of Donnachie, Kirsopp, and Lovelace[16] (DKL) and constructed the combinations[17]

$$f_{l\pm}^{0}=\tfrac{1}{3}(f_{l\pm}^{1/2}+2f_{l\pm}^{3/2}), \tag{1}$$

$$f_{l\pm}^{1}=\tfrac{1}{3}(f_{l\pm}^{1/2}-f_{l\pm}^{3/2}). \tag{2}$$

In Fig. 1 we present Argand plots for the real and imaginary parts of the amplitudes $f_{l\pm}^{0}$ and $f_{l\pm}^{1}$. The seven lowest partial waves are shown. The $I=1$ amplitudes are represented by "clean" closed circles with very little background.[18] The $I=0$ amplitudes also show

[16] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968). We thank R. Aviv of Tel-Aviv University for supplying us with the numerical values of these phase shifts, which he received from Donnachie *et al.*

[17] Our notations, conventions, and explicit kinematic relations for πN scattering are as follows. The matrix element is $M=\bar{u}(p_2)[A(\nu,t)-i\gamma\cdot QB(\nu,t)]u(p_1)$, where p_1, p_2 are the nucleon momenta, q_1, q_2 are the pion momenta, $Q=\frac{1}{2}(q_1+q_2)$, $\nu=(p_1\cdot q_1)/M$, M is the nucleon mass, $W=\sqrt{s}$ is the total c.m. energy, E is the nucleon c.m. energy,

$$A=4\pi[(W+M)(E-M)f_1-(W-M)(E+M)f_2]/(E^2-M^2),$$
$$B=4\pi[(E-M)f_1+(E+M)f_2]/(E^2-M^2),$$
$$f_1=\sum_l [f_{l+}P_{l+1}'(X)-f_{l-}P_{l-1}'(X)], \quad f_2=\sum_l (f_{l+}-f_{l-})P_l'(X),$$
$$X=\cos\theta, \quad f_{l\pm}=(\eta_{l\pm}e^{2i\delta_{l\pm}}-1)/2iq, \quad A'=A+[\nu/(1-t/4M^2)]B,$$
$$\nu=E_{\rm lab}+t/4M, \quad k\sigma_{\rm tot}(\nu)=\mathrm{Im}A'(\nu,0),$$

and k is the pion lab momentum.

[18] The S-wave amplitude is the only one in which the $I=1$ circle is both distorted and shifted from the center. This is true not only for the DKL phase shifts (Ref. 16) but also for the other sets of phase shifts that we have tried (see Sec. IV A).

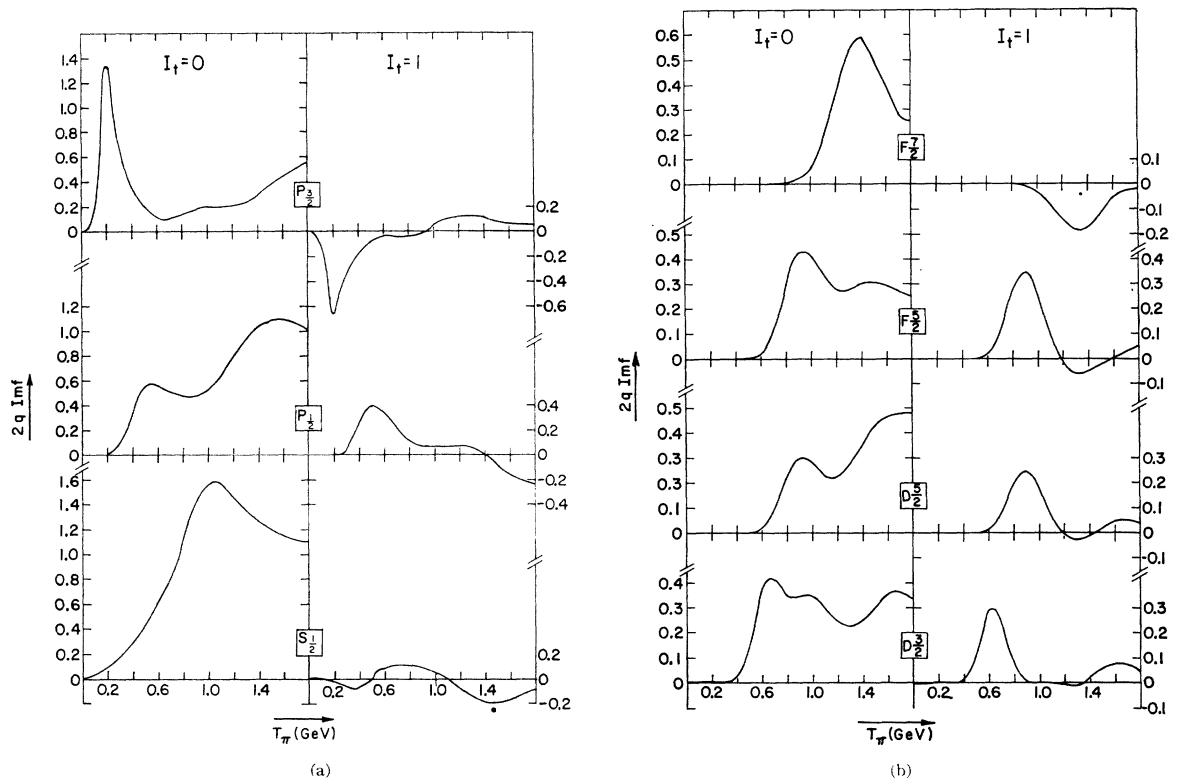

FIG. 2. Energy dependence of the imaginary part of the s-channel partial-wave amplitudes having definite t-channel isospins. The $I=1$ combinations do not indicate any significant background, while the $I=0$ amplitudes include substantial background contributions.

circles but these are accompanied by a predominantly imaginary nonresonating background.

The success of the pure-resonance description for $f_{l\pm}{}^1$ and the existence of a significant smooth imaginary background in all $f_{l\pm}{}^0$ amplitudes is even better demonstrated when we plot the imaginary parts of these amplitudes as a function of energy (Fig. 2). The contrast between the $I=0$ and $I=1$ t-channel combinations is striking, in excellent agreement with our qualitative predictions.

Another way of demonstrating these features of the $I=0$ and $I=1$ t-channel combinations is shown in Fig. 3. We use the same set of partial-wave amplitudes and present the amplitudes $A'^{(\pm)}$ and $B^{(\pm)}$ as a function of energy for various t values. The $I=0$ amplitudes $A'^{(+)}$ (and to a certain extent also $B^{(+)}$) show clear indications of nontrivial background contributions. $A'^{(-)}$ and $B^{(-)}$ are the $I=1$ t-channel combinations. They are completely dominated by resonances.

The details of our method of computing the total contribution of resonances will be discussed in Sec. IV, but none of the important features of Fig. 3 depends on these details.

IV. DETAILED MODEL FOR πN SCATTERING

Encouraged by the striking qualitative success of our model, we now proceed to discuss its quantitative details. After discussing our specific choice of resonance parameters, resonance shapes, and the Pomeranchon parameters, we test the assumption that the entire πN scattering amplitude can be described in terms of its s-channel resonances plus the contribution of the Pomeranchuk singularity. We then assume that (in the sense of FESR) the resonances alone build the contribution of the P' trajectory and we compute the parameters of this trajectory.[19] Finally, we consider the nonresonating background and try to reach conclusions concerning the properties of the Pomeranchon itself.

A. Numerical Input of Model

Four numerical ingredients are needed for an explicit formulation of the model:

(*a*) *Parameters of πN resonances.* As a starting point, we use the list of resonances given by DKL,[16] and adopt the numerical values given by these authors for their masses, widths, and elasticities. This information is summarized in Table I. We have chosen this particular partial-wave analysis simply because its details were more readily available to us. In order to test the sensitivity of our analysis to the precise values of the parame-

[19] The analysis of the P' properties as well as many of the other details of this section were previously reported in F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters **21**, 323 (1968). We are indebted to Fred Gilman for many discussions of these points.

