Two-Photon Decays of π^0, η^0, and $\eta'^0(958)$ in Broken $SU(3)$ Symmetry

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By using the current algebra and the asymptotic $SU(3)$ symmetry imposed only on the charge operator V_K, which is the $SU(3)$ raising or lowering operator in the symmetry limit, we obtain two sum rules for the two-photon decay amplitudes of the π^0, η^0, and $\eta'^0(958)$. These sum rules, which are consistent with the Gell-Mann–Okubo mass splittings of the $SU(3)$ multiplets, exhibit an interesting interplay between the physical masses, mixing and coupling constants. For example, in the first approximation, we predict $R\equiv\langle\gamma\gamma|\eta^0\rangle/\langle\gamma\gamma|\pi^0\rangle=(1/\sqrt{3})(1/\cos\alpha)[(m_{\eta'}^2-m_\pi^2)/(m_{\eta'}^2-m_\eta^2)]\simeq(1/\sqrt{3})\times1.5$ in place of the $SU(3)$ prediction $R=1/\sqrt{3}$. Here α is the η-η' mixing angle. In an improved approximation, we obtain $R\simeq(1/\sqrt{3})\times1.7$. The result indicates a significantly larger width for the $\eta^0\to2\gamma$ decay than the $SU(3)$ value—a conclusion that is consistent with the present experimental observation.

I. INTRODUCTION

EXACT $SU(3)$ symmetry predicts for the amplitudes of $\pi^0\to2\gamma$ and $\eta^0\to2\gamma$ the equality $\langle\gamma\gamma|\pi\rangle=+\sqrt{3}\langle\gamma\gamma|\eta\rangle$. If we use the experimental masses and this exact $SU(3)$ relation, which is certainly not justified in view of the large mass splittings involved, we obtain $\Gamma(\eta\to2\gamma)/\Gamma(\pi\to2\gamma)=\frac{1}{3}(m_\eta/m_\pi)^3$. From present experiment,[1] $\Gamma(\pi\to2\gamma)=7.37\pm1.5$ eV, this predicts $\Gamma(\eta\to2\gamma)=165\pm34$ eV. However, present experiment, though preliminary,[2] indicates a value $\Gamma(\eta\to2\gamma)=1.00\pm0.22$ keV, which is considerably larger than the above $SU(3)$ value. In reality we have to take into account the $\eta-\eta'$ mixing which brings the hitherto unknown $\eta'\to2\gamma$ decay amplitude into the sum rule. The usual $SU(3)$ approach (with $\eta-\eta'$ mixing angle α) assumes the sum rule[3]

$$\langle\gamma\gamma|\pi\rangle-\sqrt{3}\cos\alpha\langle\gamma\gamma|\eta\rangle+\sqrt{3}\sin\alpha\langle\gamma\gamma|\eta'\rangle=0. \quad (1)$$

However, there is some doubt as to whether this is a realistic sum rule in broken $SU(3)$ symmetry. For example, consider the vector meson $\to l+\bar{l}$ decays. The first spectral function sum rules[4,5] for the ratio of these decays are different from the ones [similar to Eq. (1)] obtained by using the conventional $SU(3)$ symmetry with ω-ϕ mixing. In this paper we discuss the following: (i) There appears to be a correction term to Eq. (1). (ii) Using a reasonable model of $SU(3)$ breaking which gives many good sum rules in other places, we can derive an additional sum rule. (iii) From these two sum rules we can predict both $\Gamma(\eta\to2\gamma)$ and $\Gamma(\eta'\to2\gamma)$ from $\Gamma(\pi\to2\gamma)$. The $\Gamma(\eta\to2\gamma)$ thus determined turns out to be considerably larger than the $SU(3)$ value. The value of $\Gamma(\eta'\to2\gamma)$ seems also reasonable. In one place of the computation we use the idea of field-current identity.[6]

