$SU(3)$ Multiplet Mixing and Unitarity*

J. J. BREHM AND L. F. COOK
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01002
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A purely S-matrix approach to $SU(3)$ multiplet mixing is given. Particular attention is devoted to the role of strong mixing in the basis states of the S matrix itself and to the role of the inelastic channels in calculating mixed representations. The dependence of the mixed wave functions on the energy separation between resonances and on the magnitude of symmetry-violating vertices is considered. The analysis is applied to the single-octet mixing of the $\frac{3}{2}^-$ baryon system in terms of a multichannel model involving the states B_8P_8, B_8V_8, and B_8V_1. The effects of ϕ-ω mixing in the basis states are included in the calculation and a dramatic improvement between the experimental and theoretical values of the branching ratios for the decays of $\Lambda(1520)$ and $\Lambda'(1700)$ into $\bar{K}N$ and $\pi\Sigma$ is obtained.

I. INTRODUCTION

In spite of the fact that $SU(3)$ symmetry must be recognized as being far from exact, it is widely accepted in particle physics and applied as a useful means of organizing the properties of hadrons. The physicist's ignorance, at the profound level, of how the symmetry is broken has not deterred him from drawing conclusions in its application. The concept of octet dominance and the mass sum rules that follow from it, for example, have provided confidence that $SU(3)$ multiplet assignments mean something. It is also reassuring that dynamical models, based on analyticity, unitarity, and crossing, and incorporating exact $SU(3)$ symmetry, have generally yielded results conforming with what is known of the $SU(3)$ systematics of the hadrons.

$SU(3)$ multiplet mixing is a phenomenon expected to be relevant when a broken symmetry is used. The properties of the nine vector mesons, the vestiges of a broken-symmetry octet and singlet, can be organized if mixing of the (hypercharge) $Y=0$, (isospin) $T=0$ members of the multiplets is invoked. Mixing occurs as the result of symmetry breaking and, moreover, is understood to be the dominant manifestation of it. None of the physical states is $SU(3)$-pure; nevertheless, it proves to be a useful first approximation to introduce the impurities only by performing a rotation in the subspace of states with common (Y,T) quantum numbers, and then to examine the implications of $SU(3)$ symmetry for the resultant set of states. The nine vector mesons have always been analyzed for their $SU(3)$ content in this way. Thus mixing, in this example ϕ-ω mixing, emerges or is isolated as the most pronounced effect of $SU(3)$-symmetry breaking. It would seem of some interest to demonstrate non-phenomenologically how this can occur for a given particle system.[1] Presumably the effect is due to the circumstance of near-degeneracy of the $SU(3)$ multiplets that occur in the hypothetical symmetric world. The aim of this paper is to choose a system, richer in its details than the 1^- mesons, which can exhibit mixing and to attempt to describe the mixing phenomenon in a precise way. The vehicle which seems most natural for explication of the problem is one based on S-matrix methods, unitarity being the primary consideration.

If we adhere to the belief that the particles should be viewed as composites, whether stable or unstable, then the partial-wave S matrix is ideally suited for their description: The particles are identified with the poles of the multichannel partial-wave amplitude. The residues of a given pole of the multichannel amplitude give the components of that particle's wave function and specify the particle's couplings. If the dynamical problem that was solved to yield this result were $SU(3)$-symmetric, then the wave function would be $SU(3)$-pure. The dynamics could than be modified to include a specific mode of $SU(3)$ violation. If the basis states of the wave function for the particle pole can be reconstructed according to their $SU(3)$ transformation properties, then the wave function would give the $SU(3)$ impurities of the particle. If the symmetric problem had yielded two poles, each with its own $SU(3)$-pure wave function, then the problem modified to include $SU(3)$ violation would give wave functions whose $SU(3)$ content should exhibit multiplet mixing as the dominant sort of impurity. The role of a near-degeneracy in the poles of the symmetric problem would then be apparent and crucial in the mixing phenomenon for the broken-symmetry problem.

For definiteness, in Sec. III we consider a model for the system of $\frac{3}{2}^-$ baryon resonances, the most likely set of states among the baryons to involve an interesting mixing effect. The dynamics of the $N^*(1518)$ is believed to be based on virtual ρ production, the coupling of πN (d-wave) and ρN (s-wave) channels.[2] The $SU(3)$-symmetric version of this problem, based on coupled P_8B_8 and V_8B_8 channels, yields octet and singlet resonant states.[3] The model is driven by the coupling of P_8B_8 to V_8B_8 due to P_8 exchange, as shown in Fig. 1. In order to describe the mixing of octet and singlet

* Research supported in part by the National Science Foundation.

[1] The vector mesons have been studied in this respect by L. F. Cook and H. L. Watson, Phys. Rev. 174, 2113 (1968).

[2] L. F. Cook and B. W. Lee, Phys. Rev. 127, 297 (1962).

[3] J. J. Brehm, Phys. Rev. 136, B216 (1964).

d-wave resonances, we introduce the V_1B_8 channel, coupled to P_8B_8 by P_8 exchange. This modification involves an $SU(3)$ violation, a nonvanishing $V_1P_8P_8$ vertex. This is the only symmetry breaking we shall assume; in particular, we shall retain the degeneracies of the P_8, V_8, and B_8 multiplets. In the spirit of ϕ-ω mixing, we shall also assume the nine vector mesons of the V_8-V_1 system to be degenerate. These assumptions allow us to isolate the baryon mixing effect neatly, although we admit at the outset that, in ignoring the mass splittings within the P_8, V_8, and B_8 multiplets, we are leaving out a substantial source of symmetry breaking that contributes to the mass splittings of the $\frac{3}{2}^-$ resonances. Thus the calculations which follow should be viewed as a simple dynamical model of multiplet mixing which yields couplings in better agreement with experiment than exact $SU(3)$ symmetry provides.

II. S-MATRIX VIEW OF MULTIPLET MIXING

Symmetry considerations, particularly those involving broken symmetries, are most often presented in terms of fields rather than in terms of the S matrix directly. However, many aspects of broken symmetry are intrinsically related to dynamical considerations, and the S matrix does lend itself readily to the construction of dynamical models which can provide some insight into various aspects of broken $SU(3)$ in the strong interactions. Our goal is to show how such dynamical considerations can be employed to understand the interrelations between two nearly degenerate $SU(3)$ multiplets in the presence of broken $SU(3)$. A crucial ingredient in such an approach is unitarity. That this is so follows from the observation that any discussion of multiplet mixing involves a consideration of several coupled channels and the principal constraint between the channels is unitarity. Further, since we wish to consider the symmetry properties of a "particle," regarded as a composite, it follows that unitarity is central to the discussion, since the existence of the particle may be viewed as the result of the unitarization of a particular force structure. Thus in what follows we will formulate a working definition of multiplet mixing within the context of the S matrix which we believe to be useful in discussing the dynamics of broken $SU(3)$.

The method by which a symmetry group is used to reduce the S matrix to disjoint sectors labeled by the eigenvalues of the Casimir operators of the group is a familiar one. The partial-wave S matrix, itself the result of such a procedure, is then spanned by states of the particle basis and, if an exact internal symmetry group exists, then the particles are assembled into multiplets and the S matrix can be reduced further. The definition of the states in these multiplets is often carried out by referring to field operators for each particle with specified internal-symmetry transformation properties. The two-body states are then decomposed into irreducible representations whose Casimir eigenvalues suffice to label the sectors of the S matrix transformed to this basis. The content of an internal symmetry then is simply that an energy-independent transformation exists which reduces the partial-wave S matrix.

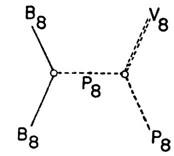

FIG. 1. P_8 exchange in $P_8B_8 \leftrightarrow V_8B_8$. Substitution of V_1 for V_8 involves $SU(3)$ violation.

