Model Construction of the Nucleon Electromagnetic-Mass-Difference Tadpole*

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Model expressions for the forward off-shell Compton scattering amplitudes with suitable high-energy behavior are given and used to compute the proton-neutron mass difference.

I. INTRODUCTION

THE notorious negative sign[1] for $\delta m = m_p - m_n$ (which felicitously licenses the stability of the hydrogen atom against weak decay) contradicts naive expectation and has defied a satisfactory theoretical explanation for decades. The hope of Feynman and Speisman[2] that the negative magnetic energy may overwhelm the positive Coulomb contribution was dashed by the eventual measurement of nucleon electromagnetic form factors. (In nature, the form factors are damped too rapidly at high photon momentum for the derivative magnetic coupling to gain ascendance.) Thus, by the beginning of this decade it was generally realized that the Born term alone is inadequate[3] and that more details of strong interaction must be considered.

So far, none of the many attempts at dealing with the strong interaction dynamics,[4] though all rather ingenious, can claim to be totally satisfactory. A somewhat less ambitious approach, proceeding within a symmetry-group context, was advocated by Coleman and Glashow in 1964.[5] They delegated all strong-interaction effects beyond the nucleon pole to an $I=1$, $I_3=0$ scalar meson capable of tadpoling into the vacuum. Two years later, Harari[6] made a significant advance by considering the high-energy behavior of the amplitudes appearing in the Cottingham formula[7]

$$\delta m = -\frac{1}{2\pi}\int_0^\infty \frac{dq^2}{q^2}\int_0^q d\nu\,(q^2-\nu^2)^{1/2}[3q^2 t_1(q^2,i\nu) - (q^2+2\nu^2)t_2(q^2,i\nu)], \quad (1)$$

where $t_{1,2}(q^2,\nu)$ are the two invariant amplitudes[8] describing the forward Compton scattering of off-shell photons:

$$T_{\mu\nu} = t_1(q^2,\nu)[q^2 g_{\mu\nu} - q_\mu q_\nu] + t_2(q^2,\nu)[\nu^2 g_{\mu\nu} + (q^2/m^2)p_\mu p_\nu + (\nu/m)(p_\mu q_\nu + p_\nu q_\mu)]. \quad (2)$$

Regge theory hints that the large-ν behavior of t_i is determined by the intercept of the leading trajectory with $I=1$, $C=1$, $G=-1$, $P=(-1)^J$, that is, the A_2 trajectory, and that $t_1(q^2,\nu) \to \nu^\alpha$ and $t_2(q^2,\nu) \to \nu^{\alpha-2}$ as $\nu \to \infty$ with q^2 fixed $[\alpha = \alpha_{A_2}(0)]$. As α seems to lie experimentally between 0 and 1, $t_1(q^2,\nu)$ can never aspire to be represented by a fixed-q^2 unsubtracted dispersion relation, while a once-subtracted dispersion relation for $t_2(q^2,\nu)$ may be dominated with a bit of good fortune by the nucleon pole alone. However, now one must deal with a basically unknown q^2-dependent subtraction constant $t_1(q^2,\nu_0)$. Harari interprets the contribution of this subtraction constant to δm as the manifestation of the tadpole of Coleman and Glashow. Thus, the tadpole need not be a physical particle in the sense that it may appear as a propagator pole, but is merely a convenient mnemonic summarizing partially the effects of the high-mass states nelgected. Harari's achievment is to correlate the postulated symmetry-group properties of the unobserved tadpole with the actual symmetry-group properties of the observed low-mass resonances.[9]

In this note, we write down for $t_i(q^2,\nu)$ explicit expressions based on the Veneziano model,[10] which exhibit suitable high-ν Regge behavior and contain the correct nucleon-pole expression, and proceed to calculate the resulting δm.[11] Although the value of any blatantly model-dependent calculation would appear to be somewhat dubious, this procedure provides us with a theoretical laboratory to observe the explicit contribution of high-lying states and their partial summary in the subtraction constant. Furthermore, this would subject the Veneziano model, which has had a measure of success in strong-interaction processes,[12] to an electromagnetic test.

