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Some Aspects of Field Symmetries. II

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The consideration of representations of operators satisfying trilinear equal-time commutation relations and their possible physical relevance is continued. A representation previously studied, when modified to some extent, yields an improved correspondence between the operators of the representation and physical particles. Several schemes of associating operators with particles are considered. The representations raise the possibility of partially eliminating some divergences. In an approximation in which spin-1 and higher-spin resonances are disregarded, the requirement that the lowest-order self-energy corrections to the propagators of spin-$\frac{1}{2}$ baryons be finite implies a set of relations between the coupling constants and mass ratios of the particles concerned.

INTRODUCTION

FROM the formalism of Schwinger's action principle[1] it is possible to derive a set of trilinear equal-time commutation relations which contains the trilinear commutation relations discussed by Green,[2] by Kibble and Polkinghorne,[3] and elsewhere[4,5] as special cases.[6] The trilinear equal-time commutation relations are[7–9]

$$[\Psi_\alpha(\mathbf{x}),[\chi_\beta(\mathbf{y}),\chi_\gamma'(\mathbf{z})]_-]_-=\delta(\mathbf{x}-\mathbf{y})(\gamma_4')_{\alpha\beta}M\chi_\gamma'(\mathbf{z})-\delta(\mathbf{x}-\mathbf{z})(\gamma_4')_{\alpha\gamma}M\chi_\beta(\mathbf{y}),\quad(1\text{a})$$

$$[\Phi_\mu(\mathbf{x}),[\chi_\nu(\mathbf{y}),\chi_\rho'(\mathbf{z})]_+]_-=i\tilde{\delta}_{\mu\nu}\delta(\mathbf{x}-\mathbf{y})N\chi_\rho'(\mathbf{z})+i\tilde{\delta}_{\mu\rho}\delta(\mathbf{x}-\mathbf{z})N\chi_\nu(\mathbf{y}).\quad(1\text{b})$$

In Eq. (1a), any two field variables may be either kinematically related or unrelated, and each variable denotes independently of the others either a field variable or its canonical conjugate. The same applies to the field operators occurring in Eq. (1b). The operators M and N, which depend on the fields in whose commutation relations they occur, are direct products of numerical matrices and appropriate "operator Kronecker δ's" introduced previously.[8] The ordered Kronecker δ, $\tilde{\delta}_{\mu\nu}$, also has been defined previously.[5]

The representations of generalized fields [i.e., operators satisfying Eqs. (1)] considered here are direct products of numerical matrices and field operators ("component fields"):

$$\Psi_i(\mathbf{x})=A_i\times\psi(\mathbf{x}),\quad\bar{\Psi}_i(\mathbf{x})=A_i'\times\bar{\psi}(\mathbf{x}),\qquad(2\text{a})$$

and

$$\Phi_i(\mathbf{x})=B_i\times\phi(\mathbf{x}),\quad\Pi_i(\mathbf{x})=B_i'\times\pi(\mathbf{x}).\qquad(2\text{b})$$

For the representations considered in this discussion, the generalized fields also satisfy the bilinear equal-time commutation relations

$$[\Psi_{i,\alpha}(\mathbf{x}),\bar{\Psi}_{i,\beta}(\mathbf{y})]_+=(\gamma_4)_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y})A_iA_i'\times\delta(\Psi_i,\bar{\Psi}_i),\quad(3\text{a})$$

$$[\Phi_{i,\mu}(\mathbf{x}),\Pi_{i,\nu}(\mathbf{y})]_-=i\delta_{\mu\nu}\delta(\mathbf{x}-\mathbf{y})B_iB_i'\times\delta(\Phi_i,\Pi_i),\quad(3\text{b})$$

which amount to a generalization of canonical field quantization (no summation over repeated indices). The operator Kronecker δ's appearing in Eqs. (3) have the same bilinear commutation behavior with respect to other component fields as the products of the component fields, respectively, occurring on the left-hand sides of the commutation relations (3).

The connection between the numerical matrix coefficients of the generalized fields and the bilinear equal-time commutation relations between the component fields has been considered in some detail.[7–9] Since the matrix coefficients specify the bilinear commutation relations between distinct component fields, the subscripts of the component fields have for simplicity been suppressed throughout this discussion. The interdependence of interactions and bilinear commutation relations of the component fields as well as the selection rules implied by the bilinear commutation relations have been discussed elsewhere.[7–13] For the representations of the generalized fields considered in this discussion, these selection rules imply that three generalized fields, which in addition to Eqs. (1) also satisfy bilinear equal-time commutation relations with each other, can enter into a trilinear interaction if the three matrices associated with the fields concerned either all commute or all anticommute with each other. In addition, the interactions are required to be Hermitian. If the generalized fields involved in a trilinear or higher-order interaction do not all satisfy bilinear equal-time commutation relations with each other, the interaction has to be suitably symmetrized. For the

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[13] H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Co., Amsterdam, 1956), p. 197.

correspondence between particles and generalized fields to be assumed in this discussion, all distinct fields which can enter into an allowed trilinear interaction either commute or anticommute for equal times.

It is expedient to reconsider a representation of the generalized fields based on the generators[7-9]

$$C_1=\begin{pmatrix}\sigma_x & 0\\ 0 & \sigma_y\end{pmatrix},\quad C_2=\begin{pmatrix}\sigma_y & 0\\ 0 & \sigma_x\end{pmatrix},\quad C_3=\begin{pmatrix}0 & \sigma_x\\ \sigma_x & 0\end{pmatrix}. \tag{4}$$

Disregarding phase and normalization factors, the matrices obtained from the generators (4) can most effectively be divided into sets in the following manner:

$$G=(I, C_1C_3C_1C_3, C_1C_2, C_1C_3C_2C_3),$$
$$K=((C_1C_3\pm C_3C_1), (C_2C_3\pm C_3C_2)),$$
$$A=(C_1, C_2, C_3C_1C_3, C_3C_2C_3, (C_1\pm C_3C_1C_3), (C_2\pm C_3C_2C_3)), \tag{5}$$

and

$$B=(C_3, C_1C_3C_1, C_1C_2C_3, C_1C_3C_2, (C_3\pm C_1C_3C_1), (C_1C_2C_3\pm C_1C_3C_2)).$$

The matrices in set G are diagonal. The matrices in set A have nonvanishing elements only in the upper left and lower right quadrants, whereas the nonvanishing elements of the matrices contained in sets B and K are in the upper right and lower left quadrants. A total of 16 linearly independent matrices are obtained if, for example, the last four matrices in set A and the first four matrices in set B are disregarded.

For the representations of the generalized fields considered, Lagrangians are 4×4 matrices. For interactions involving half-integral-spin fields, the Lagrangian densities are assumed to have the form

$$R_{abc}[\Psi_a(\mathbf{x}),\Gamma\Psi_b(\mathbf{x})]_-\Phi_c(\mathbf{x})+\text{H.c.}, \tag{6}$$

where the R's are numerical coupling matrices. However, the consideration of four-fermion Fermi-type couplings will not necessarily be excluded.

An "interaction matrix" has been defined[9] as the product of the matrices associated with the generalized fields entering an interaction. A "free-field matrix" can similarly be defined as the product of the matrices associated with a field and its adjoint. The various coupling matrices, whose elements are proportional to coupling constants, must be constructured in such a manner that for each field, the free-field and interaction Lagrangian densities, when expressed in terms of the component fields, occupy the same positions in the total matrix Lagrangian density of all fields.

For the correspondence between baryons and generalized fields to be considered, all the free-field and interaction matrices turn out to be diagonal. The free-field matrices of the generalized bosons are likewise all diagonal.

For generalized fields for which the free-field matrices are not diagonal, the simplest procedure is to define the free-field Lagrangian density as being equal to the corresponding conventional free-field density, except that generalized fields are substituted for conventional fields, multiplied by the inverse of the free-field matrix, and to require that corresponding coupling and interaction matrices S be inversely related:

$$RS=gI, \tag{7}$$

where g is the coupling constant characteristic of any interaction. The total Lagrangian density is then diagonal. Alternatively, nondiagonal free-field Lagrangian densities may be considered, provided the coupling matrices are adjusted accordingly.

In any event, the total generalized Lagrangian density of all fields in the representations to be considered contains the component Lagrangian densities of each component field four times. These four component densities occupy different positions in the 4×4 Lagrangian matrix, and they will be completely equivalent if the relative magnitudes and phases of the matrix elements of each coupling matrix are suitably chosen.

All free-field and interaction matrices are assumed to be nonsingular. Free-field matrices and the matrix associated with a neutral self-adjoint integral-spin field are required to be Hermitian. The interaction matrices associated with an interaction and the Hermitian-conjugate interaction are required to be related by Hermitian conjugation.

BARYON OCTET, PIONS, AND KAONS

The matrices of odd order in the generators (4), i.e., those matrices contained in sets A and B, will be used for the description of half-integral-spin fields, and those of even order in the generators for the description of integral-spin fields.

Any matrix of sets A and B can be associated with the proton. This matrix can, without loss of generality, be selected to be C_1. Matrices associated with a baryon field operator and its adjoint are supposed to belong to the same set.[9] Since the matrices associated with a generalized field operator and its adjoint are required to commute,[8] there are, up to phase factors and with the above choice for p, only two possible candidates for the matrix to be associated with $\bar{p}$: C_1 or $C_3C_2C_3$. The second alternative will be considered in this discussion. Since the matrices associated with p and $\bar{p}$ are contained in set A, all matrices associated with half-integral-spin fields of even strangeness must belong to set A, and those associated with half-integral-spin fields of odd strangeness must belong to set B.[9]

The kaons are supposed to couple fields with matrix coefficients in set A to fields with matrix coefficients in set B. Therefore, the matrices associated with the kaons must be contained in set K. With the generalized field representing the proton as specified above, there are several alternatives for selecting the generalized fields representing the neutron and the kaons. These various *a priori* possibilities do not all have the same implications as far as the interactions of the particles in a comprehensive system of fields are concerned. Four schemes, A–D, will be studied in this discussion. The generalized fields representing the kaons and the neutron are not the same in all four schemes.

