Calculation of the Kaon Electromagnetic Mass Difference Based on the Veneziano Representation*†

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An expression for the isovector part of the electromagnetic mass shift of the K mesons is derived from the difference of the first-order (in the fine-structure constant) electromagnetic self-energies of the positive and neutral kaons. Vector-meson dominance is used to relate this part of the mass shift to strong-interaction processes. A simplified Veneziano representation (including only one Regge trajectory in each channel) is written for these strong-interaction processes. The results are presented in terms of a cutoff on the four-momentum carried by the virtual photon producing the first-order mass shift. A relatively low cutoff value of about 3 BeV is required to yield the experimental mass difference.

I. INTRODUCTION

A WIDELY used approach for calculating the electromagnetic mass difference of hadrons within the same isotopic spin multiplet is to relate it to the forward Compton scattering amplitude for virtual photons.[1] This amplitude is then computed using techniques such as current algebra[2] or dispersion relations.[3] The fact that calculations of the kaon mass difference based on this approach have been less successful than similar calculations for pions was partially clarified by Harari,[4] who suggested that the strong-interaction processes which must be considered for $\Delta I=1$ mass differences have different high-energy behavior from those required for $\Delta I=2$ mass differences.

In the calculation reported here, the above approach is followed up to the point where the forward Compton amplitude is obtained. Then vector-meson dominance is used so that the relevant strong-interaction processes are exhibited explicitly. Finally, these processes are given an analytic form by using a simplified Veneziano representation[5] so that the high-energy behavior should be approximately correct. For completeness, the derivation of the mass-difference expression in Sec. II starts from the electromagnetic self-energy (to first order in the fine-structure constant) of the positive and neutral kaons. The numerical evaluation of the resulting integrals and the results are presented in Sec. III, and Sec. IV contains a discussion.

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[1] See, e.g., Riazuddin, Phys. Rev. **114**, 1184 (1959); also, V. Barger and E. Kazes, Nuovo Cimento **28**, 385 (1963).

[2] See, e.g., K. Tanaka, Nuovo Cimento **56A**, 764 (1968); also, C. L. Cook, L. E. Evans, M. Y. Han, N. R. Lipshutz, and N. Straumann, Phys. Rev. Letters **20**, 295 (1968).

[3] See, e.g., K. Yamamoto, Phys. Rev. **168**, 1677 (1968).

[4] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

[5] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

II. DERIVATION OF MASS-DIFFERENCE EXPRESSION

A. Notation and Kinematics

The kinematics for the two types of processes which are considered in this investigation are presented in this section. The first type of process, shown in Fig. 1, is the emission and reabsorption of a virtual photon from a kaon. The four-momenta of the kaon and photon will be denoted by p and q, respectively. The scattering of vector particles (either photons or vector mesons) off kaons, shown in Fig. 2, is the second type of process. Again the momenta will be labeled by p and q for the kaons and vector particles with subscripts 1 and 2 referring to initial and final particles, respectively. The polarization of the vector particles is denoted by λ. The kaon mass is M and the metric tensor is given by $g^{00}=1$, $g^{ii}=-1$ $(i=1, 2, 3)$, so that $p^2=M^2$.

Other kinematic variables which will be used include those introduced by Mandelstam,

$$\begin{aligned} s&=(p_1+q_1)^2=(p_2+q_2)^2\,,\\ t&=(p_1-p_2)^2=(q_1-q_2)^2\,,\\ u&=(p_1-q_2)^2=(p_2-q_1)^2\,, \end{aligned} \tag{2.1}$$

and the laboratory energy of the vector particle ν,

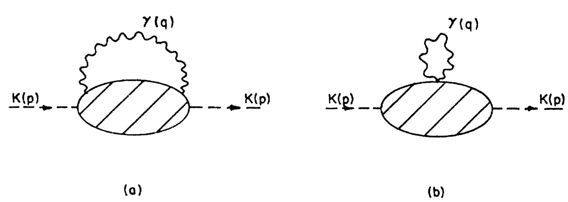

FIG. 1. Self-energy diagrams.

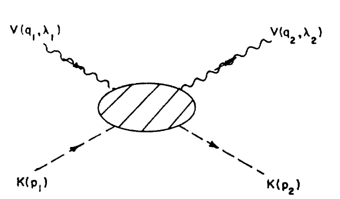

FIG. 2. Vector-particle–kaon scattering.

where

$$\nu = p_1 \cdot q_1/M = (s-u-t)/4M = (s-M^2-q^2)/2M - t/4M. \quad (2.2)$$

Thus, in the forward direction ($t=0$), we have

$$s = M^2+q^2+2M\nu, \quad u = M^2+q^2-2M\nu. \quad (2.3)$$

B. Electromagnetic Self-Energy of Kaons

The electromagnetic self-energy of mesons, to first order in e^2, is determined from the Feynman graphs of Fig. 1.[5] However, as pointed out by Das *et al.*,[6] the "contact" term (in which the virtual photon is emitted and reabsorbed at the same space-time point) can be eliminated if the calculation is done in the gauge introduced by Fried and Yennie.[7] In this gauge, only the diagram of Fig. 1(a) contributes, so that the electromagnetic self-energy of the positive (neutral) kaon is given by

$$\delta E^{+(0)} = -\frac{i}{2}\int \frac{d^4q}{(2\pi)^4} \frac{g^{\mu\nu}-4q^\mu q^\nu/q^2}{q^2+i\epsilon}[T_{\mu\nu}{}^{+(0)} - \hat{T}_{\mu\nu}], \quad (2.4)$$

where

$$T_{\mu\nu}{}^{+(0)} = -i(2\pi)^3\int d^4x \times e^{-iq\cdot x}\langle K^{+(0)}|T(J_\mu{}^\gamma(x)J_\nu{}^\gamma(0))|K^{+(0)}\rangle,$$

$$\hat{T}_{\mu\nu} = -i(2\pi)^3\int d^4x\, e^{-iq\cdot x}\langle 0|T(J_\mu{}^\gamma(x)J_\nu{}^\gamma(0))|0\rangle.$$