If the symmetry is not exact and yet a concept of approximate symmetry is still to have some meaning, then it is not obvious how to proceed. The problem of specific concern to us here is the formulation of $SU(3)$ multiplet mixing in an S-matrix framework. Therefore we must have in mind an S matrix referred to the same set of $SU(3)$ basis states that are dictated by the symmetric problem, insofar as it is valid to do this. A perturbative treatment suggests itself; nonetheless, difficulties of principle arise. Mixing must have its origin in $SU(3)$-symmetry breaking, but this breaking splits the mass degeneracies of the $SU(3)$ multiplets, with the result that previously degenerate coupled-channel thresholds split apart. The transformation used to reduce the S matrix in the $SU(3)$-symmetric problem is replaced by one which is no longer energy-independent. Moreover, it is no longer possible to form linear combinations of particle states belonging to the basis of an irreducible representation of $SU(3)$. Thus, once threshold degeneracies disappear, it would appear that contact with $SU(3)$ symmetry is immediately lost. Nevertheless, among the physical particles, $SU(3)$ remains a useful and meaningful symmetry. For example, broken multiplets satisfy mass formulas based on first-order perturbation theory (except where strong mixing is present)—in spite of the difficulties mentioned above. No doubt this $SU(3)$ remnant is due to the details of the strong-interaction dynamics and to the particulars involved in breaking the symmetry. Although this feature of broken $SU(3)$ is not thoroughly understood, it allows us to adopt a pragmatic approach to multiplet mixing. In particular, based on the spirit of a first-order perturbation theory, we are motivated to ignore the departure from $SU(3)$ purity of the particles in the basis states when none of the constituent particles shows strong mixing effects. The content of this approach is simply the following: The arguments which support broken $SU(3)$ as a useful property of the strong interactions also support the view of multiplet mixing to be considered here.

To illustrate this approach with a preparatory example, let us consider P_8B_8 scattering in the context of a dynamical model governed by a particular force structure and constrained by unitarity. Since neither the P_8 nor B_8 multiplets exhibit strong mixing, let us retain the degeneracies of these multiplets so that we

	B_8P_8	B_8V_8
B_8P_8	0	M^T non-diag.
B_8V_8	M non-diag.	0

FIG. 2. Matrix structure of the production-dominated Born term in the $SU(3)$ basis. There are two scattering sectors, B_8P_8 and B_8V_8. If $SU(3)$-symmetric couplings are not employed, then M is not diagonal in the $SU(3)$ basis.

have an elastic problem, in that the phase space is common to all channels. We shall assume that a single-exchange mechanism dominates the potential. To introduce $SU(3)$ breaking, we admit coupling constants in the potential which differ from their $SU(3)$-symmetric values, but the mass of the exchanged multiplet is kept fixed. Thus the Born matrix consists of a matrix of numbers [which violate $SU(3)$] times a single function of the energy. The partial-wave S matrix reduces to blocks labeled by (T,Y) quantum numbers; each block is spanned by the relevant particle states and these can be transformed, under the conditions given, to a basis labeled by those $SU(3)$ representations in $8\otimes 8$ which pertain to the particular (T,Y). Since the potential is $SU(3)$-violating, the blocks in the $SU(3)$ basis will exhibit transitions between different $SU(3)$ representations. Under these conditions the potential, for each (T,Y), can be diagonalized independently of energy and this same transformation diagonalizes the ND^{-1} solution of the problem. If the force structure, unitarized in this way, yields a resonance in one of the amplitudes for the problem, diagonalized for each (T,Y), then the $SU(3)$ admixture of the resonance can be read off by referring to the diagonalizing transformation. If the $SU(3)$-symmetric version of this problem had yielded two resonances, both $SU(3)$-pure and near-degenerate, then the problem modified to include the $SU(3)$ violation should exhibit mixing of two $SU(3)$ representations in the wave functions as the dominant effect.

The role of unitarity in the above model is in a sense quite minimal. We note that it is really not necessary to solve the coupled equations for the amplitudes; the diagonalization of the Born matrix itself diagonalizes the amplitudes and yields the multiplet structure of the amplitudes. In this case, the multiplet structure itself is energy-independent. This feature is a consequence of the very simple energy dependence of the Born matrix. Unitarity is only necessary to provide the connection between the force structure and the scattering amplitudes. Thus to the extent that the dynamics of a resonance may be understood in terms of a single-exchange mechanism in an elastic process, the $SU(3)$ mixing of the resonance is given finally by the diagonalization of the Born term. It is clear, however, that any alteration of the energy dependence of the symmetric Born term, e.g., internal mass shifts or incorporation of more than one exchange, will destroy this simplistic situation.

The bulk of this paper will not be concerned with purely elastic problems with fixed basis states, but with the effects of strong multiplet mixing in the basis states together with effects associated with inelastic processes. It is not difficult to construct models which illustrate these effects separately, although they are somewhat uninteresting physically. In Sec. III we present a calculation of mixing which includes both effects for multiplets of physical interest.

To illustrate the separate effects of strong multiplet mixing in the basis states, consider P_8V_8 elastic scattering and, to be definite, let us assume that the potential is dominated by u-channel P_8 exchange with $P_8P_8V_8$ coupling. For kinematic reasons alone one would expect P_8V_1 states to be important, but our dynamical assumption of the force structure excludes them since the $P_8P_8V_1$ coupling is forbidden by $SU(3)$. However, if $SU(3)$ is broken, the $T=Y=0$ elements of V_8 and V_1 mix strongly, and this mixing should affect the basis states strongly. To include such an effect we add the P_8V_1 channel to the S matrix, consider V_8 and V_1 to be degenerate, and break the symmetry by taking the coupling $P_8P_8V_1$ to be nonzero. If we invoke no other symmetry-breaking effects, then the reduction of the S matrix to (T,Y) blocks proceeds as in our previous example with the addition of the P_8V_1 channel in the (T,Y) blocks to which it can contribute. Now $SU(3)$-forbidden transitions can take place for each (T,Y) between the $|P_8V_8\rangle$ $SU(3)$ states and the $|P_8V_1\rangle$ state. Because the energy dependence of the Born matrix remains unchanged, the final diagonalization within each (T,Y) is energy-independent and also diagonalizes the ND^{-1} amplitudes. The new basis states are linear combinations of $|P_8V_8\rangle$ and $|P_8V_1\rangle$, and V_8 and V_1 have been mixed in the basis states by the diagonalization. As before, if the $SU(3)$-symmetric version of this model contained two near-degenerate resonances, then the diagonalization involving $SU(3)$ violation should exhibit mixing of these two $SU(3)$ representations in the wave functions as the dominant effect.

To illustrate the separate effect of inelasticity, consider the coupled-channel scattering problem involving B_8P_8 and B_8V_8. In this case, we have two sectors with different phase space, so that the S matrix will be reduced to (T,Y) blocks, each containing four sub-blocks which are labeled by the relevant particle states of the B_8P_8 and B_8V_8 sectors and represent the processes $B_8P_8\to B_8P_8$, $B_8V_8\to B_8V_8$, $B_8P_8\to B_8V_8$, and $B_8V_8\to B_8P_8$. Both of these sectors can be transformed to a basis labeled by $SU(3)$ representations. If the symmetry is exact, each of the four sub-blocks is diagonal (except for the multiplicity of 8 in $8\otimes 8$) and for each (T,Y) we have (again except for octet multiplicity) a number of 2×2 disjoint scattering problems labeled by $SU(3)$. If we further assume the force to be dominated by the production diagram of Fig. 1, and break the symmetry by altering the coupling constants from the $SU(3)$-symmetric values, then for each (T,Y) the Born matrix in the $SU(3)$ basis is as shown in Fig. 2. Separate transformations in each sector. i.e.,

B_8P_8 and B_8V_8, on the $SU(3)$ bases will diagonalize the off-diagonal sub-blocks of Fig. 2 and again, because of the fixed energy dependence of the symmetry violation, these transformations will reduce the ND^{-1} solution to a set of 2×2 problems. The wave function of a resonant state will then contain an admixture of $SU(3)$ states, and if two resonances are near-degenerate, the mixing should be strong. In this case, in contrast to the previous examples, there are separate transformations for the B_8P_8 and B_8V_8 states. Thus, in general, the multiplet structure of a resonance will be a function of the states to which it couples, although special circumstances may occur for which the two transformations are identical. In this example, if the off-diagonal, symmetry-violating sub-block of the Born matrix is itself a symmetric matrix (i.e., $M=M^T$), then the two transformations are identical and the multiplet structure of the resonance is independent of the basis sector. However, it may be that any such circumstance is impossible in a given problem (e.g., M in Fig. 2 would not even be square if the sectors had been B_8P_8, $B_{10}P_8$). This particular example is somewhat unphysical in that the V_8 state in the basis mixes strongly with V_1, and V_1 has not been included in the example. A reasonable physical situation can be constructed by combining the basis-mixing effect of our second example with the inelastic effect of our third example. This, in fact, is the content of Sec. III.