II. CURRENT ALGEBRA AND ASYMPTOTIC $SU(3)$ SYMMETRY

Write the electromagnetic current as $V_\mu^{\text{em}}(x)$ and the charge operator which is the $SU(3)$ raising or lowering operator in the symmetry limit as V_K. In a quark model, for example, the V_{K^0} will be the space integral of the time component of the current

$$V_\mu^{K^0}(x)=i\bar{q}(x)\gamma_\mu\tfrac{1}{2}(\lambda_6+i\lambda_7)q(x)$$

and, in this notation,

$$V_\mu^{\text{em}}(x)=V_\mu^{\pi^0}(x)+(\sqrt{3})^{-1}V_\mu^{\eta^0}(x).$$

We then notice that the following commutator is valid:

$$[V_\mu^{\text{em}}(x),V_{K^0}]=0. \quad (2)$$

We now consider a simple model of $SU(3)$ breaking.[7] Suppose that the $SU(3)$ breaking is given by H'

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[2] See Ref. 1, p. 134.

[3] Assuming $\Gamma(\pi\to2\gamma)=7.5\pm1.5$ eV, $\Gamma(\eta\to2\gamma)=0.88\pm0.22$ keV, and $\alpha=\pm10°$, Harari obtained $\Gamma(\eta'\to2\gamma)=50\pm30$ keV or 350 ± 90 keV. [H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 199.]

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[7] For example, see S. Matsuda and S. Oneda, Phys. Rev. 174, 1992 (1968). The following remark may be useful in judging the validity of our asymptotic symmetry. We consider the commutators $[\dot{V}_{K^0},V_{K^0}]=0$ and $[\dot{V}_{K^0},A_{K^0}]=0$ which are also valid in the present model. Taken between the states $\langle n(\mathbf{q})|$ and $|\Xi^0(\mathbf{q})\rangle$ with $|\mathbf{q}|=\infty$, both commutators lead to the same well-satisfied GMO mass formula for hyperons. Therefore, our broken $SU(3)$ sum rules are always compatible with the GMO mass splitting (including mixing if it exists).

$=\int \bar{q}(x)\lambda_8 q(x)d^3x$. Then $\dot{V}_{K^0}=i[V_{K^0},H']$ and the following commutator also holds:

$$[V_\mu^{\text{em}}(x),\dot{V}_{K^0}]=0. \quad (3)$$

One may forget the above model used for its derivation and assume that these commutators between the current and the $SU(3)$ charge operator, V_K, are valid in the broken $SU(3)$ world.

We now introduce our asymptotic symmetry.[7] Crudely speaking, this assumes that the vector charge operator V_K will act as an $SU(3)$ generator even in the presence of broken symmetry but *only* at the zero-momentum-transfer limit. In the real world where the mass splittings take place, this limit can only be achieved by taking an appropriate infinite-momentum limit. In this infinite limit only, we assume that V_K connects the states belonging only to the same irreducible representation of $SU(3)$ group and its matrix elements take the $SU(3)$ values. We also take into account the particle mixing in this limit.[7]

III. BROKEN $SU(3)$ SUM RULES FOR THE TWO-PHOTON DECAYS OF THE π^0, η^0, AND η'^0

We now demonstrate some of the direct consequences. Insert Eqs. (2) and (3) between, for example, the states $\langle\pi^-(\mathbf{q})|$ and $|K^{*-}(\mathbf{p})\rangle$. We obtain two equations:

$$\langle\pi^-(\mathbf{q})|V_\mu^{\text{em}}(x)|\rho^-(\mathbf{p})\rangle\langle\rho^-(\mathbf{p})|V_{K^0}|K^{*-}(\mathbf{p})\rangle$$
$$=\langle\pi^-(\mathbf{q})|V_{K^0}|K^-(\mathbf{q})\rangle\langle K^-(\mathbf{q})|V_\mu^{\text{em}}(x)|K^{*-}(\mathbf{p})\rangle, \quad (4)$$

$$\langle\pi^-(\mathbf{q})|V_\mu^{\text{em}}(x)|\rho^-(\mathbf{p})\rangle\langle\rho^-(\mathbf{p})|\dot{V}_{K^0}|K^{*-}(\mathbf{p})\rangle$$
$$=\langle\pi^-(\mathbf{q})|\dot{V}_{K^0}|K^-(\mathbf{q})\rangle\langle K^-(\mathbf{q})|V_\mu^{\text{em}}(x)|K^{*-}(\mathbf{p})\rangle. \quad (5)$$