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[9] Or equivalently the symmetry group properties of relatively high-lying Regge trajectories.
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[11] Our calculation is a more sophisticated version of the work of Y. Srivastava, Phys. Rev. Letters **20**, 232 (1968).
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II. FORWARD COMPTON AMPLITUDES WITH REGGE BEHAVIOR

The model expressions we propose for $t_i(q^2,\nu)$ are

$$t_1(q^2,\nu)=\frac{m}{\mu^2}\Gamma(-\alpha)f_1(q^2)\left[\frac{\Gamma(N(s)-\frac{1}{2})}{\Gamma(N(s)-\frac{1}{2}-\alpha)}+(\nu\to-\nu)\right],$$

$$t_2(q^2,\nu)=\frac{m}{\mu^2}\Gamma(2-\alpha)f_2(q^2)\left[\frac{\Gamma(N(s)-\frac{1}{2})}{\Gamma(N(s)+\frac{3}{2}-\alpha)}+(\nu\to-\nu)\right],$$

$$s=(p+q)^2,\quad \nu=pq/m.$$

m is the nucelon mass, and the mass μ characterizes the slope of the nucleon trajectory $N(s)=\frac{1}{2}+(s-m^2)/2\mu^2$ ($2\mu^2/m^2\equiv\tau^2\simeq1.12$, experimentally). By isospin, only an $I=\frac{1}{2}$ baryon trajectory can contribute, and so only the nucleon trajectory is assumed to enter.

$$f_1(q^2)=\frac{e^2}{4\pi}\left\{\left[\frac{G_M{}^2-G_E{}^2}{q^2+4m^2}\right]_{\text{proton}}-[\cdots]_{\text{neutron}}\right\},$$

$$f_2(q^2)=\frac{e^2}{4\pi}\left\{\left[\frac{q^2G_M{}^2+4m^2G_E{}^2}{q^2(q^2+4m^2)}\right]_{\text{proton}}-[\cdots]_{\text{neutron}}\right\}.$$

For the Sachs form factors, we take the dipole fit[13]:

$$G_{Ep}(q^2)\simeq\frac{G_{Mp}(q^2)}{\mu_p}\simeq\frac{G_{Mn}(q^2)}{\mu_n}\simeq\frac{1}{(1+q^2/q_0{}^2)^2},$$

$$G_{En}(q^2)\simeq0,$$

where $q_0{}^2\simeq0.72$ BeV^2 and $\Lambda^2\equiv q_0{}^2/m^2$. α, the intercept of the A_2 trajectory, is experimentally[14] about 0.34.

These amplitudes exhibit the assumed Regge behavior and we fix their scales by requiring that the nucleon pole appears with the correct normalization. Note $t_1(q_1{}^2\nu)$ has a $\nu=0$ pole only for $q^2=0$, conforming to the decoupling of the longitudinal mode of massless particles.[6] Models of the Veneziano type were originally intended to describe only on-shell hadronic processes. To include the off-shell photons,[15] we assume the standard expedient that only the particle-trajectory-particle coupling function depends on the masses of the external particles and not the trajectory itself. This is equivalent for our purposes to the ansatz that the nucleon-resonance electromagnetic transition form factors have shapes similar to the nucleon electromagnetic form factors, a proposition which holds for example in vector-meson-dominance models.

Experimentally, $\kappa_p+\kappa_n\simeq0$ or $\kappa_p{}^2-\kappa_n{}^2\simeq0$, where the magnetic moments are $\mu_p=1+\kappa_p$, $\mu_n=\kappa_n$. Thus, $f_i(q^2)$ is proportional to $2\kappa_p$, whence δm_1 may be referred to as the magnetic mass. On the other hand, $f_2(q^2)$ is proportional to $(1+4m^2/q^2+2\kappa_p)$, indicating that δm_2 is mainly Coulombic. Note also the emphasis on low q^2 in δm_2, causing a positive mass difference in calculation using the Born term alone. (In an obvious notation, we have referred to the contribution of t_i to δm as δm_i. $\delta m=\delta m_1+\delta m_2$.)