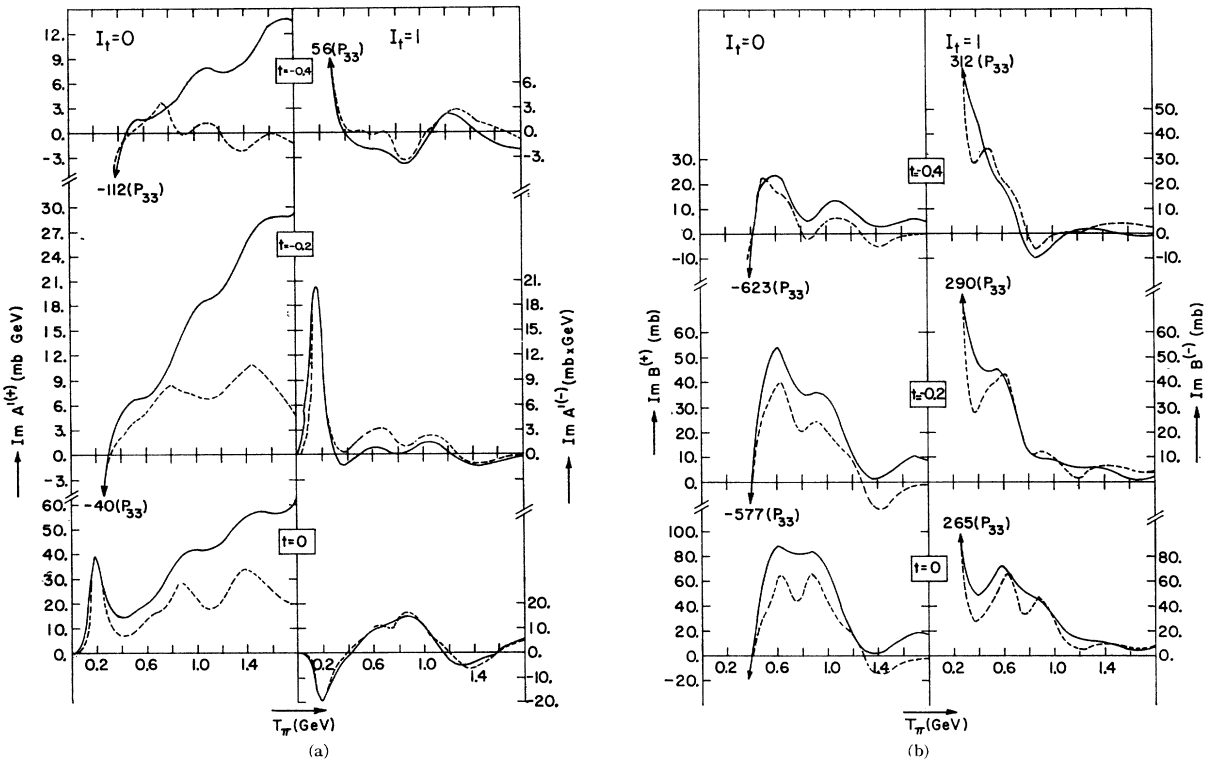

FIG. 3. Energy dependence of the amplitudes $A'(\nu,t)$, $B(\nu,t)$ for $t=0$, -0.2, -0.4 GeV^2. The solid line represents the full amplitudes given by the DKL analysis (Ref. 16). The dashed line is the total contribution of s-channel resonances computed according to the specifications of Sec. IV A. The $A'^{(-)}$ amplitude is well reproduced by the sum of resonances, while $A'^{(+)}$ requires large nonresonating contributions. The distinction is less significant for the $B^{(-)}$ and $B^{(+)}$ amplitudes.

ters of DKL we have performed the following checks:

(i) DKL found only one S_{11} resonance. Most other groups, including the 1968 CERN analysis (which is essentially an updated version of the work of DKL), find two S_{11} states: one around 1550 and the other around 1710 MeV.[20] We have reanalyzed the S-wave part of the amplitude assuming that two resonances exist and found no qualitative changes in our conclusions.

(ii) In his recent review,[1] Donnachie has suggested that the $P_{33}(1688)$, $F_{17}(1983)$, $D_{35}(1954)$, and $D_{13}(2057)$ states, as well as another possible $D_{13}(1730)$ state, be considered as being more doubtful than the other states listed in Table I. We have repeated our analysis, omitting these states, and found no significant changes.

(iii) Finally, there are differences between the DKL results and the latest Berkeley,[21] Saclay,[22] and Glasgow[23] phase shifts. We have repeated certain parts of our calculation using the Saclay and Glasgow parameters for various resonances, again with no significant modification of our conclusions.

(*b*) *Shape of resonance curves.* Another numerical factor which affects the computation of the total con-

TABLE I. Resonance parameters used in the calculations of πN amplitudes. The parameters are those of DKL (Ref. 16). For a discussion of the sensitivity of our results to the specific parameters which are chosen see Sec. IV A(a). Needless to say, we do not believe that the resonance parameters are known to the accuracy implied by the numbers in the table, which were copied from DKL.

Wave	Mass (MeV)	Width (MeV)	Elasticity
P_{33}	1235.8	125.1	1
P_{11}	1466	211	0.658
D_{13}	1541	149	0.509
S_{11}	1591	268	0.696
S_{31}	1635	177	0.284
D_{15}	1678	173	0.391
F_{15}	1687	177	0.560
P_{33}	1688	281	0.098
D_{33}	1691	269	0.137
P_{11}	1751	327	0.320
P_{13}	1863	296	0.207
F_{35}	1913	350	0.163
P_{31}	1934	339	0.229
F_{37}	1946	221	0.386
D_{35}	1954	311	0.154
F_{17}	1983	225	0.128
D_{13}	2057	293	0.260
G_{17}	2265	298	0.349

[20] See, e.g., A. Donnachie (Ref. 1).

[21] C. H. Johnson *et al.*, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley–Interscience, Inc., New York, 1968); University of California Lawrence Radiation Laboratory Report No. UCRL-18001, 1967 (unpublished).

[22] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968).

[23] A. T. Davies and R. G. Moorhouse, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).

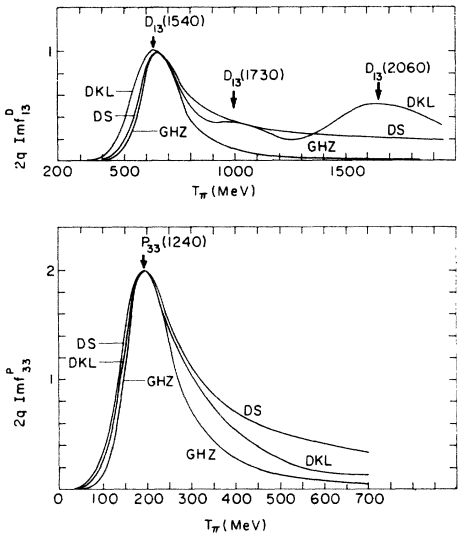

FIG. 4. Comparison of various resonance formulas for the P_{33} and D_{13} partial-wave amplitudes in πN scattering. The curve marked DKL is the full amplitude (including background and all resonances) obtained from Ref. 16. The curve marked GHZ represents the contribution of the $P_{33}(1240)$ and $D_{13}(1540)$ states as computed in Ref. 19. It consists of an uncorrected Breit-Wigner form for $E>E_R$ and a simple $(q/q_R)^{2l+1}$ correction for $E<E_R$. The curve marked DS represents the contribution of the same two resonances according to Ref. 24. In certain regions the DS curve overestimates the full amplitude even if higher resonances and background terms are ignored.

tribution of the s-channel resonances is the precise shape assumed for the resonance curves. There are many versions of correcting the simple Breit-Wigner form for threshold factors. We are not aware of any solid theoretical justification for preferring any one of these parametrizations. On the low-energy side ("left shoulder") of a resonance, one should certainly introduce a threshold factor which guarantees the proper threshold behavior of the partial-wave amplitude. The leading momentum dependence at threshold must be proportional to q^{2l+1}, where q is the c.m. momentum of the πN system. Whether we simply multiply Γ by $(q/q_R)^{2l+1}$ or introduce a more sophisticated (but not necessarily more convincing) parametrization, the results are almost the same for $E<E_R$. The only place where sophisticated parametrizations are significantly superior over the simplest q^{2l+1} factor is the left shoulder of the $P_{33}(1236)$ state. However, in this region we certainly expect very little or no background, and for the purpose of our discussion we believe that it is useless to try to find the best fit for this particular region ($T_\pi<0.2$ GeV). A much more serious problem arises with respect to the high-energy side ("right shoulder") of the resonance curves. The usual versions of introducing threshold factors damp the resonant amplitude for $E<E_R$ but enhance it for $E>E_R$. This enhancement has no physical meaning and no reasonable justification, especially for $E\gg E_R$. It is clear that threshold effects do not and should not control the resonance curve for $E\gg E_R$. Consequently, we believe that any modification of the Breit-Wigner form at such energies, which is based on threshold effects, is intrinsically meaningless.[24] It is extremely hard to find a "correct" way of parametrizing the high-energy side of a resonance curve, particularly in view of the impossibility of separating the distant high-energy contribution of a given resonance from other physical effects at the same energy (such as background contributions or the presence of other resonances in the same partial-wave amplitude). We cannot even compare a specific choice of the resonance formula with the full partial-wave amplitude, since any energy-dependent phase-shift analysis that we might use for comparison actually assumes, explicitly or implicitly, a specific resonance form as a basis for its best fit. We would then be fitting somebody's favorite resonance formula rather than experimental data.