III. EXAMPLE OF MIXING: $\frac{3}{2}^-$ HADRON SYSTEM

There is some evidence that the $\frac{3}{2}^-$ baryons form an $SU(3)$ octet and singlet in which mixing occurs between the $T=Y=0$ elements of the multiplets.[4] This is of interest here because a dynamical S-matrix model of these resonances exists[3] and, further, the properties of the model lend themselves directly to an exploration of the effects described in Sec. II. In what follows we will consider a multichannel problem with two thresholds and for which the force structure is entirely contained in the production amplitudes. Such a two-sector production model may be further specialized by specifying the details of the Born matrix. After considering the general characteristics of the model, we will specialize further in terms of Born matrices for exact and broken $SU(3)$.

A. Two-Sector Production Model

Consider a multichannel problem with two thresholds which divide the particle channels into two sectors. We will label channels in one sector by indices i or j and those in the other sector by p or q. The partial-wave S matrix will therefore consist of four blocks labeled

[4] R. D. Tripp, D. W. G. Leith, A. Minten, R. Armenteros, M. Ferro-Luzzi, R. Levi-Setti, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Nucl. Phys. **B3**, 10 (1967).

$$B(w)=\begin{array}{c|cc} & j & q \\ \hline i & 0 & \beta^T \\ p & \beta & 0 \end{array}\times f(w)$$

FIG. 3. Production-dominated Born matrix with a common energy dependence.

(i,j), (i,q), (p,j), and (p,q). We construct the S matrix by assuming a Born matrix and using ND^{-1} to satisfy unitarity. The dynamical assumption of the model is specified by a choice of the Born matrix B; we will assume for B that the diagonal blocks (i,j) and (p,q) consist of zeros and that the off-diagonal blocks, the production blocks, are given by a real matrix β which is, in general, rectangular (see Fig. 3). Further, we assume the energy dependence of $B(w)$ is given by a single function, $f(w)$, common to each nonvanishing element of B.

To impose unitarity, we employ a multichannel ND^{-1} technique for which one has the well-known system of equations

$$\begin{aligned} N(w)&=\frac{1}{\pi}\int_L\frac{dx}{x-w}[\operatorname{Im}B(x)]D(x)\,,\\ D(w)&=1-\int_R\frac{dx}{x-w}\rho(x)N(x)\,. \end{aligned}\tag{1}$$

Thus

$$N(w)=B(w)-\int_R\frac{dx}{x-w}[B(w)-B(x)]\rho(x)N(x)\,,$$

where $\rho(x)$ is a multichannel, diagonal, two-threshold, phase-space factor. Because of the assumed nature of the Born matrix, the system of equations breaks into two coupled parts, which we may write as

$$\begin{aligned} N_{11}(w)&=-\beta^T\int_R\frac{dx}{x-w}[f(w)-f(x)]\rho_2(x)N_{21}(x)\,,\\ N_{21}(w)&=\beta f(w)-\beta\int_R\frac{dx}{x-w}[f(w)-f(x)]\rho_1(x)N_{11}(x) \end{aligned}\tag{2}$$

and

$$\begin{aligned} N_{12}(w)&=\beta^Tf-\beta^T\int_R\frac{dx}{x-w}[f(w)-f(x)]\rho_2(x)N_{22}(x)\,,\\ N_{22}(w)&=-\beta\int_R\frac{dx}{x-w}[f(w)-f(x)]\rho_1(x)N_{12}(x)\,, \end{aligned}\tag{3}$$

where indices (11), (12), (21), and (22) refer to the blocks (i,j), (i,q), (p,j), and (p,q), respectively, of the Born matrix.

To reduce the coupled equations in Eqs. (2) and (3) to manageable form, we "diagonalize" the problem. Consider the N_{11},N_{21} system. We have, by substituting

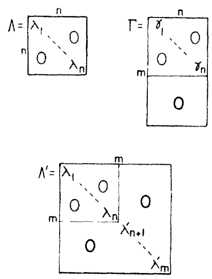

FIG. 4. Structure of the matrices Λ, Λ', and Γ as a result of the diagonalization. Note that $\gamma_i^2=\lambda_i$.

N_{11} into the equation for N_{21} in Eq. (2),

$$N_{21}(w)=\beta f(w)+\beta\beta^T\int dy \times\left(\int dx\frac{f(x)-f(y)}{x-y}\,\frac{f(x)-f(w)}{x-w}\rho_1(x)\right)\times\rho_2(y)N_{21}(y)$$
$$=\beta f(w)+\beta\beta^T\int dy\,\rho_2(y)K_1(y,w)N_{21}(y)\,, \tag{4}$$

where the second equation serves to define $K_1(y,w)$. To reduce Eq. (4), set

$$N_{21}(w)=\beta n(w)\,, \tag{5}$$

where $n(w)$ is some new square matrix, and observe that if $n(w)$ satisfies

$$n(w)=f(w)I+\beta^T\beta\int dy\,\rho_2(y)K_1(y,w)n(w)\,, \tag{6}$$

where I is the identity matrix, then $N_{21}(w)$, as given in Eq. (5), satisfies Eq. (4). Since $\beta^T\beta$ is real and symmetric, it can be diagonalized by an orthogonal transformation, which we can write as

$$U\beta^T\beta U^T=\Lambda\,. \tag{7}$$

Furthermore, U is independent of w. Defining $n(w)$ in the new basis as

$$\tilde{n}(w)=Un(w)U^T\,,$$

and transforming Eq. (6), we have

$$\tilde{n}(w)_{ij}=f(w)\delta_{ij}+\lambda_i\int dy\,\rho_2(y)K_1(y,w)\tilde{n}(y)_{ij}\,. \tag{8}$$

If we exclude accidental solutions where the λ_i, the eigenvalues of Λ, are the eigenvalues of the homogeneous integral equation, then we know that $\tilde{n}(w)$ must be diagonal and is given by the solutions of

$$\tilde{n}(w)_i=f(w)+\lambda_i\int dy\,\rho_2(y)K_1(y,w)\tilde{n}(y)_i\,. \tag{9}$$

The solutions of Eq. (9) will yield,

$$(N_{21}(w))_{pj}=\beta_{pk}[U^T\tilde{n}(w)U]_{kj}\,,$$

$$(N_{11}(w))_{ij}=\left(\beta^T\beta U^T\int\frac{dx}{x-w}\times[f(x)-f(w)]\rho_2(x)\tilde{n}(x)U\right)_{ij}$$
$$=(\beta^T\beta)_{ik}[U^TI_n(w)U]_{kj}\,, \tag{10}$$

which defines the diagonal matrix $I_n(w)_i$.