Here we have taken a limit $|\mathbf{p}|=\infty$ and $|\mathbf{q}|=\infty$ but $s=-(p-q)^2$ is kept arbitrary. In this limit we have used our asymptotic symmetry for the matrix elements of V_{K^0}. For example, write

$$\langle\pi^-(\mathbf{q})|V_\mu^{\text{em}}(0)|\rho^-(\mathbf{p})\rangle=(2q_0 2p_0)^{-1/2} \times g_{\rho^-\pi^-}(s)\epsilon_{\mu\alpha\beta\gamma}\epsilon_\alpha{}^\rho q_\beta p_\gamma,$$

where $\epsilon_\alpha{}^\rho$ is the ρ-meson polarization vector. After summing over the intermediate spin states, we obtain from Eq. (4)

$$g_{\rho^-\pi^-}(s)=g_{K^{*-}K^-}(s).$$

This implies that the $\rho^-\to\pi^-+\gamma$ and $K^{*-}\to K^-+\gamma$ coupling ($s=0$ for these processes) satisfy the same sum rule as in exact $SU(3)$ symmetry under our asymptotic symmetry.[8] In this way we can derive sum rules for other electromagnetic processes such as the baryon magnetic moment in broken symmetry. This will be discussed elsewhere. We now consider Eq. (5). Equation (5) is compatible with Eq. (4) only when $E_\rho(\mathbf{p})-E_{K^*}(\mathbf{p})=E_\pi(\mathbf{q})-E_K(\mathbf{q})$ at $|\mathbf{p}|=\infty$ and $|\mathbf{q}|=\infty$, which implies the $SU(6)$ mass formula,

$$m_{K^*}{}^2-m_\rho{}^2=m_K{}^2-m_\pi{}^2.$$

This is obtained without assuming $SU(6)$ symmetry. In a similar way we can derive many intermultiplet mass formulas.[9] Another example: Insert Eqs. (2) and (3) between the states $\langle\pi^0(\mathbf{q})|$ and $|\bar{K}^{*0}(\mathbf{p})\rangle$ and take the same limit as above. Now ω and ϕ appear as the intermediate states. However, if we assume that $\Gamma(\phi\to\pi+\gamma)\simeq 0$ from experiment, these two equations are consistent only if $m_\rho{}^2\simeq m_\omega{}^2$, which is also well satisfied experimentally. Conversely, if we use $m_\rho=m_\omega$ as an experimental input these equations predict that $\Gamma(\phi\to\pi+\gamma)\simeq 0$. Therefore, our asymptotic symmetry and the model characterized by the commutator (3) look very reasonable,[9] and we proceed to the problem of two-photon decays.

Insert Eqs. (2) and (3) between the photon state $\langle\gamma(\mathbf{q})|$ and the $|\bar{K}^0(\mathbf{p})\rangle$ and again consider the same limit $|\mathbf{q}|=\infty$ and $|\mathbf{p}|=\infty$ but $s=-(p-q)^2$ is arbitrary. We write here the photon state with the understanding that it will be identified with the appropriate hadron states, i.e., the vector-meson states according to the idea of field-current identity.[6] We then obtain two equations:

$$\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}(x)|\pi(\mathbf{p})\rangle\langle\pi(\mathbf{p})|V_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$+\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}(x)|\eta(\mathbf{p})\rangle\langle\eta(\mathbf{p})|V_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$+\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}|\eta'(\mathbf{p})\rangle\langle\eta'(\mathbf{p})|V_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$=\sum_n\langle\gamma(\mathbf{q})|V_{K^0}|n(\mathbf{q})\rangle\langle n(\mathbf{q})|V_\mu^{\text{em}}(x)|\bar{K}^0(\mathbf{p})\rangle, \quad (6)$$

$$\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}(x)|\pi(\mathbf{p})\rangle\langle\pi(\mathbf{p})|\dot{V}_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$+\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}(x)|\eta(\mathbf{p})\rangle\langle\eta(\mathbf{p})|\dot{V}_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$+\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}|\eta'(\mathbf{p})\rangle\langle\eta'(\mathbf{p})|\dot{V}_{K^0}|\bar{K}^0(\mathbf{p})\rangle$$
$$=\sum_n\langle\gamma(\mathbf{q})|\dot{V}_{K^0}|n(\mathbf{q})\rangle\langle n(\mathbf{q})|V_\mu^{\text{em}}(x)|\bar{K}^0(\mathbf{p})\rangle. \quad (7)$$

