the masses of B_1^* and l_1, other parameters, except possibly coupling constants, remaining the same if corresponding interactions of l_1 and l_4 are similar, and if it is assumed that the masses of l_2 and l_3 vanish. If the masses of B_1^* and B_4^*, which are determined at least in part by strong interactions, are not equal, the masses of l_1 and l_4 can also be expected to be different. If l_1 and l_4 are assumed to correspond to the electron and muon or vice versa, the degeneracy between them could conceivably be removed in this manner.

Further investigations along the lines suggested in this discussion are in progress.

Recently, another attempt has been made to relate representations of operators satisfying trilinear or higher-order commutation relations to the internal symmetries of elementary particles.[20] It is also proper to mention that several years ago renewed interest in higher-order commutation relations was stimulated by Volkov.[21,22]

[20] A. B. Govorkov, Zh. Eksperim. i Teor. Fiz. **54**, 1785 (1968) [English transl.: Soviet Phys.—JETP **27**, 960 (1968)].
[21] D. V. Volkov, Zh. Eksperim. i Teor. Fiz. **36**, 1560 (1959) [English transl.: Soviet Phys.—JETP **9**, 1107 (1959)].
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Current Algebra, Field-Current Identity, the $K_2^0K_1^0$ Electromagnetic Transition, and the Decays $K^0\rightarrow\pi\pi e^+e^-$†

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A general discussion is given of the matrix element for the decays $K^0\rightarrow\pi\pi e^+e^-$; then, using the results of the vector-meson-dominance model to predict the $K_2^0K_1^0$ electromagnetic transition radius, partial conservation of axial-vector current and current algebra to predict the $K_2^0\rightarrow\pi^+\pi^-\gamma$ transition rate, and the experimental value of the CP-violating parameter ϵ, we find the following branching ratios: $\Gamma(K_L^0\rightarrow\pi^+\pi^-e^+e^-)/\Gamma(K_L^0\rightarrow \text{all})=1.7\times10^{-7}$, $\Gamma(K_L^0\rightarrow\pi^0\pi^0e^+e^-)/\Gamma(K_L^0\rightarrow \text{all})=0.2\times10^{-7}$, and $\Gamma(K_S^0\rightarrow\pi^+\pi^-e^+e^-)/\Gamma(K_S^0\rightarrow \text{all})=2.0\times10^{-5}$.

I. INTRODUCTION

In this paper, we propose to study the decays of the neutral K mesons into two pions and an electron-positron pair. In some of these decays the contribution from the bremsstrahlung diagram is absent or very much suppressed, so that the decay amplitudes are governed by the structure-dependent terms in the $K\rightarrow2\pi\gamma$ vertex and the $K_2^0K_1^0$ electromagnetic transition amplitude. From the theoretical point of view, both these processes are important. The value of the $K_2^0K_1^0\gamma$ transition radius is predicted by the theory proposed by Kroll, Lee, and Zumino,[1] who assume that the entire hadronic electromagnetic current operator is to be identified with a linear combination of the renormalized field operators for the neutral vector mesons[2] ρ^0, ω^0, and ϕ^0. Accurate data for these decays may even distinguish between predictions of the mass mixing and the current mixing models of the ω-ϕ system. Evaluation of the $K\rightarrow2\pi\gamma$ structure-dependent terms from these decays will check the models proposed in the literature.[3] Finally, these decays offer a good opportunity to study the various consequences of CP-violating effects, because both weak and electromagnetic interactions play a role in the decay amplitude.[4]

The plan of this paper is as follows. In Sec. II, we begin by studying the general structure of the matrix elements for $K^0\rightarrow\pi\pi\gamma$ and $K^0\rightarrow\pi\pi e^+e^-$ decays. The CP properties of the various terms are discussed. Then we calculate the decay spectra in the dipion and dilepton invariant masses and the decay rates are expressed in terms of form factors which are taken to be constants. In Sec. III, we indicate how the different

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[1] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).
[2] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).
[3] H. Chew, Nuovo Cimento **26**, 1109 (1962); S. V. Pepper and Y. Ueda, *ibid.* **33**, 1614 (1964); S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. **136**, 1064 (1964); C. S. Lai and B. L. Young, Nuovo Cimento **52A**, 83 (1967).
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structure-dependent terms are evaluated. Section IV is devoted to our results and some remarks about the present experimental situation.