It is expedient to consider superpositions of the fields corresponding to the Σ^0 and Λ particles and to define[14]

$$Y \equiv (\Lambda+\Sigma^0)/\sqrt{2} \tag{8a}$$

and

$$Z \equiv (\Lambda-\Sigma^0)/\sqrt{2}\,. \tag{8b}$$

The full octet can be generated by multiplying the matrices associated with the proton and neutron by the matrices associated with the kaons and repeating this procedure for the fields so generated. In this fashion, the fields corresponding to the octet are obtained, with the matrix coefficients and phase factors as tabulated in Tables I–IV for schemes A–D, respectively.

In each scheme two chains of octet fields are generated in this manner:

$$\begin{tikzcd} & \Sigma^+ \arrow[dr] & \\ p \arrow[ur] \arrow[dr] & & \Xi^0 \\ & Y \arrow[ur] & \end{tikzcd} \tag{9a}$$

and

$$\begin{tikzcd} & Z \arrow[dr] & \\ n \arrow[ur] \arrow[dr] & & \Xi^-. \\ & \Sigma^- \arrow[ur] & \end{tikzcd} \tag{9b}$$

Strangeness-conserving kaonic transitions can occur between the particles in each chain, as indicated by the arrows. However, kaonic transitions between a spin-$\frac{1}{2}$ field in one chain and a field in the other chain are not allowed by the selection rules.

Once the octets have been generated in this way, it is a simple matter to decide which matrices must in each scheme be associated with the pions. Each field in one chain can enter into a pionic interaction with itself and with one field in the other chain.

The operators representing the Σ^0 and Λ particles are superpositions of generalized fields. For scheme A,

$$\Lambda = (Y+Z)/\sqrt{2} = \tfrac{1}{2}(C_3 - C_1C_3C_1)\times\psi - \tfrac{1}{2}i(C_3 + C_1C_3C_1)\times\psi', \tag{10a}$$

$$\bar{\Lambda} = (\bar{Y}+\bar{Z})/\sqrt{2} = \tfrac{1}{2}i(C_1C_2C_3 - C_1C_3C_2)\times\bar{\psi} + \tfrac{1}{2}(C_1C_2C_3 + C_1C_3C_2)\times\bar{\psi}', \tag{10b}$$

$$\Sigma^0 = (Y-Z)/\sqrt{2} = \tfrac{1}{2}(C_3 - C_1C_3C_1)\times\psi + \tfrac{1}{2}i(C_3 + C_1C_3C_1)\times\psi', \tag{10c}$$

$$\bar{\Sigma}^0 = (\bar{Y}-\bar{Z})/\sqrt{2} = \tfrac{1}{2}i(C_1C_2C_3 - C_1C_3C_2)\times\bar{\psi} - \tfrac{1}{2}(C_1C_2C_3 + C_1C_3C_2)\times\bar{\psi}'. \tag{10d}$$

Hence,

$$\bar{\Lambda}\Lambda = \tfrac{1}{2}iC_1C_2\times\bar{\psi}\psi - \tfrac{1}{2}iC_1C_2\times\bar{\psi}'\psi' - \tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}\psi' + \tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}'\psi, \tag{11a}$$

$$\bar{\Sigma}^0\Sigma^0 = \tfrac{1}{2}iC_1C_2\times\bar{\psi}\psi - \tfrac{1}{2}iC_1C_2\times\bar{\psi}'\psi' + \tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}\psi' - \tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}'\psi, \tag{11b}$$

TABLE I. Prentki-d'Espagnat Hamiltonian. Bilinear equal-time commutation relations between the component fields (scheme A).

$i[(C_2C_3-C_3C_2)/\sqrt{2}]\times\phi^\dagger$: K^-	−	+	+	−	−	+	+	−	+	−	−	+	+	−	−	+
$[(C_2C_3+C_3C_2)/\sqrt{2}]\times\phi$: K^+	+	−	−	+	+	−	−	+	−	+	+	−	−	+	+	−
$[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi^\dagger$: $\bar{K}^0$	−	+	+	−	+	−	−	+	−	+	+	−	+	−	−	+
$i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi$: K^0	+	−	−	+	−	+	+	−	+	−	−	+	−	+	+	−
$C_2C_1\times\phi$: $\pi^\pm$	+	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
$I\times\phi$: π^0	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
	p	$\bar{p}$	n	$\bar{n}$	Σ^+	$\bar{\Sigma}^-$	Y	$\bar{Y}$	Z	$\bar{Z}$	Σ^-	$\bar{\Sigma}^+$	Ξ^0	$\bar{\Xi}^0$	Ξ^-	$\bar{\Xi}^+$
$C_1\times\psi$: p	+	+	−	−	−	+	−	+	+	−	+	−	−	−	+	+
$C_3C_2C_3\times\bar{\psi}$: $\bar{p}$	+	+	−	−	+	−	+	−	−	+	−	+	−	−	+	+
$C_2\times\psi$: n	−	−	+	+	−	+	−	+	+	−	+	−	+	+	−	−
$C_3C_1C_2\times\bar{\psi}$: $\bar{n}$	−	−	+	+	+	−	+	−	−	+	−	+	+	+	−	−
$i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^+	−	+	−	+	+	+	−	−	+	+	−	−	−	+	−	+
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^-$	+	−	+	−	+	+	−	−	+	+	−	−	+	−	+	−
$[(C_3-C_1C_3C_1)/\sqrt{2}]\times\psi$: Y	−	+	−	+	−	−	+	+	−	−	+	+	−	+	−	+
$i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{Y}$	+	−	+	−	−	−	+	+	−	−	+	+	+	−	+	−
$-i[(C_3+C_1C_3C_1)/\sqrt{2}]\times\psi$: Z	+	−	+	−	+	+	−	−	+	+	−	−	+	−	+	−
$[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{Z}$	−	+	−	+	+	+	−	−	+	+	−	−	−	+	−	+
$[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^-	+	−	+	−	−	−	+	+	−	−	+	+	+	−	+	−
$-i[(C_3-C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^+$	−	+	−	+	−	−	+	+	−	−	+	+	−	+	−	+
$-C_2\times\psi$: Ξ^0	−	−	+	+	−	+	−	+	+	−	+	−	+	+	−	−
$C_3C_1C_3\times\bar{\psi}$: $\bar{\Xi}^0$	−	−	+	+	+	−	+	−	−	+	−	+	+	+	−	−
$C_1\times\psi$: Ξ^-	+	+	−	−	−	+	−	+	+	−	+	−	−	−	+	+
$-C_3C_2C_3\times\bar{\psi}$: $\bar{\Xi}^+$	+	+	−	−	+	−	+	−	−	+	−	+	−	−	+	+

[14] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

$$\bar{\Lambda}\Sigma^0=\tfrac{1}{2}iC_1C_2\times\bar{\psi}\psi+\tfrac{1}{2}iC_1C_2\times\bar{\psi}'\psi' +\tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}\psi'+\tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}'\psi, \quad (11c)$$

$$\bar{\Sigma}^0\Lambda=\tfrac{1}{2}iC_1C_2\times\bar{\psi}\psi+\tfrac{1}{2}iC_1C_2\times\bar{\psi}'\psi' -\tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}\psi'-\tfrac{1}{2}C_1C_3C_2C_3\times\bar{\psi}'\psi. \quad (11d)$$

The lack of Hermiticityof the right-hand sides of Eqs. (11a) and (11b) is only apparent, because it must be remembered that under the generalized Hermitian conjugation, a generalized field goes over into its generalized adjoint.

Expressions (11a) and (11b) cannot be coupled to neutral pions because in both equations the component fields occurring in the last two terms do not have the bilinear equal-time commutation behavior required for such a coupling. This argument does not apply to the

TABLE II. Prentki-d'Espagnat Hamiltonian. Bilinear equal-time commutation relations between the component fields (scheme B).

$i[(C_2C_3-C_3C_2)/\sqrt{2}]\times\phi^\dagger$: K^-	−	+	−	+	−	+	+	−	−	+	+	−	+	−	+	−
$[(C_2C_3+C_3C_2)/\sqrt{2}]\times\phi$: K^+	+	−	+	−	+	−	−	+	+	−	−	+	−	+	−	+
$[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi^\dagger$: $\bar{K}^0$	−	+	−	+	+	−	−	+	+	−	−	+	+	−	+	−
$i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi$: K^0	+	−	+	−	−	+	+	−	−	+	+	−	−	+	−	+
$C_1C_3C_1C_3\times\phi$: $\pi^\pm$	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$I\times\phi$: π^0	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
	p	$\bar{p}$	n	$\bar{n}$	Σ^+	$\bar{\Sigma}^-$	Y	$\bar{Y}$	Z	$\bar{Z}$	Σ^-	$\bar{\Sigma}^+$	Ξ^0	$\bar{\Xi}^0$	Ξ^-	$\bar{\Xi}^+$
$C_1\times\psi$: p	+	+	−	−	−	+	−	+	+	−	+	−	−	−	+	+
$C_3C_2C_3\times\bar{\psi}$: $\bar{p}$	+	+	−	−	+	−	+	−	−	+	−	+	−	−	+	+
$C_3C_1C_3\times\psi$: n	−	−	+	+	+	−	+	−	−	+	−	+	+	+	−	−
$C_2\times\bar{\psi}$: $\bar{n}$	−	−	+	+	−	+	−	+	+	−	+	−	+	+	−	−
$i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^+	−	+	+	−	+	+	−	−	−	−	+	+	−	+	+	−
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^-$	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
$[(C_3-C_1C_3C_1)/\sqrt{2}]\times\psi$: Y	−	+	+	−	−	−	+	+	+	+	−	−	−	+	+	−
$i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{Y}$	+	−	−	+	−	−	+	+	+	+	−	−	+	−	−	+
$-i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\psi$: Z	+	−	−	+	−	−	+	+	+	+	−	−	+	−	−	+
$[(C_3-C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{Z}$	−	+	+	−	−	−	+	+	+	+	−	−	−	+	+	−
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\psi$: Σ^-	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
$-i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^+$	−	+	+	−	+	+	−	−	−	−	+	+	−	+	+	−
$-C_2\times\psi$: Ξ^0	−	−	+	+	−	+	−	+	+	−	+	−	+	+	−	−
$C_3C_1C_3\times\bar{\psi}$: $\bar{\Xi}^0$	−	−	+	+	+	−	+	−	−	+	−	+	+	+	−	−
$-C_3C_2C_3\times\psi$: Ξ^-	+	+	−	−	+	−	+	−	−	+	−	+	−	−	+	+
$C_1\times\bar{\psi}$: $\bar{\Xi}^+$	+	+	−	−	−	+	−	+	+	−	+	−	−	−	+	+