For the N_{12},N_{22} system, the reduction proceeds in an analogous way. In Eq. (3), substitute N_{22} into the equation for N_{12} to obtain

$$N_{12}(w)=\beta^Tf(w)+\beta^T\beta\int dy\times\left(\int dx\frac{f(x)-f(y)}{x-y}\,\frac{f(x)-f(w)}{x-w}\rho_2(x)\right)\times\rho_1(y)N_{12}(y)$$
$$=\beta^Tf(w)+\beta^T\beta\int dy\,\rho_1(y)K_2(y,w)N_{12}(y)\,. \tag{11}$$

Set

$$N_{12}(w)=\beta^Tm(w)\,, \tag{12}$$

where $m(w)$ is a new square matrix which satisfies

$$m(w)=f(w)I+\beta\beta^T\int dy\,\rho_1(y)K_2(y,w)m(y)\,. \tag{13}$$

We diagonalize $\beta\beta^T$ by an orthogonal transformation V such that

$$V\beta\beta^TV^T=\Lambda'\,; \tag{14}$$

then in the new basis we have

$$\tilde{n}(w)=Vm(w)V^T\,, \tag{15}$$

where $\tilde{m}(w)$ must be diagonal, like $\tilde{n}$, and satisfies

$$\tilde{m}(w)_p=f(w)+\lambda_p'\int dy\,\rho_1(y)K_2(y,w)\tilde{m}(y)_p\,. \tag{16}$$

The solutions of Eq. (16) will yield

$$(N_{12}(w))_{iq}=\beta_{ir}{}^T[V^T\tilde{m}(w)V]_{rq}\,,$$

$$(N_{22}(w))_{pq}=(\beta\beta^T)_{pr}\left(V^T\int dx\frac{f(x)-f(w)}{x-w}\rho_1(x)\tilde{m}(x)V\right)_{rq}$$
$$=(\beta\beta^T)_{pr}[V^TI_m(w)V]_{rq}\,, \tag{17}$$

which defines the diagonal matrix $I_m(w)_p$.

Using the results of Eqs. (10) and (17), we have for the N matrix

$$N(w)=\begin{pmatrix}\beta^T\beta U^T I_n(w)U & \beta^T V^T\tilde{m}V\\ \beta U^T\tilde{n}U & \beta\beta^T V^T I_m V\end{pmatrix},$$

which yields, when transformed to the new basis,

$$\hat{N}(w)=\begin{pmatrix}U & 0\\ 0 & V\end{pmatrix}N(w)\begin{pmatrix}U^T & 0\\ 0 & V^T\end{pmatrix}$$
$$=\begin{pmatrix}\Lambda I_n(w) & \Gamma^T\tilde{m}(w)\\ \Gamma\tilde{n}(w) & \Lambda' I_m(w)\end{pmatrix}. \tag{18}$$

In (18) we have defined

$$\Gamma=V\beta U^T. \tag{19}$$

By the use of Eq. (1), one finds the D matrix in the new basis

$$\hat{D}(w)=\begin{pmatrix}1-\Lambda J_n(w) & -\Gamma^T H_m(w)\\ -\Gamma H_n(w) & 1-\Lambda' J_m(w)\end{pmatrix}, \tag{20}$$

where we have defined the diagonal matrices

$$J_n(w)=\int\frac{dx}{x-w}\rho_1 I_n(x)\,,\quad J_m(w)=\int\frac{dx}{x-w}\rho_2 I_m(x)\,,$$
$$H_n(w)=\int\frac{dx}{x-w}\rho_2\tilde{n}(x)\,,\quad H_m(w)=\int\frac{dx}{x-w}\rho_1\tilde{m}(x)\,. \tag{21}$$

The general reduction and solution of the multichannel, two-sector model is nearly complete as given by Eqs. (18) and (20), together with the solutions of Eqs. (9) and (16). The details of the structure of the matrix Γ, in Eq. (19), remain to be discussed. If Γ were diagonal, then the multichannel problem would be reduced to a series of disjoint 2×2 problems, each characterized by the eigenvalues of Λ and Λ'. In general, however, Γ cannot be diagonal since it is in general not square. Nevertheless, it is "partially" diagonal, as we now show.

From Eqs. (7), (14), and (19) we have

$$\Gamma^T\Gamma=\Lambda \quad\text{and}\quad \Gamma\Gamma^T=\Lambda' \tag{22}$$

and

$$\Gamma\Lambda=\Lambda'\Gamma. \tag{23}$$

Writing this out in components and using the fact that Λ and Λ' are diagonal, we obtain

$$\Gamma_{pj}(\lambda_j-\lambda_p')=0$$

for each (p,j). We assume that none of the λ_j is degenerate and similarly for the λ_p'. Say that $j=1,2,\cdots,n$ and $p=1,2,\cdots,m$, so that Γ is an $m\times n$ matrix. Assume that $m>n$. Now pick a λ_p'; either there exists a λ_j such that $\lambda_j=\lambda_p'$ or there does not. If $\lambda_j\neq\lambda_p'$ for any j, then $\Gamma_{pj}=0$ for all j; i.e., the pth row of Γ vanishes. If there exists one of the λ_j's such that $\lambda_j=\lambda_p'$, then $\Gamma_{pj}=0$ for all $j\neq p$. By passing through the various values of p and reordering the indices $p=1$ to m, we see that Γ consists of an $n\times n$ diagonal matrix and an $(m-n)\times n$ block of zeros, i.e., Γ is partially diagonalized. Of course, the diagonal elements of Γ need not be nonzero, but, in the case where each λ_j is matched by a λ_p' and $m>n$, we will have the situation in Fig. 4. We label the diagonal elements of Γ by γ_i and note that $\gamma_i^2=\lambda_i$. This yields the transformed N and D functions $\hat{N}$ and $\hat{D}$ as shown in Fig. 5. As expected, with $m>n$, some of the inelastic channels are decoupled from the elastic channels and we are left with n 2×2 problems. If we consider only the coupled amplitudes, then we have for the transformed $(2n)\times(2n)$ system

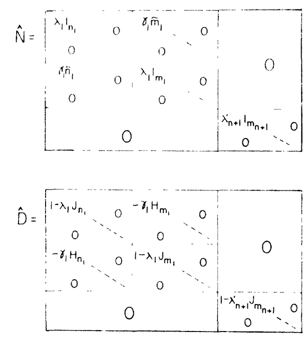

FIG. 5. Structure of the N and D matrices, transformed to the basis defined by U and V to yield a set of disjoint 2×2 problems.

$$\hat{M}=\hat{N}\hat{D}^{-1}$$
$$=\begin{pmatrix}F & G\\ \bar{G} & F'\end{pmatrix}, \tag{24}$$

where

$$F=\Lambda[I_n(1-\Lambda J_m)+H_n\tilde{m}]Z^{-1},$$
$$G=\Gamma[\tilde{m}(1-\Lambda J_n)+\Lambda I_n H_m]Z^{-1},$$
$$\bar{G}=\Gamma[\tilde{n}(1-\Lambda J_m)+\Lambda H_n I_m]Z^{-1},$$
$$F'=\Lambda[I_m(1-\Lambda J_n)+H_m\tilde{n}]Z^{-1},$$

and

$$Z=(1-\Lambda J_n)(1-\Lambda J_m)-\Lambda H_n H_m.$$

In these equations all matrices are $n\times n$ and diagonal. Finally, one may show that $G=\bar{G}$, so that $\hat{M}$ is symmetric. Thus Eq. (24) completes the construction of the reduced, unitarized amplitudes for the given Born matrix (Fig. 3).

This model illustrates some of the remarks made in Sec. II, particularly those regarding energy-independent transformations. In particular, we note that the reduction given in Eq. (24) is energy-independent and that this property stems from the particular form of the Born matrix, viz., the presence of a common energy dependence. Further, we note that the diagonalizing transformations given by U and V [see Eqs. (7) and (14)] certainly need not be identical, which implies that the mixing parameters differ between the two sectors. On the other hand, if the Born matrix is such that

$$\beta=\beta^T \quad (\text{for } \beta \text{ square}),$$

then U and V are identical transformations, which in fact diagonalize β.