In these expressions, the superscript γ indicates that the current is an electromagnetic one. In the calculation of the mass difference, the vacuum expectation value of the current drops out, and therefore it will not be considered further.

The isoscalar and isovector components of the electromagnetic mass shift are defined by

$$\delta M_{K^+} = \delta M_S + \delta M_V, \quad \delta M_{K^0} = \delta M_S - \delta M_V.$$

If it is assumed that the positive and neutral kaon masses are degenerate in the absence of electromagnetic interactions, then the mass difference is given by

$$\Delta M^2 = 2M(\delta M_{K^+} - \delta M_{K^0}) = 2E(\delta E^+ - \delta E^0) = 4M\delta M_V.$$

It can be shown by using the Wigner-Eckart theorem that the isovector mass shift depends only on the combination of an isovector component and an isoscalar component of the electromagnetic current, so that (2.4) becomes

$$\Delta M^2 = -i\int \frac{d^4q}{(2\pi)^4}\frac{g^{\mu\nu}-4q^\mu q^\nu/q^2}{q^2+i\epsilon}T_{\mu\nu}{}^V, \quad (2.5)$$

where

$$T_{\mu\nu}{}^V = -2iE(2\pi)^3\int d^4x \times e^{-iq\cdot x}\{\langle K^+|T(J_\mu{}^V(x)J_\nu{}^S(0))|K^+\rangle + \langle K^+|T(J_\mu{}^S(x)J_\nu{}^V(0))|K^+\rangle\} = -4iE(2\pi)^3\int d^4x\,\langle K^+|(J_\mu{}^V(x)J_\nu{}^S(0))|K^+\rangle.$$

The superscripts on the currents refer to the isospin component and $\epsilon^\mu T_{\mu\nu}{}^V\epsilon^{\nu*}$ is the invariant amplitude for the forward Compton scattering[8] of virtual photons off kaons where one photon is an isovector one and the other is an isoscalar one.

If $\epsilon^\mu \bar{T}_{\mu\nu}{}^V\epsilon^{\nu*}$ is defined as the invariant amplitude for Compton scattering (not necessarily in the forward direction as indicated by the bar), then it can be expanded in terms of tensor amplitudes:

$$\bar{T}_{\mu\nu}{}^V = \bar{A}_1{}^\gamma P_\mu P_\nu + \bar{A}_2{}^\gamma(P_\mu Q_\nu + Q_\mu P_\nu) + \bar{A}_3{}^\gamma Q_\mu Q_\nu + \bar{A}_4{}^\gamma g_{\mu\nu}, \quad (2.6)$$

where

$$P = \tfrac{1}{2}(p_1+p_2), \quad Q = \tfrac{1}{2}(q_1+q_2), \quad (2.7)$$

and the $\bar{A}_i{}^\gamma$ are functions of q^2, ν, and t. In the forward direction, $\bar{T}_{\mu\nu}{}^V$ is equal to $T_{\mu\nu}{}^V$, so that

$$T_{\mu\nu}{}^V = A_1{}^\gamma p_\mu p_\nu + A_2{}^\gamma(p_\mu q_\nu + q_\mu p_\nu) + A_3{}^\gamma q_\mu q_\nu + A_4{}^\gamma q_{\mu\nu}, \quad (2.8)$$

with

$$A_i{}^\gamma(q_z,\nu) = \bar{A}_i{}^\gamma(q^2,\nu,t=0).$$

Time reversal and parity invariance were used in these expansions and now gauge invariance is invoked,[9] so that (2.8) becomes

$$T_{\mu\nu}{}^V = (M^2/q^2)A_1{}^\gamma(q^2,\nu) \times[\nu^2 g_{\mu\nu} + (q^2/M^2)p_\mu p_\nu - (\nu/M)(p_\mu q_\nu + q_\mu p_\nu)] - A_3{}^\gamma(q^2,\nu)[q^2 g_{\mu\nu} - q_\mu q_\nu]. \quad (2.9)$$

Before this is inserted into the mass-difference expression (2.5), an approximate relation for the tensor amplitudes $A_1{}^\gamma$ and $A_3{}^\gamma$ will be derived using vector-meson dominance and the Veneziano model.

C. Vector-Meson Dominance

The theory of vector-meson dominance (VMD) of the electromagnetic interaction[10] will now be used to relate the Compton scattering to the strong interactions

[6] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).

[7] H. M. Fried and D. R. Yennie, Phys. Rev. 112, 1391 (1958).