III. NUMERICAL EVALUATION

Once expressions for t_i have been given, only the integration in the Cottingham formula need be discussed. We venture to describe the evaluation in some details, hoping to indicate the relative contribution of the different terms involved. As the possibility of an exact evaluation appears somewhat remote, we resort to a judicious mixture of approximate analytical and machine integration.

Let us treat δm_2 first. Write an unsubtracted dispersion relation for t_2, or in other words, express t_2 as a sum of poles. Integrate over ν and define for convenience $u\equiv q^2/m^2$. Then

$$\delta m_2=\frac{e^2}{4\pi^2}m\left(\frac{q_0}{m}\right)^8\Gamma(2-\alpha)\left(\frac{-\sin\pi\alpha}{\pi}\right)\times\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha-1)}{\Gamma(n+1)}[(\mu_p{}^2-\mu_n{}^2)B_n+4\tilde{B}_n],$$

where

$$B_n\equiv\tfrac{1}{4}\pi\int_0^\infty du\frac{1}{u+4}\frac{1}{(u+\Lambda^2)^4}\left[\left(1-\frac{(u+n\tau^2)^2}{2u}\right)\times[(u+n\tau^2)^2+4u]^{1/2}+\frac{(u+n\tau^2)^3}{2u}\right]$$

$$\equiv\tfrac{1}{4}\pi\int_0^\infty du\, b_n(u)$$

and

$$\tilde{B}_n\equiv\tfrac{1}{4}\pi\int_0^\infty du\,\tilde{b}_n(u)\equiv\tfrac{1}{4}\pi\int_0^\infty du\, b_n(u)\frac{1}{u}.$$

The nth term in the series then represents the contribution of the nth resonance lying on the nucleon trajectory, with $n=0$ designating the nucleon pole. As a function of photon mass squared, $b_n(u)$ rises from zero at $u=0$ to a positive peak at $q^2\approx0.2m^2$ before being damped by the form factor. Great care must be taken if $b_n(u)$ is evaluated on a machine, as for large n and small u it involves the small difference between two numbers several orders of magnitude larger. Note that $B_n>0$

[13] More sophisticated fits differ in high-q^2 behavior, but presumably the dominant effect comes from the low-q^2 region. A recent reference on form factors is D. H. Coward *et al.*, Phys. Rev. Letters **20**, 292 (1968).

[14] For the value of α, see R. J. N. Philips and W. Rarita, Phys. Rev. **140**, B200 (1966); V. Barger and M. Olsson, *ibid.* **146**, 1080 (1966); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 294 (1967); K. V. L. Sarma and G. H. Renninger, *ibid.* **20**, 399 (1968); S. Y. Chu and D. P. Roy, *ibid.* **20**, 958 (1968); **21**, 57 (1968); V. Barger and M. Olsson, *ibid.* **18**, 294 (1967). The value of α is by no means certain. We take $\alpha=0.34\pm0.03$ as given in the last reference cited above.

[15] The usual fixed pole coming from the current commutator does not exist here, of course, while the discussion of H. D. I. Abarbanel and S. Nussinov [Phys. Rev. **158**, 1462 (1967)] is irrelevant for the mass difference.

and $\tilde{B}_n>0$. Taking into account the γ-function factors, we see that each of the resonances contributes a negative value to δm_2 while the nucleon contribution is positive. In this model, all the higher resonances join in working against the nucleon pole. Although B_n may be evaluated exactly, the ensuing sum over n can only be done by machine. Instead, we choose to approximate the u integral by taking the first Taylor expansion in u of the square bracket appearing in $b_n(u)$ (the $n=0$ nucleon term must be treated separately). Then $B_n \sim 1/n$ and the series may be evaluated exactly:

$$\Gamma(2-\alpha)\left(\frac{-\sin\pi\alpha}{\pi}\right)\sum_{n=1}^{\infty}\frac{\Gamma(n+\alpha-1)}{\Gamma(n+1)}\frac{1}{n}=-\gamma-\psi(2-\alpha)$$
$$=-0.75 \quad \text{for } \alpha=0.34$$
$$=-2(1-\ln 2) \quad \text{for } \alpha=\tfrac{1}{2},$$

where ψ is the digamma function and γ is the Euler-Mascheroni constant. We then evaluate the first few B_n on a machine to correct for the Taylor approximation. As the sum converges rapidly, this procedure yields a reliable answer. The Taylor approximation is even more justified for $\tilde{B}_n$, and $\tilde{B}_n$ dominates over B_n. The result is

$$\delta m_2=\delta m_2{}^{\text{Born}}+\delta m_2{}^{\text{resonance}}\simeq 1.1\text{–}0.28 \text{ MeV}.$$

Although the resonance contribution lies in the right direction, it does not overwhelm the Born term. From our discussion above, it may be seen readily that a larger α would favor the resonances.

