$\pi^- p$ Charge-Exchange Differential Cross Section and Polarization in a Regge-Pole Model*

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Regge-pole fits to $\pi^- p$ charge-exchange differential-cross-section and polarization data have been given at moderately high energies and over a wide range of momentum transfers. Besides the ρ trajectory, whose slope and intercept are determined by exchange degeneracy, a conspiring ρ' trajectory of the same slope but intercept near zero has been taken into account. In the spirit of the Veneziano formula, the residues are assumed to be of the form const/$\Gamma(\alpha)$. A four-parameter fit gives good agreement with the experimental data.

1. INTRODUCTION

It is well known that the exchange of the ρ Regge pole in the t channel is for the most part responsible for the high-energy $\pi^- p$ charge-exchange reaction. There are, however, two minor details which need further consideration: (a) The ρ trajectory alone predicts zero polarization, in contrast to the experimental data,[1] and (b) exchange degeneracy of the ρ and A_2 trajectories and their residues would require that the differential cross section should vanish at $t \approx -0.6$ where $\alpha_\rho = 0$,[2,3] whereas the experimental data have only a dip there. Both of these difficulties can be resolved if in addition to the ρ exchange there is some other contribution from another trajectory (the ρ') or a cut. The purpose of this article is to consider the first alternative and obtain a fit to the differential cross section and polarization over a wide range of energies and momentum transfers. In the spirit of the Veneziano representation[4] we take the residues to be of the form constant/$\Gamma(\alpha)$. The ρ trajectory is determined from the ρ and A_2 masses. The conspiring ρ' trajectory is assumed to have approximately zero intercept and to be parallel to the ρ trajectory. We are then fitting four independent parameters. In the Sec. 2 the problem is formulated and the results are given.

2. FORMALISM AND RESULTS

The exchange-degenerate ρ-A_2 trajectory, passing through the $\rho(760)$ and $A_2(1310)$, is given by

$$\alpha_\rho = 0.5 + at\,, \quad a = 0.9\ \text{GeV}^{-2}. \tag{1}$$

The helicity-nonflip and helicity-flip parts of the ρ contribution are given by

$$A_\rho' = [\beta_\rho{}^n/\Gamma(\alpha_\rho)]\xi_\rho(as)^{\alpha_\rho}, \tag{2}$$

$$B_\rho = [\beta_\rho{}^f/\Gamma(\alpha_\rho)]\xi_\rho(as)^{\alpha_\rho - 1}, \tag{3}$$

where $\xi = (1 - e^{-i\pi\alpha})/\sin\pi\alpha$, and β^n (β^f) is the helicity-nonflip (-flip) residue constant.

Igi[5] has given a Veneziano type formula for pion-nucleon scattering. In order to find a crossing symmetric formula with the correct signature factor he finds that the trajectories in the t channel (e.g., the ρ trajectory) should have the same slope as the trajectories in the s and u channels (e.g., the nucleon trajectory). Igi's formulas in fact reduce (effectively) to Eqs. (2) and (3) for the physical s channel (large s). In the same spirit as Eqs. (2) and (3) the contributions of a conspiring ρ' trajectory are given by

$$A_{\rho'}' = [t\beta_{\rho'}{}^n/\Gamma(\alpha_{\rho'})]\xi_{\rho'}(as)^{\alpha_{\rho'}}, \tag{4}$$

$$B_{\rho'} = [\beta_{\rho'}{}^f/\Gamma(\alpha_{\rho'})]\xi_{\rho'}(as)^{\alpha_{\rho'} - 1}. \tag{5}$$

In Eqs. (4) and (5) the ρ' trajectory is assumed to have the same slope as the ρ trajectory. Simplicity is the motivation for this assumption. One can imagine a Veneziano formula in which this ρ' trajectory combines with appropriate baryon trajectories having the same slope. If the ρ' trajectory has the same slope as the ρ trajectory then one can make the simplifying assumption that the ρ' trajectory coexists in a Veneziano formula with the ordinary baryon trajectories—in the same way as the ρ trajectory does. In other words, the nucleon, Δ, etc., in the s and u channels are shared between the ρ and ρ'. In this way the ρ' must have the same slope as the ρ and one avoids introducing new baryon trajectories of nonuniversal slope.

A possible Lorentz pole classification of exchange-degenerate and conspiring trajectories has been given by Ahmadzadeh[3] and by Ahmadzadeh and Jacob.[6] In this classification the exchange-degenerate π-B trajectory conspires with the exchange-degenerate A'_2-ρ' trajectory, where A'_2 is the co-conspirator of the pion and ρ' is the co-conspirator of the B trajectory. Therefore in this formalism π, B, ρ', and A'_2 all have the same intercept. But the π-B trajectory has the same universal slope as the ρ. Therefore, the ρ' trajectory would in fact be degenerate with the π-B trajectory and *it would be very interesting if a new particle at the mass*

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[2] A. Ahmadzadeh and C. H. Chan, Phys. Letters **22**, 692 (1966).