TABLE III. Prentki-d'Espagnat Hamiltonian. Bilinear equal-time commutation relations between the component fields (scheme C).

$i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi^\dagger$: K^-	+	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
$[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi$: K^+	−	+	−	+	+	−	+	−	+	−	+	−	−	+	−	+
$[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi^\dagger$: $\bar{K}^0$	−	+	−	+	+	−	+	−	+	−	+	−	−	+	−	+
$i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi$: K^0	+	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
$C_1C_3C_1C_3\times\phi$: $\pi^\pm$	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$I\times\phi$: π^0	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
	p	$\bar{p}$	n	$\bar{n}$	Σ^+	$\bar{\Sigma}^-$	Y	$\bar{Y}$	Z	$\bar{Z}$	Σ^-	$\bar{\Sigma}^+$	Ξ^0	$\bar{\Xi}^0$	Ξ^-	$\bar{\Xi}^+$
$C_1\times\psi$: p	+	+	−	−	−	+	+	−	+	−	−	+	+	+	−	−
$C_3C_2C_3\times\bar{\psi}$: $\bar{p}$	+	+	−	−	+	−	−	+	−	+	+	−	+	+	−	−
$C_3C_1C_3\times\psi$: n	−	−	+	+	+	−	−	+	−	+	+	−	−	−	+	+
$C_2\times\bar{\psi}$: $\bar{n}$	−	−	+	+	−	+	+	−	+	−	−	+	−	−	+	+
$i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^+	−	+	+	−	+	+	−	−	−	−	+	+	−	+	+	−
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^-$	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
$-[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\psi$: Y	+	−	−	+	−	−	+	+	+	+	−	−	+	−	−	+
$-i[(C_3-C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{Y}$	−	+	+	−	−	−	+	+	+	+	−	−	−	+	+	−
$-i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\psi$: Z	+	−	−	+	−	−	+	+	+	+	−	−	+	−	−	+
$[(C_3-C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{Z}$	−	+	+	−	−	−	+	+	+	+	−	−	−	+	+	−
$-[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^-	−	+	+	−	+	+	−	−	−	−	+	+	−	+	+	−
$i[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^+$	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
$-C_1\times\psi$: Ξ^0	+	+	−	−	−	+	+	−	+	−	−	+	+	+	−	−
$-C_3C_2C_3\times\bar{\psi}$: $\bar{\Xi}^0$	+	+	−	−	+	−	−	+	−	+	+	−	+	+	−	−
$C_3C_1C_3\times\psi$: Ξ^-	−	−	+	+	+	−	−	+	−	+	+	−	−	−	+	+
$C_2\times\bar{\psi}$: $\bar{\Xi}^+$	−	−	+	+	−	+	+	−	+	−	−	+	−	−	+	+

TABLE IV. Prentki-d'Espagnat Hamiltonian. Bilinear equal-time commutation relations between the component fields (scheme D).

$i[(C_2C_3-C_3C_2)/\sqrt{2}]\times\phi^\dagger$: K^-	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$	$-$
$[(C_2C_3+C_3C_2)/\sqrt{2}]\times\phi$: K^+	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$-$	$+$
$[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi^\dagger$: $\bar{K}^0$	$-$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$-$
$i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi$: K^0	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$	$+$
$\pm I\times\phi$: $\pi^\pm$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$-I\times\phi$: π^0	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
	p	$\bar{p}$	n	$\bar{n}$	Σ^+	$\bar{\Sigma}^-$	Y	$\bar{Y}$	Z	$\bar{Z}$	Σ^-	$\bar{\Sigma}^+$	Ξ^0	$\bar{\Xi}^0$	Ξ^-	$\bar{\Xi}^+$
$C_1\times\psi$: p	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$-$	$-$
$C_3C_2C_3\times\bar{\psi}$: $\bar{p}$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
$C_1\times\psi$: n	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$-$	$-$
$-C_3C_2C_3\times\bar{\psi}$: $\bar{n}$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
$i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Σ^+	$-$	$+$	$-$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$-$	$+$	$-$	$+$
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^-$	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$-$
$[(C_3-C_1C_3C_1)/\sqrt{2}]\times\psi$: Y	$-$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$
$i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{Y}$	$+$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$
$-i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi$: Z	$-$	$+$	$-$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$-$	$+$	$-$	$+$
$[(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi}$: $\bar{Z}$	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$-$
$-[(C_3-C_1C_3C_1)/\sqrt{2}]\times\psi$: Σ^-	$-$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$
$i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi}$: $\bar{\Sigma}^+$	$+$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$
$C_2\times\psi$: Ξ^0	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$+$
$-C_3C_1C_3\times\bar{\psi}$: $\bar{\Xi}^0$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$
$C_2\times\psi$: Ξ^-	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$+$
$C_3C_1C_3\times\bar{\psi}$: $\bar{\Xi}^+$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$

expression

$$\bar{\Lambda}\Sigma^0+\bar{\Sigma}^0\Lambda=iC_1C_2\times\bar{\psi}\psi+iC_1C_2\times\bar{\psi}'\psi'. \quad (12)$$

This means that, in the scheme considered, $\Sigma^0\Sigma^0\pi^0$ and $\Lambda\Lambda\pi^0$ couplings cannot occur, but that the $\Sigma^0\Lambda\pi^0$ coupling is allowed. Analogous conclusions can be reached for scheme D.

According to the correspondences between particles and generalized fields given in Tables I–IV, all the processes corresponding to the various terms in the Prentki-d'Espagnat Hamiltonian are allowed in the four schemes. At least in schemes A and D, the converse is true: In the context of trilinear, Hermitian, pseudoscalar, nonderivative Yukawa-type couplings, all processes not allowed by the Prentki-d'Espagnat Hamiltonian are forbidden according to Tables I and IV, respectively. Actually, the selection rules discussed here do not depend on the precise covariant character of the interaction, as long as it is trilinear.

In the four schemes, the interaction matrices for kaonic interactions are proportional to the unit matrix. The pionic interaction matrices for baryons of even and odd strangeness are, respectively, proportional to the diagonal Hermitian matrices $C_1C_3C_2C_3$ and iC_1C_2.

If for scheme A the coupling constants for the processes corresponding to any term in the Prentki-d'Espagnat Hamiltonian are all in phase, multiplication of the matrix coefficients of the relevant generalized fields, as tabulated in Table I, ensures in a natural manner that the phase relations between the various processes turn out in agreement with invariance of the interactions under rotations in isotopic-spin space, e.g., the proton and the neutron, as well as the Σ^+ and Σ^- particles, couple to the neutral pion with opposite phase, etc. This agreement of the phase relations with charge independence of the strong interactions does not settle the question of the relative magnitudes of the various relevant coupling constants.

For schemes B and D, analogous conclusions can be obtained except that in the two terms of the Prentki-d'Espagnat Hamiltonian, conventionally written as

$$g[\bar{p}p\pi^0-\bar{n}n\pi^0+\sqrt{2}(\bar{p}n\pi^++\bar{n}p\pi^-)] \quad (13a)$$

and

$$g'[\bar{\Xi}^0\Xi^0\pi^0-\bar{\Xi}^+\Xi^-\pi^0+\sqrt{2}(\bar{\Xi}^0\Xi^-\pi^++\bar{\Xi}^+\Xi^0\pi^-)], \quad (13b)$$

the signs of the square roots are reversed.

For scheme C, the signs of the square roots are reversed in the interaction (13a) and in the term conventionally written as

$$g''[\sqrt{2}(\bar{\Xi}^0\Sigma^+K^--\bar{\Xi}^+\Sigma^-\bar{K}^0)-\bar{\Xi}^+\Sigma^0K^--\bar{\Xi}^0\Sigma^0\bar{K}^0]. \quad (13c)$$

The selection rules corresponding to the Prentki-d'Espagnat Hamiltonian, as obtained from Tables I, II, and IV, are independent of charge conservation. In other words, direct processes like

$$p\rightarrow p+\pi^\pm \quad (14a)$$

and

$$n\rightarrow\Sigma^-+K^0, \quad (14b)$$

etc., cannot occur in the absence of electromagnetic interactions or if charge were not conserved. As a consequence of charge conservation, corresponding fields in the two chains (9a) and (9b) differ in charge by one unit. In this connection, it may be worthwhile to recall that the matrices C_1 and C_2 or C_1 and $C_3C_2C_3$ are completely equivalent. If, therefore, in Tables I–IV the substitution $p \leftrightarrow n$ is made, with corresponding substitutions for the other fields, completely equivalent schemes are, of course, obtained.

