Compton Scattering and Regge Cuts

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Multiple Pomeranchuk-trajectory exchange is studied in nucleon Compton scattering as an alternative means to a wrong-signature fixed pole in order to obtain a constant total asymptotic cross section and avoid paradoxes involving the vector-meson-dominance model. It is argued that the multi-Pomeranchon branch points interfere constructively, and an estimate is made of their contributions. One finds that the inclusion of up to five-Pomeranchon branch points leads to some 40% of the total nucleon Compton cross section. The implications of these results for the Pomeranchuk trajectory are discussed.

I. INTRODUCTION

THE application of Regge-pole theory to processes involving photons, in particular Compton scattering, has led to difficulties which are essentially caused by the crossing properties of zero-mass particles.[1,2] For example, one would naively expect the Pomeranchuk trajectory to contribute to nucleon (or pion) Compton scattering, thus obtaining a constant total cross section at large energies. However, when one crosses the s-channel helicity amplitudes (the s channel is given by $\gamma N \rightarrow \gamma N$, t by $\gamma\gamma \rightarrow N\bar{N}$), which contribute to forward scattering into t-channel helicity amplitudes,[3] one finds a nonsense zero at $\alpha(t)=1$.[1,2] This, of course, implies that a Pomeranchuk trajectory with $\alpha(0)=1$ does not contribute to the forward scattering of real photons on nucleons.[4]

The above result, however, is true only if the effects of the third double spectral function (ρ_{su}) are negligible. Inclusion of these effects would allow for the existence of a fixed pole at $J=1$ in a wrong-signature amplitude,[5] which in itself would not contribute to the high-energy behavior because of the signature factor. In particular, it can occur in the residue of the Pomeranchuk trajectory, thereby allowing it to dominate the forward amplitude, and then the high-energy cross section would tend to a constant.

The existence of such a fixed pole is incompatible with the vector-meson-dominance model, since, for example, the photoproduction of transversely polarized ρ^0 mesons does not need a singular residue function for the Pomeranchuk trajectory.[2,6] A way out of this paradox is to include the effects of the third double spectral function by examining the contribution of branch points due to multiple Pomeranchuk-trajectory exchange.

In Sec. II a discussion is given of how the need for third double spectral function effects arises and how their inclusion by means of fixed poles or Regge cuts can give rise to a nonvanishing[7] asymptotic total cross section for Compton scattering.

Section III contains a discussion of the relative signs of the multiple Pomeranchuk-trajectory branch-point contributions by using a Reggeon diagram technique,[8] as well as a suggestion for avoiding the previously mentioned paradox from vector-meson dominance.

In Sec. IV, a numerical estimate is attempted of the effect of the Pomeranchuk branch points by means of an optical prescription. Finally, in Sec. V, the conclusions are summarized and discussed.

II. EFFECT OF THIRD DOUBLE SPECTRAL FUNCTION

The purpose of this section is to review briefly the difficulties encountered in the coupling of the Pomeranchuk trajectory to the two-photon system and show how the inclusion of a fixed pole or a branch point avoids them. For our purpose, it is sufficient to consider Compton scattering on pions. We have two s-channel helicity amplitudes $\langle 1|T^s|1\rangle$ and $\langle 1|T^s|-1\rangle$, of which only the former survives at $t=0$. Crossing into the t channel is very simple; indeed, the photon helicity is simply flipped:

$$\langle 1|T^s|1\rangle = \langle 0|T^t|1-1\rangle \equiv T_{+-}{}^t. \tag{2.1}$$

If one makes a partial-wave expansion of $T_{+-}{}^t$,[3] one finds that, since the leading terms of $d_{2,0}{}^J(\theta_t)$ vanish for $J=1$, it appears that a Pomeranchuk trajectory with $\alpha(0)=1$ cannot contribute to the total cross section. Then the total $\gamma\pi$ cross section goes to zero with increasing energy. However, as a consequence of the absence of bilinear unitarity for processes to lowest order in weak or electromagnetic interactions, one may have fixed poles at nonsense values of the angular momentum.[9] Also, for a nonzero third double spectral