[8] This amplitude is related to the S matrix for Compton scattering by $S_{fi} = \delta_{fi} - ie^2(2\pi)^4\delta^4(p_1+q_1-p_2-q_2)\epsilon^\mu T_{\mu\nu}{}^V\epsilon^{\nu*}$.

[9] The author is indebted to Professor R. T. Torgerson for pointing out that gauge invariance for off-shell photons is valid within certain models such as the algebra-of-fields model.

[10] Y. Nambu, Phys. Rev. 106, 1366 (1957); J. J. Sakurai, Ann. Phys. (N. Y.) 11, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

$\rho+K\rightarrow\varphi+K$ and $\rho+K\rightarrow\omega+K$ as shown diagrammatically in Fig. 3. If $T_{\mu\nu}{}^{v}$ (where v refers to φ or ω) is defined by

$$T_{\mu\nu}{}^{v}=-2iE(2\pi)^3\int d^3x\, e^{-iq\cdot x}\langle K|T(J_\mu{}^\rho(x)J_\nu{}^v(0))|K\rangle\,,$$

then VMD says[11]

$$T_{\mu\nu}{}^{V}=2(C^{(\varphi)}T_{\mu\nu}{}^{\varphi}+C^{(\omega)}T_{\mu\nu}{}^{\omega})\,, \tag{2.10}$$

where $C^{(\varphi)}$ and $C^{(\omega)}$ are model-dependent.

For this calculation, the "current-mixing" model for the system is used because it presently seems to agree with experiment better than various "mass-mixing" models.[12] Thus, $C^{(\varphi)}$ and $C^{(\omega)}$ are given by

$$\begin{aligned} C^{(\varphi)}&=\frac{e^2}{2f_Yf_\rho}\cos\theta_Y\frac{m_\rho{}^2}{q^2-m_\rho{}^2}\frac{m_\varphi{}^2}{q^2-m_\varphi{}^2}\,,\\ C^{(\omega)}&=-\frac{e^2}{2f_Yf_\rho}\sin\theta_Y\frac{m_\rho{}^2}{q^2-m_\rho{}^2}\frac{m_\omega{}^2}{q^2-m_\omega{}^2}\,, \end{aligned} \tag{2.11}$$

where θ_Y is an angle which is used to relate the source currents for the φ and ω fields to the hypercharge current.

The couplings of these currents to a K meson, which will be needed later in the consideration of the kaon pole term, is also given in this model. The coupling of the ρ current to kaons, $F_{KK}{}^\rho(q^2)$, is considered first. It is broken up into the coupling at zero momentum transfer $F_{KK}{}^\rho(0)$ and form factor $f_{KK}{}^\rho(q^2)$:

$$F_{KK}{}^\rho(q^2)=F_{KK}{}^\rho(0)f_{KK}{}^\rho(q^2)\,,$$

with

$$f_{KK}{}^\rho(0)=1\,.$$

Now, as shown by Kroll, Lee, and Zumino,[11] $F_{KK}{}^\rho(0)$ is proportional to the isospin of the states to which it is connected; thus

$$F_{KK}{}^\rho(0)=g_{\rho KK}I_z(K)\,.$$

Finally, by universality, the proportionality constant is

$$g_{\rho KK}=f_\rho\,,$$

so that, for the positive kaon,

$$F_{K^+K^+}{}^\rho(q^2)=\tfrac{1}{2}f_\rho f_{KK}{}^\rho(q^2)\,. \tag{2.12a}$$

In the same fashion, the φ and ω currents are related to the hypercharge and baryon currents, so that

$$F_{KK}{}^\varphi(q^2)=f_Y\frac{\cos\theta_N}{\cos(\theta_Y-\theta_N)}f_{KK}{}^\varphi(q^2) \tag{2.12b}$$

and

$$F_{KK}{}^\omega(q^2)=-f_Y\frac{\sin\theta_N}{\cos(\theta_Y-\theta_N)}f_{KK}{}^\omega(q^2)\,, \tag{2.12c}$$

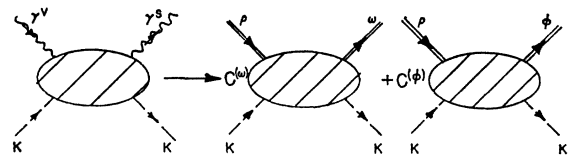

FIG. 3. Relation of electromagnetic and strong interactions by vector-meson dominance.

where θ_N relates the φ and ω field source currents to the baryon current in the same way that θ_Y related them to the hypercharge current.

Finally, we note that if the invariant amplitude for the strong interaction $\rho+K\rightarrow v+K$ is expanded in terms of tensor amplitudes:

$$\bar{T}_{\mu\nu}{}^{v}=\bar{A}_1{}^vP_\mu P_\nu+\bar{A}_2{}^v(P_\mu Q_\nu+Q_\mu P_\nu)+\bar{A}_3{}^vQ_\mu Q_\nu+\bar{A}_4{}^vg_{\mu\nu}\,, \tag{2.13}$$

then the tensor amplitudes for Compton scattering can be related to these by VMD. Comparison of Eqs. (2.8), (2.10), and (2.13) shows

$$\bar{A}_i{}^\gamma(q^2,\nu,t)=2[C^{(\varphi)}(q^2)\bar{A}_i{}^\varphi(q^2,s,t)+C^{(\omega)}(q^2)\bar{A}_i{}^\omega(q^2,s,t)]\,. \tag{2.14}$$