In the absence of a better choice we have therefore decided to use the uncorrected Breit-Wigner formula for $E>E_R$, and the simplest threshold correction to the width, $(q/q_R)^{2l+1}$, for $E<E_R$.

We have checked, however, the possible effects of replacing our choice by other resonance shapes. None of our conclusions would have been modified had we replaced our simple threshold factor for $E<E_R$ by other threshold corrections—such as $(q/q_R)^{2l+1}D_l(q_R\cdot R)/D_l(q\cdot R)$, where $D_l(x)$ is the polynomial of degree $2l$ used in Ref. 24, or $(q/q_R)^{2l+1}\{[(q_R\cdot R)^2+1]/[(q\cdot R)^2+1]\}^l$, as used in Ref. 23.

We found, however, that had we extended the domain of applying the threshold factors for a given resonance to all energies above E_R, significant quantitative

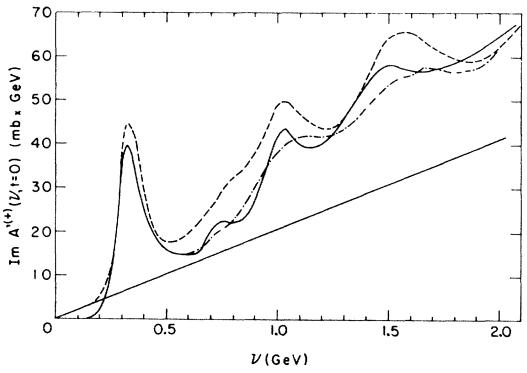

FIG. 5. Comparison of Pomeranchon + resonances model with experimental πN scattering data at $t=0$. Solid lines represent the extrapolated Pomeranchon contribution $[\sigma_\infty(\pi N)=21$ mb] and the experimental values for $\mathrm{Im}A'^{(+)}(\nu,0)$ deduced from the πN total cross sections listed by Höhler *et al.*, Z. Physik **180**, 430 (1964). The dashed-dotted line is $\mathrm{Im}A'^{(+)}(\nu,0)$ as computed from the DKL phase shifts (Ref. 16) and the dashed line is our prediction, based on the parameters mentioned in Sec. IV A.

[24] In this category we include the parametrization used by D. R. Dance and G. Shaw [Phys. Letters **28B**, 182 (1968)], who applied their threshold correction to all values of E, including the $E\gg E_R$ region. Figure 4 demonstrates that such a procedure is likely to lead to a gross overestimate of the resonance contribution, as was indeed the case in the work of Dance and Shaw.

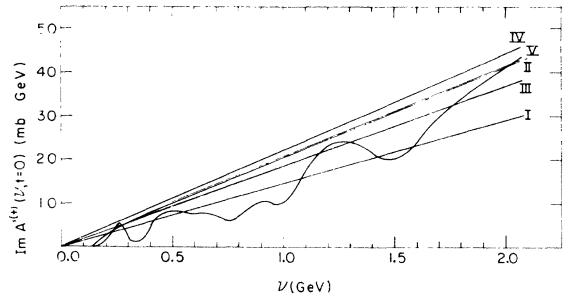

FIG. 6. Comparison of various extrapolations of the Pomeranchon contribution with the background contribution to $\mathrm{Im}A'^{(+)}(\nu,0)$. Curves I, II, III, and IV describe the Pomeranchon term according to the $\sigma_\infty(\pi N)$ values of Refs. 27, 25, 28, and 29, respectively. Curve V is obtained by subtracting the DKL resonances (Ref. 16) from the values of $\mathrm{Im}A'^{(+)}(\nu,0)$ computed from the DKL phase shifts. The bumps in curve V presumably give a good measure of our helplessness in finding the "correct" resonance shapes.

changes in our sum of resonances would occur, and the background term (defined as the difference between the full amplitude and the sum of resonances) would become much smaller. We believe, however, that this procedure (of using the threshold correction for $E\gg E_R$) is totally unjustified and we do not see any reason to adopt it. Figure 4, in which we have plotted various resonance formulas for the P_{33} and D_{13} partial-wave amplitudes, illustrates this point. The application of threshold corrections of the $P_{33}(1236)$ state to $E\gg E_R$ leads to a resonance contribution (at $T_\pi=700$ MeV) which is larger than the full amplitude by a factor of 2.5. The $\Delta(1236)$ contribution at the same energy, as computed from the uncorrected Breit-Wigner curve, gives 50% of the full amplitude. This is a reasonable number in view of the existence of a possible second P_{33} state a few hundred MeV later and the probable presence of some background contribution. A similar pattern is observed in the D_{13} amplitude. The contribution of the $D_{13}(1540)$ state computed by the expression of Ref. 24 exceeds the full amplitude in the region of the possible 1730-MeV D_{13} state, without allowing for any contribution of this state or any background.

(*c*) *Nature of Pomeranchuk singularity.* The simplest assumption that we could have made for the Pomeranchuk singularity would be to assume that it is a simple pole. The possibility of associating a more complicated singularity with the Pomeranchon would lead to nontrivial changes in some of our numerical results, since we are extrapolating the Pomeranchon's contribution to extremely low energies, and even logarithmic factors may become important. In the analysis presented in this section we assume a simple pole behavior.

(*d*) *Parameters of Pomeranchon.* We assume that $\alpha_P(0)=1$ and that the slope $\alpha_P'(t)$ is smaller or equal to 0.5 BeV^{-2}. We find that our results are relatively insensitive to variations of $\alpha_P'(t)$ between 0 and 0.5 BeV^{-2}. For $\beta_P(0)$ we have chosen the value given by Barger and Phillips,[25] corresponding to $\sigma_\infty(\pi N)=21$ mb. We have not applied any threshold corrections to the Pomeranchon's contribution simply because we are not aware of any unique procedure of introducing such effects. Replacing ν by $(\nu^2-\nu_{\mathrm{thresh}}^2)^{1/2}$ would not change our results significantly. Note that the general effect of the threshold factors would be to decrease the Pomeranchon's contribution at low energies. We return to these questions in Sec. VII.

B. Pomeranchon + Resonances Model

Using the input discussed above, we now proceed to assume that, at all energies, the πN amplitude is given by a sum of two terms: the Pomeranchon contribution and the combined contribution of all s-channel resonances.[26] The comparison of this model with the πN data at $t=0$ and $\nu<2$ GeV is shown in Fig. 5. The agreement is satisfactory, although a lower value for the parameter $\sigma_\infty(\pi N)$ would certainly yield a better fit.

Another way of testing the model would be to compare the $t=0$ background amplitude with the extrapolated Pomeranchon contribution. This is done in Fig. 6,[27–29] and we see that the background amplitude actually corresponds to a Pomeranchon term characterized by $\sigma_\infty(\pi N)\sim 15$ mb rather than the more popular value of 21 mb.

Finally, we may compare the total contributions of all s-channel resonances to $\nu\ \mathrm{Im}A'^{(+)}(\nu,0)$ with the extrapolated P'-exchange term. We plot $\nu\ \mathrm{Im}A'^{(+)}$ rather than $\mathrm{Im}A'^{(+)}$ itself, since the first function is the one which

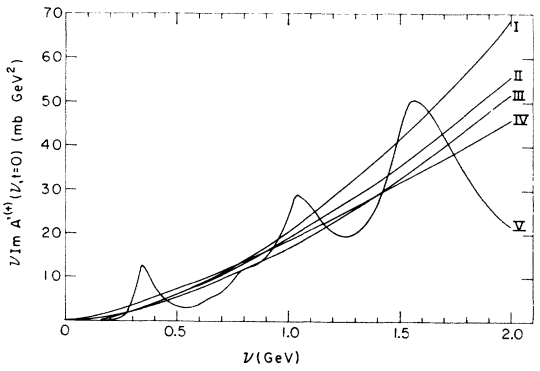

FIG. 7. Comparison of various extrapolations of the P'-exchange contribution with the resonance contribution to $\mathrm{Im}A'^{(+)}(\nu,0)$. Curves I, II, III, and IV show the P' term according to Refs. 27, 25, 28, and 29, respectively. Curve V is obtained by summing all DKL resonances (Ref. 16) with the resonance shape described in Sec. IV A.