For scheme C (Table III) the selection rules are not independent of charge conservation. In the absence of electromagnetic interactions, several octet baryons would become identical, and if charge were not conserved, strangeness-nonconserving transitions could occur.

Instead of the identifications (8a) and (8b), it is possible to consider the correspondences

$$Y \equiv \Sigma^0 \quad \text{or} \quad \Lambda \tag{8c}$$

and

$$Z \equiv \Lambda \quad \text{or} \quad \Sigma^0. \tag{8d}$$

However, Eqs. (8c) and (8d) give rise to a less favorable agreement with the Prentki-d'Espagnat Hamiltonian than Eqs. (8a) and (8b) do.

DECIMET

If the complications encountered in the quantization of higher-spin fields are disregarded, such fields can be included in this discussion. Using the octet as a guide, the symmetry of the decimet can be generated in a straightforward manner. The simplest procedure is to start with the singlet of strangeness -3 and to multiply the matrices associated with the Ω field by the matrices associated with the K^0 and K^+ operators, and similarly to multiply the matrices associated with $\bar{\Omega}$ by the matrices associated with the $\bar{K}^0$ and K^- operators. This procedure can then be repeated for the Ξ^* doublet so obtained to generate the Σ^* triplet (Table V), etc. This procedure can, in fact, be continued indefinitely to generate multiplets of baryons of positive strangeness. Two such quintuplets of "X particles" of strangeness $+1$ have been included in Table V. An inspection of these hypothetical quintuplets shows that they give rise to strangeness-nonconserving transitions. There appears to be no obvious reason, for example, why direct $\Delta S = \pm 4$ pionic or electromagnetic transitions from X_1^- to Ω_1 or from X_2^- to Ω_2 cannot occur if such quintuplets were to exist.

There are a number of different generalized fields which can *a priori* be associated with the singlet of strangeness -3 and which would give rise to a corresponding number of different sequences of fields. The matrix coefficients of the Ω particle must be contained in set B [Eq. (5)]. The most obvious eight candidates for the matrices to be associated with the Ω field are [the matrix coefficient associated with a generalized

TABLE V. Sequences of fields generated from a singlet by kaonic transitions (scheme A).[a]

$i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{\Omega}_1^+$	$-(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{\Omega}_2^+$
$(C_1C_2C_3-C_1C_3C_2)\times\psi$: Ω_1^-	$i(C_1C_2C_3+C_1C_3C_2)\times\psi$: Ω_2^-
$iC_2\times\bar{\psi}$: $\bar{\Xi}_1^{+*}$	$-C_3C_2C_3\times\bar{\psi}$: $\bar{\Xi}_2^{+*}$
$iC_3C_1C_3\times\psi$: Ξ_1^{-*}	$C_1\times\psi$: Ξ_2^{-*}
$C_3C_1C_3\times\bar{\psi}$: $\bar{\Xi}_1^{0*}$	$-iC_1\times\bar{\psi}$: $\bar{\Xi}_2^{0*}$
$-C_2\times\psi$: Ξ_1^{0*}	$-iC_3C_2C_3\times\psi$: Ξ_2^{0*}
$(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{\Sigma}_1^{+*}$	$-i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{\Sigma}_2^{+*}$
$i(C_1C_2C_3+C_1C_3C_2)\times\psi$: Σ_1^{-*}	$(C_1C_2C_3-C_1C_3C_2)\times\psi$: Σ_2^{-*}
$i(C_1C_2C_3-C_1C_3C_2)\times\bar{\psi}$: $\bar{\Sigma}_1^{0*}$	$-(C_1C_2C_3+C_1C_3C_2)\times\bar{\psi}$: $\bar{\Sigma}_2^{0*}$
$(C_3-C_1C_3C_1)\times\psi$: Σ_1^{0*}	$i(C_3+C_1C_3C_1)\times\psi$: Σ_2^{0*}
$(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{\Sigma}_1^{-*}$	$-i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{\Sigma}_2^{-*}$
$i(C_1C_2C_3+C_1C_3C_2)\times\psi$: Σ_1^{+*}	$(C_1C_2C_3-C_1C_3C_2)\times\psi$: Σ_2^{+*}
$C_3C_2C_3\times\bar{\psi}$: $\bar{\Delta}_1^+$	$-iC_2\times\bar{\psi}$: $\bar{\Delta}_2^+$
$C_1\times\psi$: Δ_1^-	$iC_3C_1C_3\times\psi$: Δ_2^-
$-iC_1\times\bar{\psi}$: $\bar{\Delta}_1^0$	$C_3C_1C_3\times\bar{\psi}$: $\bar{\Delta}_2^0$
$iC_3C_2C_3\times\psi$: Δ_1^0	$C_2\times\psi$: Δ_2^0
$C_3C_2C_3\times\bar{\psi}$: $\bar{\Delta}_1^-$	$-iC_2\times\bar{\psi}$: $\bar{\Delta}_2^-$
$C_1\times\psi$: Δ_1^+	$iC_3C_1C_3\times\psi$: Δ_2^+
$-iC_1\times\bar{\psi}$: $\bar{\Delta}_1^{--}$	$C_3C_1C_3\times\bar{\psi}$: $\bar{\Delta}_2^{--}$
$iC_3C_2C_3\times\psi$: Δ_1^{++}	$C_2\times\psi$: Δ_2^{++}
$-i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_1^+$	$(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_2^+$
$-(C_1C_2C_3-C_1C_3C_2)\times\psi$: X_1^-	$-i(C_1C_2C_3+C_1C_3C_2)\times\psi$: X_2^-
$(C_1C_2C_3+C_1C_3C_2)\times\bar{\psi}$: $\bar{X}_1^0$	$-i(C_1C_2C_3-C_1C_3C_2)\times\bar{\psi}$: $\bar{X}_2^0$
$i(C_3+C_1C_3C_1)\times\psi$: X_1^0	$(C_3-C_1C_3C_1)\times\psi$: X_2^0
$-i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_1^-$	$(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_2^-$
$-(C_1C_2C_3-C_1C_3C_2)\times\psi$: X_1^+	$-i(C_1C_2C_3+C_1C_3C_2)\times\psi$: X_2^+
$(C_1C_2C_3+C_1C_3C_2)\times\bar{\psi}$: $\bar{X}_1^{--}$	$-i(C_1C_2C_3-C_1C_3C_2)\times\bar{\psi}$: $\bar{X}_2^{--}$
$i(C_3+C_1C_3C_1)\times\psi$: X_1^{++}	$(C_3-C_1C_3C_1)\times\psi$: X_2^{++}
$-i(C_3-C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_1^{---}$	$(C_3+C_1C_3C_1)\times\bar{\psi}$: $\bar{X}_2^{---}$
$-(C_1C_2C_3-C_1C_3C_2)\times\psi$: X_1^{+++}	$-i(C_1C_2C_3+C_1C_3C_2)\times\psi$: X_2^{+++}

[a] Some normalization factors $1/\sqrt{2}$ have for simplicity been omitted.

TABLE VI. Octet and decimet (scheme A).[a]