[3] A. Ahmadzadeh, Phys. Rev. Letters **20**, 1125 (1968).

[4] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

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of the B meson were found with the quantum numbers of the ρ meson. In the past the co-conspirators have been assumed to have very small slope to "explain" the fact that no particles belonging to these trajectories have been found experimentally. From the present point of view these particles may be hidden underneath other (known) resonances.[7]

The invariant amplitudes are given therefore in this model by

$$A' = A_\rho' + A_{\rho'}', \tag{6}$$

$$B = B_\rho + B_{\rho'}, \tag{7}$$

where A_ρ', B_ρ, $A_{\rho'}'$, and $B_{\rho'}$ are given by Eqs. (3)–(5).

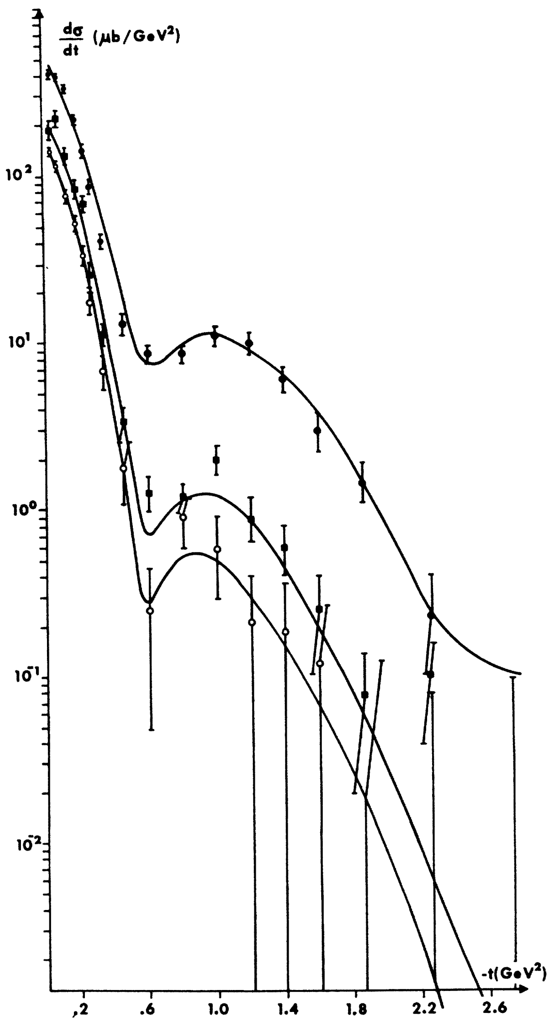

FIG. 1. Fit of differential cross sections at 5.9-, 13.3-, and 18.2-GeV/c lab momenta.

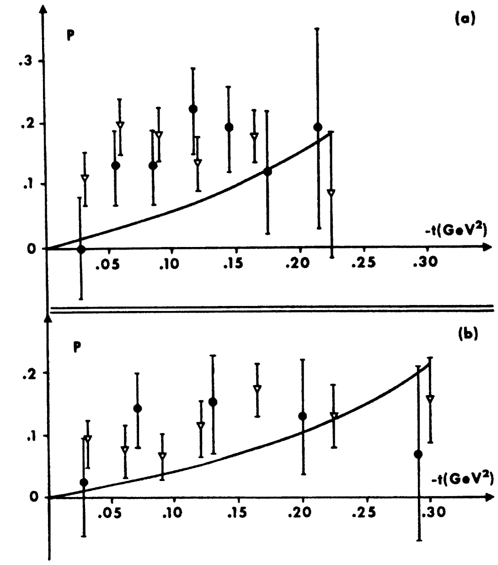

FIG. 2. Fit of polarization at (a) 5.9 GeV/c, (b) 11.2 GeV/c. The circles are data due to Bonamy *et al.*; the triangles are data due to Schneider *et al.*

The differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}\left\{ (4M^2 - t)|A'|^2 + \frac{t}{4M^2 - t}[4\mu^2 M^2 - ts - (s - M^2 - \mu^2)^2]|B|^2 \right\} \tag{8}$$

and polarization is

$$P = \frac{-\sin\theta \,\mathrm{Im}(A'B^*)}{16\pi(\sqrt{s})d\sigma/dt}. \tag{9}$$

It is seen from the above equations that we are left with four free parameters $\beta_\rho{}^n$, $\beta_\rho{}^f$, $\beta_{\rho'}{}^n$ and $\beta_{\rho'}{}^f$.