B. $\frac{3}{2}^-$ System: Exact $SU(3)$

The previous model forms the basis for a dynamical model of the $\frac{3}{2}^-$ baryon system when we specify the two sectors as $|B_8P_8\rangle$ and $|B_8V_8\rangle$ and the Born matrix to be that resulting from the diagram in Fig. 1 with $SU(3)$ couplings. The details of this model with exact $SU(3)$ have been given elsewhere,[3] and we therefore confine ourselves to summarizing the results.

The diagonalization procedure of the previous section is simplified, since the channels $|B_8P_8\rangle$ and $|B_8V_8\rangle$ have the same $SU(3)$ structure; thus $\beta=\beta^T$, so that $U=V$. U is found in two steps, the first being consultation of the tabulated $SU(3)$ Clebsch-Gordan coefficients.[5] The transformation of β to the $SU(3)$ basis of de Swart must be followed by a 45° rotation in the octet $(8_s,8_a)$ plane to yield the following diagonal elements of Γ:

$$\begin{aligned}\gamma_1&=6f,\\ \gamma_{8_1}&=3f+5^{1/2}(1-f),\\ \gamma_{8_2}&=3f-5^{1/2}(1-f),\\ \gamma_{10}&=-\gamma_{\overline{10}}=2(f-1),\\ \gamma_{27}&=-2f.\end{aligned} \tag{25}$$

The quantity f is the Yukawa F/D parameter for the $B_8B_8P_8$ vertex. Note that f enters the description of the B_8^* and B_1^* $\frac{3}{2}^-$ baryons via the model (Fig. 1). It should not be confused with the F/D parameter for the $B_8^*B_8P_8$ vertex. This parameter—call it f^*—is determined[3] from the model by the 45° rotation required to get Eqs. (25). To be specific, the rotation

$$\begin{pmatrix}8_1\\8_2\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}8_s\\8_a\end{pmatrix}$$

implies that $f^*=(\sqrt{5})/(3+\sqrt{5})=0.428$. To illustrate how a given (T,Y) block is spanned in the new basis, we have for $\Gamma(T=0,\ Y=0)$

$$\Gamma(0,0)=\begin{array}{c} \begin{array}{cccc} |B_8P_8,27\rangle & |8_1\rangle & |8_2\rangle & |1\rangle \end{array} \\ \left[\begin{array}{cccc} -2f & 0 & 0 & 0\\ 0 & 3f+(\sqrt{5})(1-f) & 0 & 0\\ 0 & 0 & 3f-(\sqrt{5})(1-f) & 0\\ 0 & 0 & 0 & 6f \end{array}\right] \end{array} \begin{array}{c} \langle B_8V_8,27| \\ \langle 8_1| \\ \langle 8_2| \\ \langle 1| \end{array}. \tag{26}$$

The over-all coupling strengths associated with the two relevant vertices, scaled by $g(NN\pi)$ and $g(\rho\pi\pi)$, are such that resonances can be produced for appropriate values of f. If we choose $f\simeq 0.40$, then singlet and octet (8_1) resonances are produced. For $f=0.40$, the singlet resonance is at a somewhat higher energy than the octet resonance; if $f=0.428$ [i.e., $f=(\sqrt{5})/(3+\sqrt{5})$], the two resonances are degenerate; if $f>0.428$, the singlet resonant energy is below that of the octet.

Although we do not wish to argue that this model represents all of the dynamical aspects of the $\frac{3}{2}^-$ baryon system, it does yield the dominant features of the data, viz., octet and singlet resonant states in the spin-parity state $\frac{3}{2}^-$. In addition, it provides a relatively clean example of the effects discussed in Sec. II. Thus a calculation of $SU(3)$ mixing based on the basis-state impurities associated with the vector mesons should provide some dynamical understanding of the broken $\frac{3}{2}^-$ multiplets.

C. $\frac{3}{2}^-$ System: Broken $SU(3)$

To break $SU(3)$ for the $\frac{3}{2}^-$ hadron system we consider only one aspect of the broken symmetry, viz., the $SU(3)$ impurity of the vector mesons in the basis states $|B_8V_8\rangle$. To accomplish this we proceed as we did in Sec. II by introducing the additional $SU(3)$ channel $|B_8V_1\rangle$. We consider the V_8 and V_1 multiplets to be degenerate and thus by kinematical arguments we would expect the $|B_8V_1\rangle$ channel to be an important part of the $\frac{3}{2}^-$ dynamics. In order to introduce this channel into the coupled $|B_8P_8\rangle-|B_8V_8\rangle$ model and still maintain the relative simplicity of the production model in Sec. III A, we include in the Born matrix terms arising from Fig. 1 when the V_8 multiplet is replaced by the V_1. This breaks $SU(3)$ because of the presence of the vertex $P_8P_8V_1$, which is $SU(3)$-violating. Since we maintain isospin and hypercharge conservation, this amounts to only the one new vertex $K\bar{K}V_1$. The symmetry breaking, and subsequent mixing, is thus characterized by a single parameter γ, related to the $K\bar{K}V_1$ coupling constant. We define

$$g(K\bar{K}V_1)=\gamma g(K\bar{K}V_{800}), \tag{27}$$

where V_{800} is the $T=Y=0$ member of V_8.

With the addition of the $|B_8V_1\rangle$ channel, β is no longer a square matrix. We will label its columns by the $SU(3)$ representations of $|B_8P_8\rangle$ appropriate for a given choice of (T,Y), while its rows contain, in addition to $|B_8V_8\rangle$, the octet state $|B_8V_1\rangle$. Thus, in contrast to

[5] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

Eq. (26), we find for $\beta(0,0)$ in this basis

$$\hat{\Gamma}(0,0)=\begin{array}{c} \begin{array}{cccc} |B_8P_8,27\rangle & |8_1\rangle & |8_2\rangle & |1\rangle \end{array} \\ \left[\begin{array}{cccc} \gamma_{27} & 0 & 0 & 0 \\ 0 & \gamma_{8_1} & 0 & 0 \\ 0 & 0 & \gamma_{8_2} & 0 \\ 0 & 0 & 0 & \gamma_1 \\ \dfrac{3\sqrt{(30)}}{20}\gamma\gamma_{27} & -\dfrac{1}{\sqrt{(10)}}\gamma\gamma_{8_1} & -\dfrac{1}{\sqrt{(10)}}\gamma\gamma_{8_2} & -\dfrac{1}{2\sqrt{2}}\gamma\gamma_1 \end{array}\right] \end{array} \begin{array}{c} \langle B_8V_8,27| \\ \langle 8_1| \\ \langle 8_2| \\ \langle 1| \\ \langle B_8V_1,8| \end{array} . \tag{28}$$

The caret on $\hat{\Gamma}(0,0)$ indicates that the basis states are $SU(3)$ configurations. The matrix $\Gamma(0,0)$, whose structure is described in Sec. III A, is obtained from $\hat{\Gamma}(0,0)$ via transformations U and V which will be transformations on the $SU(3)$ basis states.

In order to obtain the multiplet mixing, it is necessary to perform the transformations U and V which partially diagonalize the matrix in Eq. (28) in the sense of Eq. (19). Since we are referring the $SU(3)$ breaking directly to the $SU(3)$ basis, we will use β and $\hat{\Gamma}$ interchangeably. Once this diagonalization is accomplished, we may read off the mixed wave function from the transformation matrices directly.