	$i(C_3-C_1C_3C_1)\times\bar\psi:\ \bar\Omega_1^+$	$-(C_3+C_1C_3C_1)\times\bar\psi:\ \bar\Omega_2^+$
	$(C_1C_2C_3-C_1C_3C_2)\times\psi:\ \Omega_1^-$	$i(C_1C_2C_3+C_1C_3C_2)\times\psi:\ \Omega_2^-$
$-C_3C_2C_3\times\bar\psi:\ \bar\Xi^+$	$\big(iC_2\times\bar\psi:\ \bar\Xi_1^{+*}$	$-C_3C_2C_3\times\bar\psi:\ \bar\Xi^{+*}$
$C_1\times\psi:\ \Xi^-$	$iC_3C_1C_3\times\psi:\ \Xi_1^{-*}\big)$	$C_1\times\psi:\ \Xi^{-*}$
$C_3C_1C_3\times\bar\psi:\ \bar\Xi^0$	$C_3C_1C_3\times\bar\psi:\ \bar\Xi^{0*}$	$\big(-iC_1\times\bar\psi:\ \bar\Xi_2^{0*}$
$-C_2\times\psi:\ \Xi^0$	$-C_2\times\psi:\ \Xi^{0*}$	$-iC_3C_2C_3\times\psi:\ \Xi_2^{0*}\big)$
$-i(C_3-C_1C_3C_1)\times\bar\psi:\ \bar\Sigma^+$	$(C_3+C_1C_3C_1)\times\bar\psi:\ \bar\Sigma_1^{+*}$	$-i(C_3-C_1C_3C_1)\times\bar\psi:\ \bar\Sigma^{+*}$
$(C_1C_2C_3-C_1C_3C_2)\times\psi:\ \Sigma^-$	$i(C_1C_2C_3+C_1C_3C_2)\times\psi:\ \Sigma_1^{-*}$	$(C_1C_2C_3-C_1C_3C_2)\times\psi:\ \Sigma^{-*}$
$(C_1C_2C_3+C_1C_3C_2)\times\bar\psi:\ \bar Z$		$-(C_1C_2C_3+C_1C_3C_2)\times\bar\psi:\ \bar Z^*$
$-i(C_3+C_1C_3C_1)\times\psi:\ Z$		$i(C_3+C_1C_3C_1)\times\psi:\ Z^*$
$i(C_1C_2C_3-C_1C_3C_2)\times\bar\psi:\ \bar Y$	$i(C_1C_2C_3-C_1C_3C_2)\times\bar\psi:\ \bar Y^*$	
$(C_3-C_1C_3C_1)\times\psi:\ Y$	$(C_3-C_1C_3C_1)\times\psi:\ Y^*$	
$(C_3+C_1C_3C_1)\times\bar\psi:\ \bar\Sigma^-$	$(C_3+C_1C_3C_1)\times\bar\psi:\ \bar\Sigma^{-*}$	$-i(C_3-C_1C_3C_1)\times\bar\psi:\ \bar\Sigma_2^{-*}$
$i(C_1C_2C_3+C_1C_3C_2)\times\psi:\ \Sigma^+$	$i(C_1C_2C_3+C_1C_3C_2)\times\psi:\ \Sigma^{+*}$	$(C_1C_2C_3-C_1C_3C_2)\times\psi:\ \Sigma_2^{+*}$
	$C_1\times\bar\psi:\ \bar\Delta^+$	$\big(iC_3C_1C_3\times\bar\psi:\ \bar\Delta_2^+$
	$C_3C_2C_3\times\psi:\ \Delta^-$	$-iC_2\times\psi:\ \Delta_2^-\big)$
$C_3C_1C_3\times\bar\psi:\ \bar n$	$\big(iC_3C_2C_3\times\bar\psi:\ \bar\Delta_1^0$	$C_2\times\bar\psi:\ \bar\Delta^0$
$C_2\times\psi:\ n$	$-iC_1\times\psi:\ \Delta_1^0\big)$	$C_3C_1C_3\times\psi:\ \Delta^0$
$C_3C_2C_3\times\bar\psi:\ \bar p$	$C_1\times\bar\psi:\ \bar\Delta^-$	$\big(iC_3C_1C_3\times\bar\psi:\ \bar\Delta_2^-$
$C_1\times\psi:\ p$	$C_3C_2C_3\times\psi:\ \Delta^+$	$-iC_2\times\psi:\ \Delta_2^+\big)$
	$\big(iC_3C_2C_3\times\bar\psi:\ \bar\Delta_1^{--}$	$C_2\times\bar\psi:\ \bar\Delta^{--}$
	$-iC_1\times\psi:\ \Delta_1^{++}\big)$	$C_3C_1C_3\times\psi:\ \Delta^{++}$
	$-i(C_3-C_1C_3C_1)\times\bar\psi:\ \bar X_1^-$	$(C_3+C_1C_3C_1)\times\bar\psi:\ \bar X_2^-$
	$-(C_1C_2C_3-C_1C_3C_2)\times\psi:\ X_1^+$	$-i(C_1C_2C_3+C_1C_3C_2)\times\psi:\ X_2^+$
	$(C_1C_2C_3+C_1C_3C_2)\times\bar\psi:\ \bar X_1^{--}$	$\big(-i(C_1C_2C_3-C_1C_3C_2)\times\bar\psi:\ \bar X_2^{--}$
	$i(C_3+C_1C_3C_1)\times\psi:\ X_1^{++}$	$C_3-C_1C_3C_1)\times\psi:\ X_2^{++}\big)$

[a] Some normalization factors $1/\sqrt{2}$ have for simplicity been omitted.

field χ is denoted by $M(\chi)$]

$$M(\bar\Omega)=\pm i(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}\,,\quad \pm i(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}\,,\quad \pm(C_3+C_1C_3C_1)/\sqrt{2}\,,\quad \pm(C_3-C_1C_3C_1)/\sqrt{2} \qquad (15a)$$

and, correspondingly,

$$M(\Omega)=(C_3+C_1C_3C_1)/\sqrt{2}\,,\quad (C_3-C_1C_3C_1)/\sqrt{2}\,,\quad i(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}\,,\quad i(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}\,. \qquad (15b)$$

The matrix coefficients of the Ω fields as given in Table V have been selected in such a manner that for scheme A, a kaonic transition from Ξ^0 to Ω_1 and from Ξ^- a kaonic transition to Ω_2 are possible. The corresponding sequences for schemes B–D can readily be constructed but are not explicitly reproduced.

An inspection of Table V shows that direct strangeness-nonconserving transitions from Δ^- to Ω are possible. Corresponding transitions can occur in the other schemes. These transitions can be eliminated by the simple device of interchanging the matrices associated with each quadruplet field and its adjoint. Direct pionic and electromagnetic transitions from the strangeness-zero doublet to the quadruplet are still possible, but the quadruplet fields cannot enter directly into kaonic transitions. This device cannot be used to eliminate the X^--Ω transition. For scheme A, the decimet as listed in Table VI is then obtained by selecting for consideration those fields from the two sequences of Table V which can be reached from the octet via electromagnetic, pionic, or kaonic transitions. The decimets for schemes B–D can be obtained in a similar manner but again are not explicitly reproduced. Those fields in Table VI which are enclosed in large parentheses cannot be reached from the octet via direct transitions. Table VI also includes some generalized fields suitable for the description of positive-strangeness baryons. Such fields (pK^0 and pK^+ resonances), which cannot be classified according to the simple quark model, are believed to have been observed and have been discussed in the literature.[15] The positive-strangeness baryons included in Table VI of course do not necessarily have to have spin $\frac{3}{2}$.

In this discussion, it is assumed that the generalized field corresponding to the Ω particle is a superposition of two independent, presumably degenerate fields.

In a previous discussion,[9] the field assigned to the Ω particle consisted of a single field rather than a superposition of two fields. However, as a consequence the fields corresponding to Y and Z were identical in their interactions, and the prospects of removing the Y-Z degeneracy were poor.

The assignment of the same matrices to different

[15] O. W. Greenberg and C. A. Nelson, Phys. Rev. Letters **20**, 604 (1968); A. S. Carroll *et al.*, *ibid.* **21**, 1282 (1968); B. R. Martin, *ibid.* **21**, 1286 (1968); A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **41**, 109 (1969); O. W. Greenberg and C. A. Nelson, Phys. Rev. **179**, 1354 (1969).

fields, such as Δ^+ and Δ^-, can be circumvented by utilizing some of those matrices from sets A and B which have not been used.[9] For the decimet, the selection rules are explicitly dependent on charge conservation, because in all four schemes strangeness-nonconserving transitions, such as from Ω to p or n, could occur if charge were not conserved. The free-field and interaction matrices are again all diagonal, kaonic interaction matrices being proportional to the unit matrix.

For the correspondences between generalized fields and baryons considered, the relation

$$M(\Psi)=(M(\bar{\Psi}))^\dagger \tag{16}$$

is not valid. Instead, one obtains for baryons of even strangeness

$$M(\Psi)=\pm C_1C_3C_2C_3M(\bar{\Psi})=\pm C_1C_3C_2C_3(M(\bar{\Psi}))^\dagger, \tag{17a}$$

whereas for baryons of odd strangeness,

$$M(\Psi)=\pm iC_1C_2M(\bar{\Psi})=\pm iC_1C_2(M(\bar{\Psi}))^\dagger. \tag{17b}$$

Furthermore, for each generalized field considered,

$$M(\chi)=\pm(M(\chi))^\dagger. \tag{17c}$$

LEPTONS AND INTERMEDIATE BOSONS

The guiding consideration in selecting generalized fields to represent the leptons and intermediate bosons is that the proper selection rules be satisfied: In the context of trilinear interactions, no baryon should be allowed to decay into a lepton, and the intermediate bosons $W^\pm$ should have the decay modes

$$W^{\cdot}\begin{array}{l}\nearrow\; l_1+l_2\\ \searrow\; l_3+l_4\end{array},$$

where l_1 and l_2 are electronic and l_3 and l_4 are muonic leptons, or vice versa. Furthermore, in the context of trilinear interactions, no muonic lepton should be able to interact directly with an electronic lepton. Moreover, the generalized fields representing the intermediate bosons should not be able to cause undesirable, e.g., $\Delta S=\pm2$, transitions in the hadronic systems discussed above.

There are several *a priori* possibilities of selecting generalized fields to represent the particles under discussion. Several alternatives, which have different consequences, will be mentioned. In this connection it is to be expected that the difference between the four hadronic schemes A–D will become manifest, since these schemes differ, in particular, in the generalized fields representing the kaons and the charged pions. By combining the various hadronic schemes with the leptonic correspondences to be discussed, comprehensive systems of fields are obtained which differ in some allowed weak processes and in some couplings involving only integral spin fields.

TABLE VII. Some bilinear equal-time commutation relations between the component fields for the assignments given in Eqs. (18) and (19).

W^-	—	—	—	—	—	—	—	—
W^+	—	—	—	—	—	—	—	—
	l_1	$\bar{l}_1$	l_2	$\bar{l}_2$	l_3	$\bar{l}_3$	l_4	$\bar{l}_4$

The hadronic and leptonic schemes considered in this discussion should not be construed as exhausting all possible correspondences between particles and generalized fields.

For schemes A and B, the following correspondence between generalized fields and leptons gives a qualitatively correct description of the selection rules satisfied by leptons, assuming that all their fundamental couplings are trilinear in the fields concerned:

$$\begin{array}{ll}
l_1\colon\ C_1\times\psi, & \bar{l}_1\colon\ C_1\times\bar{\psi},\\
l_2\colon\ C_3C_2C_3\times\psi, & \bar{l}_2\colon\ C_3C_2C_3\times\bar{\psi},\\
l_3\colon\ C_3C_1C_3\times\psi, & \bar{l}_3\colon\ C_3C_1C_3\times\bar{\psi},\\
l_4\colon\ C_2\times\psi, & \bar{l}_4\colon\ C_2\times\bar{\psi},\\
W^+\colon\ C_1C_3C_2C_3\times\phi_\mu, & W^-\colon\ C_1C_3C_2C_3\times\phi_\mu{}^\dagger.
\end{array} \tag{18}$$

Some bilinear equal-time commutation relations for the correspondence (18) are given in Table VII.