D. Veneziano Model Applied to $\rho+K\rightarrow v+K$

A model recently introduced by Veneziano[5] for the scattering of physical particles has been successful in explaining some physical scattering results.[13] In order to write down this representation of the amplitudes, their asymptotic behavior must be known. Therefore, the general scattering case is considered and written in terms of tensor amplitudes (as in the Compton-scattering case). Thus,

$$\bar{T}_{\mu\nu}{}^{v}=\bar{A}_1{}^vP_\mu P_\nu+\bar{A}_2{}^v(P_\mu Q_\nu+Q_\mu P_\nu)+\bar{A}_3{}^vQ_\mu Q_\nu+\bar{A}_4{}^vg_{\mu\nu}\,, \tag{2.15}$$

where P and Q are defined as in the Compton case [see Eq. (2.7)]. Using the method of de Alfaro *et al.*,[14] it can be shown that these amplitudes have the following asymptotic properties as $s\rightarrow\infty$:

$$\begin{aligned} &\bar{A}_1{}^v(s,t)\sim s^{\alpha_L(t)-2}\,,\\ &\bar{A}_2{}^v(s,t)\sim s^{\alpha_L(t)-1}\,,\\ &\bar{A}_3{}^v(s,t),\quad \bar{A}_4{}^v(s,t)\sim s^{\alpha_L(t)}\,, \end{aligned}$$

where $\alpha_L(t)$ is the leading Regge trajectory in the crossed (t) channel. The determination of the asymptotic t behavior is accomplished by writing the amplitude for $\rho(q_1)+v(-q_2)\rightarrow K(p_2)+\bar{K}(-p_1)$ as

$$T_{\mu\nu}{}'=B_1(P_\mu{}'P_\nu{}'+Q_\mu{}'Q_\nu{}')+B_2P_\mu{}'Q_\nu{}'+B_3Q_\mu{}'P_\nu{}'+B_4g_{\mu\nu}\,,$$

[11] J. J. Sakurai, *Currents and Mesons* (The University of Chicago Press, Chicago, 1969), p. 48; N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[12] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

[13] See, e.g., C. Lovelace, Phys. Letters **28B**, 264 (1968).

[14] V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, Phys. Letters **21**, 576 (1966); V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, Ann. Phys. (N. Y.) **44**, 165 (1967).

where

$$P'=\tfrac{1}{2}(p_1+q_2)\,,$$
$$Q'=\tfrac{1}{2}(p_2+q_1)\,.$$

The relations between the $\bar{A}_i^v$ and the B_i which are determined by expanding P, Q, P', and Q' in terms of p_1, q_1, and q_2 are

$$\begin{aligned}\bar{A}_1^v&=\tfrac{1}{4}(2B_1+B_2+B_3)\,,\\ \bar{A}_2^v&=\tfrac{1}{4}(2B_1+3B_2-B_3)\,,\\ \bar{A}_3^v&=\tfrac{1}{4}(-6B_1+9B_2+B_3)\,,\\ \bar{A}_4^v&=B_4\,.\end{aligned}$$

For large t, the B_i's behave similarly to the $\bar{A}_i^v$'s for large s, and since the leading term dominates, it is apparent that for large t,

$$\bar{A}_1^v(s,t),\ \bar{A}_2^v(s,t),\ \bar{A}_3^v(s,t),\ \bar{A}_4^v(s,t)\sim t^{\alpha_L(s)}\,,$$

where $\alpha_L(s)$ is the leading s-channel trajectory.

The Veneziano representation of the amplitudes should include all trajectories which can contribute in the various channels. An investigation of the crossed reaction $\rho+v\rightarrow K+\bar{K}$ shows that any intermediate state in the t channel must have the quantum numbers $P=(-)^J$ and $I^G=1^-$. Since the only known meson satisfying these requirements is the A_2 with $J^P=2^+$, only one Regge trajectory [which is denoted by $\alpha_A(t)$] seems necessary in this channel. However, in the s channel, there are several possible contributors and at this point an approximation is made: *Only the trajectory on which the kaon lies will be considered.* We also use the customary approximation that this trajectory is real and rises linearly with s:

$$\alpha_K(s)=\alpha_K(0)+\alpha_K's=\alpha_K'(s-M^2)\,.$$

Only the two tensor amplitudes $\bar{A}_1^v$ and $\bar{A}_3^v$ are needed in this calculation; we therefore write

$$\begin{aligned}\bar{A}_1^v(s,t)=(\bar{\beta}_1^v/\pi)[&\mathrm{B}(-\alpha_K(s),\,2-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(u),\,2-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(s),\,-\alpha_K(u))]\,,\\ \bar{A}_3^v(s,t)=(\bar{\beta}_3^v/\pi)[&\mathrm{B}(-\alpha_K(s),\,-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(u),\,-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(s),\,-\alpha_K(u))]\,,\end{aligned}\tag{2.16}$$

where $\mathrm{B}(x,y)$ is the Euler beta function and $\bar{\beta}_i^v$ is a reduced residue function, which is independent of s and t. The value of this residue function is determined by comparing the Veneziano representation and the perturbation expansion at a particular point, namely, the s-channel kaon pole. The perturbation amplitude with a vector-meson–kaon–kaon interaction Hamiltonian, given by