Thus we seek the diagonalization of $\beta\beta^T$ and of $\beta^T\beta$, where $\beta=\hat{\Gamma}$ is given in the $SU(3)$ basis. The matrices involved are large enough to be unwieldy, and, since the $SU(3)$ violation is presumed to be small, we calculate only to $O(\gamma^2)$. The matrices to be diagonalized, e.g., $\beta\beta^T$, are always of the form, in the $SU(3)$ basis,

$$\begin{bmatrix} x_1 & & & y_1 \\ & \ddots & 0 & \vdots \\ & 0 & & y_{N-1} \\ y_1 & \cdots & y_{N-1} & x_N \end{bmatrix},$$

where y_i is $O(\gamma)$, x_i is $O(1)$, $i\neq N$, and x_N is $O(\gamma^2)$. Diagonalization of $\beta\beta^T$ consists of finding Λ' and V. For the eigenvalues, one obtains, to $O(\gamma^2)$,

$$\lambda_{i\neq N}'=x_i+y_i^2/x_i\,,\quad \lambda_N'=0\,. \tag{29}$$

The eigenvectors corresponding to the λ_i' form the columns of the matrix V^T according to Eq. (14). The result is

$$V^T=\begin{bmatrix} 1-\dfrac{y_1^2}{2x_1^2} & -\dfrac{y_2}{x_2}\dfrac{y_1}{x_1-x_2} & \cdots & -\dfrac{y_1}{x_1} \\ -\dfrac{y_1}{x_1}\dfrac{y_2}{x_2-x_1} & 1-\dfrac{y_2^2}{2x_2^2} & \cdots & -\dfrac{y_2}{x_2} \\ \vdots & \vdots & & \vdots \\ -\dfrac{y_1}{x_1}\dfrac{y_{N-1}}{x_{N-1}-x_1} & -\dfrac{y_2}{x_2}\dfrac{y_{N-1}}{x_{N-1}-x_2} & \cdots & -\dfrac{y_{N-1}}{x_{N-1}} \\ \dfrac{y_1}{x_1} & \dfrac{y_2}{x_2} & \cdots & 1-\frac{1}{2}\sum\limits_{i\neq N}\left(\dfrac{y_i}{x_i}\right)^2 \end{bmatrix}. \tag{30}$$

The rows and columns are labeled by $SU(3)$ states in the $|B_8V\rangle$ sector. We will refer to the eigenvectors, the columns of (30), as $v^{(i)}$. Of course, if $\gamma\to 0$, then $v_j^{(i)}\to\delta_{ij}$; furthermore, the analysis which leads to Eq. (30) assumes that $x_i\neq x_j$ for $i\neq j$, i.e., degeneracy is excluded. We note that the eigenvectors in Eq. (30) have exactly the properties one would expect in nondegenerate systems. In particular, there is very little mixing, i.e., only $O(\gamma^2)$, unless two of the γ_i of Eqs. (25) are near one another. If $x_j-x_i\sim y_j$, then the representations (j) and (i) will mix strongly: $O(\gamma)$ rather than $O(\gamma^2)$. The occurrence of this $O(\gamma)$ effect in the resonant eigenvectors is the principal result of this paper. Mass splitting occurs and can only be studied by looking into the details of the energy dependence. This, however, follows the eigenvalue shifts which are $O(\gamma^2)$. Thus to $O(\gamma)$, the originally degenerate resonant octet remains degenerate and, to this order, mixing of the

$T=Y=0$ members of the singlet and octet emerges as the dominant effect. If $x_j-x_i\sim O(\gamma^2)$, then the analysis leading to Eq. (30) is invalid and the effects of quasi-degeneracy must be included.

The comments on mixing in the previous paragraph pertain to mixing of $SU(3)$ representations in the BV sector. What we learn about mixing as seen in the BP sector is of more interest, since these are in general the resonance decay channels. We note again that the manifestations of mixing should distinguish between the particle sectors to which the mixed state is coupled. Mixing in the BP sector is determined by the matrix U. To find U, we first obtain Γ from $\Lambda'=\Gamma\Gamma^T$. The sign of the root in the determination of Γ is such that Γ is near $\hat{\Gamma}=\beta$. Once Γ is found, U^T is obtained from $U^T\Gamma^T=\beta^TV^T$. The result is

$$U^T{}_{ip}=(1/\gamma_p)\beta^T{}_{iq}V^T{}_{qp}\,, \tag{31}$$

where γ_p refers to the "diagonal" elements of Γ in the sense of Fig. 4. We will refer to the columns of U^T as $u^{(i)}$; these give the BP wave functions in the $SU(3)$ basis.

In the $\frac{3}{2}^-$ hadron system, resonant scattering occurs in the singlet and octet configurations. We give below the wave functions of the resonating states for the pertinent (T,Y) quantum numbers for both the BP and BV sectors, referred to as $u^{(i)}(T,Y)$ and $v^{(i)}(T,Y)$, respectively.

$T=0$, $Y=0$; Y_0^*:

$$u^{(1)}(0,0)=\frac{1}{\eta^{(1)}(0,0)}\begin{pmatrix} \dfrac{3\sqrt{(15)}}{40}\,\dfrac{\gamma^2\gamma_1/\gamma_{27}}{1-(\gamma_1/\gamma_{27})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2\gamma_1/\gamma_{8_1}}{1-(\gamma_1/\gamma_{8_1})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2\gamma_1/\gamma_{8_2}}{1-(\gamma_1/\gamma_{8_2})^2} \\ \eta^{(1)}(0,0) \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](1)\end{matrix}\,,\qquad v^{(1)}(0,0)=\begin{pmatrix} \dfrac{3\sqrt{(15)}}{40}\,\dfrac{\gamma^2}{1-(\gamma_1/\gamma_{27})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2}{1-(\gamma_1/\gamma_{8_1})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2}{1-(\gamma_1/\gamma_{8_2})^2} \\ 1/\eta^{(1)}(0,0) \\ -\frac{1}{4}\sqrt{2}\gamma \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](1)\\(8)\end{matrix}\,,$$

where $\eta^{(1)}(0,0)=1+\frac{1}{10}\gamma^2$;

$$u^{(8)}(0,0)=\frac{1}{\eta^{(8)}(0,0)}\begin{pmatrix} \dfrac{3\sqrt{3}}{20}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_{27}}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ \eta^{(8)}(0,0) \\ -\dfrac{1}{10}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_{8_2}}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_1}{1-(\gamma_{8_1}/\gamma_1)^2} \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](1)\end{matrix}\,,\qquad v^{(8)}(0,0)=\begin{pmatrix} \dfrac{3\sqrt{3}}{20}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ 1/\eta^{(8)}(0,0) \\ -\dfrac{1}{10}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ -\dfrac{1}{4\sqrt{5}}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_1)^2} \\ -\gamma/\sqrt{(10)} \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](1)\\(8)\end{matrix}\,,$$

where $\eta^{(8)}(0,0)=1+\gamma^2/20$;

$T=\frac{1}{2}$, $Y=1$; N^*:

$$u^{(8)}(\tfrac{1}{2},1)=\frac{1}{\eta^{(8)}(\frac{1}{2},1)}\begin{pmatrix} -3\dfrac{\sqrt{5}-1}{20\sqrt{2}}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_{27}}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ \eta^{(8)}(\frac{1}{2},1) \\ \dfrac{1}{10}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_{8_2}}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ \dfrac{\sqrt{5}-1}{4\sqrt{(10)}}\,\dfrac{\gamma^2\gamma_{8_1}/\gamma_{\overline{10}}}{1-(\gamma_{8_1}/\gamma_{\overline{10}})^2} \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](\overline{10})\end{matrix}\,,\qquad v^{(8)}(\tfrac{1}{2},1)=\begin{pmatrix} -3\dfrac{\sqrt{5}-1}{20\sqrt{2}}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ 1/\eta^{(8)}(\frac{1}{2},1) \\ \dfrac{1}{10}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ \dfrac{\sqrt{5}-1}{4\sqrt{(10)}}\,\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{\overline{10}})^2} \\ \dfrac{\sqrt{5}-1}{\sqrt{(40)}}\gamma \end{pmatrix}\begin{matrix}(27)\\[2ex](8_1)\\[2ex](8_2)\\[2ex](\overline{10})\\[2ex](8)\end{matrix}\,,$$

where $\eta^{(8)}(\frac{1}{2},1)=1+[(3-\sqrt{5})/40]\gamma^2$;