Here $\langle\gamma(\mathbf{q})|V_\mu^{\text{em}}(x)|\pi^0(\mathbf{p})\rangle$ with $s=0$, for example, can be identified with the amplitude of $\pi\to 2\gamma$ decay (to order α), $\langle\gamma\gamma|\pi^0\rangle$. We write to the first order in symmetry breaking

$$\langle\eta(\mathbf{p})|=\cos\alpha\,\langle\eta_8(\mathbf{p})|+\sin\alpha\,\langle\eta_1(\mathbf{p})|$$

and

$$\langle\eta'(\mathbf{p})|=\cos\alpha\,\langle\eta_1(\mathbf{p})|-\sin\alpha\,\langle\eta_8(\mathbf{p})|$$

in the limit $|\mathbf{p}|=\infty$. [$\eta\to\eta_8$ and $\eta'-\eta_1$ in the $SU(3)$ limit.] Using the asymptotic symmetry ($|\mathbf{p}|\to\infty$), we obtain

$$\langle\eta_8(\mathbf{p}')|V_{K^0}|\bar{K}^0(\mathbf{p})=(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')(\sqrt{\tfrac{3}{2}})$$

and

$$\langle\eta_1(\mathbf{p}')|V_{K^0}|\bar{K}^0(\mathbf{p})\rangle=0,$$

etc.

[8] Depending on the processes, our broken $SU(3)$ sum rules for physical couplings occasionally take the same form as exact $SU(3)$ sum rules. However, usually the sum rules involve the physical masses and mixing angle. See, for review, S. Matsuda and S. Oneda, Nucl. Phys. **B9**, 55 (1969).

[9] We can also derive $m_K{}^2-m_\pi{}^2\simeq m_{K^{**}}{}^2-m_{A_2}\simeq m_{K_A}{}^2-m_{A_1}{}^2$, etc. Here K^{**} and K_A are the kaons of 2^+- and 1^+-meson octets and A_2 and A_1 are the $(I=1)$ 2^+ and 1^+ mesons, respectively.

Equation (6) (with $s=0$) can be written as[10]

$$\langle\gamma\gamma|\pi^0\rangle-\sqrt{3}\cos\alpha\,\langle\gamma\gamma|\eta^0\rangle+\sqrt{3}\sin\alpha\,\langle\gamma\gamma|\eta'\rangle =\sqrt{2}\sum_n \langle\gamma(\mathbf{q})|V_{K^0}|n(\mathbf{q})\rangle\langle n(\mathbf{q})|V_\mu^{\rm em}(x)|\bar{K}^0(\mathbf{p})\rangle. \quad (8)$$

Comparing with Eq. (1), we see that the right-hand side of Eq. (8) represents the modification of the conventional sum rule (1). To estimate this contribution, we use the field-current identity[6]

$$V_\mu^{\pi^0}(x)=-(m_\rho^2/g_\rho)\rho_\mu(x)$$

and

$$V_\mu^{\eta^0}(x)=-\tfrac{1}{2}g_Y^{-1}[\cos\theta_Y\, m_\phi^2\phi_\mu(x)-\sin\theta_Y\, m_\omega^2\omega_\mu(x)].$$

We note that for the isovector photon γ^V,

$$(2q_0)^{1/2}\langle\gamma^V(\mathbf{q})|=\langle 0|\int V_\mu^{\pi^0}(x)e^{iqx}d^4x=-(m_\rho^2/g_\rho)\langle 0|\times\int\rho_\mu(x)e^{iqx}d^4x=-(m_\rho^2/g_\rho)\frac{1}{q^2+m_\rho^2}\langle 0|\times\int j_\mu^\rho(x)e^{iqx}d^4x\equiv-\frac{1}{g_\rho}\langle\tilde{\rho}(\mathbf{q})|,$$

since $q^2=0$. Here $j_\mu^\rho(x)$ is the source of the ρ-meson field, and $\langle\tilde{\rho}(\mathbf{q})|$ denotes the ρ-meson state with its mass extrapolated to zero. In a similar way we can replace the isoscalar photon state $\langle\gamma^S(\mathbf{q})|$ by