[25] V. Barger and R. J. N. Phillips, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).

[26] Calculations based on this general philosophy were previously carried out by several authors. See, e.g., F. N. Dikmen, Phys. Rev. Letters **22**, 624 (1969); T. Lasinski, R. Levi-Setti, and E. Predazzi, Phys. Rev. **179**, 1426 (1969).

[27] W. Rarita *et al.*, Phys. Rev. **165**, 1615 (1968).

[28] C. B. Chiu *et al.*, Phys. Rev. **161**, 1563 (1967).

[29] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967).

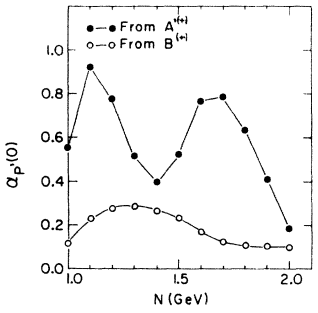

FIG. 8. $\alpha_{P'}(0)$ determined from Eqs. (5)–(8) as a function of the cutoff value of N.

should be averaged by the P' contribution, according to the appropriate FESR. The comparison (Fig. 7) indicates reasonable agreement with various high-energy parametrizations of the P' term.

C. Properties of P' Trajectory

Our next test of the model will be to use FESR and the resonance-dominance approximation in order to calculate the parameters $\alpha(t)$, $\beta^{A'}(t)$, and $\beta^{B}(t)$ of the P' trajectory.[19] The P' contribution to the amplitudes A' and B is written in the form

$$A_{P'}(\nu,t)=-\beta_{P'}{}^{A'}(t)\frac{1+e^{-i\pi\alpha_{P'}(t)}}{\sin\pi\alpha_{P'}(t)}\nu^{\alpha_{P'}(t)}, \tag{3}$$

$$B_{P'}(\nu,t)=-\beta_{P'}{}^{B}(t)\alpha_{P'}(t)\frac{1+e^{-i\pi\alpha_{P'}(t)}}{\sin\pi\alpha_{P'}(t)}\nu^{\alpha_{P'}(t)-1}. \tag{4}$$

The relevant FESR are

$$S_{2n+1}{}^{P'}(t)\equiv\frac{1}{N^{2n}}\int_0^N \nu^{2n+1}\,\mathrm{Im}A'_{\mathrm{RES}}{}^{(+)}(\nu,t)d\nu$$
$$=\beta_{P'}{}^{A'}(t)\frac{N^{\alpha_{P'}(t)+2}}{\alpha_{P'}(t)+2n+2}, \tag{5}$$

$$S_{2n}{}^{P'}(t)\equiv\frac{1}{N^{2n}}\int_0^N \nu^{2n}\,\mathrm{Im}B_{\mathrm{RES}}{}^{(+)}(\nu,t)d\nu$$
$$=\beta_{P'}{}^{B}(t)\alpha_{P'}\frac{N^{\alpha_{P'}(t)}}{\alpha_{P'}(t)+2n}. \tag{6}$$

We first try to compute $\alpha_{P'}(0)$. We can do this by using either one of the equations

$$\alpha_{P'}=(2S_1{}^{P'}-4S_3{}^{P'})/(S_3{}^{P'}-S_1{}^{P'}), \tag{7}$$

$$\alpha_{P'}=2S_2{}^{P'}/(S_0{}^{P'}-S_2{}^{P'}). \tag{8}$$

Our results for $\alpha_{P'}(0)$ are fairly sensitive to the choice of cutoff N. In Fig. 8 we display $\alpha_{P'}(0)$ as a function of N, as obtained from Eqs. (7) and (8). Considering the reliability of similar FESR calculations of α (in cases in which no specific models are assumed), the fact that $\alpha_{P'}(0)$ is consistently between 0.1 and 0.9 can be regarded as a satisfactory result. The average value that we obtain for $\alpha_{P'}(0)$ is 0.4, while most high-energy parametrizations oscillate between 0.5 and 0.6. For t values in the vicinity of $t=0$, the results for $\alpha_{P'}(t)$ are similar to the $t=0$ results, but as we approach the region $t\sim-0.5$ BeV^2 the calculation becomes more and more sensitive to the choice of N and no significant conclusions can be drawn.

In order to compute $\beta_{P'}{}^{A'}(t)$ and $\beta_{P'}{}^{B}(t)$, we have therefore constrained $\alpha_{P'}(t)$ to some average value of the

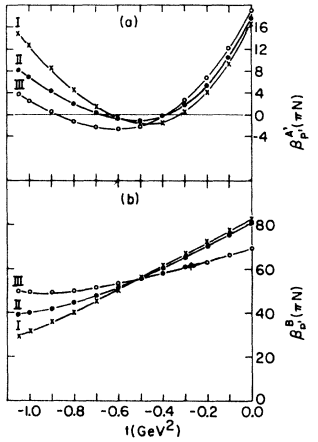

FIG. 9. Predicted t dependence of the P' residue functions for πN amplitudes. $\alpha_{P'}(t)=0.5+t$ is assumed. $N(t)=N(0)+t/4M$. $\beta_{P'}{}^{A,B}$ is defined as in Eqs. (3) and (4). (a) $\beta_{P'}{}^{A}(t)$: I, $N(0)=1$ GeV; II, $N(0)=1.4$ GeV; III, $N(0)=1.8$ GeV. (b) $\beta_{P'}{}^{B}(t)$: I–III, same as in (a).

commonly accepted high-energy parametrizations:

$$\alpha_{P'}(t)=0.5+t. \tag{9}$$

Using Eqs. (5) and (6) for $n=0$, we then obtained the functions shown in Fig. 9.

$\beta_{P'}{}^{A'}$ has two (possibly coinciding) zeros in the region $-1\leq t\leq 0$. Since $\beta_{P'}{}^{A'}(t)$ *has* to vanish at least linearly for $\alpha_{P'}=0$, we are left here with two possibilities: (a) We have a double zero of $\beta_{P'}{}^{A'}$ at $\alpha_{P'}=0$, corresponding to the "no-compensation" ghost-eliminating mechanism.[28] (b) We have one zero at $\alpha_{P'}=0$ and another "dynamical" zero elsewhere (probably around $t=-0.2$ or -0.3). In this case we would have either the Chew[30] or the Gell-Mann[31] mechanism. The accuracy of our analysis of the $A'^{(+)}$ amplitude is certainly not sufficient to distinguish between possibilities (a) and (b). The ambiguity can be resolved, however, by looking at the t behavior of $\beta_{P'}{}^{B}$. The Chew and no-compensation mechanisms demand that $\beta_{P'}{}^{B}$ [as defined in Eq. (4)] vanish for $\alpha_{P'}=0$, while in the Gell-Mann mechanism it does not. Figure 9(b) shows that the Gell-Mann mechanism is definitely favored and $\beta_{P'}{}^{B}(t)$ does not vanish anywhere in the region of interest. We therefore

[30] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).

[31] M. Gell-Mann, in *Proceedings of the Eleventh International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962).

propose that the P' trajectory actually chooses to follow the Gell-Mann mechanism with an extra zero in the $\beta_{P'}{}^{A'}(t)$ residue function.[32] It is interesting to add that the same mechanism is also favored for the A_2 trajectory in processes such as $\pi N \to \eta N$,[33] $KN \to K\Delta(1236)$,[34] and $KN \to KN$.[35] At least for $KN \to KN$, FESR predict[35] an extra zero in the A' amplitude. In view of the $SU(3)$ relation between the P' and A_2 trajectories, it would be embarrassing if they followed different ghost-eliminating mechanisms. Our conclusions with respect to P' are in accord with the requirement of a similar behavior for the two trajectories.[36]

D. Properties of Pomeranchon

In order to study the Pomeranchon parameters we repeat the analysis of the previous section, replacing the resonance contributions by the background term. The background (BG) contributions to $A'^{(+)}(\nu,t)$ and $B^{(+)}(\nu,t)$ were obtained by first subtracting the resonance terms from every one of the partial-wave amplitudes of DKL and then constructing the contributions of these amplitudes to A' and B. The calculation of α_P region to $\nu^3\,\mathrm{Im}A'^{(+)}(\nu,0)$ and therefore to S_3. Small ambiguities in the parameters of the resonances in this region are sufficient to change $\alpha_P(0)$ dramatically.