A modification of the correspondence (18) can be used in conjunction with schemes B and C:

$$\begin{array}{ll}
l_1\colon\ C_1\times\psi, & \bar{l}_1\colon\ C_1\times\bar{\psi},\\
l_2\colon\ C_3C_2C_3\times\psi, & \bar{l}_2\colon\ -C_3C_2C_3\times\bar{\psi},\\
l_3\colon\ C_3C_1C_3\times\psi, & \bar{l}_3\colon\ -C_3C_1C_3\times\bar{\psi},\\
l_4\colon\ C_2\times\psi, & \bar{l}_4\colon\ C_2\times\bar{\psi},\\
W^+\colon\ iC_1C_3C_2C_3\times\phi_\mu, & W^-\colon\ iC_1C_3C_2C_3\times\phi_\mu{}^\dagger.
\end{array} \tag{19}$$

Table VII also applies to the correspondence (19).

In conjunction with all four hadronic schemes, the following two correspondences can be considered:

$$\begin{array}{ll}
l_1\colon\ C_1\times\psi, & \bar{l}_1\colon\ [(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi},\\
l_2\colon\ i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi, & \bar{l}_2\colon\ -C_3C_2C_3\times\bar{\psi},\\
l_3\colon\ C_2\times\psi, & \bar{l}_3\colon\ -i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi},\\
l_4\colon\ -[(C_3-C_1C_3C_1)/\sqrt{2}]\times\psi, & \bar{l}_4\colon\ C_3C_1C_3\times\bar{\psi},\\
W^\pm\colon\ iC_1C_2\times\phi_\mu, & W^\mp\colon\ -C_1C_3C_2C_3\times\phi_u{}^\dagger
\end{array} \tag{20}$$

and

$$\begin{aligned}
&l_1\colon\ C_1\times\psi, && \bar{l}_1\colon\ [(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi},\\
&l_2\colon\ i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\psi, && \bar{l}_2\colon\ -C_3C_2C_3\times\bar{\psi},\\
&l_3\colon\ C_3C_1C_3\times\psi, && \bar{l}_3\colon\ -i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\bar{\psi},\\
&l_4\colon\ -[(C_3+C_1C_3C_1)/\sqrt{2}]\times\psi, && \bar{l}_4\colon\ C_2\times\bar{\psi},\\
&W^{\pm}\colon\ iC_1C_2\times\phi_\mu, && W^{\mp}\colon\ -C_1C_3C_2C_3\times\phi_\mu{}^{\dagger}.
\end{aligned}\tag{21}$$

Some bilinear equal-time commutation relations for the last two assignments are summarized in Table VIII.

In conjunction with scheme C, the following correspondence can be considered:

$$\begin{aligned}
&l_1\colon\ C_1\times\psi, && \bar{l}_1\colon\ [(C_3+C_1C_3C_1)/\sqrt{2}]\times\bar{\psi},\\
&l_2\colon\ C_2\times\psi, && \bar{l}_2\colon\ i[(C_1C_2C_3-C_1C_3C_2)/\sqrt{2}]\times\bar{\psi},\\
&l_3\colon\ C_3C_1C_3\times\psi, && \bar{l}_3\colon\ -[(C_3-C_1C_3C_1)/\sqrt{2}]\times\bar{\psi},\\
&l_4\colon\ C_3C_2C_3\times\psi, && \bar{l}_4\colon\ i[(C_1C_2C_3+C_1C_3C_2)/\sqrt{2}]\times\bar{\psi},\\
&W^{-}\colon\ [(C_2C_3+C_3C_2)/\sqrt{2}]\times\phi_\mu, && W^{+}\colon\ i[(C_2C_3-C_3C_2)/\sqrt{2}]\times\phi_\mu{}^{\dagger}.
\end{aligned}\tag{22}$$

Table VIII also applies to this correspondence.

Other assignments can be obtained from the ones given above by making suitable substitutions, such as $C_1\leftrightarrow C_2$, for example. These substitutions lead in some cases to leptonic assignments with different consequences once the hadronic fields have been selected in a definite way, such as in schemes A–D.

As already mentioned, the four hadronic schemes, when considered in conjunction with the leptonic assignments, do not all have the same implications concerning the interactions of the various particles. The intermediate bosons of assignment (19) can, for example, be directly coupled to the pions, as represented in all four hadronic schemes, whereas the intermediate bosons of the other schemes cannot be coupled in this manner. In some combinations of assignments, the intermediate bosons cannot be coupled to any baryon, whereas in other cases, processes like

$$\Sigma^- \to Y + W^- \tag{23a}$$

and

$$\Sigma^+ \to Z + W^+ \tag{23b}$$

are possible.

In all the assignments (18)–(22), Hermiticity requirements and bilinear equal-time commutation relations permit the coupling of the four leptons, so that, at least in a first approximation, a Fermi-type coupling can be considered in the study of muon decay. For a suitable assignment of the four l_i's to the leptons, the proton and neutron can be coupled to two leptons in some combinations of schemes but not in others, i.e., in some but not all cases a Fermi-type interaction can be considered, at least as an approximation, in the study of the β decay of the neutron.

Some other differences in the implications of the various schemes will be mentioned below.

In this discussion, the purely leptonic interactions (e.g., μ decay) are assumed to be of the current-current form in the limit of an infinitely massive intermediate boson. However, if four fermion interactions are excluded, then in any one of the schemes under consideration in which the intermediate boson cannot be coupled directly to any baryon, the weak decays of hadrons must proceed via intermediate couplings involving three or more integral-spin fields (cf. next section). This is hardly consistent with the conventional current-type weak interaction. However, whereas in the limit of an infinitely massive intermediate vector boson the current-current-type weak interaction appears to be well established for purely leptonic processes, there appears to be room for reasonable doubt in this respect in connection with the decays of hadrons.

The absence of direct couplings between baryons and intermediate bosons is also consistent with the negative results of experiments on intermediate boson production. If such bosons cannot interact directly with baryons, the only remotely realistic experiments confirming their existence would have to proceed via electromagnetic or neutrino-charged lepton interactions.

OTHER BOSONS AND DECAYS OF HADRONS

In order to correlate some elementary particle phenomena even qualitatively, it is obviously necessary to introduce more integral-spin fields into this discussion. One way of accomplishing this is to consider the neutral, self-adjoint, scalar or pseudoscalar gen-

TABLE VIII. Some bilinear equal-time commutation relations between the component fields for the assignments given in Eqs. (20)–(22).

$W^{\mp}$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$
$W^{\pm}$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
	l_1	$\bar{l}_1$	l_2	$\bar{l}_2$	l_3	$\bar{l}_3$	l_4	$\bar{l}_4$

eralized fields

$$B_1=[(C_1C_3+C_3C_1)/\sqrt{2}]\times\phi\,, \tag{24a}$$

$$B_2=i[(C_1C_3-C_3C_1)/\sqrt{2}]\times\phi\,, \tag{24b}$$

$$B_3=i[(C_2C_3-C_3C_2)/\sqrt{2}]\times\phi\,, \tag{24c}$$

$$B_4=[(C_2C_3+C_3C_2)/\sqrt{2}]\times\phi\,. \tag{24d}$$

The corresponding fields for spin 1 are denoted by B_i^*.

For $J^P=0^-$ these generalized fields could, on the basis of the selection rules which they satisfy, conceivably be correlated with the η, η', and $E(1420)$ resonances, whereas for $J^P=1^-$ some of the B_i^* could represent the ω and ϕ resonances.

In none of the hadronic schemes discussed can the bosons (24) interact directly (trilinearly) with any baryons. If, therefore, one of the generalized fields (24) is to represent the η resonance, the meson-baryon coupling considered in this discussion is different from the corresponding coupling in the conventional octet model. The fields (24) can be coupled directly to kaons, and in schemes B–D they can interact directly not only with neutral but also with charged pions. Moreover, all the leptonic assignments (18)–(22) permit a direct coupling of the fields (24) to leptons. Such a coupling would, of course, give rise to neutral lepton currents. The ϕ (kaonic) resonance, for example, reportedly has an empirically observed small but finite branching ratio of decaying into electron-positron pairs.[16–18] This decay could conceivably be due to a direct coupling.

The four fields (24) differ from each other in their allowed couplings, and therefore they presumably are nondegenerate. In schemes A and B, for example, the fields B_3^* and B_4^* can be coupled directly to charged kaons, whereas the fields B_1^* and B_2^* cannot. The fields B_3^* and B_4^* (as well as B_1^* and B_2^*) differ from each other in their possible leptonic decays. For the assignment (21), the following hypothetical decays are allowed:

$$B_i^*\to l_i+\bar{l}_i\,,\quad i=1,\ldots,4. \tag{25}$$

The leptonic assignments (18)–(22) differ from each other in their interactions with the fields (24). For the assignment (22), the allowed decays are not given by Eq. (25). Instead,

$$B_1^*\to l_1+\bar{l}_1 \quad\text{or}\quad l_3+\bar{l}_3\,, \tag{26a}$$

$$B_2^*\to l_2+\bar{l}_2 \quad\text{or}\quad l_4+\bar{l}_4\,. \tag{26b}$$

B_3^* and B_4^* cannot decay directly into leptons in this case.

The fields (24) thus do not necessarily interact with the leptons in a symmetric manner. It is therefore possible that in one of the leptonic assignments (18)–(22), or in a similar assignment, the electron-muon degeneracy may be removed by an asymmetric inter-

[16] R. G. Astvacaturov *et al.*, Phys. Letters **27B**, 45 (1968).
[17] D. M. Binnie *et al.*, Phys. Letters **27B**, 106 (1968).
[18] U. Becker *et al.*, Phys. Rev. Letters **21**, 1504 (1968).