[1] V. D. Mur, Zh. Eksperim. i Teor. Fiz. **44**, 2173 (1963); **45**, 1051 (1963) [English transls.: Soviet Phys.—JETP **17**, 1458 (1963); **18**, 727 (1964)].
[2] H. K. Shepard, Phys. Rev. **159**, 1331 (1967); H. D. I. Abarbanel and S. Nussinov, *ibid.* **158**, 1462 (1967).
[3] T. L. Trueman and G. C. Wick, Ann. Phys. (N.Y.) **26**, 322 (1964); I. J. Muzinich, J. Math. Phys. **5**, 1418 (1964); M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959).
[4] This can also be seen by noting that Yang's theorem forbids the decay of a spin-1 object into two photons.
[5] S. Mandelstam and L-L. Wang, Phys. Rev. **160**, 1490 (1967).
[6] J. J. Sakurai, SLAC Report No. TN-68-11 (unpublished).
[7] Up to possible energy-dependent logarithmic factors.
[8] V. M. Gribov, Zh. Eksperim. i Teor. Fiz. **53**, 654 (1967) [English transl.: Soviet Phys.—JETP **26**, 414 (1968)].
[9] A. H. Mueller and T. L. Trueman, Phys. Rev. **160**, 1296 (1967); **160**, 1396 (1967); H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, *ibid.* **160**, 1329 (1967).

function, one may have, in the presence of suitable cuts, fixed poles for strong-interaction processes at nonsense wrong-signature points.[5]

The total photon cross section is given by

$$\sigma_T = \tfrac{1}{4} s \,\mathrm{Im}(T_{+-}{}^t)_{t=0}\,, \tag{2.2}$$

and normal Regge behavior implies

$$\mathrm{Im}T_{+-}{}^t \to \beta(t)[\alpha(t)-1]s^{\alpha(t)-2}. \tag{2.3}$$

For the Pomeranchuk trajectory to contribute, it is sufficient for its residue to be singular at $J=1$. Thus we would have

$$\beta(t) = \gamma(t)/(J-1) \tag{2.4}$$

for the residue function of the Jth partial wave near $J=\alpha(t)=1$, where, of course, $\gamma(0)\neq 0$. Hence the presence of a multiplicative fixed pole at $J=1$ allows the Pomeranchuk trajectory to contribute to the forward amplitude.

However, the effect of the third double spectral function need not manifest itself only through the existence of a fixed pole at a nonsense wrong-signature point, but can also arise through the consideration of the Regge-cut contributions due to multiple Pomeranchuk-trajectory exchange. Indeed, it is known that a high-energy behavior for the scattering amplitude corresponding to a moving branch point in the complex angular momentum plane is obtained from the examination of nonplanar perturbation-theory diagrams[10] or Reggeon diagrams.[8] These graphs have a nonzero third double spectral function and are characteristic of a fully relativistic crossing-symmetric theory.

The contribution of a Regge pole near $J=\alpha(t)$ to a signatured partial-wave amplitude of a given parity is of the form

$$T_{+-}{}^t(J) = \frac{\beta(t)[\alpha(t)-1]}{[J-\alpha(t)]}, \tag{2.5}$$

whereas a Regge cut from $J=-\infty$ to $J=\alpha_c(t)$ would have the form

$$T_{+-}{}^t(J) = 2\pi i \int_{-\infty}^{\alpha_c(t)} d\alpha' \frac{\gamma(t,\alpha')[\alpha'(t)-1]}{J-\alpha'(t)} \tag{2.6}$$

with a contribution to the high-energy limit like

$$T_{+-}{}^t \to \int_{-\infty}^{\alpha_c(t)} d\alpha \frac{\beta_c(t,\alpha)}{\sin\pi\alpha}(1\pm e^{-i\pi\alpha})s^{\alpha}. \tag{2.7}$$