$$\langle\gamma^S(\mathbf{q})|=-\tfrac{1}{2}g_Y^{-1}[\cos\theta_Y\,\langle\tilde{\phi}(\mathbf{q})|-\sin\theta_Y\,\langle\tilde{\omega}(\mathbf{q})|],$$

where $\langle\tilde{\phi}|$ and $\langle\tilde{\omega}|$ are the states of ϕ and ω meson with zero mass. We now define[5] the couplings of the ρ, ω, and $\phi\rightarrow l+\bar{l}$ decays, G_ρ, G_ω, and G_ϕ, by, for example,

$$(2q_0)^{1/2}\langle 0|V_\mu^{\pi^+}(0)|\rho^-(\mathbf{q})\rangle=G_\rho\epsilon_\mu{}^\rho.$$

We then obtain

$$G_\rho=\sqrt{2}(m_\rho^2/g_\rho),$$
$$G_\phi=(\sqrt{3}/2)(m_\phi^2/g_Y)\cos\theta_Y,$$

and

$$G_\omega=(-\sqrt{3}/2)(m_\omega^2/g_Y)\sin\theta_Y.$$

The ρ, ϕ, ω, and K^* form a nonet. The field-current identity implies as above that we can simultaneously extrapolate the masses of the ρ, ϕ, and ω to zero to identify the states, $\tilde{\rho}$, $\tilde{\phi}$, and $\tilde{\omega}$ with the photon state. Correspondingly, the K^*-meson state will also be extrapolated to zero mass [and denoted by $\langle\tilde{K}^*(\mathbf{q})|$] according to the Gell-Mann–Okubo mass formula[11] which completes the nonet vector-meson states with zero mass. Therefore, according to the spirit of our asymptotic symmetry, the state $|n(\mathbf{q})\rangle$ which can be reached from the $\langle\gamma(\mathbf{q})|$, i.e., from the $\langle\tilde{\rho}(\mathbf{q})|$, $\langle\tilde{\phi}(\mathbf{q})|$ and $\langle\tilde{\omega}(q)|$ states, via the operator V_K in our asymptotic limit $|\mathbf{q}|\rightarrow\infty$, will be $\langle\tilde{K}^*(\mathbf{q})|$. Namely, the most important contribution on the right-hand side of Eqs. (6) and (7) will come from the state represented by the $|\tilde{K}^*(\mathbf{q})\rangle$ state. We write to the first order in symmetry breaking

$$|\phi(\mathbf{q})\rangle=\cos\theta|\phi_8(\mathbf{q})\rangle+\sin\theta|\phi_1(\mathbf{q})\rangle$$

and

$$|\omega(\mathbf{q})\rangle=\cos\theta|\phi_1(\mathbf{q})\rangle-\sin\theta|\phi_8(\mathbf{q})\rangle$$

in the limit $|\mathbf{q}|\rightarrow\infty$. [In the $SU(3)$ limit $\phi\rightarrow\phi_8$ and $\omega\rightarrow\phi_1$.] With the θ defined above, we have previously obtained[5] the following sum rules from our asymptotic symmetry:

$$G_\phi=(1/\sqrt{2})G_\rho(m_\phi/m_\rho)\cos\theta,$$
$$G_\omega=-(1/\sqrt{2})G_\rho(m_\omega/m_\rho)\sin\theta.$$

This leads to the first spectral function sum rule

$$G_\rho/m_\rho^2=2(G_\omega^2/m_\omega^2+G_\phi^2/m_\phi^2).$$

We see that θ_Y is related to the θ by

$$\tan\theta_Y=(m_\phi/m_\omega)\tan\theta.$$

Thus one can express $\langle\gamma(q)|=\langle\gamma^V(\mathbf{q})|+(\sqrt{3})^{-1}\langle\gamma^S(\mathbf{q})|$ in terms of the G_ρ and θ (in the limit $|\mathbf{q}|=\infty$),

$$\langle\gamma(\mathbf{q})|=\frac{G_\rho}{\sqrt{2}}\frac{1}{m_\rho^2}\langle\tilde{\rho}(\mathbf{q})|+\frac{G_\rho}{\sqrt{6}}\frac{1}{m_\rho}\frac{\cos\theta}{m_\phi}\langle\tilde{\phi}(\mathbf{q})|-\frac{G_\rho}{\sqrt{6}}\frac{1}{m_\rho}\frac{\sin\theta}{m_\omega}\langle\tilde{\omega}(\mathbf{q})|. \quad (9)$$