$$\mathcal{H}_I=F_{KK^v}\mathcal{V}^\mu K\overleftrightarrow{\partial}_\mu K^*\,,$$

has an s-channel kaon pole term[15]

$$T_{\mu\nu}{}^P=\frac{4F_{KK^\rho}F_{KK^v}}{s-M^2}[p_{1\mu}p_{1\nu}+p_{1\mu}q_{1\nu}]\,,$$

and in terms of the momenta p_1, q_1, and q_2, the amplitude in (2.15) becomes

$$\begin{aligned}\bar{T}_{\mu\nu}{}^v=\bar{A}_1^vp_{1\mu}p_{1\nu}&+\tfrac{1}{2}(\bar{A}_1^v+\bar{A}_2^v)p_{1\mu}q_{1\nu}+\tfrac{1}{2}(-\bar{A}_1^v+\bar{A}_2^v)q_{2\mu}p_{1\mu}\\&+\tfrac{1}{4}(-\bar{A}_1^v+\bar{A}_3^v)q_{2\mu}q_{1\nu}+\bar{A}_4^vg_{\mu\nu}\,,\end{aligned}$$

where the conservation of the vector-meson fields has been used (i.e., $q_{1\mu}=q_{2\nu}=0$). Thus, we have

$$\begin{aligned}\bar{A}_1^v(s=M^2,\,t)&=\frac{\bar{\beta}_1^v}{\pi}[2\Gamma(\alpha_K(s)=0)]\\&=-\frac{2\bar{\beta}_1^v}{\pi}\frac{1}{\alpha_K(s)}=-\frac{2\bar{\beta}_1^v}{\pi\alpha_K'}\frac{1}{s-M^2}\,,\\ \bar{A}_3^v(s=M^2,\,t)&=(\bar{\beta}_3^v/\pi)[2\Gamma(\alpha_K(s)=0)]\,,\end{aligned}$$

so that

$$\bar{\beta}_1^v=\bar{\beta}_3^v\equiv\bar{\beta}^v=-2\pi\alpha_K'F_{KK^\rho}F_{KK^v}\,.\tag{2.17}$$

Thus the Veneziano representation of the amplitudes $\bar{A}_1^v(s,t)$ and $\bar{A}_3^v(s,t)$ becomes

$$\begin{aligned}\bar{A}_1^v(s,t)=-2\alpha_K'F_{KK^\rho}F_{KK^v}[&\mathrm{B}(-\alpha_K(s),\,2-\alpha_K(t))\\&+\mathrm{B}(-\alpha_K(u),\,2-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(s),\,-\alpha_K(u))]\,,\\ \bar{A}_3^v(s,t)=-2\alpha_K'F_{KK^\rho}F_{KK^v}[&\mathrm{B}(-\alpha_K(s),\,-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(u),\,-\alpha_A(t))\\&+\mathrm{B}(-\alpha_K(s),\,-\alpha_K(u))]\,.\end{aligned}\tag{2.18}$$

E. Application of the Cottingham Procedure

The work of the previous sections shows that the squared mass difference can be written as

$$\begin{aligned}\Delta M^2=-2i\int\frac{d^4q}{(2\pi)^4}\frac{g^{\mu\nu}-4q^\mu q^\nu/q^2}{q^2+i\epsilon}&\\ \times\{C^{(\varphi)}[A_1{}^\varphi(q^2,s)\mathfrak{F}_{1,\mu\nu}(q^2,\nu)-A_3{}^\varphi(q^2,s)\mathfrak{F}_{3,\mu\nu}(q^2,\nu)]&\\ +C^{(\omega)}[A_1{}^\omega(q^2,s)\mathfrak{F}_{1,\mu\nu}(q^2,\nu)&\\ -A_3{}^\omega(q^2,s)\mathfrak{F}_{3,\mu\nu}(q^2,\nu)]\}\,,&\end{aligned}\tag{2.19}$$

where $A_i^v(q^2,s)=\bar{A}_i^v(q^2,\,s,\,t=0)$ and the coupling constants F_{KK^ρ} and F_{KK^v} are given a q^2 dependence as in (2.12). The factors $\mathfrak{F}_{1,\mu\nu}(q^2,\nu)$ and $\mathfrak{F}_{3,\mu\nu}(q^2,\nu)$ are taken from Eq. (2.9):

$$\begin{aligned}\mathfrak{F}_{1,\mu\nu}(q^2,\nu)&=(M^2/q^2)[\nu^2g_{\mu\nu}+(q^2/M^2)p_\mu p_\nu\\&\qquad-(\nu/M)(p_\mu q_\nu+q_\mu p_\nu)]\,,\\ \mathfrak{F}_{3,\mu\nu}(q^2,\nu)&=q^2g_{\mu\nu}-q_\mu q_\nu\,.\end{aligned}\tag{2.20}$$

[15] The vertex function is given by $\langle K(p)|J_\mu{}^\rho|K(p)\rangle=(2\pi)^{-3}\times(1/2\sqrt{EE'})(p+p')_\mu F_{KK^\rho}(q^2)$.