In Eq. (2.6), γ is a spectral function that depends on the details of the dynamics. It is clear that in spite of the nonsense zero in the integrand of Eq. (2.6), there is no *a priori* reason to expect Eq. (2.6) or β_c in Eq. (2.7) to vanish for $t=0$ if $\alpha(0)=1$. Thus we expect that a cut generated by multiple Pomeranchuk-trajectory exexchange gives a nonzero contribution to forward Compton scattering. The high-energy behavior is generally given by the position of the leading branch point which, for the case of the Pomeranchuk trajectory, is expected to coincide with the pole at $t=0$.[8,10–12] The multi-Pomeranchon branch point is also expected to have the same signature as the Pomeranchuk trajectory itself. We shall later return to these points. Thus we see that in the latter fashion we can also obtain a constant asymptotic total cross section for Compton scattering.[7]

The difficulty with the vector-meson-dominance model can be seen to arise in the following fashion.[6] Vector-meson dominance relates invariant amplitudes for vector-meson photoproduction to those for Compton scattering. Crossing from the s to t channel for the case of vector-meson photoproduction does not flip the vector-meson helicity. Then one does not obtain a nonsense zero for Pomeranchuk-trajectory exchange at $t=0$ for the photoproduction of transversely polarized neutral vector mesons. Thus one does not need a singular residue function in contrast with Compton scattering. It can also be shown that the fixed-pole approach leads to a definite total photon hadron cross section[9] once the Pomeranchuk-trajectory slope is known, and, in particular, one finds that the total $\gamma\Lambda$ cross section vanishes. This, on the basis of vector-meson dominance, would imply the surprising result that the $\rho\Lambda$ cross section also vanishes.

If, on the other hand, one considers the contribution of multiple Pomeranchuk-trajectory branch points to Compton scattering, no such inconsistencies are obtained. We shall examine how this occurs in Sec. III.

III. CONTRIBUTIONS OF MULTIPLE POMERANCHUK-TRAJECTORY BRANCH POINTS

The purpose of this section is to examine the relative contributions of the multiple Pomeranchuk-trajectory branch points to a process, such as nucleon Compton scattering, for which single Pomeranchuk-trajectory exchange vanishes at $t=0$.

Normally, in a Regge-pole model for elastic scattering, one prefers the orthodox picture of a single simple Regge pole with intercept unity together with the associated branch points. The importance of the branch points should be emphasized in view of the mysterious nature of the Pomeranchuk trajectory[12]; moreover, it has also been speculated that cuts may actually dominate elastic scattering.[13] Also, since experimentally one finds a nearly flat Pomeranchuk trajectory,[14] one

[10] S. Mandelstam, Nuovo Cimento **30**, 1127 (1963); **30**, 1148 (1963); J. C. Polkinghorne, J. Math. Phys. **4**, 1396 (1963).

[11] H. Rothe, Phys. Rev. **159**, 1471 (1967).

[12] J. H. Schwarz, Phys. Rev. **167**, 1342 (1968).

[13] E. J. Squires, Phys. Letters **26B**, 461 (1968).

[14] $\alpha_p'=0\pm0.1$ was obtained from superconvergence relations by V. Barger and R. J. N. Phillips, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).

would expect multiple Pomeranchuk-trajectory branch points to be important.

Information about Regge cuts on physical sheets can be obtained from models involving the analysis of Feynman or Reggeon diagrams which contain third double spectral functions.[8,10,11] The signature and the phase within a factor π of a cut can be obtained. Moreover, if the same Regge trajectory is involved, the relative sign of the discontinuities for n and $n+1$ Regge-pole exchange is determined.[8] Let us see how the above results are obtained by using the Reggeon-diagram technique suggested by Gribov[8] and then examine how they are modified if the single Reggeon contribution vanishes at $t=0$.

All results for Regge cuts are obtained by considering the diagram exhibited in Fig. 1, in which one assumes that all particles involved are scalars or are treated as such. The asymptotic form of the graph in Fig. 1 is calculated for large s and fixed t in terms of the asymptotic form of the amplitudes $f(k_1,k,k_2)$ and $f'(p_1-k_1, q-k, p_2-k_2)$ represented by the bubbles in Fig. 1. The method of Sudakov is used[15] for calculating the asymptotic behavior. This method consists of suitably decomposing the internal momenta into vectors lying in the plane of the incident momenta and perpendicular to it:

$$k=\alpha p_2'+\beta p_1'+k_\perp,\quad k_1=\alpha_1 p_2'+\beta_1 p_1'+k_{1\perp},\quad k_2=\alpha_2 p_2'+\beta_2 p_1'+k_{2\perp}, \tag{3.1}$$

where

$$p_1'=p_1-(m^2/s)p_2,\quad p_2'=p_2-(m^2/s)p_2,\quad s=-(p_1+p_2)^2,\quad t=-q^2=-(p_1-p_3)^2. \tag{3.2}$$