TABLE IX. Some bilinear equal-time commutation relations between component fields for scheme D combined with the leptonic assignment (21).

		Δ^{++}	$\bar{\Delta}^{--}$	Δ^+	$\bar{\Delta}^-$	Δ^0	$\bar{\Delta}^0$	Δ^-	$\bar{\Delta}^+$	p	$\bar{p}$	n	$\bar{n}$	Σ^+ Σ^{+*}	$\bar{\Sigma}^-$ $\bar{\Sigma}^{-*}$	Y Y^*	$\bar{Y}$ $\bar{Y}^*$	Z Z^*	$\bar{Z}$ $\bar{Z}^*$	Σ^- Σ^{-*}	$\bar{\Sigma}^+$ $\bar{\Sigma}^{+*}$	Ξ^0 Ξ^{0*}	$\bar{\Xi}^0$ $\bar{\Xi}^{0*}$	Ξ^- Ξ^{-*}	$\bar{\Xi}^+$ $\bar{\Xi}^{+*}$	Ω_1	$\bar{\Omega}_1$	Ω_2	$\bar{\Omega}_2$	l_1	$\bar{l}_1$	l_2	$\bar{l}_2$	l_3	$\bar{l}_3$	l_4	$\bar{l}_4$
$W^\mp$		−	−	−	−	−	−	−	−	−	−	−	−	+	+	+	+	+	+	+	+	−	−	−	−	+	+	+	+	−	+	+	−	−	+	+	−
$W^\pm$		+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+	−	−	−	−	+	−	−	+	+	−	−	+
B_4^*	B_4	−	+	−	+	−	+	−	+	+	−	+	−	+	−	−	+	+	−	−	+	−	+	−	+	+	−	−	+	+	−	+	−	+	+	−	−
B_3^*	B_3	+	−	+	−	+	−	+	−	−	+	−	+	−	+	+	−	−	+	+	−	+	−	+	−	−	+	+	−	−	+	−	+	−	−	+	+
B_2^*	B_2	−	+	−	+	−	+	−	+	+	−	+	−	−	+	+	−	−	+	+	−	−	+	−	+	−	+	+	−	+	+	−	−	+	−	+	−
B_1^*	B_1	+	−	+	−	+	−	+	−	−	+	−	+	+	−	−	+	+	−	−	+	+	−	+	−	+	−	−	+	−	−	+	+	−	+	−	+
K^{-*}	K^-	+	−	+	−	+	−	+	−	−	+	−	+	−	+	+	−	−	+	+	−	+	−	+	−	−	+	+	−	−	+	−	+	−	−	+	+
K^{+*}	K^+	−	+	−	+	−	+	−	+	+	−	+	−	+	−	−	+	+	−	−	+	−	+	−	+	+	−	−	+	+	−	+	−	+	+	−	−
$\bar{K}^{0*}$	$\bar{K}^0$	+	−	+	−	+	−	+	−	−	+	−	+	+	−	−	+	+	−	−	+	+	−	+	−	+	−	−	+	−	−	+	+	−	+	−	+
K^{0*}	K^0	−	+	−	+	−	+	−	+	+	−	+	−	−	+	+	−	−	+	+	−	−	+	−	+	−	+	+	−	+	+	−	−	+	−	+	−
$\rho^\pm$	$\pi^\pm$	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
ρ^0	π^0	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
γ		−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−

action of the leptons with generalized fields like B_i^* (cf. next section).

The intermediate bosons of assignments (20) and (21) cannot be coupled directly to baryons. In combination with schemes A and B, the leptonic strangeness-changing decays of hadrons could proceed via the virtual interactions

$$K^{\pm} \to W^{\pm} + B_1 \text{ or } B_2. \qquad (27)$$

Such decays conform with the $\Delta S = \Delta Q$ rule.

Superpositions of neutral kaons of the form

$$K^0 + e^{i\alpha}\bar{K}^0 = [(C_1C_3 + C_3C_1) + ie^{i\alpha}(C_1C_3 - C_3C_1)] \times \phi \quad (28)$$

can be coupled to neutral pions in scheme A and to neutral and charged pions in schemes B–D. As has been discussed previously,[9] the component fields of K^0 and $\bar{K}^0$ lose their separate identities when these fields are coupled to other bosons. Therefore, only one component field has to be considered in such couplings, as indicated in Eq. (28). If the phase α is equal to zero or to a multiple of π, the matrix coefficient on the right-hand side of Eq. (28) is singular, only two elements being different from zero. This implies that for these values of the phase, the fundamental equivalence of the four Lagrangian densities of the component fields (cf. Introduction) is destroyed. The equivalence of the four Lagrangians can be maintained if α is equal to an odd-integral multiple of $\frac{1}{2}\pi$, in which case the four nonvanishing matrix elements differ only in phase. Other values of α and superpositions more general than (28) may have to be considered. For the interaction under consideration, the interaction matrix is non-Hermitian, at least in the representation discussed.

Table IX summarizes some bilinear equal-time commutation relations between the component fields for scheme D and the leptonic assignment (21). Tables of this type faciliate the checking of allowed and forbidden processes. It must be remembered that in addition to the bilinear commutation requirements of component fields entering allowed interactions, the latter must be Hermitian. There may also be other factors inhibiting some interactions. The commutation relations of the fermion-fermion and boson-boson component fields are not tabulated, but these can readily be obtained. Some trilinear or higher-order interactions involving only integral-spin fields also have to be considered.

POSSIBLE CANCELLATIONS OF SOME DIVERGENCES

There appears to be no obvious reason why the conventional S-matrix formalism cannot be applied to generalized fields.

As an example of the possible cancellations of some divergences, it is expedient to study the lowest-order corrections to fermion propagators (Fig. 1).

It is assumed that both pions and kaons are pseudoscalars. The lowest-order correction to spin-$\frac{1}{2}$ baryon propagators due to pseudoscalar interactions is contained in the second-order S-matrix element

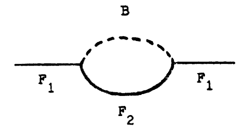

FIG. 1. Lowest-order corrections to fermion propagators. The labels refer to particles.

$$S^{(2)} = \left(\frac{-iG}{\hbar c}\right)^2 \frac{1}{2!} \int d^4x_1 \int d^4x_2 T\{N(\bar{\Psi}_a(x_1)\gamma_5\Psi_b(x_1)) \times N(\bar{\Psi}_b(x_2)\gamma_5\Psi_a(x_2))\} T(\Phi^\dagger(x_1)\Phi(x_2)). \quad (29)$$

The field operators occurring in Eq. (29) are generalized fields.

As an approximation, only the interactions of spin-$\frac{1}{2}$ baryons with each other and with pions and kaons will be considered. Electromagnetic effects and loops due to higher-spin fields will in this approximation be disregarded.

The standard calculation yields the usual result that the coefficient of the logarithm is proportional to G^2M, where M is the mass of the intermediate baryon F_2 (Fig. 1). In addition, the contractions of the generalized fields Ψ_b and $\bar{\Psi}_b$ and of $\Phi^\dagger$ and Φ [cf. Eq. (3)] give rise to the factors

$$M(\Psi_b)M(\bar{\Psi}_b) \times \delta(\Psi_b, \bar{\Psi}_b) \qquad (30a)$$

and

$$M(\Phi^\dagger)M(\Phi) \times \delta(\Phi^\dagger, \Phi), \qquad (30b)$$

respectively, where the δ's are operator Kronecker δ's. The factors (30) commute with all the generalized fields.

If the external lines are proton lines, the four matrix factors, as obtained from Eqs. (30) for scheme A, associated, respectively, with the four allowed loops are

$$\begin{aligned}
S_1 &\equiv M(p)M(\bar{p})M(\pi^0)M(\pi^0) \times \delta(p,\bar{p})\delta(\pi^0,\pi^0) \\
&= C_1C_3C_2C_3 \times \delta(p,\bar{p})\delta(\pi^0,\pi^0), \\
S_2 &\equiv M(n)M(\bar{n})M(\pi^-)M(\pi^+) \times \delta(n,\bar{n})\delta(\pi^-,\pi^+) \\
&= C_2C_3C_1C_3(-I) \times \delta(n,\bar{n})\delta(\pi^-,\pi^+), \\
S_3 &\equiv M(\Sigma^+)M(\bar{\Sigma}^-)M(\bar{K}^0)M(K^0) \\
&\quad \times \delta(\Sigma^+,\bar{\Sigma}^-)\delta(\bar{K}^0,K^0) = iC_1C_2iC_1C_3C_1C_3 \\
&\quad \times \delta(\Sigma^+,\bar{\Sigma}^-)\delta(\bar{K}^0,K^0), \\
S_4 &\equiv M(Y)M(\bar{Y})M(K^-)M(K^+) \times \delta(Y,\bar{Y})\delta(K^-,K^+) \\
&= iC_1C_2iC_2C_3C_2C_3 \times \delta(Y,\bar{Y})\delta(K^-,K^+).
\end{aligned} \qquad (31)$$

Since

$$\delta(p,\bar{p})\delta(\pi^0,\pi^0) = \delta(n,\bar{n})\delta(\pi^-,\pi^+) = \delta(\Sigma^+,\bar{\Sigma}^-)\delta(\bar{K}^0,K^0) = \delta(Y,\bar{Y})\delta(K^-,K^+), \quad (32)$$

matrix multiplication yields the result that

$$S_1 = S_2 = -S_3 = -S_4, \qquad (33)$$

i.e., up to phase factors, the matrices associated with the four loops are equal, and the matrices associated with the kaonic loops are 180° out of phase with the matrices associated with the pionic (and electromagnetic) loops, although both pions and kaons are assumed to be pseudoscalars. A similar result is obtained if the external lines are associated with any one of the other octet baryons. Moreover, these results are valid in all four hadronic schemes A–D. The 180° phase difference is due solely to the matrix structure of the generalized fields.