The evaluation of δm_1 is considerably trickier as t_1 has slower convergence properties. We write a once-subtracted dispersion relation for t_1 in the variable $q^2+2\nu m$ and subtracted at $q^2+2\nu m=2\mu^2$. After integration over ν,

$$\delta m_1=-\left(\frac{e^2}{4\pi^2}\right)\left(\frac{3m}{\pi}\right)\left(\frac{m^2}{2\mu^2}\right)\Lambda^8(\mu_p{}^2-\mu_n{}^2-1)$$
$$\times\Gamma(-\alpha)\frac{\sin\pi\alpha}{\pi}\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha)}{\Gamma(n+1)}A_n$$

and

$$A_n=\tfrac{1}{2}\pi\int_0^\infty du\frac{1}{u+4}\frac{1}{(u+\Lambda^2)^2}$$
$$\times u\left(1-\frac{n\tau^2}{2u}\{[(u+(n-1)\tau^2)^2+4u]^{1/2}-u-(n-1)\tau^2\}\right)$$
$$\equiv\tfrac{1}{2}\pi\int_0^\infty du\, a_n(u).$$

The nth term in the series represents the contribution of the $(n-1)$th resonance (conforming with the nomenclature already introduced) lying on the nucleon trajectory. The $n=1$ term corresponds to the nucleon pole itself and the $n=0$ term is the subtraction. (It must be emphasized that we do not disperse at fixed q^2, in contrast to Harari.) A Taylor approximation for the A_n may be verified explicitly to be unreasonable, particularly since the sum converges rather slowly. We have integrated A_n carefully on a machine. Extreme caution must be exercised as the integrand again involves the near cancellation among several large numbers. The integrand $a_n(u)$ starts out negative and attains a deep negative peak at $u\sim 0.1$ (or photon mass squared $q^2\sim 0.1m^2$), then turns positive around $u\sim 1$ and again peaks at $u\sim 2$ before being damped by the form factor. It is remarkable that the positions of the peaks and the point of sign change hardly move at all as n varies. The heights of the peaks go down as $1/n$ roughly, however, and the positive peak flattens out considerably. Thus, although the positive-peak height is only about $\frac{1}{10}$ the height of the negative peak, its contribution almost cancels the contribution of the negative peak to give the integral a rather small negative number. This unpleasant feature virtually assures that a rough first-trial machine calculation would produce misleading results. Typical values for $(2/\pi)A_n\times 10^3$ are ($n=0$, $+50.8$), ($n=1$, -8.9), ($n=2$, -3.8), ($n=10$, -0.4), ($n=20$, -0.16), ($n=101$, -1.96×10^{-5}), ($n=200$, -8.8×10^{-6}), ($n=1001$, -2.5×10^{-6}). Unfortunately, the large positive subtraction term successfully resists the hordes of small negative nucleon and resonance terms to yield $\delta m_1\simeq +0.5$ MeV.

Finally, we may of course integrate directly with the γ functions on a machine, without transforming first into a sum. Besides losing insights into the relative contribution of the various terms, we have to laboriously compile γ function for complex arguments. We have carried out such a calculation roughly, and the order of magnitude and the sign agree with what are concluded above. For example, in the expression for δm_1, the integral over ν turns out to be negative for small q^2 but crosses over to positive values too soon at $q^2\sim 0.2m^2$. The result is a positive δm_1.