One can then take advantage of the fact that the asymptotics is determined in a number of cases by two-dimensional integrals.[10] Furthermore, the assumption is made that only certain regions in the integration are important. This is a consequence of assuming that the amplitudes f, f' are large only in regions where one would expect them to be large on the basis of a Regge-like behavior. It also leads to considerable simplifications in calculating the asymptotic behavior.

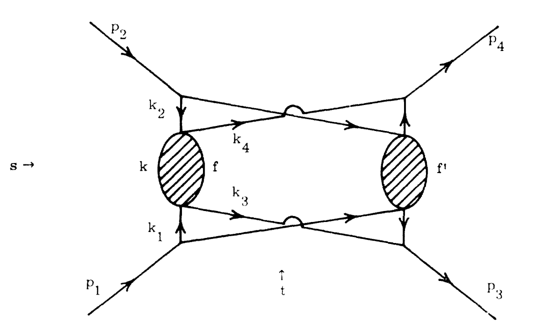

FIG. 1. Diagram giving rise to branch points on the physical sheet.

[15] V. V. Sudakov, Zh. Eksperim. i Teor. Fiz. **30**, 87 (1956) [English transl.: Soviet Phys.—JETP **3**, 65 (1956)].

The amplitudes f and f' are also assumed to factorize under asymptotic conditions

$$f(k_1,k,k_2)=g_1(k_1^2,(k-k_1)^2,k^2)g_2(k_2^2,(k_1+k_2)^2,k^2)\times G(k^2,2k_1\cdot k_2), \tag{3.3}$$

$$\begin{aligned} f'(p_1-k_1,\, q-k,\, p_2-k_2) &= g_1'((p_1-k_1)^2,\, (p_1-k_1-q+k)^2,\, (q-k)^2) \\ &\times g_2'((p_2-k_2)^2,\, (p_2-k_2+q-k)^2,\, (q-k)^2) \\ &\times G'((q-k)^2,\, 2(p_1-k_1)\cdot(p_2-k_2)). \end{aligned} \tag{3.4}$$

One then obtains, for the graph in Fig. 1,[8]

$$f_c=\frac{i\pi}{2|s|}\int\frac{dl_1}{2i\pi}\int\frac{dl_2}{2i\pi}\xi_{l_1}\xi_{l_2}\int\frac{d^2k_\perp}{(2\pi)^2}N_{l_1l_2}{}^2(q,k_\perp)G_{l_1}(k_\perp{}^2)\times G_{l_2}'((q-k_\perp)^2)s^{l_1+l_2}, \tag{3.5}$$

where

$$\begin{aligned} N_{l_1l_2}(q,k_\perp) &= \frac{1}{4(4\pi)^{1/2}}\int\frac{d^2k_{1\perp}d\beta_1 d\alpha_1 d\alpha}{(2\pi)^4}s^2\lambda^2 g_1 g_1' \\ &\times\beta_1{}^{l_1}(1-\beta_1)^{l_2}\{(k_1{}^2+m^2)[(p_1-k_1)^2+m^2] \\ &\times[(k_1-k)^2+m^2][(p_1-k_1+k-q)^2+m^2]\}^{-1}, \end{aligned} \tag{3.6}$$

and λ is a coupling constant associated with the vertices in Fig. 1. The G_l are defined by the Sommerfeld-Watson integrals

$$G(k^2,2k_1\cdot k_2)=-\int\frac{dl_1}{4i}\xi_{l_1}G_{l_1}(k^2)(\alpha_2\beta_1 s)^{l_1}, \tag{3.7}$$

$$G'((q-k)^2,2(p_1-k_1)\cdot(p_2-k_2)) = -\int\frac{dl_2}{4i}\xi_{l_2}G_{l_2}'((q-k)^2)[(1-\alpha_2)(1-\beta_1)s]^{l_2}. \tag{3.8}$$