$T=\frac{1}{2}$, $Y=-1$; Ξ^*:

$$u^{(8)}(\tfrac{1}{2},-1)=\frac{1}{\eta^{(8)}(\frac{1}{2},-1)}\left[\begin{array}{c} 3\dfrac{\sqrt{5}+1}{20\sqrt{2}}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{27}}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ \eta^{(8)}(\frac{1}{2},-1) \\ \dfrac{1}{10}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{8_2}}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ -\dfrac{\sqrt{5}+1}{4\sqrt{(10)}}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{10}}{1-(\gamma_{8_1}/\gamma_{10})^2} \end{array}\right]\begin{array}{c} (27) \\ (8_1) \\ (8_2) \\ (10) \end{array},\qquad v^{(8)}(\tfrac{1}{2},-1)=\left[\begin{array}{c} 3\dfrac{\sqrt{5}+1}{20\sqrt{2}}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ 1/\eta^{(8)}(\frac{1}{2},-1) \\ \dfrac{1}{10}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ -\dfrac{\sqrt{5}+1}{4\sqrt{(10)}}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{10})^2} \\ -\dfrac{\sqrt{5}+1}{\sqrt{(40)}}\gamma \end{array}\right]\begin{array}{c} (27) \\ (8_1) \\ (8_2) \\ (10) \\ (8) \end{array},$$

where $\eta^{(8)}(\frac{1}{2},-1)=1+[(3+\sqrt{5})/40]\gamma^2$;

$T=1$, $Y=0$; Y_1^*:

$$u^{(8)}(1,0)=\frac{1}{\eta^{(8)}(1,0)}\left[\begin{array}{c} -\dfrac{\sqrt{3}}{10}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{27}}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ \eta^{(8)}(1,0) \\ -\dfrac{1}{10}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{8_2}}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ \dfrac{1}{2\sqrt{(10)}}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{10}}{1-(\gamma_{8_1}/\gamma_{10})^2} \\ \dfrac{1}{2\sqrt{(10)}}\dfrac{\gamma^2\gamma_{8_1}/\gamma_{\overline{10}}}{1-(\gamma_{8_1}/\gamma_{\overline{10}})^2} \end{array}\right]\begin{array}{c} (27) \\ (8_1) \\ (8_2) \\ (10) \\ (\overline{10}) \end{array},\qquad v^{(8)}(1,0)=\left[\begin{array}{c} -\dfrac{\sqrt{3}}{10}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{27})^2} \\ 1/\eta^{(8)}(1,0) \\ -\dfrac{1}{10}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{8_2})^2} \\ \dfrac{1}{2\sqrt{(10)}}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{10})^2} \\ \dfrac{1}{2\sqrt{(10)}}\dfrac{\gamma^2}{1-(\gamma_{8_1}/\gamma_{\overline{10}})^2} \\ \gamma/\sqrt{(10)} \end{array}\right]\begin{array}{c} (27) \\ (8_1) \\ (8_2) \\ (10) \\ (\overline{10}) \\ (8) \end{array},$$

where $\eta^{(8)}(1,0)=1+\gamma^2/20$.

The superscripts (1) and (8) on u and v refer to new broken-symmetry wave functions which are close to the pure **1** and $\mathbf{8}_1$, respectively. The wave functions u and v are normalized to $O(\gamma^2)$.

These wave functions yield the particle configurations in the two sectors when a specific choice of f and γ is made. The Yukawa mixing parameter f is commonly assigned a value $f\simeq 0.4$. Although this choice of f in the symmetric model leads to resonating octet and singlet multiplets, the octet configuration resonates $\sim$10 MeV below the singlet. Since the data indicate that the singlet is at a lower energy, the symmetric model must be viewed as representing the dynamics in only an approximate manner. Since the relative positions of the two resonating states depend on the specific choice of f, we will view f as a parameter to yield a reasonable ordering of the resonating states. In fact, $f=0.5$ yields a resonating singlet $\sim$50 MeV below the resonating octet. This arrangement is probably not far removed from a correct description of the symmetric dynamics, so that we will use $f=0.5$ as an artifice for ordering the symmetric levels **1** and **8**, and proceed with our calculations of the mixing.

The choice of γ is of a different nature. It is not directly accessible to experiment, and its value is dependent on the proper description of ϕ-ω mixing. However, it is reasonable to assume that it is bounded by $0<\gamma<1$, and, corresponding to this range, we can obtain a range of mixing angles for the $T=Y=0$ $\frac{3}{2}^-$ hadrons. In addition, the validity of the mixing calculation given here imposes an upper bound on γ; in particular, the diagonalization to $O(\gamma^2)$ is only valid if $y_j\leq x_j-x_i$. For $f=0.5$, γ can be as large as $\frac{2}{3}$ by this criterion. We will examine the results for $0<\gamma<1$, keeping this bound in mind.

We confine ourselves to the (T,Y) block where mixing is strong, i.e., (0,0), and consider only singlet-octet mixing. We define a mixing angle θ by

$$\begin{aligned} |\Lambda'(1700)\rangle &= \cos\theta|\Lambda^{(8)}\rangle_{BP}+\sin\theta|\Lambda^{(1)}\rangle_{BP}, \\ |\Lambda(1520)\rangle &= -\sin\theta|\Lambda^{(8)}\rangle_{BP}+\cos\theta|\Lambda^{(1)}\rangle_{BP}, \end{aligned} \tag{32}$$

where, from the wave functions $u^{(8)}(0,0)$ and $u^{(1)}(0,0)$ above,

$$\tan\theta=-\frac{1}{4\sqrt{5}}\frac{\gamma^2\gamma_{8_1}/\gamma_1}{1-(\gamma_{8_1}/\gamma_1)^2}. \tag{33}$$

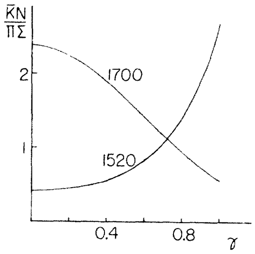

FIG. 6. Branching ratios $\Gamma(\bar{K}N)/\Gamma(\pi\Sigma)$ for $\Lambda(1520)$ and $\Lambda'(1700)$ as a function of γ.

With $f=0.5$, we have

$$\tan\theta = -0.41\gamma^2.$$

Thus we find that

$$\begin{aligned} \text{for}\quad \gamma&=0.2, & \theta&=-1°;\\ &=0.4, & &=-4°;\\ &=0.6, & &=-8°;\\ &=0.8, & &=-15°;\\ &=1.0, & &=-22°. \end{aligned} \tag{34}$$

For the bound $\gamma\sim\frac{2}{3}$, we have $\theta\sim-11°$. These mixing angles are obtained on the basis of a dynamical study of the $\frac{3}{2}^-$ hadrons, and they also span the values obtained by phenomenological analyses.[4,6] Figure 6 shows the branching ratio of $|\bar{K}N\rangle$ to $|\pi\Sigma\rangle$ decays [phase-space corrections as $(P_{\bar{K}N}/P_{\pi\Sigma})^5$] as a function of γ for $\Lambda(1520)$ and $\Lambda'(1700)$. The dramatic departure from the $SU(3)$-symmetric branching ratios *toward* the experimental results[7] is the most striking conclusion to observe from the figure.[8] We note, in particular, that the sign of θ obtained from our model is negative and that this is crucial to correct the $SU(3)$ results in the right direction. Present experimental knowledge[7] of these branching ratios is as follows:

$$\left.\frac{\Gamma(\bar{K}N)}{\Gamma(\pi\Sigma)}\right|_{\Lambda(1520)}=\frac{45\pm4}{45\pm4}$$

and

$$\left.\frac{\Gamma(\bar{K}N)}{\Gamma(\pi\Sigma)}\right|_{\Lambda'(1700)}=\frac{25}{35}.$$

In our analysis of $\frac{3}{2}^-$ mixing, we have considered γ to be a parameter which characterizes $SU(3)$ mixing in the basis states, but in fact γ is calculable, at least in principle, from a study of ϕ-ω mixing.[1] In such a calculation, γ characterizes ϕ-ω mixing and vector-meson mass shifts. Remarkably enough, the characteristics of the vector mesons can be reproduced when $\gamma=0.78$, a value which reproduces both branching ratios rather well.