II. DECAYS $K^0 \to \pi\pi\gamma$ AND $K^0 \to \pi\pi e^+e^-$

Several authors[3,4] have discussed the general structure of the $K^0 \to \pi^+\pi^-\gamma$ decay. There is also an upper limit[5] on the decay rate for $K_L{}^0 \to \pi^+\pi^-\gamma$ of 7.5×10^3 sec^{-1}. (We denote the CP eigenstates of K mesons by $K_1{}^0$ and $K_2{}^0$, while $K_S{}^0$ and $K_L{}^0$ refer to the physical particles.) In these decays the photon is on its mass shell and cannot be coupled to an intermediate $K_2{}^0K_1{}^0$ state, so there are two structure-dependent terms and the usual inner bremsstrahlung contribution. When the photon produces a Dalitz pair, e.g., in the decay $K^0 \to \pi\pi e^+e^-$, the photon is off its mass shell and the $K_2{}^0K_1{}^0\gamma$ transition can make an important contribution. From Low's theorem[6] we know that the amplitude for $K^0 \to \pi\pi\gamma$ contains a bremsstrahlung contribution (of order k^{-1}) and two contributions (of order k) which are parity-conserving and parity-violating, respectively. The fact that there is no term of order zero follows from a theorem due to Chew and to Pestieau.[7] When the photon is off its mass shell, one more amplitude contributes to the matrix element and it must be of higher order in k and constructed out of $k\cdot\epsilon$ and k^2. Thus, if we use the notation $K^0(P) \to \pi^+(q_1)+\pi^-(q_2)+\gamma(k)$, the off-shell amplitude, which is linear in the photon polarization vector ϵ_μ, can be written as $M_\mu\epsilon_\mu$, where

$$M_\mu = ef[A_1(k^2P_\mu - P\cdot kk_\mu)+B_1(P\cdot kq_{1\mu}-q_1\cdot kP_\mu) +B_2\epsilon_{\mu\nu\rho\sigma}k_\nu q_{1\rho}P_\sigma + C(q_{1\mu}/q_1\cdot k - q_{2\mu}/q_2\cdot k)]. \quad (2.1)$$

Here f is the $K^0 \to \pi\pi$ decay constant and e the electric charge. We denote by C the bremsstrahlung coupling constant (retaining only the infrared divergent terms), A_1 the coefficient of the K^0 pole term (Fig. 1), B_1 the coefficient of the parity-violating direct emission amplitude, and B_2 the coefficient of the parity-conserving direct emission amplitude. The form factors A_1, B_1, and B_2 are functions of k^2, $P\cdot k$, and $(q_1-q_2)\cdot k$, while C is a constant determined entirely by the corresponding nonradiative vertex, namely, $K^0 \to \pi^+\pi^-$. When the photon is real, there are only two invariants $P\cdot k$ and $(q_1-q_2)\cdot k$, and the A_1 term makes no contribution.

In the c.m. frame of the two pions, $P\cdot k$ is spherically symmetric and $(\mathbf{q}_1-\mathbf{q}_2)\cdot\mathbf{k} = 2|\mathbf{q}_1||\mathbf{k}|\cos\theta$, where θ is the angle between $\mathbf{q}_1$ and $\mathbf{k}$. CP invariance implies that B_1 is an odd function of $\cos\theta$, while B_2 is even. Hence, the dominant term among the structure-dependent contributions is expected to be the magnetic dipole emission term B_2 with the two pions in a p wave. B_1 contributes to electric dipole CP-violating and electric quadrupole CP-conserving radiation and is expected to be smaller than B_2 by roughly one order of magnitude. Time-reversal-violating effects can be observed in these decays by finding a large asymmetric distribution of e^+ and e^- with respect to the plane defined by the pion momenta; cf. the discussion by Dolgov and Ponomarev.[4]

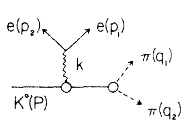

FIG. 1. Kaon pole model for the decay $K^0 \to \pi\pi e^+e^-$.