ξ_{l_1} and ξ_{l_2} are signature factors given by

$$\begin{aligned} \xi_l &= \zeta^{-1}\exp\{-\tfrac{1}{2}\pi i[l+\tfrac{1}{2}(1-P)]\}, \\ \zeta_l &= \sin(\tfrac{1}{2}\pi l),\quad P=1 \\ &= \cos(\tfrac{1}{2}\pi l),\quad P=-1 \end{aligned} \tag{3.9}$$

P being the signature which we shall take to be positive, since we shall restrict ourselves to vacuum-trajectory exchange in all our considerations.

We may also write $f_c(s,q^2)$ in the form of a Sommerfeld-Watson integral:

$$f_c(s,q^2)=-\int\frac{dj}{4i}\xi_j f_j(q^2)s^j, \tag{3.10}$$

where $f_j(q^2)$ is determined by an integral over the absorptive part which has the following form near the singularity:

$$f_j(q^2)=\frac{2}{\pi}\int_{s0}^{\infty}ds'\,(s')^{-(j+1)}\,\mathrm{Im}f_c(s',q^2) \tag{3.11}$$

$$=-i(2\pi)^3\sin(\tfrac{1}{2}\pi j)\int\frac{dl_1}{2i\pi}\frac{dl_2}{2i\pi}\frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\times\delta(q-k_1-k_2)\delta(j+1-l_1-l_2)N_{l_1l_2}{}^2g_{l_1}g_{l_2}', \tag{3.12}$$

where $g_l = G_l/\zeta_l$, and Eq. (3.12) has been obtained from Eq. (3.11) by using Eq. (3.5). Three-Reggeon exchange may be calculated by taking for f' a form such as Eq. (3.5) rather than a simple form as given by Eqs. (3.4) and (3.8). Similarly, using such an iteration procedure, one can obtain, for the n trajectory exchange,[8]

$$f_j(q^2) = (-1)^{n-1} i(2\pi)^3 \sin(\tfrac{1}{2}\pi j) \int \frac{dl_1}{2i\pi}\cdots\frac{dl_n}{2i\pi} \times \frac{d^2k_1}{(2\pi)^2}\cdots\frac{d^2k_n}{(2\pi)^2}\delta(q-\sum_i k_i)\delta(j+n-1-\sum_i l_i) \times N_{l_1\cdots l_n}{}^2(k_1\cdots k_n) g_{l_1}(k_1{}^2) g_{l_2}(k_2{}^2)\cdots g_{l_n}(k_n{}^2), \quad (3.13)$$

where $N_{l_1\cdots l_n}{}^2$ is the n Reggeon generalization of $N_{l_1 l_2}{}^2$. From the above, if $g_{l_i}(k_i{}^2) = \zeta_{l_i}{}^{-1}[l_i - \alpha_i(k_i{}^2)]^{-1}$, one can obtain the discontinuities across the singularities of f_j:

$$\Delta f_j = (-1)^n \pi(\sin\tfrac{1}{2}\pi j) \int \frac{d^2k_1}{(2\pi)^2}\cdots\frac{d^2k_n}{(2\pi)^2}(2\pi)^2\delta(q-\sum_i k_i) \times \delta(j+n-1-\sum_i \alpha_i(k_i{}^2)) N_{\alpha_1\cdots\alpha_n}{}^2 / \prod_i \zeta_{\alpha_i}. \quad (3.14)$$

We immediately note the alternation in sign for the discontinuities. Let us see how this arises. From the product of the n signature factors, we have

$$(\xi_l)^n = \exp[-\tfrac{1}{2}i\pi(\sum_i l_i)]\prod_i (\zeta_i)^{-1} = (-i)^{n-1}\exp(-\tfrac{1}{2}i\pi j)\prod_i (\zeta_i)^{-1}. \quad (3.15)$$

The iteration of Eq. (3.5) $n-1$ times gives a factor $(i)^{n-1}$ and the relative sign of the one Reggeon and cut contributions a factor $(-1)^{n-1}$. Thus, we obtain the factor $(-1)^{n-1}$ in Eq. (3.13).