Now we note that [using Eq. (2.2)]

$$q^\mu \mathfrak{F}_{1,\mu\nu} = q^\mu \mathfrak{F}_{3,\mu\nu} = 0\,,$$

so that the second term in the virtual photon propagator may be dropped. Another way of looking at this is to note that the Veneziano model (at least in the approximation of real trajectories) generates intermediate states that are physical (i.e., on the mass shell). Since the kaons are also physical, the current must be conserved; thus we had from the beginning

$$q^\mu J_\mu = 0.$$

Without this second term in the propagator, Eq. (2.21) is very similar to the expression used by Cottingham[16] in the calculation of the neutron-proton mass difference. Thus, we use his method in which the integration contour of the energy variable is rotated 90° in the complex-energy plane so that it extends from $-\infty$ to $+\infty$ along the imaginary axis. The major advantage in this approach is that the integration then includes only spacelike virtual photons, so that the singularities introduced by VMD are avoided. The use of this technique on Eq. (2.19) yields

$$\Delta M^2 = \frac{1}{4\pi^3}\int_{-\infty}^{0}\frac{dq^2}{q_2}\left\{C^{(\varphi)}\int_{-\surd(-q^2)}^{\surd(-q^2)} d\nu(-q^2-\nu^2)^{1/2}\right.$$
$$\times\left[M^2\left(1-\frac{2\nu_2}{q_2}\right)A_1{}^{\varphi}(q^2,\bar{s}) - 3q^2 A_3{}^{\varphi}(q^2,\bar{s})\right]$$
$$+C^{(\omega)}\int_{-\surd(-q^2)}^{\surd(-q^2)} d\nu(-q^2-\nu^2)^{1/2}$$
$$\left.\times\left[M_2\left(1-\frac{2\nu_2}{q_2}\right)A_1{}^{\omega}(q^2,\bar{s}) - 3q^2 A_3{}^{\omega}(q^2,\bar{s})\right]\right\}.$$

The bar over a variable (i.e., over s in this equation) means that ν should be replaced by $i\nu$ in the expansion of the variable; thus

$$\bar{s} = M^2 + q^2 + 2iM\nu\,,$$
$$\bar{u} = M^2 + q^2 - 2iM\nu\,.$$

Expanding the amplitudes and collecting terms yields

$$\Delta M^2 = -\frac{\alpha\alpha_K{}'}{2\pi^2}\int_{-\infty}^{0}\frac{dq^2}{q^2}\,\frac{m_\rho{}^2 f_{KK}{}^{\rho}(q^2)}{q^2-m_\rho{}^2}\left[\frac{m_\rho{}^2 f_{KK}{}^{\varphi}(q^2)}{q^2-m_\varphi{}^2}\,\frac{\cos\theta_Y\cos\theta_N}{\cos(\theta_Y-\theta_N)} + \frac{m_\omega{}^2 f_{KK}{}^{\omega}(q^2)}{q^2-m_\omega{}^2}\,\frac{\sin\theta_Y\sin\theta_N}{\sin(\theta_Y-\theta_N)}\right]$$
$$\times\int_{-\surd(-q^2)}^{\surd(-q^2)} d\nu(-q^2-\nu^2)^{1/2}\left\{M^2\left(1-\frac{2\nu^2}{q_2}\right)[\mathrm{B}(-\alpha_K(\bar{s}),\,2-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{u}),\,2-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{s}),\,-\alpha_K(\bar{u}))]\right.$$
$$\left.-3q^2[\mathrm{B}(-\alpha_K(\bar{s}),\,-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{u}),\,-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{s}),\,-\alpha_K(\bar{u}))]\right\}\,,\qquad(2.21)$$

where $\alpha = e^2/4\pi$ is the fine-structure constant. This is the final expression for the squared electromagnetic mass difference of the kaons in this model.

III. EVALUATION OF MASS DIFFERENCE

The expression for the squared mass difference derived in Sec. II is rewritten slightly for convenience as

$$\Delta M^2 = -\frac{\alpha\alpha_K{}'m_\rho{}^2}{2\pi^2\cos(\theta_Y-\theta_N)}(m_\varphi{}^2\cos\theta_Y\cos\theta_N\Delta M_\varphi{}^2 + m_\omega{}^2\sin\theta_Y\sin\theta_N\Delta M_\omega{}^2)\,,\qquad(3.1)$$

where

$$\Delta M_v{}^2 = \int_{-\infty}^{0}\frac{dq^2}{q^2}f^{(v)}(q^2)\int_{-\surd(-q^2)}^{\surd(-q^2)} d\nu(-q^2-\nu^2)^{1/2}$$
$$\times\left\{M^2\left(1-\frac{2\nu^2}{q^2}\right)[\mathrm{B}(-\alpha_K(\bar{s}),\,2-\alpha_A(0))\right.$$
$$+\mathrm{B}(-\alpha_K(\bar{u}),\,2-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{s}),\,-\alpha_K(\bar{u}))]$$
$$-3q^2[\mathrm{B}(-\alpha_K(\bar{s}),\,-\alpha_A(0))+\mathrm{B}(-\alpha_K(\bar{u}),\,-\alpha_A(0))$$
$$\left.+\mathrm{B}(-\alpha_K(\bar{s})-\alpha_K(\bar{u}))]\right\}\qquad(3.2)$$

and

$$f^{(v)}(q^2) = \frac{f_{KK}{}^{\rho}(q^2)}{q^2-m_\rho{}^2}\,\frac{f_{KK}{}^{v}(q^2)}{q^2-m_v{}^2}\,.\qquad(3.3)$$