In evaluating the contributions from the bremsstrahlung diagrams, one immediately finds that CP invariance plays an important role. The diagram where the photon radiates from the charged pion lines in the CP-allowed vertex $K_1{}^0 \to \pi^+\pi^-$ is expected to dominate over the structure-dependent terms. However, for the $K_2{}^0$ decay there is no analogous diagram if CP is strictly conserved. Even allowing for the possibility of CP violation in the decay $K_L{}^0 \to \pi^+\pi^-\gamma$, we expect the structure-dependent terms to be larger than the bremsstrahlung contribution. We note that the decay mode into neutral pions has no bremsstrahlung term at all. Thus, the decays where the bremsstrahlung term is suppressed or absent allow one to study the structure-dependent terms. In fact, this gives us the possibility of measuring the $K_2{}^0K_1{}^0\gamma$ coupling constant which enters into A.

Let us now review briefly the previous theoretical work on these decays. Chew[3] calculated the internal conversion factor $\Gamma(K_2{}^0 \to \pi^+\pi^-e^+e^-)/\Gamma(K_2{}^0 \to \pi^+\pi^-\gamma)$ taking only the B_2 term into account and using a final-

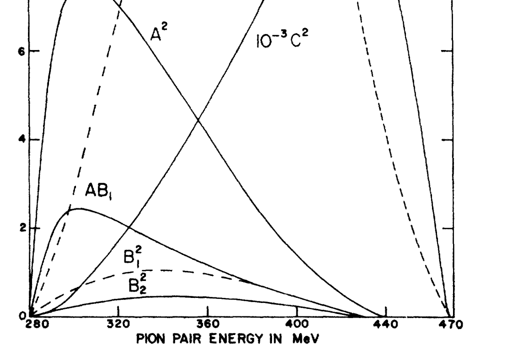

FIG. 2. Pion-pair energy spectrum. Note that two of the curves have been rescaled. After changing units to $M_K = 1$, the area under each curve is the coefficient of the corresponding term in Eq. (2.8).

[5] R. C. Thatcher, A. Abashian, R. J. Abrams, D. W. Carpenter, R. E. Mischke, B. M. K. Nefkens, J. H. Smith, L. J. Verhey, and A. Wattenberg, Phys. Rev. **174**, 1674 (1968).

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[7] H. Chew, Phys. Rev. **123**, 377 (1961); J. Pestieau, *ibid.* **160**, 1555 (1967).

state interaction between the pions. He found values for this factor between 0.64×10^{-2} and 0.52×10^{-2} depending on the values of the final-state phase shifts. Kondratyuk, Pononarev, and Zakharov[8] recently considered the same decay taking the A and B_2 terms into account. However, they did not consider the problem of evaluating the coupling constants from the theoretical point of view and made only very rough estimates. In this paper, we plan to keep all four terms in the matrix element and make a systematic evaluation of the $K_S{}^0$ and $K_L{}^0$ spectra and decay rates. Incidentally, we note that analogous considerations also hold for the decay $\eta\to\pi^+\pi^-e^+e^-$ where there is no inner bremsstrahlung term (the decay $\eta\to\pi^+\pi^-$ is forbidden because it violates parity) and only the B_2 term need be retained in the structure-dependent amplitude.