Before studying our particular case, let us note a few points. The above alternation in signs is what one would expect on the basis of absorption corrections to single Regge-pole exchange.[16] The signs would not alternate had the corrections been due to rescattering.[16] These corrections differ insofar as the Pomeranchuk trajectory is considered to consist of different parts of the amplitude.[16] Let us further note that in the usual Regge picture one expects the discontinuities for n-Reggeon exchange to become less and less important as n becomes larger.

Let us examine a situation for which the single-trajectory-exchange contribution vanishes in the forward direction. We shall assume that the above hierarchy structure for cuts is still true. Then the sign of the cut contribution must be positive because of the optical theorem, in contrast with the usual case for which the single-trajectory exchange does not vanish at $t=0$. One can then have a positive relative sign of the one-Reggeon exchange input f and the cut contribution f_c. As a result, the factor $(-1)^{n-1}$ on the right-hand sides of Eqs. (3.13) and (3.14) will disappear.

From the above considerations, we note that the total amplitude f_T does not coincide with f, since f_T also includes the branch-point contributions. If, as in the usual case, the amplitudes alternate in sign, one may speculate that the two-Reggeon exchange may be cancelled by the three-Reggeon exchange the four- by the five-, and so on. Thus $f_T \sim f$, that is, single-trajectory exchange may be a suitable approximation to the amplitude. If, on the other hand, f does not contribute to f_T, then the total amplitude in the forward direction will consist of the branch-point contributions obtained by iterating f.

Let us apply the above considerations to a relevant physical situation. In the photoproduction of transversely polarized vector mesons (or the scattering of transversely polarized vector mesons) off nucleons, the Pomeranchuk trajectory can contribute in the forward direction. One may then repeat Gribov's procedure and find that the branch-point contributions alternate in sign, and one is practically left with single Pomeranchuk-trajectory exchange. On the other hand, for nucleon Compton scattering the Pomeranchuk trajectory does not contribute to the total amplitude in the forward direction. One is then left with the branch-point contributions. We can try to estimate them by suitably iterating the neutral-vector-meson photoproduction amplitude.[17] As we have discussed, we expect the branch points to interfere constructively, in which case the total contribution will be the same as if the Pomeranchuk trajectory contributed to forward nucleon Compton scattering. We also note that the signature of the multiple Pomeranchuk-trajectory branch point is the same as that of the Pomeranchuk trajectory itself.

In Sec. IV, we discuss in detail how the branch-point contributions can be estimated and our choice of the photoproduction amplitude for iteration.

IV. ESTIMATE OF POMERANCHUK BRANCH-POINT CONTRIBUTIONS

In Sec. III, we found how the inconsistency between a fixed pole in Compton scattering and vector-meson photoproduction can be avoided by considering multiple Pomeranchuk-trajectory exchange. The purpose of this section is to estimate the magnitude of a few of the branch-point contributions.

As we have mentioned, the examination of nonplanar graphs leads to a high-energy behavior for the amplitude on the physical sheet of the s plane corresponding to a moving branch point. This branch point is analogous to the one found on the unphysical sheet by Amati,

[16] M. Jacob and J. Finkelstein, Nuovo Cimento **56**, 681 (1968).

[17] By this we mean that we iterate the strong-interaction part of the photoproduction amplitude; thus each branch-point contribution is of the same order of magnitude in the electromagnetic interactions. See, in particular, Sec. IV.

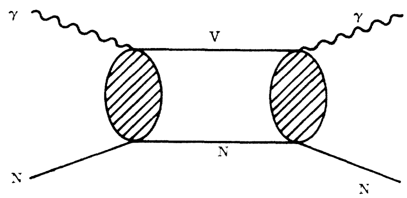

FIG. 2. Diagram giving rise to AFS cuts in nucleon Compton scattering.

Fubini, and Stanghellini (AFS)[18] for planar graphs. One may expect the coefficients of the two graphs to be similar, since they are related to a contracted diagram which is topologically the same for both graphs.[10,11,19] Thus we may attempt to estimate the Regge-cut contributions by calculating the discontinuity across the two-particle cut.[20,21] The two-particle intermediate states in our case are a nucleon and a vector meson, which means that we shall need the vector-meson photoproduction amplitudes[17] (see Fig. 2).