IV. CLOSING REMARKS

We summarize the situation. In δm_2, the major contribution comes from $\tilde{B}_n$ with its emphasis on the low-q^2 region. Each of the resonance has a sign opposite to that of the nucleon. A glance at the expressions given above reveals that $\tilde{b}_0(u)$ differs fundamentally from $\tilde{b}_{n\neq 0}(u)$. For small u, $\tilde{b}_0(u)\sim u^{-1/2}$ while $\tilde{b}_{n\neq 0}(u)\sim$ constant. The nucleon pole thus dominates δm_2. For δm_1, the nucleon and the resonances all contribute negatively. Here, however, a subtraction term looms large and positive. A calculation of the present sort is not totally insensitive to the value of α and the form factors. As noted above, a large α would favor the resonances. In conclusion, we are led to suggest that a model of the Veneziano type fails to describe the effect of the higher resonances in the nucleon electromagnetic mass difference. It damps them too severely.

We make a closing remark about the "tadpole" term of Harari coming from the subtraction at $q^2=0$ of a dispersion relation in ν. Thus,

$$\delta m_1^{\mathrm{Harari}} = -\tfrac{3}{8}\int_0^\infty dq^2 q^2 t_1(q^2,0)\,.$$

Harari proves that

$$\lim_{q^2\to 0} q^2 t_1(q^2,0) = -\frac{\alpha}{\pi}\frac{1}{m}(\mu_p{}^2-\mu_n{}^2-1)>0$$

(which is satisfied by the present model of course) and then conjectures that $h(q^2)$, defined by

$$q^2 t_1(q^2,0) = -\frac{\alpha}{\pi}\frac{1}{m}(\mu_p{}^2-\mu_n{}^2-1)h(q^2)\,,$$

is a positive and rapidly decreasing function. In our specific model,

$$q^2 t_1(q^2,0) = \frac{2m}{\mu^2}\Gamma(-\alpha)q^2 f_1(q^2)\frac{\Gamma(q^2/2\mu^2)}{\Gamma(q^2/2\mu^2-\alpha)}$$

and crosses over quite soon to negative values at $q^2=2\alpha\mu^2$, before the damping from the form factor has set in. Finally, we need not emphasize again that the calculation presented here is specifically model-dependent and serves only as an illustration.

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Duality and the Pomeranchuk Singularity*

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The conjecture on the role played by the Pomeranchuk singularity in finite-energy sum-rule (FESR) calculations and within the duality framework is reviewed and subjected to various experimental tests. It is assumed that in the FESR sense the Pomeranchon is built from nonresonating background contributions, while all other trajectories are constructed from s-channel resonances. Previous results based on this conjecture are reviewed first. A detailed model for πN elastic scattering is then compared with experiment. All $I=1$ t-channel amplitudes for πN scattering are entirely accounted for by the N^*-resonance contributions, while the $I=0$ t-channel amplitudes require significant nonresonating background. This background is predominantly imaginary, and is presumably associated with the Pomeranchon-exchange term. The residue functions of the P and P' trajectories are calculated, using FESR and assuming our conjecture. The calculated functions are then used to predict high-energy differential cross sections and polarizations for πN scattering, in reasonable agreement with experiment. The P' trajectory seems to favor the Gell-Mann ghost-eliminating mechanism both in πN and in KN elastic scattering. Inelastic processes such as $K^+n\to K^0p$, $KN\to K\Delta$, and $KN\to K^*N$ are predicted to have purely real amplitudes at large s and small t. Various phenomenological models are shown to be consistent with this prediction. The paper concludes with a few remarks concerning various properties of the Pomeranchon, the connection of the model with multiparticle production and to photon initiated reactions, and the (only) failure of the model in baryon-antibaryon scattering.

I. INTRODUCTION

In the absence of a better understanding of strong-interaction dynamics, it is customarily assumed that, in many cases, direct-channel resonances dominate the strong-scattering amplitudes over a wide energy range. This working hypothesis has gained new popularity in the last year or two. Empirically, the ever-increasing number of experimentally identified hadronic states[1] seems to support this idea. Theoretically, it is now believed that the duality between direct-channel resonances and crossed-channel exchanges[2] allows the resonance-dominance assumption to coexist with the usual exchange mechanisms or even to replace them in some cases. In practice, many model calculations actually

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[1] See, e.g., the rapporteur talks of B. French, A. Donnachie, R. D. Tripp, and H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).