The two parameters of the Regge trajectories which are required are taken to be[17]

$$\alpha_K{}' = 0.8\ \mathrm{BeV}^{-2},\quad \alpha_A(0) = \tfrac{1}{2}.$$

Now, we would like to write the Euler beta functions in terms of their integral representations, but since this representation is valid only if the real part of each argument is positive, the two beta functions having $-\alpha_A(0)$ for an argument must be changed. This is done by using the Γ-function representation; thus, for example,

$$\mathrm{B}(-\alpha_K(\bar{s}),\,-\tfrac{1}{2}) = \frac{\Gamma(-\alpha_K(\bar{s}))\Gamma(-\frac{1}{2})}{\Gamma(-\frac{1}{2}-\alpha_K(\bar{s}))}$$
$$=2[\alpha_K(\bar{s})+\tfrac{1}{2}]\frac{\Gamma(-\alpha_K(\bar{s}))\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}-\alpha_K(\bar{s}))}$$
$$=2[\alpha_K(\bar{s})+\tfrac{1}{2}]\mathrm{B}(-\alpha_K(\bar{s}),\,\tfrac{1}{2}).$$

[16] W. N. Cottingham, Ann. Phys. (N. Y.) 25, 424 (1963).

[17] M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969).

Now, the integral representation of the beta function,

$$\mathrm{B}(x,y)=\int_0^1 dt\, t^{x-1}(1-t)^{y-1},$$

can be used for all the beta functions in Eq. (3.2). Thus we have

$$\Delta M_v{}^2=\int_{-\infty}^{0}\frac{dq^2}{q^2}f^{(v)}(q^2)\int_{-\surd(-q^2)}^{\surd(-q^2)}d\nu(-q^2-\nu^2)^{1/2}$$
$$\times\Bigg\{M^2\left(1-\frac{2\nu^2}{q^2}\right)\left[\int_0^1 dt(1-t)^{1/2}t^{-\alpha_K'q^2-1}(t^{-2i\alpha_K'M\nu}+t^{2i\alpha_K'M\nu})+\int_0^1 dt\, t^{-\alpha_K q^2-1-2i\alpha_K'M\nu}(1-t)^{-\alpha_K'q^2-1+2i\alpha_K'M\nu}\right]$$
$$-3q^2\Bigg[2(\alpha_K'q^2+2i\alpha_K'M\nu+\tfrac{1}{2})\int_0^1 dt(1-t)^{-1/2}t^{-\alpha_K'q^2-1-2i\alpha_K'M\nu}+2(\alpha_K'q^2-2i\alpha_K'M\nu+\tfrac{1}{2})$$
$$\times\int_0^1 dt(1-t)^{-1/2}t^{-\alpha_K'q^2-1+2i\alpha_K'M\nu}+\int_0^1 dt\, t^{-\alpha_K'q^2-1-2i\alpha_K'M\nu}(1-t)^{-\alpha_K'q^2-1+2i\alpha_K'M\nu}\Bigg]\Bigg\},$$

where the first two terms within the first square bracket have been combined. Now the order of the ν and t integrations are interchanged and the ν integration is performed. This yields (with a change of variables[18])

$$\Delta M_v{}^2=-\int_0^\infty\frac{d\sigma^2}{\sigma^2}f^{(v)}(\sigma^2)\left[M^2(K_1+K_2)+\frac{2M^2}{\sigma^2}(K_3+K_4)+3\sigma^2(K_5+K_6+K_7)\right],\quad(3.4)$$

where

$$K_1(\sigma)=\frac{2\pi\sigma}{b}\int_0^\infty\frac{dz}{z}e^{-az}(1-e^{-z})^{1/2}J_1(b\sigma z),$$

$$K_2(\sigma)=\frac{2\pi\sigma}{b}\int_0^\infty\frac{dw}{w}\frac{e^{-aw}}{(1+e^{-w})^{2a}}J_1(b\sigma w),$$

$$K_3(\sigma)=2\pi\sigma^4\int_0^\infty dz e^{-az}(1-e^{-z})^{1/2}\times\left[\frac{J_1(b\sigma z)}{b\sigma z}-\frac{3J_2(b\sigma z)}{(b\sigma z)^2}\right],$$

$$K_4(\sigma)=2\pi\sigma^4\int_0^\infty dw\frac{e^{-aw}}{(1+e^{-w})^{2a}}\times\left[\frac{J_1(b\sigma w)}{b\sigma w}-\frac{3J_2(b\sigma w)}{(b\sigma w)^2}\right],\quad(3.5)$$

$$K_5(\sigma)=\frac{4\pi\sigma}{b}(\tfrac{1}{2}-a)\int_0^\infty\frac{dz}{z}(1-e^{-z})^{-1/2}e^{-az}J_1(b\sigma z),$$

$$K_6(\sigma)=-4\pi\sigma^2\int_0^\infty\frac{dz}{z}(1-e^{-z})^{-1/2}e^{-az}J_2(b\sigma z),$$

$$a=-\alpha_K'q^2=\alpha_K'\sigma^2,\quad b=2M\alpha_K',$$

and $J_n(x)$ is the Bessel function of the first kind and order n.