We may write the neutral-vector-meson photoproduction amplitudes[22] in the following form:

$$A_1(s,t)=i\beta(0)e^{at}(s/s_0)\,, \qquad (4.1)$$

where $\beta(0)$ is real. Equation (4.1) states that the Pomeranchuk trajectory contributes to the photoproduction of vector mesons[2] and all t dependence has been put in the residue function. We note that the integrated cross sections, with the above form for the amplitude, agree with the quoted channel cross sections (except for ω photoprodudtion for which π exchange is fairly important at measured energies), indicating predominance of the diffraction mechanism.[23] A last point worth noting about the above form, Eq. (4.1), is that it corresponds to a fixed-pole Pomeranchuk trajectory. This, of course, implies the existence of shielding cuts[24]; however, let us note that our conclusions are unaltered if the Pomeranchuk trajectory has a small enough slope.[14]

We have the following form for the magnitude of the n-Pomeranchuk-trajectory exchange[21] contribution to Compton scattering:

$$A_n(s,t)=\frac{i}{16\pi^2 s}\left(\frac{f_V}{e}\right)^{n-2}\sum_{j=1}^{n}\int_{-s}^{0}dt' \times\int_{-s}^{0}dt''A_j(s,t')A_{n-j}{}^*(s,t'')\frac{\theta(K)}{\sqrt{K}}, \qquad (4.2)$$

where

$$K(t,t',t'')=-(t^2+t'^2+t''^2)+2(tt'+t't''+tt'')+4tt't''/s\,,$$

and f_V is related to the photon–neutral-vector-meson vertex. On substituting Eq. (4.1) into (4.2), one finds that the integrals can be performed exactly for A_2 and approximately for A_3, A_4, and A_5.[21,25] The results are, for $t=0$,

$$A_2=\frac{i[\beta(0)]^2}{32\pi a s_0}\left(\frac{s}{s_0}\right), \qquad (4.3)$$

$$A_3=\frac{i[\beta(0)]^3}{384\pi^2 s_0{}^2 a^2}\left(\frac{s}{s_0}\right)\frac{f_V}{e}, \qquad (4.4)$$

$$A_4=\frac{i[\beta(0)]^4}{a^3 8^3(2\pi s_0)^3}\left(\frac{s}{s_0}\right)\left[\frac{161}{132}\right]\frac{f_V{}^2}{e^2}, \qquad (4.5)$$

$$A_5=\frac{i[\beta(0)]^5}{a^4 8^4(2\pi s_0)^4}\left(\frac{s}{s_0}\right)\left[\frac{388}{149}\right]\frac{f_V{}^3}{e^3}. \qquad (4.6)$$

As input, we may use the results obtained from neutral-vector-meson photoproduction data at a primary photon energy of 4.5–5.8 GeV.[23] These are:

For ρ photoproduction,

$$\beta=80.4\ (\text{GeV}^2\,\mu\text{b})^{1/2},\quad a=3.95\ \text{GeV}^{-2}; \qquad (4.7)$$

for ω photoproduction,

$$\beta=37.8\ (\text{GeV}^2\,\mu\text{b})^{1/2},\quad a=3.8\ \text{GeV}^{-2}; \qquad (4.8)$$

for φ photoproduction,

$$\beta=9.0\ (\text{GeV}^2\,\mu\text{b})^{1/2},\quad a=1.75\ \text{GeV}^{-2}. \qquad (4.9)$$

In Eqs. (4.7)–(4.9), we have omitted the errors, which are always at least 10%. We may also use $e^2/f_\rho{}^2:e^2/f_\omega{}^2:e^2/f_\varphi{}^2=9:1:2$ and $(e^2/f_\rho{}^2)\sim 1/340$.[6]

On substituting the above in Eqs. (4.3)–(4.6), we obtain the following result for the total photon cross section on protons:

$$\sigma_T=[(16.2+8.3+5.8+5.8)_\rho+(3.75+2.7+1.5+1.5)_\omega +(0.9+0.3+0.1+\text{negligible})_\varphi] =47\ \mu\text{b}\,, \qquad (4.10)$$

where the different vector-meson contributions are identified and we have written the two-, three-, four-, and five-Pomeranchuk-trajectory contributions in order. It is clear that the dominant contribution is given by the ρ term.