If $\alpha_P(0)=1$, we have $S_3{}^P=0.6S_1{}^P$. An error of $\pm 10\%$ in the calculated value of $S_3{}^P$ would change $\alpha_P(0)$ between 0.3 and 1.9. The ambiguities in the resonance parameters above 1.6 BeV are at least of that order of magnitude. Consequently, we cannot reach any conclusions concerning $\alpha_P(0)$.

Assuming that $\alpha_P(t)=1$ or $\alpha_P(t)=1+0.5t$, we have computed $\beta_P{}^{A'}(t)$ and $\beta_p{}^B(t)$. The results are shown in Fig. 10.

An interesting conclusion can be drawn by plotting $\mathrm{Im}A'_{\mathrm{BG}}{}^{(+)}(\nu,t)$ as a function of t for various values of ν (Fig. 11). A slow shrinkage is observed when ν increases, and at $\nu<2$ GeV the slope parameter for $A'^{(+)}$ is smaller than its high-energy values. This may hint that the slope of the Pomeranchuk trajectory is actually nonvanishing (if it is a simple Regge pole and if our conjecture is assumed to be true).

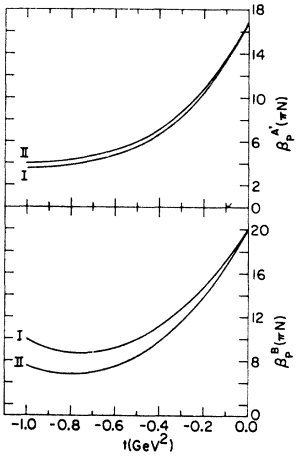

FIG. 10. Predicted t dependence of the Pomeranchon residue functions for πN scattering: I, $\alpha_P(t)=1$; II, $\alpha_P(t)=1+0.5t$.

turns out to be much more sensitive to details than that of $\alpha_{P'}$. Any value between -1 and $+2$ can be obtained for $\alpha_P(0)$, depending on N. The sensitivity follows here from the dominant contribution of the 1.6–1.9-GeV

E. Predictions for High-Energy πN Scattering

In the previous two sections we have obtained predictions for the residue functions of the P and P' trajectories. These were derived from the low-energy data using our conjecture and FESR. We may now take the same residue functions (Figs. 9 and 10), together with similar ones obtained for the ρ trajectory from FESR and low-energy resonances,[37] and predict the cross sections for $\pi^{\pm}p$ elastic scattering at high energies. Our

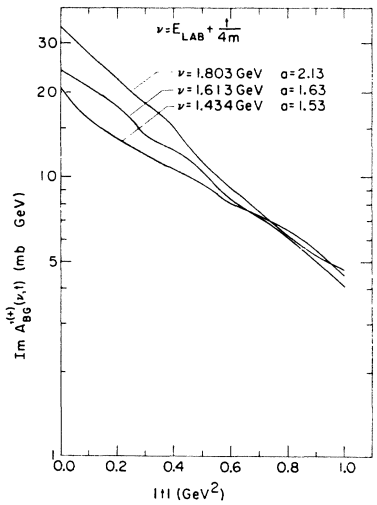

FIG. 11. t dependence of the background contributions to $\mathrm{Im}A'^{(+)}(\nu,t)$ at various values of ν. A slow shrinkage of the slope is observed when ν is increased. All slope values (denoted by a in the figure) below $\nu=2$ GeV are consistently smaller than the corresponding high-energy slopes. If the background is identified with the Pomeranchon, a nonvanishing slope is implied for $\alpha_P(t)$.

[32] The same mechanism is favored by the analysis of Barger and Phillips (Ref. 25), although their "dynamical zero" is at a different value of t.

[33] The absence of a dip in $\pi^-p \to \eta n$ hints that the correct mechanism is that of Gell-Mann.

[34] M. Krammer and U. Maor, Nuovo Cimento **52A**, 308 (1967).

[35] K. Igi and S. Matsuda, Phys. Rev. Letters **19**, 928 (1967).

[36] Several recent papers dealing with high-energy Regge fits have claimed that the no-compensation mechanism is preferred for P', and that a double zero at $\alpha_{P'}=0$ probably exists in $\beta_{P'}{}^A$. Our result is numerically very close to this possibility, since our two different zeros of $\beta_{P'}{}^A$ are not very far apart. It is only $\beta_{P'}{}^B$ (about which we know very little at high energies, experimentally) that tells us to prefer the Gell-Mann mechanism.

[37] The ρ parameters were computed using the recipe of Sec. IV A from the DKL resonances (Ref. 16). We used $\alpha_\rho(t)=0.57+0.96t$, as given by G. Höhler, J. Baack, and G. Eisenbeiss, Phys. Letters **22**, 203 (1966).

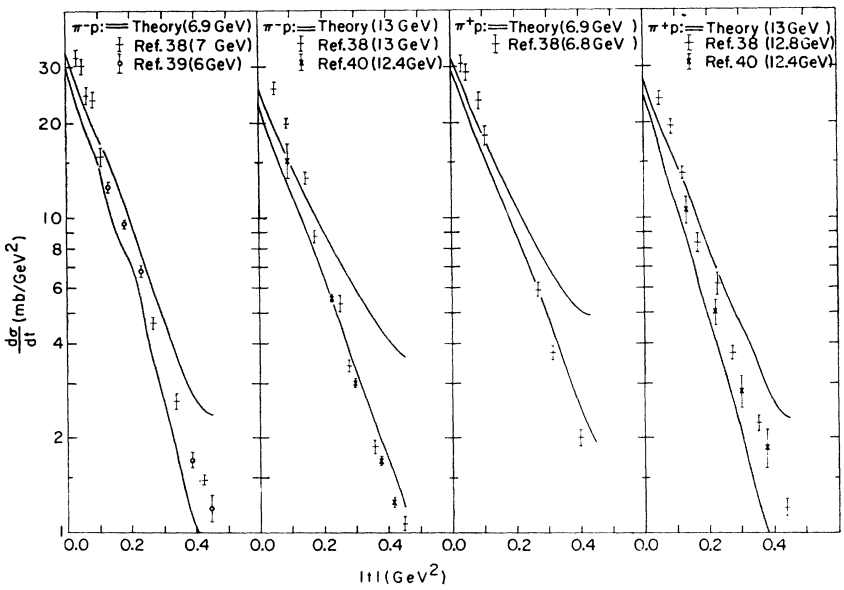

FIG. 12. Comparison with experiment of our predicted angular distributions for π^+p and π^-p elastic scattering. The "input" includes $\alpha_{P'}(t)=0.5+t$, $\alpha_\rho(t)=0.57+0.96t$, $\alpha_P(t)=1+\alpha' t$, the residue functions of Figs. 9 and 10, and similar residue functions obtained for ρ exchange from FESR and the resonance approximation. The two lines in every figure represent the upper and lower limits of our predictions using $0\leq\alpha'\leq 0.5$ GeV^{-2} and all the values of $\bar{N}$ appearing in Fig. 9.

predictions for $d\sigma/dt$ are compared with experiment in Fig. 12.[38–40] The polarization predictions are given in Fig. 13.[41] The agreement is certainly respectable, considering the approximations and extrapolations involved.

V. P' TRAJECTORY IN KN ELASTIC SCATTERING

The next process to which we would like to apply our model is elastic KN scattering. Here the ambiguities in the low-energy side of the FESR are much larger than in the πN case, and the errors encountered in calculating the nonresonating background by subtracting the resonance contributions from the total amplitude are significant. This is caused by the absence of an accurate phase-shift analysis for KN scattering, by the large errors in the low-energy total cross sections, and by our poor knowledge of the unphysical contributions below the KN threshold.