The matrix element for $K^0\to\pi\pi e^+e^-$ is

$$M=(e/k^2)\bar{v}(p_2)\gamma_\mu u(p_1)M_\mu\,, \tag{2.2}$$

where M_μ is given in Eq. (2.1). A straightforward sum over spins and trace calculation will not be reported here. In view of the fact that the decay rates were very small, we do not discuss angular distributions but give the double distribution in x, the square of the mass of the pion pair; and in y, the square of the mass of the lepton pair. The other three angles necessary to specify the orientation of the specific particles have been integrated out using invariant-phase-space techniques. Throughout this calculation we have neglected all terms in the trace proportional to the electron mass. The branching ratio is found to be

$$\frac{\Gamma(K^0\to\pi\pi e^+e^-)}{\Gamma(K_S{}^0\to\pi^+\pi^-)}=\frac{\alpha}{64\pi^3\lambda^{1/2}(1,\mu^2,\mu^2)}\int_{4m^2}^{(1-2\mu)^2}\frac{dy}{y^3}\lambda^{1/2}(y,m^2,m^2)\times\int_{4\mu^2}^{(1-\sqrt{y})^2}dx\,\lambda^{1/2}(1,x,y)F(x,y)\,, \tag{2.3}$$

where m and μ are the electron and pion masses, respectively, normalized to the kaon mass

$$\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2xz \tag{2.4}$$

and

$$\begin{aligned}F(x,y)={}&\frac{1}{3}(A_1{}^2+A_1B_1)\frac{y^2}{x}\lambda(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2)+\frac{1}{18}B_2{}^2\frac{y}{x^2}\lambda(1,x,y)\lambda^{3/2}(x,\mu^2,\mu^2)\\&+\frac{1}{18}B_1{}^2\frac{y}{x}\lambda^{1/2}(x,\mu^2,\mu^2)\left([\lambda(1,x,y)+6y]\lambda(x,\mu^2,\mu^2)+\frac{2y}{x}\lambda(1,x,y)(x-\mu^2)\right)\\&+\frac{8}{3}B_1Cy\left(-\lambda^{1/2}(x,\mu^2,\mu^2)+\frac{xy+2\mu^2(1-x-y)}{\lambda^{1/2}(1,x,y)}\ln L\right)\\&+\frac{8}{3}C^2y\left[-\frac{16\lambda^{1/2}(x,\mu^2,\mu^2)x\mu^2}{[x^2(x+y-1)^2-\lambda(1,x,y)\lambda(x,\mu^2,\mu^2)]}+\frac{(x-2\mu^2)}{(x+y-1)\lambda^{1/2}(1,x,y)}\ln L\right]\end{aligned} \tag{2.5}$$

with the form factors A_1, B_1, B_2, and C defined in Eq. (2.1) and

$$L=\frac{(x+y-1)x+\lambda^{1/2}(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2)}{(x+y-1)x-\lambda^{1/2}(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2)}\,. \tag{2.6}$$

As expected, the interference between terms of different parity vanishes. The term in A_1C does not vanish in the trace but reduces to zero after integration over the angles. (Cf. the general theorem of Burnett and Kroll[9] that interference between bremsstrahlung and anomalous magnetic moment terms gives no contribution to the total cross section. The theorem has been extended to particles with spin by Van Royen and Bell.[10]) We also note that the $B_2{}^2$ term checks with the result of Jarlskog and Pilkuhn[11] for η decay. The matrix element for Fig. 1 requires a kaon pole term linking the $K_2{}^0K_1{}^0\gamma$ vertex with the $\pi^+\pi^-$ vertex, so A_1 contains an extra factor proportional to $(\frac{1}{2}k^2-P\cdot k)^{-1}=2(1-x)^{-1}$. This has been subsequently incorporated into the two terms $A_1{}^2$ and A_1B_1 by defining

$$A_1=2A/(1-x)\,. \tag{2.7}$$

The spectra in x and y for the charged decay mode are plotted in Figs. 2 and 3 assuming a cutoff on x at 30 MeV. We notice that the spectra in y are strongly peaked in the region of small y, an effect not seen in the spectra reported by the authors of Ref. 8. A double integration over x and y yields the following values for the branching ratios:

$$\begin{aligned}\Gamma(K^0\to\pi^+\pi^-e^+e^-)/\Gamma(K^0\to\pi^+\pi^-)\\=1.68\times10^{-8}A^2+4.52\times10^{-9}AB_1+4.55\\\times10^{-7}B_1C+2.84\times10^{-9}B_1{}^2+1.35\\\times10^{-9}B_2{}^2+3.048\times10^{-5}C^2\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}\Gamma(K^0\to\pi^0\pi^0e^+e^-)/\Gamma(K^0\to\pi^0\pi^0)\\=1.98\times10^{-8}A^2+5.49\times10^{-9}AB_1\\+3.48\times10^{-9}B_1{}^2+1.66\times10^{-9}B_2{}^2.\end{aligned} \tag{2.9}$$