The results in Eq. (34), as we have indicated, ignore contributions from **27** and $\mathbf{8}_2$. This is consistent with the spirit of the analysis here, and, in particular, these contributions represent corrections to mixing from the nonresonant background. Since we have already neglected such nonresonant background in the form of neglected states irrelevant to the dynamics, we must also neglect background contributions from **27** and $\mathbf{8}_2$. Nevertheless, to indicate the size of the effects involved we give, numerically, the two wave functions of interest using $f=0.5$:

$$\begin{aligned} u^{(1)}(0,0)&=\frac{1}{1+0.06\gamma^2}\begin{array}{c}\begin{bmatrix}0.11\gamma^2\\0.41\gamma^2\\0.02\gamma^2\\1\end{bmatrix}\end{array}\begin{array}{c}(27)\\(8_1)\\(8_2)\\(1)\end{array},\\ u^{(8)}(0,0)&=\frac{1}{1+0.05\gamma^2}\begin{array}{c}\begin{bmatrix}0.11\gamma^2\\1\\0.02\gamma^2\\-0.41\gamma^2\end{bmatrix}\end{array}\begin{array}{c}(27)\\(8_1)\\(8_2)\\(1)\end{array}. \end{aligned} \tag{35}$$

Thus we see that **27** is a 10% correction to singlet-octet mixing, while $\mathbf{8}_2$ is completely negligible.

IV. CONCLUSION

We have endeavored in this paper to develop an S-matrix approach to $SU(3)$ multiplet mixing. Although certain aspects of such a development are scattered throughout the literature, generally in other contexts, little effort has been devoted to the role of inelastic states or to strong mixing effects among the particles in basis states of the S matrix itself. Both of these effects play a crucial role in S-matrix discussions of multiplet mixing, and both of them can lead to large mixing effects in resonating states without producing large mass shifts. The principal goal, then, of such a discussion of mixing is to understand the roles played by various aspects of broken $SU(3)$ in the structure of resonances and ultimately to calculate branching ratios on the basis of dynamical models.

In practical applications of the notion of particle mixing, there is a tendency to speak rather loosely of the multiplet structure of a resonance to the extent that the resonance or particle itself is endowed with an intrinsic mixture of $SU(3)$ representations. The composite view of particles implies that this usage is incorrect. We have treated an example in which compositeness calls for more than a single sector of scattering states in order to explicate what we believe to be a more accurate description. Once the symmetry is broken, a resonance or particle has no intrinsic multiplet structure, and it cannot be described by a single mixing angle even when one neglects all but the dominant mixing

[6] E. Golowich, Phys. Rev. **177**, 2295 (1969).
[7] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).
[8] A similar effect has been obtained in the symmetric quark model analysis of D. R. Divgi and O. W. Greenberg, Phys. Rev. **175**, 2024 (1968). See, in particular, D. R. Divgi, *ibid.* **175**, 2027 (1968).

effects (say, octet-singlet mixing). The composite particle has a wave function with components in several sectors. The multiplet mixing varies from one sector to another, e.g., B_8P_8 and B_8V_8, and each sector must be separately specified. It is true that many resonances can only decay, because of energetics, into but one open sector, and in this case a resonance can be represented by a single mixing angle to determine branching ratios. Generally, however, this is not the case, and there is no reason to suppose that such a simple description holds for the decays of high-mass baryons, for which there are several open sectors.

As indicated above, one of our reasons for formulating an S-matrix approach to mixing was the availability of dynamical models for various resonances. The details of the $\frac{3}{2}^-$ system are given in Sec. III, but two points should be emphasized. First, without including any effects other than strong mixing in the basis states in an intrinsic, coupled-sector problem, we have been able to produce a dramatic improvement over pure $SU(3)$ in the agreement between the theoretical and experimental branching ratios for the $T=Y=0$ elements. Second, the use of a mixing angle for these elements must be motivated in a more logical way than the usual phenomenological methods provide. To be specific, if we assume, as we did in Sec. III, that the resonance positions, in exact $SU(3)$, are separated by $\sim$50 MeV and observe that the full widths for both are 40 MeV, we see there is very little overlap of the resonances. A conventional description based on the diagonalization of a two-level Hamiltonian leads to a mixing angle, but this method presupposes a quasidegeneracy of the two levels. When we are confronted with nonoverlapping resonances, it is clear that mixing, parametrized by a single angle, must be defended in some other way. We have developed such a parametrization for the coupling of the $\Lambda(1520)$, $\Lambda'(1700)$ system to the B_8P_8 states as a consequence of the nature of the force structure (Fig. 3). The energy dependence exhibited in Fig. 3 was a natural dynamical assumption to make. Such a Born matrix admits energy-independent transformations of the basis, and a description of the coupling to B_8P_8 in terms of a single mixing angle then emerges as the dominant effect. That a different angle is relevant for the B_8V_8 sector is an inescapable consequence of our procedure.

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Mass Relations for Mixed $SU(3)$ Supermultiplets

L. GOMBEROFF AND V. TOLMACHEV*
Department of Physics, Faculty of Science, University of Chile, Santiago, Chile
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It is shown that mixing among $SU(3)$ supermultiplets leads to new mass relations. These mass relations could be useful for the interpretation of some experiments and for further particle assignments to $SU(3)$ supermultiplets. In this context, some possibilities are discussed for the $\frac{5}{2}^-$, $\frac{5}{2}^+$ baryons and 1^+ mesons.

I. INTRODUCTION

THE idea of mixing of some $SU(3)$ supermultiplets arose originally in connection with the so-called ϕ-ω mixing. These two particles seem to belong to the mixed $\{8\}+\{1\}$ nonet of vector mesons 1^-.[1–3] It has been suggested that a similar mixing could take place in the case of 2^+ mesons and that the $\frac{3}{2}^-$ baryons could belong to either $\{8\}+\{1\}$, $\{8\}+\{10\}$, or $\{8\}+\{27\}$ mixed representations of $SU(3)$.[4] More recently, the mixing of two octets has been proposed for the assignment of the 1^+ mesons.[5]

The necessity to incorporate mixing in the scheme of unitary symmetry is dictated primarily by our wish to avoid the difficulties which exist in the theory concerning the assignment of all mesons and baryons to definite $SU(3)$ supermultiplets. The present situation seems to be very satisfactory for the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryons and 0^-, 1^- mesons. It also seems to be satisfactory for the particles with spin-parity $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{7}{2}^-$, $\frac{7}{2}^+$, and 2^+, but very poor for $\frac{1}{2}^-$ and 1^+.

The experimental situation[6] is such that it is impossible to think of just one $SU(3)$ supermultiplet to which all well-established isospin multiplets of $\frac{1}{2}^-$, $\frac{3}{2}^-$, and 1^+ particles could separately belong. Furthermore, in the case of $\frac{1}{2}^-$ baryons, for example, it is necessary to think in terms of at least three $SU(3)$ supermultiplets.

We stress two points which in our opinion indicate unavoidably the necessity to exploit the idea of mixing in order to understand some present, and possible future, experiments.

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* On leave of absence from Moscow University, Moscow, U.S.S.R.
[1] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).
[2] S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).
[3] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).
[4] Y. Ne'eman, *Algebraic Theory of Particle Physics* (W. A. Benjamin, Inc., New York, 1967), p. 108.
[5] H. J. Lipkin, Phys. Rev. **176**, 1709 (1968).
[6] A. H. Rosenfeld *et al.*, University of California Lawrence Radiation Laboratory Report No. UCRL-8030, 1968 (unpublished).