We have therefore concluded that the only meaningful analysis that we can perform here is to predict the properties of the P' trajectory, using the proposed relation between the low-energy resonances and the ordinary trajectories. We consider the imaginary part of the $C=+1$, $I=0$, t-channel amplitudes $A'^{(+)}$ and $B^{(+)}$. The amplitude $A'^{(+)}$ is defined by

$$A'^{(+)}(\nu,t)=\tfrac{1}{4}[A_{K^+p}'(\nu,t)+A_{K^-p}'(\nu,t)+A_{K^+n}'(\nu,t)+A_{K^-n}'(\nu,t)]. \quad (10)$$

A similar expression defines $B^{(+)}$. At $t=0$ we have

$$\mathrm{Im}A'^{(+)}(\nu,0)=\tfrac{1}{4}(\nu^2-m_K^2)^{1/2}[\sigma_t{}^{K^+p}(\nu)+\sigma_t{}^{K^-p}(\nu)+\sigma_t{}^{K^+n}(\nu)+\sigma_t{}^{K^-n}(\nu)]. \quad (11)$$

In order to predict the P' residue functions, we use FESR similar to Eqs. (5) and (6) for $n=0$. The input data on the left-hand side of the FESR include the ΛKN and ΣKN coupling constants, the $Y_0^*(1405)$ and $Y_1^*(1385)$ unphysical-region contributions, and the contributions of all Y_0^* and Y_1^* resonances above the KN threshold. The following ambiguities were taken into account in the calculations:

(i) We have used the coupling constants given by Kim[42] ($g_{\Lambda KN}{}^2=16.0\pm2.5$ and $g_{\Sigma KN}{}^2=0.3\pm0.5$) as well as those of Zovko[43] ($g_{\Lambda KN}{}^2=6.8\pm2.9$ and $g_{\Sigma KN}{}^2=2.1\pm0.9$). Our error bars (Fig. 14) include the effect of the variation of these constants in this range.

(ii) For the $Y_0^*(1405)$ contribution we have used $\varsigma_{Y_0^*KN}/4\pi=0.32$ as given by the analysis of Warnock and Frye.[44] For $Y_1^*(1385)$ we used[44] $g_{Y_1^*KN}{}^2/4\pi=1.9$,

TABLE II. Resonance parameters used in the calculation of KN amplitudes. The parameters are taken from Ref. 45. The elasticities of the $Y_0^*(1670)$, $Y_1^*(1660)$, and $Y_1^*(1700)$ resonances were changed within the limits given in the table.

Wave	Mass (MeV)	Width (MeV)	Elasticity
D_{03}	1520	16	0.45
S_{01}	1670	18	0.1–0.2
D_{03}	1695	40	0.20
F_{05}	1815	75	0.70
D_{05}	1830	80	0.08
G_{07}	2100	140	0.30
D_{13}	1660	50	0.025–0.2
P_{13}?	1700	110	0.05–0.2
D_{15}	1765	100	0.45
F_{15}	1910	60	0.08
F_{17}	2030	120	0.10

[38] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 425 (1963).
[39] C. T. Coffin *et al.*, Phys. Rev. **159**, 1169 (1967).
[40] D. Harting *et al.*, Nuovo Cimento **38**, 60 (1965).
[41] N. Borghini *et al.*, Phys. Letters **24B**, 77 (1967).
[42] J. W. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967).
[43] N. Zovko, Phys. Letters **23**, 143 (1966).
[44] R. L. Warnock and G. Frye, Phys. Rev. **138B**, 947 (1965).

but arbitrarily allowed for a $\pm 25\%$ error in our error bars (Fig. 14).

(iii) Above the KN threshold we used the resonance parameters listed in Table II. These are taken from the tables of the Particle Data Group.[45] We have checked the sensitivity of our results to the detailed resonance parameters by replacing the parameters of Table II by those of Conforto *et al.*[46] The obtained variation is also included in the errors of Fig. 14. Finally, we tried the parameters given by Tripp[1] and found only slight changes in the values of the residue functions. The resonance shape used here is the same as in the previous section [see Sec. IV A(b)].

The t dependence of the P' residue functions $\beta_{P'}{}^{A'}(KN)$ and $\beta_{P'}{}^{B}(KN)$, assuming $\alpha_{P'}(t)=0.5+t$, is given in Fig. 14. We obtain $\beta_{P'}{}^{A'}(0)=4.5\pm 1.0$. The errors reflect the combined effect of the ambiguities discussed above, as well as the dependence upon the cutoff energy $[1.1\leq N(0)\leq 1.9$ BeV$]$. This value for $\beta_{P'}{}^{A'}(0)$ should be compared with typical high-energy values such as $\beta_{P'}{}^{A'}(0)=5.5\pm 1.3$ [47] or 4.5 ± 0.1.[48]

The situation with respect to the ghost-eliminating mechanism chosen by the P' in this case is obscured by the large errors. Still, the results indicate a preference of the Gell-Mann mechanism with an extra zero in $\beta_{P'}{}^{A'}(t)$, consistent with our conclusions in the πN case.

VI. PROCESSES HAVING PURELY REAL AMPLITUDES

Processes having exotic quantum numbers in the s channel and nonvanishing quantum numbers in the t channel are predicted by our model to have purely real amplitudes.[3] The Pomeranchon contribution to the imaginary part of such amplitudes is absent because of the nonvanishing t-channel quantum numbers. The contribution of all other (ordinary) trajectories to the imaginary part is presumably vanishing as a result of the absence of s-channel resonances. The entire imaginary part is therefore predicted to vanish. The real part of the amplitude may not vanish, however, because of the effect of distant (u-channel) resonances.[49] The simplest examples of such processes are the reactions[3]

$$K^+n \leftrightarrow K^0p\,, \tag{12}$$

$$pn \leftrightarrow np\,, \tag{13}$$

$$KN \rightarrow K\Delta\,, \tag{14}$$

$$KN \rightarrow K^*\Delta\,, \tag{15}$$

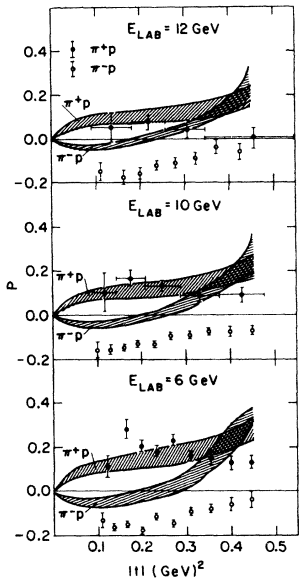

FIG. 13. Comparison with experiment of our predicted polarizations. Theoretical input is the same as in Fig. 12. Data taken from Ref. 41.

$$pp \rightarrow p\Delta\,, \tag{16}$$

$$KN \rightarrow K_N{}^*N\,, \tag{17}$$

where K^* is any $S=1$, $B=0$, $I=\frac{1}{2}$ resonance, Δ is any $S=0$, $B=1$, $I=\frac{3}{2}$ state, and $K_N{}^*$ is a natural-parity $[(-1)^J=P]$ K^* state. The first five reactions have $I=1$ in the t channel and $Y=2$ in the s channel, thus satisfying our criteria. The last reaction has $Y=2$ in the s channel and involves the exchange of unnatural parity at small t, thus preventing the Pomeranchon from contributing.[50]

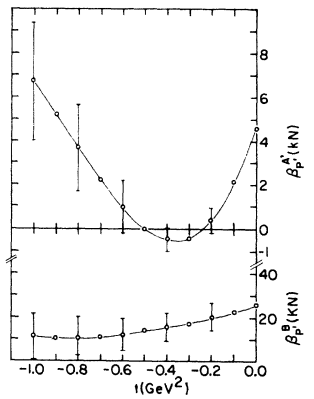

FIG. 14. Predicted t dependence of the P' residue functions for KN amplitudes. $\alpha_{P'}(t)=0.5+t$ is assumed. $N(t)=N(0)+t/4m$. $\beta_{P'}{}^{A',B}$ is defined in Eqs. (5) and (6). (a) $\beta_{P'}{}^{A'}(t)$; (b) $\beta_{P'}{}^{B}(t)$. Error bars indicate the combined effect of ambiguities in the resonance parameters, the coupling constants $g_{\Lambda KN}$, $g_{\Sigma KN}$, $g_{Y_0{}^*KN}$, and $g_{Y_1{}^*KN}$, and the variation of $N(0)$ in the region $1.1\leq N(0)\leq 1.9$ GeV.