[8] L. A. Kondratyak, L. A. Ponomarev, and V. I. Zakharov, Phys. Letters **27B**, 655 (1968).
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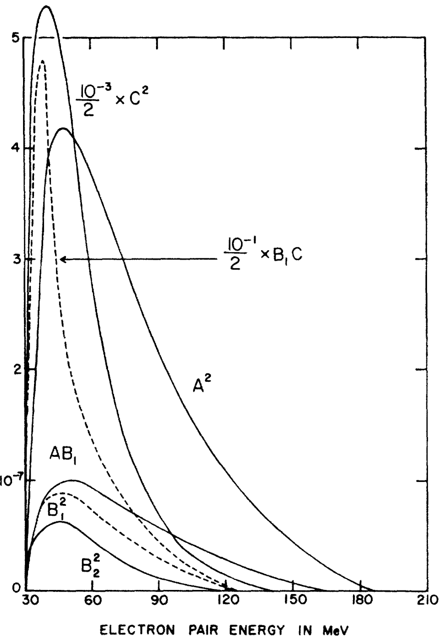

FIG. 3. Lepton-pair energy spectrum. Note that two of the curves have been rescaled. After changing units to $M_K=1$, the area under each curve is the coefficient of the corresponding term in Eq. (2.8).

Terms in C do not contribute to Eq. (2.9) because there is no inner bremsstrahlung in the neutral decay mode.

III. NUMERICAL ESTIMATES OF A, B_1, AND B_2

In this section, we consider the evaluation of the decay coupling constants. Let us first consider A. From Lorentz invariance and current conservation, we can write the matrix element of the electromagnetic current between K^0 meson states in the following form:

$$\begin{aligned}\langle K^0(p')|J_\mu{}^{\mathrm{e.m.}}(0)|K^0(p)\rangle \\ =[k^2p_\mu-(p\cdot k)k_\mu]F(k^2,p'^2,p^2)\,. \quad (3.1)\end{aligned}$$

Here F is the off-mass-shell electromagnetic form factor of the neutral kaon. When the kaons are on mass shell, this reduces to

$$\langle K^0(p')|J_\mu{}^{\mathrm{e.m.}}(0)|K^0(p)\rangle=(p+p')_\mu\tfrac{1}{2}k^2F(k^2) \quad (3.2)$$

and vanishes for $k^2=0$. Now we know that

$$\langle K^0|J_\mu|K^0\rangle=-\langle\bar{K}^0|J_\mu|\bar{K}^0\rangle=\mathrm{Re}\langle K_1{}^0|J_\mu|K_2{}^0\rangle\,, \quad (3.3)$$

where $K_1{}^0$ and $K_2{}^0$ are the CP eigenstates with eigenvalues $+1$ and -1, respectively. The mean-square radius of the K^0 charge distribution is, by definition, given by

$$R^2(K^0)=-3F(k^2)|_{k^2=0}\,, \quad (3.4)$$

and the corresponding charge radius of $\bar{K}^0$ is given by $R^2(K^0)=-R^2(\bar{K}^0)$. These mean-square radii can be evaluated by assuming vector meson dominance of the electromagnetic form factors. This is beautifully realized in the hypothesis of Kroll, Lee, and Zumino,[1] who found that, corresponding to different models of the ω, ϕ system, $R^2(K^0)$ is given by

$$\begin{aligned}R^2(K^0)&=-7.6\times10^{-28}\text{ cm}^2 &&\text{for the current-mixing model}\\ &=-7.0\times10^{-28}\text{ cm}^2 &&\text{for the mass-mixing model I}\\ &=-6.1\times10^{-28}\text{ cm}^2 &&\text{for the mass-mixing model II}. \quad (3.5)\end{aligned}$$

Assuming F to be constant, the term A in Eq. (2.7) can now be easily calculated. We observe that

$$2A/M_K{}^2=F=-\tfrac{1}{3}R^2(K^0)\,, \quad (3.6)$$

so $A=0.080$, 0.074, and 0.064 for the three models, respectively.