Therefore, one can evaluate[12] $\langle\gamma(\mathbf{q})|V_{K^0}|\text{anti-}\tilde{K}^{*0}(\mathbf{q})\rangle$ using the asymptotic symmetry for the nonet $\langle\tilde{\rho}|$, $\langle\tilde{\phi}|$, $\langle\tilde{\omega}|$ and $\langle\tilde{K}^*|$, i.e., with $|\mathbf{q}|=\infty$,

$$\langle\gamma(\mathbf{q})|V_{K^0}|\bar{K}^{*0}(\mathbf{q})\rangle=(\tfrac{1}{2}G_\rho)\frac{1}{m_\rho^2}\left[1-\left(\frac{m_\rho m_\phi}{m_8^2}\right)\cos^2\theta-\left(\frac{m_\rho m_\omega}{m_1^2}\right)\sin^2\theta\right]. \quad (10)$$

m_1 and m_8 denote the masses of the ϕ_1 and ϕ_8, respec-

[10] Using $[V_{K^0},\dot{V}_{K^0}]=0$ and asymptotic symmetry, the α defined above takes the usual value $\sin^2\alpha=\frac{1}{3}(3m_\eta^2-4m_K^2+m_\pi^2)\times(m_{\eta'}^2-m_\eta^2)^{-1}$.

[11] Consider $\langle\tilde{K}^{*0}(\mathbf{q})|[V_{K^0},\dot{V}_{K^0}]|$antiparticle of $\tilde{K}^{*0}(\mathbf{q})\rangle=0$ with $|\mathbf{q}|=\infty$.

[12] For example, from $\omega_\mu(x)=\cos\theta\,\phi_{1\mu}(x)-\sin\theta\,\phi_{8\mu}(x)$, we obtain

$$(q^2+m_\omega^2)^{-1}\langle B(\mathbf{p}')|j_\mu{}^\omega(x)|A(\mathbf{p})\rangle=\cos\theta\,(q^2+m_1^2)^{-1}\times\langle B(\mathbf{p}')|j_\mu{}^{\phi_1}(x)|A(\mathbf{p})\rangle-\sin\theta\,(q^2+m_8^2)^{-1}\langle B(\mathbf{p}')|j_\mu{}^{\phi_8}(x)|A(\mathbf{p})\rangle.$$

A and B are the appropriate arbitrary states and the j_μ's are the source currents of the vector mesons. $q_\mu=(p-p')_\mu$. Let us consider the limit $|\mathbf{q}|=|\mathbf{p}|=\infty$ so that $q^2=0$; then we get

$$m_\omega^{-2}\langle B|j_\mu|A\rangle=\cos\theta\,m_1^{-2}\langle B|J_\mu{}^{\phi_1}|A\rangle-\sin\theta\,m_8^{-2}\langle B|J_\mu{}^{\phi_8}|A\rangle.$$

Multiplying both sides by $[i/(2q_0)^{1/2}]e^{-iqx}$ and integrating over d^4x, we obtain

$$m_\omega^{-2}\langle B(\mathbf{p}')|\tilde{\omega}(\mathbf{q})A(\mathbf{p})\rangle=\cos\theta\,m_1^{-2}\langle B(\mathbf{p}')|\tilde{\phi}_1(\mathbf{q})A(\mathbf{p})\rangle-\sin\theta\,m_8^{-2}\langle B(\mathbf{p}')|\tilde{\phi}_8(\mathbf{q})A(\mathbf{p})\rangle.$$