The remaining two integrations have been done numerically on an IBM 360 computer using existing quadrature subroutines.[19] Two functional forms for the form factors $f_{KK}{}^{\rho}(\sigma^2)$ and $f_{KK}{}^{v}(\sigma^2)$ were used. In the first case, both form factors were set equal to unity and in the second case, they were both described by

$$f_{KK}{}^{\rho}(\sigma^2)=f_{KK}{}^{v}(\sigma^2)=\Lambda^2/(\sigma^2+\Lambda^2).$$

The integrands K_5 and K_6 produce a divergence in the first case so that a cutoff was required. This cutoff was used as an upper limit for all seven integrals and the kaon mass difference was calculated for various values of it. The experimental kaon mass difference[20] of -3.93 MeV was obtained with a cutoff about 2.38 BeV. For the second case, all the integrands produce convergent integrals, so that the upper limit was taken well above the point where the plot of the integrand indicated a possible contribution. The value of Λ was used as a parameter for this case, and for $\Lambda\approx 2.82$ the experimental value of the mass difference was obtained.

IV. DISCUSSION

In the approach to electromagnetic mass differences used in this calculation, a cutoff is generally needed. An exception was the pion mass difference calculation by Das *et al.*,[6] which was done in the soft-pion limit. Later,

[18] The new variables are related to the old ones by: $\sigma^2=-q^2$, $z=-\ln t$, and $w=-\ln[t/(1-t)]$.

[19] These subroutines are described in *IBM Application Program H20-0205-3; System/360 Scientific Subroutine Pacakge (360A-CM-03X); Version III Programmer's Manual* (IBM Technical Publications Department, White Plains, N. Y., 1968), 4th ed.

[20] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Whol, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).

however, the same problem done with hard-pion techniques did require a cutoff.[21]

One possible physical explanation for the cutoff is given in the vector-dominance model, which says that the strong-interaction vertex (of the vector meson connecting to the hadron line) has an unknown form factor associated with it. Even though the concept of a cutoff (or effective cutoff Λ) is accepted, any derivation of its value from first principles is lacking. The empirical "double-pole" form of the nucleon form factors would suggest an effective cutoff in the range of the vector-meson masses (i.e., 0.7–0.8 BeV) for that case. Since this is smaller than the hadronic mass to which it connects, it has also been suggested that a cutoff in the neighborhood of the hadronic mass is reasonable.

The above discussion would suggest that, for K mesons, a reasonable cutoff would be in the range 0.5–0.8 BeV. However, previous calculations of the kaon electromagnetic mass difference have typically required cutoffs in the range of 20 BeV and higher.[22] Therefore, the present calculation shows a significant improvement since the cutoff has been reduced to 3.0–3.5 BeV. One might wonder if the inclusion of other s-channel trajectories such as the ones on which the K^* or K_A mesons lie would further decrease this value. Since the present calculation has a change of sign of the mass difference for a cutoff of about 1.5 BeV, it seems doubtful that the inclusion of these other trajectories would reduce the required cutoff to a "reasonable" value.

[21] M. B. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967).

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[22] The value needed by K. Tanaka in Ref. 10, for example, was about 18 BeV.

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Theory of Deep-Inelastic Lepton-Nucleon Scattering and Lepton-Pair Annihilation Processes.* I

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The structure functions for deep-inelastic lepton processes including (along with other hadron charges and SU_3 quantum numbers) $e^-+p\to e^-+$"anything," $e^-+e^+\to p+$"anything," $\nu+p\to e^-+$"anything," $\bar{\nu}+p\to e^++$"anything" are studied in the Bjorken limit of asymptotically large momentum and energy transfers, q^2 and $M\nu$, with a finite ratio $w\equiv 2M\nu/q^2$. A "parton" model is derived from canonical field theory for all these processes. It follows from this result that all the structure functions depend only on w, as conjectured by Bjorken for the deep-inelastic scattering. To accomplish this derivation it is necessary to introduce a transverse momentum cutoff so that there exists an asymptotic region in which q^2 and $M\nu$ can be made larger than the transverse momenta of all the partons that are involved. Upon crossing to the e^+e^- annihilation channel and deriving a parton model for this process, we arrive at the important result that the deep-inelastic annihilation cross section to a hadron plus "anything" is very large, varying with colliding e^-e^+ beam energy at fixed w in the same way as do point-lepton cross sections. General implications for colliding-ring experiments and ratios of annihilation to scattering cross sections and of neutrino to electron inelastic scattering cross sections are computed and presented. Finally, we discuss the origin of our transverse momentum cutoff and the compatibility of rapidly decreasing elastic electromagnetic form factors with the parton model constructed in this work.

I. INTRODUCTION

THE structure of the hadron is probed by the vector electromagnetic current in the physically observable processes of inelastic electron scattering and of inelastic electron-positron pair annihilation

(i) $e^-+p\to e^-+$"anything,"

(ii) $e^-+e^+\to p+$"anything."

It is also probed by the weak (vector and axial-vector) current in inelastic neutrino or antineutrino scattering

(iii) $\nu_l+p\to +$"anything," $l\equiv e$ or μ

(iv) $\bar{\nu}_l+p\to \bar{l}+$"anything."

In process (i), the scattered electron is detected at a fixed energy and angle, and "anything" indicates the sum over all possible hadron states. The two structure functions summarizing the hadron structure in (i) are

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