[45] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Matts Roos, A. H. Rosenfeld, Paul Söding, and C. G. Wohl, Rev. Mod. Phys. **41**, 109 (1969).

[46] B. Conforto *et al.*, Nucl. Phys. **B8**, 265 (1969).

[47] R. J. N. Phillips and W. Rarita, Phys. Rev. **139B**, 1336 (1965).

[48] G. V. Dass, C. Michael, and R. J. N. Phillips, Nucl. Phys. **B9**, 549 (1969).

[49] See also our remarks in Sec. II A.

[50] Empirically the Pomeranchon does not seem to connect natural-parity states to unnatural-parity states.

Our model predicts that all of these processes possess purely real amplitudes. In the case of $K^+n \to K^0p$ and $pn \to np$ this can be tested, and the agreement with experiment is good. In order to see this we consider the isospin relation

$$\mathrm{Im}(K^+n \to K^0p) = \mathrm{Im}(K^+p \to K^+p) - \mathrm{Im}(K^+n \to K^+n), \quad (18)$$

which, via the optical theorem, gives at $t=0$

$$\mathrm{Im}(K^+n \to K^0p) \propto \sigma_t(K^+p) - \sigma_t(K^+n). \quad (19)$$

The total K^+p and K^+n cross sections are equal to each other within experimental errors from 1 to 20 GeV, leading to a vanishing imaginary part for the $K^+n \to K^0p$ amplitude. A similar situation occurs for $pn \to np$, where the almost equal $\sigma_t(pp)$ and $\sigma_t(pn)$ lead to small imaginary part of the forward $pn \to np$ amplitude. At this point we must add, however, that the success of these predictions was essentially guaranteed once the total K^+p, K^+n, pp, and pn cross sections were found to have a flat energy dependence.[51] We cannot really claim here that our present successful test is independent of our prediction for flatness of $\sigma_t(K^+N)$ and $\sigma_t(NN)$.

It is more difficult to test the vanishing of the imaginary parts of the amplitudes for the processes (14)–(17). The optical theorem cannot be used here in order to separate the real part from the imaginary part. The only way of testing our hypothesis (indirectly) is to ask whether the explicit amplitudes given by various phenomenological models for these processes satisfy our prediction.

We now briefly discuss the situation with respect to every one of the processes (14)–(17), restricting our attention to $K^*(890)$ and $\Delta(1236)$ in the final state. The data on higher K^* or Δ resonances are poorer and we do not discuss them here.

A. $KN \to K\Delta$

Only $I=1$ can be exchanged in the t channel. The obvious candidates are the ρ and A_2 trajectories. The situation is similar to that of the $K^+n \to K^0p$ case, in which the ρ and A_2 contributions cancel in the imaginary part because of the required exchange degeneracy. We know from the KN and NN analysis that the ρ and A_2 intercepts as well as their couplings to $K\bar{K}$ and $N\bar{N}$ are approximately equal.[52] What remains to be seen here is whether the $\bar{N}\Delta$ couplings of these two trajectories are also the same. Krammer and Maor[34] have actually discussed this reaction and presented two solutions of similar (satisfactory) χ^2 values. One of these solutions involved exchange-degenerate residues. The data are clearly too meager to reach any final conclusion concerning the success of our prediction for this case.

B. $KN \to K^*\Delta$

Here the situation is similar, except for the possibility of a π-exchange term. Such a term will necessarily be almost purely real and, by itself, will satisfy our prediction.[53] We do not know whether the ρ and A_2 terms cancel in the imaginary part, since their couplings to the KK^* system are essentially unknown. In any event the π-exchange term seems to dominate this reaction[54] and it is plausible that the amplitude is predominantly real.

If the entire $KN \to K^*\Delta$ amplitude is purely real, the density-matrix elements $\rho_{\frac{3}{2},-\frac{1}{2}}$ and $\rho_{\frac{1}{2},-\frac{1}{2}}$ for the produced Δ should vanish.[55] However, these matrix elements do not appear in the expressions for the decay distributions of the Δ. They can be measured, in principle, only from the spin correlations.

C. $p\bar{p} \to p\Delta$

Here, again, the π-exchange contribution is presumably dominant,[54] leading to a real amplitude at small values of t. The data are not sufficiently accurate to enable us to study the possible contributions of other trajectories such as ρ and A_2.

D. $KN \to K^*N$

The charge-exchange mode ($K^+n \to K^{*0}p$) is dominated by π exchange and seems to satisfy our prediction.[56] In the non-charge-exchange case ($K^+p \to K^{*+}p$), a strong $I=0$ component is exchanged in addition to the pion. Here the phenomenological situation is not clear at all. The energy dependence of $\sigma(K^+p \to K^{*+}p)$ seems to be inconsistent with an $\alpha(0) \sim 0.5$ intercept for the leading $I=0$ trajectory[56] (P' or ω). Only exchange-degenerate contributions of P' and ω can guarantee a real amplitude in this case, but the data do not clearly demand the presence of either the P' or the ω.

It seems that although we cannot prove that the amplitudes for reactions (14)–(17) are purely real, we find that in all cases our prediction is (at least) not inconsistent with the currently accepted phenomenological models.

At this point we may add that a large number of additional reactions (such as $K^-p \to \pi\Lambda$, $K^-p \to \pi\Sigma$,

[51] If $\sigma_t(K^+N)$ is flat, its $\nu=\infty$ value is already reached at relatively low energy. Since the Pomeranchuk theorem predicts $\sigma_\infty(K^+p)=\sigma_\infty(K^+n)$, the flatness condition leads to $\sigma_t(K^+p)=\sigma_t(K^+n)$ at all energies, and hence to $\mathrm{Im}(K^+n \to K^0p)=0$.

[52] This is actually required in order to ensure the flatness of $\sigma_t(KN)$ and $\sigma_t(NN)$.

[53] An elementary pion-exchange picture gives a purely real amplitude. Absorption corrections may introduce a small imaginary part but, in most cases, the amplitude at small values of t remains predominantly real. An evasive Reggeized pion will possess a real amplitude around $t=0$. A conspiring pion as well as its parity-doublet conspirator will also produce predominantly real contributions.

[54] See, e.g., B. Haber, U. Maor, G. Yekutieli, and E. Gotsman, Phys. Rev. **168**, 1773 (1968).

[55] This matrix element can be shown to be purely imaginary. See, e.g., K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

[56] See, e.g., G. V. Dass and C. D. Froggatt, Nucl. Phys. **B10**, 151 (1969).

etc.) are predicted by duality diagrams[57] to have purely real amplitudes. These predictions involve, however, additional input assumptions and we do not wish to discuss them here.

VII. CONCLUDING REMARKS

We close our discussion with some general remarks concerning the significance of our model and the basic nature of the Pomeranchuk singularity.

A. How Seriously Can One Take the Model?

It is clear that the model involves a gross oversimplification of the actual situation. We have completely ignored the contributions of branch cuts in angular-momentum plane. Such cuts may or may not be treated as ordinary contributions, and their construction in the FESR sense is not clear at all. We have also avoided all trajectories other than the Pomeranchon, the pion, and the vector and tensor nonets. We cannot know, at present, whether or not there are additional "special" trajectories, fixed poles, or any other complications with relatively low values of α which might introduce modifications in our model. Even the precise meaning of the resonance concept is currently under much discussion,[58] and the various tests and morals of our model depend on what we mean when we talk about resonances.[59]

Another quantitative point which we must consider is the possibility that exotic states (like the Z_0 baryon) may exist, as well as the fact that exchange degeneracy is not an exact relation. These are two typical examples of corrections which may be necessary when and if we try to build a more realistic (and complicated) version of the model.[60] However, as a first approximation, such effects, even if they exist, may be neglected.

All of these points convince us that the physical picture discussed in this article should be regarded as a relevant regularity of strong-interaction amplitudes rather than as a theory in the full sense of the word. We believe, however, that the long list of successes of this picture hints that something deeper may be buried behind it.