Thus, with this infinite-momentum limit in mind, we can write

$$|\tilde{\omega}\rangle=(m_\omega/m_1)^2\cos\theta\,|\tilde{\phi}_1\rangle-(m_\omega/m_8)^2\sin\theta\,|\tilde{\phi}_8\rangle$$

and

$$|\tilde{\phi}\rangle=(m_\phi/m_8)^2\cos\theta\,|\tilde{\phi}_8\rangle+(m_\phi/m_1)^2|\tilde{\phi}_1\rangle.$$

tively (see Ref. 12). Next we reexpress

$$\langle \text{anti-}\tilde{K}^{*0}(\mathbf{q})|\,V_\mu{}^{\text{em}}(x)\,|\bar{K}^0(\mathbf{p})\rangle .$$

Consider

$$\langle\tilde{\rho}^0(\mathbf{q})|\,[V_{K^0},V_\mu{}^{\text{em}}(x)]\,|\bar{K}^0(\mathbf{p})\rangle=0$$

with $|\mathbf{p}|=\infty$, $|\mathbf{q}|=\infty$, and $s=-(q-p)^2=0$. Our asymptotic symmetry leads to

$$\begin{aligned}\langle \text{anti-}\tilde{K}^{*0}(\mathbf{q})|\,V_\mu{}^{\text{em}}(x)\,|\bar{K}^0(\mathbf{p})\rangle&=\langle\tilde{\rho}^0(\mathbf{q})|\,V_\mu{}^{\text{em}}(x)\,|\pi(\mathbf{p})\rangle\\&-\sqrt{3}\cos\alpha\langle\tilde{\rho}^0(\mathbf{q})|\,V_\mu{}^{\text{em}}(x)\,|\eta(\mathbf{p})\rangle\\&+\sqrt{3}\sin\alpha\langle\tilde{\rho}^0(\mathbf{q})|\,V_\mu{}^{\text{em}}(x)\,|\eta'(\mathbf{p})\rangle .\end{aligned}$$

Noting that $\langle\gamma^V(\mathbf{q})|=-(\sqrt{2}m_\rho{}^2)^{-1}G_\rho\langle\tilde{\rho}(\mathbf{q})|$ and, for example, $\langle\tilde{\rho}|\,V_\mu{}^{\text{em}}(x)\,|\pi\rangle\propto\langle\gamma\gamma^V|\pi\rangle$, Eq. (8) can finally be written in the form

$$\langle\gamma\gamma|\pi\rangle-\sqrt{3}\cos\alpha\langle\gamma\gamma|\eta\rangle+\sqrt{3}\sin\alpha\langle\gamma\gamma|\eta'\rangle=XY\,,\quad(11)$$

where

$$X=[1-(m_\rho m_\phi/m_8{}^2)\cos^2\theta-(m_\rho m_\omega/m_1{}^2)\sin^2\theta]\,,\quad(12)$$

$$Y=[\langle\gamma\gamma^V|\pi\rangle-\sqrt{3}\cos\alpha\langle\gamma\gamma^V|\eta\rangle+\sqrt{3}\sin\alpha\langle\gamma\gamma^V|\eta'\rangle]\,.\quad(13)$$

In the $SU(3)$ limit ($\theta=0$, $m_\rho=m_\phi=m_8$, and $\alpha=0$), $X=0$. Thus Eq. (11) reproduces the $SU(3)$ limit. In the broken world, we find that X is still not very large. Using the commutator $[V_{K^0},\dot{V}_{K^0}]=0$ and the asymptotic symmetry, we can show[7] that the value of θ can be given by the usual ω-ϕ mixing angle determined from the vector-meson mass formula. We then find $X=0.10$, which seems reasonable as a first-order breaking effect. We now return to Eq. (7). We obtain

$$\begin{aligned}(m_{\pi^0}{}^2-m_{K^0}{}^2)\langle\gamma\gamma|\pi\rangle&-\sqrt{3}(m_{\eta^0}{}^2-m_{K^0}{}^2)\cos\alpha\langle\gamma\gamma|\eta\rangle\\&+\sqrt{3}(m_{\eta'}-m_{K^0}{}^2)\sin\alpha\langle\gamma\gamma|\eta'\rangle=0.\end{aligned}\quad(14)$$