Experiments by Augustin *et al.*[12] on e^+e^- annihilation into pions and kaons seems to favor the current-mixing model over the mass-mixing models. However the evidence is not conclusive so we accept a value $A=0.07$, i.e., $A^2=5\times10^{-3}$. The authors of Ref. 8 assumed that the $K_2{}^0K_1{}^0$ charge radius is equal to the pion electromagnetic radius which is known from the experiment of Akerlof *et al.*[13] to be $r_\pi=0.80\pm0.10$ F. This is an order of magnitude larger than the number predicted from the theory of Kroll, Lee, and Zumino (KLZ). Since all available experimental evidence quantitatively supports the vector-meson-dominance model[14] we prefer to use the value of $R^2(K^0)$ predicted by KLZ. We may further add that if one makes a very simple model assuming the $K_2{}^0K_1{}^0\gamma$ vertex to be dominated by the ϕ meson with the ϕ subsequently decaying into an electron-positron pair and uses the known decay rates of the ϕ meson, one gets a value of A^2 similar to that quoted above.

We now turn to the value of B_2, which is expected to be the dominant term in the $K_L{}^0\to\pi^+\pi^-\gamma$ decay mode. The present upper limit on the decay rate is 7.5×10^3 sec^{-1}. Using the expression for the decay rate

$$\Gamma(K_L{}^0\to\pi^+\pi^-\gamma)=\frac{M}{64\pi^3}\left(\frac{\mu}{M}\right)^5\tfrac{1}{3}f^2B_2{}^2\times\left[\frac{1}{\mu^5}\int_0^{\bar{k}}dk\,\frac{(\bar{k}-k)^{3/2}k^3}{(\frac{1}{2}M-k)^{1/2}}\right],$$

where $\bar{k}=\frac{1}{2}M-2\mu^2/M$, the numerical value of the integral in brackets, which is 0.0846, and taking f^2 from the $K_s{}^0\to\pi^+\pi^-$ decay rate, we find $B_2{}^2=2.3$. The authors of Ref. 8 quote $B_2{}^2=1.2$ because they used a preliminary value of the decay rate which has since been revised.

[12] J. E. Augustin, D. Benaksas, J. C. Bizot, J. Buon, B. Delcourt, V. Gracco, J. H. Haissinski, J. Jeanjean, D. Lalanne, F. Laplanche, J. Le Francois, P. Lehmann, P. Mavin, H. Nguyen Ngot, J. Perez-y-Jorba, F. Richard, F. Rumpf, E. Silva, S. Tavenier, and D. Treille, Phys. Letters **28B**, 503 (1969).

[13] C. W. Akerlof, W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Rev. **163**, 1482 (1967).

[14] J. G. Asbury, V. Becker, W. K. Bertram, M. Binkley, E. Coleman, C. L. Jordan, M. Rohde, A. J. S. Smith, and S. C. C. Ting, Phys. Rev. Letters **20**, 227 (1968).

We therefore expect B_2^2 to be smaller than 2.3 and proceed to take the reduction factor from the predictions of current algebra.

Lai and Young[3] considered the decay $K_2^0 \to \pi^+\pi^-\gamma$ and used the hypothesis of partially conserved axial-vector current (PCAC) to obtain the decay amplitude where both pions are soft. Using two different models of the weak Hamiltonian (written either in current-current form or as a combination of scalar and pseudoscalar densities), they applied current commutation relations.

The pions are in a relative p wave, so the final answer for the decay matrix element is related to the amplitude for $K_2^0 \to \gamma +$(isovector photon). $SU(3)$ symmetry relates this matrix element to that of $K_2^0 \to 2\gamma$ which is known experimentally.[15] Although their final answer contains an extra term proportional to μ^4, which should be deleted, this does not alter any of their numerical estimates. The result of this calculation indicates that B_2^2 should be reduced by a factor of 4, so we choose $B_2^2 = 0.5$.