Let us now discuss the results. Figure 1 shows the least-squares fit to representative differential-cross-section data[8] for $P_{\text{lab}} \geqslant 5.85$ GeV/c. The data of Sonderegger *et al.* were used for the fit because they include a wide range of momentum transfer and are consistent with the other available data. For the 51 data points, the χ^2 is 54. The best values of the residue constants are $\beta_\rho{}^n = 18.5$ GeV^{-1}, $\beta_\rho{}^f = 211$ GeV^{-2}, $\beta_{\rho'}{}^n = -46.8$ GeV^{-3}, and $\beta_{\rho'}{}^f = -239$ GeV^{-2}. For these values, the predicted polarization is compared with experimental polarization in Fig. 2. The χ^2 for the 37 available data points is 74.

It should perhaps be pointed out that, since in the present model the ρ' does not contribute at the forward direction, the result obtained by one of the present authors[9] in fitting meson-nucleon cross sections at the

[7] The possibility of trajectories passing through integers without actually materializing as physical particles has also been considered by various authors [e.g., Shu-Yuan Chu, Chung-I Tan, and P. D. Ting, Phys. Rev. **181**, 2079 (1969)].

[8] P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966); A. V. Stirling *et al.*, Phys. Rev. Letters **14**, 763 (1965); I. Mannelli *et al.*, *ibid.* **14**, 408 (1965); M. A. Wahlig and I. Mannelli, Phys. Rev. **168**, 1515 (1968).

[9] Jane C. Jackson, Phys. Rev. **174**, 2098 (1968).

forward direction with the ρ and A_2 alone will still hold. Also we would like to mention that Sertorio and Toller[10] have also considered a model consisting of the ρ and a conspiring ρ' to fit the π-p charge-exchange data. The present model differs from theirs mainly in the universal slope of the trajectories, the $t=0$ ρ' intercept, and the behavior of the residues. Also recently Delbourgo and Salam[11] have considered a Regge-pole supermultiplet theory whereby due to the higher assumed symmetry the helicity-flip and -nonflip residues are related. They consider the ρ contribution alone, however, and therefore their model gives zero polarization and predicts zero differential cross section (instead of dips) where α_ρ is a negative integer or zero.

In conclusion we point out that the present model, in the same spirit as in Ref. 3, is an attempt to tie up several ideas (conspiracy, exchange degeneracy, and now Veneziano-type residues, etc.) in a consistent way. The widely considered Regge-pole treatment of photoproduction[12] and np charge exchange[13] is in terms of the pion conspiracy idea. On the other hand, exchange degeneracy implies that the B trajectory should also conspire (if the pion trajectory does). The co-conspirator of the B trajectory is the ρ' which we have utilized here. Finally, we point out that the present ideas imply that a conspiring A_2' (which is the co-conspirator of the pion) of intercept near zero, together with the A_2 trajectory, should be capable of explaining the $\pi^- p \to \eta n$ data. This reaction is the subject of a future investigation.

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[11] R. Delbourgo and A. Salam, Phys. Letters **28B**, 497 (1969).
[12] See, for example, A. Ahmadzadeh, R. J. Jacob, and B. P. Nigam, Phys. Rev. **178**, 2284 (1969).
[13] See, for example, F. Arbab and J. Dash, Phys. Rev. **163**, 1603 (1967).

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Two-Variable Expansion of the Scattering Amplitude for any Mass and Crossing Symmetry for Partial Waves

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A two-variable expansion of the scattering amplitude for the process $a+b \to c+d$ is proposed, where a, b, c, and d are spinless particles of arbitrary mass. It is diagonal in angular momentum, displays the threshold and pseudothreshold behavior of partial waves, and leads to sum rules which contain a *finite* number of partial waves due to the crossing symmetry of the collision amplitude. The results of our previous work are recovered when the masses are equal. The reaction $\pi+N \to \pi+N$ is treated with the inclusion of nucleon spin. The expansion is valid over the Dalitz plot for a decay amplitude. A simple method to derive sum rules which relate a finite number of partial waves without the use of the two-variable expansion is also outlined.

I. INTRODUCTION

THIS paper formulates a generalization of some previous work on two-variable expansions of scattering amplitudes[1] to processes which involve spinless particles of arbitrary mass. The original investigation dealt with a system where the masses of the four particles were equal. The amplitude was expressed as a sum of polynomials of the variables s, t, and u which were orthogonal and complete for a suitable scalar product over the Mandelstam triangle. The basis was diagonal in angular momentum, revealed the existence of an infinite sequence of finite dimensional "crossing matrices" for partial waves, and displayed their threshold behavior.

The investigation in Ref. 1 relies on the observation that there is a partial differential operator $\mathcal{O}$ in the variables s, t, and u which commutes with the angular mo-

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