The mass factors come from the time derivatives. On the right-hand side of Eq. (7), $\langle\gamma|\,\dot{V}_{K^0}|\text{anti-}\tilde{K}^{*0}\rangle$ vanishes in our limit since both the γ and $\tilde{K}^{*0}$ have zero mass. If we tentatively assume $X=0$ in Eq. (11), we obtain from (11) and (14) (taking $\alpha\simeq10^0$)

$$\langle\gamma\gamma|\eta\rangle/\langle\gamma\gamma|\pi\rangle=\frac{1}{\sqrt{3}}\frac{1}{\cos\alpha}\left(\frac{m_{\eta'}{}^2-m_\pi{}^2}{m_{\eta'}{}^2-m_\eta{}^2}\right)\simeq\frac{1}{\sqrt{3}}\times1.5\,.\quad(15)$$

Therefore, we see that the interesting interplay of the mass splitting of pseudoscalar mesons tends to make this ratio considerably larger (about 50%) than the $SU(3)$ value. We can make a better estimate. The term X is already of the first order in $SU(3)$ breaking and is small ($\simeq0.10$). Therefore, in the evaluation of the term Y, we may assume exact $SU(3)$ symmetry if we tolerate an error of the order 10–20%. We note the relations

$$\langle\gamma\gamma^V|\pi^0\rangle=\tfrac{1}{2}\langle\gamma\gamma|\pi^0\rangle\,,\quad\langle\gamma^V\gamma^V|\eta\rangle=-3\langle\gamma^S\gamma^S|\eta\rangle\,,$$

and

$$\langle\gamma^V\gamma^V|\eta'\rangle=3\langle\gamma^S\gamma^S|\eta'\rangle\,,\quad\text{etc.,}$$

in exact $SU(3)$ symmetry. Therefore, in Eq. (11) we can write

$$XY=0.10\times\{\tfrac{1}{2}\langle\gamma\gamma|\pi\rangle-\tfrac{3}{2}\sqrt{3}\cos\alpha\,\langle\gamma\gamma|\eta\rangle+\tfrac{3}{4}\sqrt{3}\sin\alpha\langle\gamma\gamma|\eta'\rangle\}\,.$$

Combining with Eq. (14), this leads to

$$\langle\gamma\gamma|\eta\rangle/\langle\gamma\gamma|\pi\rangle\simeq(\sqrt{3})^{-1}\times1.7\,.$$

Therefore, $\Gamma(\eta\to\gamma\gamma)$ will be larger by about a factor 3 than the $SU(3)$ value, i.e., $\Gamma(\eta\to\gamma\gamma)\simeq400$–$600$ eV. This is not very far from present preliminary experimental value. For the $\eta'\to2\gamma$ decay we obtain $\Gamma(\eta'\to2\gamma)\simeq5$ keV. This also seems to be a reasonable value.[13] Although we cannot make an absolute estimate of the $\Gamma(\pi^0\to2\gamma)$ in our approach, Adler[14] recently was able to derive $\Gamma(\pi^0\to2\gamma)\simeq9.7$ eV by using the hypothesis of pion partially conserved axial-vector current. However, neglecting η-η' mixing and assuming $F_\pi=F_\eta$ and also the validity of soft-η extrapolation, Adler found the standard $SU(3)$ prediction for the $\langle\gamma\gamma|\eta\rangle$, i.e.,

$$\langle\gamma\gamma|\eta\rangle/\langle\gamma\gamma|\pi\rangle=(\sqrt{3})^{-1}\,.$$

It is very interesting to notice that a similar result is also obtained in our approach if we neglect η-η' mixing, namely, if $\alpha=0$, Eqs. (11) and (14) lead to

$$\langle\gamma\gamma|\eta^0\rangle/\langle\gamma\gamma|\pi^0\rangle\simeq(\sqrt{3})^{-1}\times1.1\,.$$

This indicates an important role played by the η-η' mixing. Though the η-η' mixing angle is not large, the large mass of η' plays a significant role. Combined with Adler's result for $\pi\to2\gamma$ decay, the present work seems to imply that the two-photon decays of pseudoscalar mesons are no longer very mysterious.

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[13] This corresponds to $\langle\gamma\gamma|\eta'\rangle\simeq1.4\langle\gamma\gamma|\pi\rangle$; i.e., all three decays have comparable coupling constants, which seems reasonable. Compare with Ref. 3.

[14] S. L. Adler, Phys. Rev. **177**, 2426 (1969).