Experimentally, the total proton cross section is of the order of magnitude $\sim$110 μb. Thus we have obtained some 40% of the total cross section by limiting ourselves to branch points caused by two- to five-Pomeranchuk-trajectory exchange. It is not unreasonable to expect that the higher branch-point contributions will yield most of the remaining total cross section.

[18] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962).

[19] R. Oehme, in *Strong Interactions and High-Energy Physics*, edited by R. G. Moorhouse (Plenum Press, Inc., New York, 1964), p. 129.

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One need not be concerned about the apparent lack of convergence of the ρ or ω contributions in Eq. (4.10) since these results are sensitive to the input values of a and β. The values that we have used are not the asymptotic values; moreover, the evaluation of the integrals is only approximate, and we are only interested in an order-of-magnitude estimate. The approximation that we have used may be regarded as an optical prescription for the calculation of the effect of cuts in a Regge model.

V. CONCLUSIONS

The examination of the problem of obtaining a constant total cross section for nucleon Compton scattering consistently with vector-meson dominance has led to some observations about the Pomeranchuk trajectory. The reason for this is that because of the particular nature of the two-photon Pomeranchuk-trajectory coupling, we have a situation for which the trajectory does not couple in the forward direction,[1,2] unless third double spectral function effects are taken into consideration.

As has been pointed out, one way of doing this is to include the effect of fixed poles at wrong-signature points[9]; however, it may be more interesting to observe that Regge cuts are intimately connected with the third double spectral function. This is particularly interesting because of the unknown nature of the Pomeranchuk trajectory. Indeed, various speculations about this trajectory have been made in order to explain the nonshrinking of the diffraction peak. It has been suggested to be a fixed pole with a suitable shielding cut[24] in order to avoid difficulties with unitarity or a self-reproducing branch point.[12] Alternatively, it has been suggested that if it is a Regge pole, the effect of cuts obtained from absorption must also be included. The latter picture is the more orthodox one, especially if one believes in the importance of cuts for elastic processes.[13]

Once one follows the above line of thought, it is natural to examine the situation in Compton scattering with respect to cuts and see what can be stated about their properties. One observes that in cases for which the Pomeranchuk trajectory can behave like a pole, that is, the coupling of a Regge pole with $\alpha(0)=1$ does not vanish at $t=0$, the multiple trajectory-exchange contributions alternate in sign and can be expected to give a negligible contribution. If, on the other hand, a Regge pole with $\alpha(0)=1$ cannot couple in the forward direction, as is the case in Compton scattering, then the multiple-exchange contributions can interfere constructively and lead to the same effect as the pole itself. Thus there is no need for singular residue functions. At this point, one sees that the difficulties with vector-meson dominance are avoided by the apparent pole-cut duality for the Pomeranchuk trajectory.

On the basis of the above considerations, we have examined the multiple Pomeranchuk-trajectory branch-point contributions to the total Compton cross section. The estimate of these contributions is made by what is essentially an optical prescription for calculating the effects of Regge cuts and the identification of the position and magnitude of the Regge cut with the second sheet cut of AFS.[21] One finds that a rough estimate of the contributions of the two- to five-Pomeranchuk-trajectory exchange cuts, using as input the vector-meson photoproduction data,[23] leads to some 40% of the total proton Compton cross section. It is not unreasonable to expect that the higher multiple trajectory exchanges contribute most of the remaining cross section.

Note added in proof. Actually one can obtain a closed form for the contribution to the total cross section of an infinite number of Pomeranchuk-trajectory branch points. The result one arrives at depends on what part of the total amplitude is identified with the Pomeranchuk trajectory [see P. J. O'Donovan, Phys. Rev. (to be published)]. Thus the results in Sec. IV can be made more definite. This and other points will be discussed elsewhere.

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