Unfortunately, we are completely ignorant of the value of B_1, and tentatively assume $B_1 = 0.1 \times B_2$. At this level B_1 can be safely neglected from further consideration but we leave its contribution in Eqs. (2.8) and (2.9) in case it turns out to be needed at some future date.

IV. RESULTS

Our final answers for the rates depend on the value of C which in turn depends on the specific decay under consideration. If we first take $K_S^0 \to \pi^+\pi^-e^+e^-$, then $C=1$ and the bremsstrahlung contribution completely dominates over all the other terms, yielding

$$\Gamma(K_S^0 \to \pi^+\pi^-e^+e^-)/\Gamma(K_S^0 \to \pi^+\pi^-) = 3.0\times 10^{-5}.$$

Actually, the corrections to the inner bremsstrahlung term are much less than one percent. In the decay $K_L^0 \to \pi^+\pi^-e^+e^-$, C is given by ϵ, the CP impurity in the $K_S^0K_L^0$ states,[16] which we know[17] to be approximately 1×10^{-3}. The inner bremsstrahlung contribution is therefore smaller than both the B_2^2 and A^2 terms but is nevertheless significant. Thus, we find

$$(K_L^0 \to \pi^+\pi^-e^+e^-)/\Gamma(K_S^0 \to \pi^+\pi^-) = 8.0\times 10^{-10}.$$

The decays into neutral pions have no bremsstrahlung contribution and, although we cannot find B_2^2 from current algebra because vector-meson decays into $2\pi^0$ are strictly forbidden, we expect the A^2 structure-dependent term to be equal in magnitude to the corresponding value for the decays into charged pions. We therefore estimate on the basis of an A^2 term alone that

$$\begin{aligned}(K_S^0 \to \pi^0\pi^0e^+e^-)/\Gamma(K_L^0 \to \pi^0\pi^0) \\ = \Gamma(K_L^0 \to \pi^0\pi^0e^+e^-)/\Gamma(K_S^0 \to \pi^0\pi^0) = 1\times 10^{-10}.\end{aligned}$$

In particular $\Gamma(K_S^0 \to \pi^0\pi^0e^+e^-)/\Gamma(K_S^0 \to \text{all})$ is vanishingly small compared to the ratio $\Gamma(K_S^0 \to \pi^0 2\gamma)/\Gamma(K_S^0 \to \pi^0\pi^0) \simeq 6\times 10^{-10}$ predicted by Della Selva *et al.*[18] Both decay rates are proportional to α^2. Also $\Gamma(K_L^0 \to \pi^0\pi^0e^+e^-)/\Gamma(K_L^0 \to \text{all}) = 0.2\times 10^{-7}$ is smaller than the branching ratio $\Gamma(K_L^0 \to \pi^0 2\gamma)/\Gamma(K_L^0 \to \text{all})$ which has been estimated by Della Selva *et al.* and Fäldt *et al.*[19] to range between 10^{-4} and 10^{-7}.

If the value assumed for the $K_2^0K_1^0$ transition radius is too small, then the rate for the decay $K_L^0 \to \pi^+\pi^-e^+e^-$ will vary accordingly. In fact, it would be very interesting to place an upper limit for this decay because it is present in the background of all η_{+-} experiments, but will be difficult to distinguish from the decay $K_L^0 \to \pi^+\pi^-\pi^0$ where only one Dalitz pair from the π^0 decay is identified. The total branching ratio $\Gamma(K_L^0 \to \pi^+\pi^-e^+e^-)/\Gamma(K_2^0 \to \text{all}) = 1.7\times 10^{-7}$ is rather small but should soon be within reach of experiment. The total branching ratio for the corresponding decay of the short-lived neutral kaon is $\Gamma(K_S^0 \to \pi^+\pi^-e^+e^-)/\Gamma(K_S^0 \to \text{all}) = 2.0\times 10^{-5}$.

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