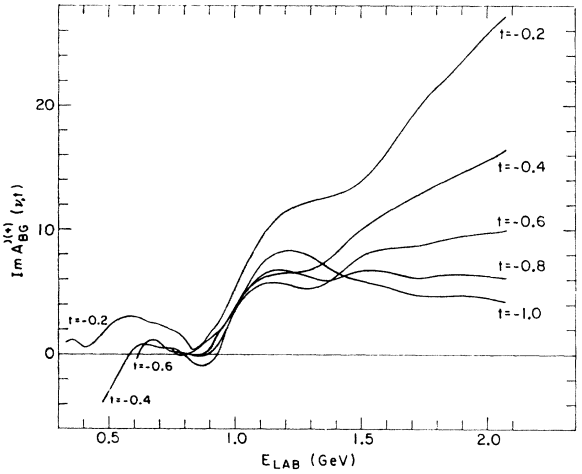

FIG. 15. Energy dependence of the background contribution to $\mathrm{Im}A'^{(+)}(\nu,t)$ for $t=-0.2$, -0.4, -0.6, -0.8, and -1 GeV^2. The $t=0$ case is displayed in Fig. 6. The curve for each value of t starts at the threshold energy for which this value of t is physical.

B. What Do We Learn about the Pomeranchon?

Assuming that our model is a reasonable approximation of πN scattering, we can learn something about the properties of the Pomeranchon.

The t dependence of the background contribution to $A'^{(+)}$ exhibits a shrinkage of the forward peak when ν increases (Fig. 11). This hints that the Pomeranchon is not flat. Our model also predicts that at small t, the imaginary parts of the pp and K^+p elastic scattering amplitudes are entirely given by the Pomeranchon, while in πN, $\bar{p}p$, and K^-p other trajectories are important. It is interesting to note that pp and K^+p scattering are the only two elastic processes indicating clear shrinkage of the forward peak. This seems to support the assumption that the Pomeranchon is not entirely flat.

Another peculiar regularity is observed when we plot $\mathrm{Im}A'_{BG}{}^{(+)}(\nu,t)$ as a function of E_{lab} for different values of t (Fig. 15). We find that for $|t|\geq 0.2$ the background (or Pomeranchon exchange) term is negligible below $E_{\mathrm{lab}}\sim 0.9$ GeV, and then it starts rising significantly. It is hard to tell whether this is an insignificant coincidence[61] or whether we learn here something about the threshold behavior of the diffractive contributions to elastic scattering.

If we consider the background terms in every one of the partial-wave amplitudes $f_{l\pm}{}^0$ [see Eq. (1) and Figs. 1 and 2], we may learn something about the partial-

[57] H. Harari, Phys. Rev. Letters **22**, 562 (1969); J. L. Rosner, *ibid.* **22**, 689 (1969).

[58] Much of the discussion between people who believe in the duality assumption and those who prefer various versions of interference-type models involves the precise definitions of the resonant part of the amplitude. For a (biased) review of the situation see, e.g., H. Harari (Ref. 1).

[59] In a very recent Glasgow Report (unpublished), A. Donnachie and R. G. Kirsopp show that they can fit the πN data with an interference model if they modify the resonance parameters. They have completely ignored the important contribution of u-channel trajectories. Their analysis deals with partial-wave amplitudes having definite s-channel isospins, and the distinction between $I=1$ exchange and $I=0$ exchange is lost. We believe that our Figs. 1 and 2 should convince anyone that at least for $I=1$ in the t channel the ρ exchange should *not* be added to the s-channel resonances. However, the Donnachie-Kirsopp paper as well as the Dance-Shaw paper (Ref. 24) are good examples of the flexibility in the choice of resonance shape and parameters and the conflicting results that may be obtained by various choices.

[60] In fact, the deviation from exchange degeneracy in KN scattering should be related to the contributions of Z-type resonances. The data are too poor to test this quantitatively, at present.

[61] Amusingly enough, the old paper of D. Amati, S. Fubini, A. Stanghellini, and M. Tonin [Nuovo Cimento **22**, 569 (1961)] gave arguments predicting that the diffractive contributions should begin in πN scattering precisely in this energy region.

wave decomposition of the Pomeranchon contribution. It might be interesting to compare this to typical diffraction or strong absorption models. All that we could conclude from such an analysis is that even at $\nu \sim 2$ GeV the Pomeranchon term is far from saturating the unitarity limit in the lowest partial waves. This is partly explained by the fact that the Pomeranchon's contribution to $B^{(+)}$ seems to be fairly small, in contrast with its contribution to $A'^{(+)}$ (Fig. 3). If we accept a naive optical or geometrical picture of the Pomeranchon, we should not be surprised to learn that the helicity-flip amplitude $B^{(+)}$ has a small contribution from such a mechanism.

C. Is the Pomeranchon Connected to "Ordinary" Trajectories?

In our model the Pomeranchon term and the ordinary trajectories are assumed to be two parts of the scattering amplitude. It seems plausible to assume that unitarity is the one ingredient which we have not used at all and which could tie together these two parts of the amplitude. Whether or not the Pomeranchon contribution can actually be constructed from the ordinary trajectories through the unitarity relation is still an open question. So far no one has succeeded in building an $\alpha_P(0)=1$ term from the ordinary vector and tensor nonets.[62]

D. Multiparticle Final States

We can apply our model to processes having three or more particles in the final state. It seems, however, that all our predictions involve statements about the imaginary parts of the multiparticle amplitudes and those are, of course, extremely hard to isolate. We do not see much point in presenting here such a list of untestable predictions.

One feature which does follow, however, from our basic philosophy is the requirement that a vertex of two Pomeranchons and one meson is not allowed in the multiperipheral Regge approach. Assuming that the Pomeranchon is a special diffractivelike entity which has very little to do with resonances or ordinary trajectories, it is hard to see how two Pomeranchons can be tied together in one vertex.

E. Pomeranchon and Photon-Initiated Reactions

Three different applications of our model to photon-initiated reactions may be interesting.

(a) We predict that $\sigma_t(\gamma p \to \text{hadrons})$ will decrease to its asymptotic constant value like all other total hadron cross sections. This is consistent with the present data,[63] but the accuracy is still not sufficient.

(b) The 1920- and 2420-MeV $I=\frac{3}{2}$ Δ resonances are observed in $\gamma p \to \rho^0 p$.[63] If we assume that the actual resonance contribution to this process at these energies is only the excess of events above the extrapolated Pomeranchon contribution, we predict relatively small cross sections for $\sigma(\gamma n \to \rho^- p)$ at these energies. For example, in the 1920-MeV region we would expect $\sigma(\gamma n \to \rho^- p) \sim 2\,\mu$b. If, on the other hand, part or most of the Pomeranchon term is built by resonances, the $\Delta(1920)$ contribution to $\sigma(\gamma p \to \rho^0 p)$ may be much larger and $\sigma(\gamma n \to \rho^- p)$ may be as large as 5 or 10 μb. γd experiments can settle this point in the near future.

(c) The total $\gamma p \to$ hadrons cross section for virtual photons is measured in inelastic electron scattering experiments. Here again we predict a decreasing σ_t for all values of q^2, in agreement with the preliminary SLAC-MIT results.[64] Another conjecture has been made with respect to the possible difference between the q^2 behavior of the Pomeranchon term and that of the contributions of the ordinary trajectories (or the N^* resonances).[65]

F. Difficulty in BB Scattering

We cannot ignore the serious difficulty posed by the failure of our model in baryon-antibaryon scattering.[12] The duality-diagrams prescription[57] "explains" why the model fails in this case, but it is not clear whether the failure stems from the strict duality assumption, from our model, or from ignoring all the exotic terms. It is interesting to note in this connection that the only place in which the quark structure of the baryons appears explicitly in the duality diagrams is the $B\bar{B}$ scattering case. All other results and conclusions of the diagrams would remain unchanged if baryons were made from one quark and a "remainder," independent of whether this remainder is a two-quark state or something else. Only in the case of the $B\bar{B}$ diagram does the three-quark structure explicitly show up. We wonder whether the $B\bar{B}$ failure tries to tell us something about the algebraic structure of baryon states.

ACKNOWLEDGMENTS

We thank our colleagues at the Weizmann Institute for many helpful remarks. We are indebted to Fred Gilman for collaborating with us in the first stages of the analysis presented in Secs. IV and V and for many interesting discussions.

[62] See, however, the interesting remarks of P. G. O. Freund, Phys. Rev. Letters **22**, 565 (1969); G. Veneziano (unpublished); P. G. O. Freund and R. J. Rivers, University of Chicago Report No. COO-264-491 (unpublished).

[63] See, e.g., S. C. C. Ting, rapporteur talk, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 43.

[64] W. K. H. Panofsky, rapporteur talk, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 23.

[65] H. Harari, Phys. Rev. Letters **22**, 1078 (1969).