For spin-$\frac{1}{2}$ baryons of even strangeness, all the loops are associated with the matrices $\pm C_1C_3C_2C_3$, whereas for external baryon lines of odd strangeness the loops are associated with the matrices $\pm iC_1C_2$.

The 180° phase difference between the matrices associated with the kaonic and pionic loops raises the possibility of canceling out the logarithmic divergence associated with the lowest-order self-energy correction to the propagators. For each one of the eight baryons, an equation involving the baryon masses and coupling constants is obtained by equating the algebraic sum of the G^2M terms to zero. The eight resulting equations contain a considerable number of "unknowns" (i.e., mass and coupling-constant ratios). An assumption concerning the relative magnitudes of coupling-constant ratios, such as charge independence of nuclear forces, would, of course, reduce the number of unknowns. It is also conceivable that from other requirements and conditions, additional equations can be obtained for the same and possibly other unknowns.

The following eight equations for the baryon masses and strong-interaction coupling constants are derived without making assumptions such as charge independence [the notation is obvious: $G_{11}=G(pp\pi^0)$, $G_{12}=G(pn\pi^\pm)$, etc.]:

$$F_1=\ p:\ G_{11}^2M_p+G_{12}^2M_n=G_{13}^2M_{\Sigma^+}+G_{14}^2M_Y\,, \tag{34a}$$

$$F_1=\ n:\ G_{21}^2M_n+G_{22}^2M_p=G_{23}^2M_Z+G_{24}^2M_{\Sigma^-}\,, \tag{34b}$$

$$F_1=\Sigma^+:\ G_{31}^2M_{\Sigma^+}+G_{32}^2M_Z=G_{33}^2M_p+G_{34}^2M_{\Xi^0}\,, \tag{34c}$$

$$F_1=\ Y:\ G_{41}^2M_Y+G_{42}^2M_{\Sigma^-}=G_{43}^2M_p+G_{44}^2M_{\Xi^0}\,, \tag{34d}$$

$$F_1=\ Z:\ G_{51}^2M_Z+G_{52}^2M_{\Sigma^+}=G_{53}^2M_n+G_{54}^2M_{\Xi^-}\,, \tag{34e}$$

$$F_1=\Sigma^-:\ G_{61}^2M_{\Sigma^-}+G_{62}^2M_Y=G_{63}^2M_n+G_{64}^2M_{\Xi^-}\,, \tag{34f}$$

$$F_1=\Xi^0:\ G_{71}^2M_{\Xi^0}+G_{72}^2M_{\Xi^-}=G_{73}^2M_{\Sigma^+}+G_{74}^2M_Y\,, \tag{34g}$$

$$F_1=\Xi^-:\ G_{81}^2M_{\Xi^-}+G_{82}^2M_{\Xi^0}=G_{83}^2M_Z+G_{84}^2M_{\Sigma^-}. \tag{34h}$$

These equations are valid for all four hadronic schemes discussed.

The generalized fields Y and Z and their masses have been used in Eqs. (34) rather than Σ^0 and Λ and their masses.

The coupling constants in the above equations are not all independent:

$$G_{12}^2=G_{22}^2\,,\quad G_{13}^2=G_{33}^2\,,\ \text{etc.} \tag{35}$$

Assuming charge independence of pionic interactions and setting $M_{\Sigma^+}=M_Y$, $M_p=M_n$, and $G_{13}^2=2G_{14}^2$, one obtains from Eq. (34a) by naively substituting the physical values for masses and coupling constants in the equation

$$G_{13}^2/G_{11}^2\approx M_p/M_{\Sigma^+}\approx 0.8\,. \tag{36}$$

This ratio of the kaon and pion coupling constants, though smaller than unity, is too large to conform with present estimates. Nevertheless, in view of the approximation made in disregarding loops due to higher-spin fields, the value obtained for the coupling constants ratio (36) is not entirely unrealistic.

The 180° phase difference between matrix elements of interactions of an external line with different fields introduced by the matrix structure of the generalized fields can conceivably also be used to eliminate the quadratic divergence of the lowest-order weak self-energy of leptons arising from the emission and reabsorption of charged intermediate vector bosons. As will be explained, the removal of the divergence may also remove the electron-muon degeneracy. Since the couplings of vector bosons to leptons are less well understood than pseudoscalar meson-baryon couplings, this part of the discussion is necessarily speculative and incomplete.

In addition to hypothetical interactions with intermediate bosons, leptons are known to interact with other vector bosons such as the ϕ [16–18] and ρ^0 [19] resonances (and possibly also with the ω resonance). Just as for the case of meson-baryon interactions discussed above, the matrices associated with all the loops (due to allowed interactions with bosons) modifying any external lepton line are equal up to phase factors. By way of illustration, it is instructive to consider the correspondence (21). The loops due to interactions of the lepton l_1 with the ρ^0 and $B_1{}^*$ resonances are associated with the matrix $+(C_1C_3+C_3C_1)$, whereas the matrix associated with the loop due to emission and reabsorption of intermediate bosons is $-(C_1C_3+C_3C_1)$. If the interactions of l_1 with the $B_1{}^*$ and ρ^0 resonances also give rise to quadratic divergences, and if the 180° phase difference due to the matrix structure of the fields is not compensated by phase differences due to the form of the interactions between l_1 and W on one hand, and between l_1 and ρ^0 and $B_1{}^*$ on the other hand, then the algebraic sum of the coefficients of all the quadratic divergences can be equated to zero to yield an equation presumably containing the masses of ρ^0, $B_1{}^*$, W, and l_1 and the relevant coupling constants. The ρ^0 resonance is assumed to be associated with the unit matrix (cf. Table IX). A similar analysis applies to the lepton l_4, for example, except that according to Eq. (25) this lepton can interact with $B_4{}^*$ instead of with $B_1{}^*$. In this case, the coefficient of the quadratic divergence will be a function of the masses of $B_4{}^*$ and l_4 instead of

[19] See, for example, S. C. C. Ting, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968).

the masses of B_1^* and l_1, other parameters, except possibly coupling constants, remaining the same if corresponding interactions of l_1 and l_4 are similar, and if it is assumed that the masses of l_2 and l_3 vanish. If the masses of B_1^* and B_4^*, which are determined at least in part by strong interactions, are not equal, the masses of l_1 and l_4 can also be expected to be different. If l_1 and l_4 are assumed to correspond to the electron and muon or vice versa, the degeneracy between them could conceivably be removed in this manner.

Further investigations along the lines suggested in this discussion are in progress.

Recently, another attempt has been made to relate representations of operators satisfying trilinear or higher-order commutation relations to the internal symmetries of elementary particles.[20] It is also proper to mention that several years ago renewed interest in higher-order commutation relations was stimulated by Volkov.[21,22]

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Current Algebra, Field-Current Identity, the $K_2{}^0K_1{}^0$ Electromagnetic Transition, and the Decays $K^0\rightarrow\pi\pi e^+e^-$†

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A general discussion is given of the matrix element for the decays $K^0\rightarrow\pi\pi e^+e^-$; then, using the results of the vector-meson-dominance model to predict the $K_2{}^0K_1{}^0$ electromagnetic transition radius, partial conservation of axial-vector current and current algebra to predict the $K_2{}^0\rightarrow\pi^+\pi^-\gamma$ transition rate, and the experimental value of the CP-violating parameter ϵ, we find the following branching ratios: $\Gamma(K_L{}^0\rightarrow\pi^+\pi^-e^+e^-)/\Gamma(K_L{}^0\rightarrow\text{all})=1.7\times10^{-7}$, $\Gamma(K_L{}^0\rightarrow\pi^0\pi^0e^+e^-)/\Gamma(K_L{}^0\rightarrow\text{all})=0.2\times10^{-7}$, and $\Gamma(K_S{}^0\rightarrow\pi^+\pi^-e^+e^-)/\Gamma(K_S{}^0\rightarrow\text{all})=2.0\times10^{-5}$.

I. INTRODUCTION

In this paper, we propose to study the decays of the neutral K mesons into two pions and an electron-positron pair. In some of these decays the contribution from the bremsstrahlung diagram is absent or very much suppressed, so that the decay amplitudes are governed by the structure-dependent terms in the $K\rightarrow 2\pi\gamma$ vertex and the $K_2{}^0K_1{}^0$ electromagnetic transition amplitude. From the theoretical point of view, both these processes are important. The value of the $K_2{}^0K_1{}^0\gamma$ transition radius is predicted by the theory proposed by Kroll, Lee, and Zumino,[1] who assume that the entire hadronic electromagnetic current operator is to be identified with a linear combination of the renormalized field operators for the neutral vector mesons[2] ρ^0, ω^0, and ϕ^0. Accurate data for these decays may even distinguish between predictions of the mass mixing and the current mixing models of the ω-ϕ system. Evaluation of the $K\rightarrow 2\pi\gamma$ structure-dependent terms from these decays will check the models proposed in the literature.[3] Finally, these decays offer a good opportunity to study the various consequences of CP-violating effects, because both weak and electromagnetic interactions play a role in the decay amplitude.[4]

The plan of this paper is as follows. In Sec. II, we begin by studying the general structure of the matrix elements for $K^0\rightarrow\pi\pi\gamma$ and $K^0\rightarrow\pi\pi e^+e^-$ decays. The CP properties of the various terms are discussed. Then we calculate the decay spectra in the dipion and dilepton invariant masses and the decay rates are expressed in terms of form factors which are taken to be constants. In Sec. III, we indicate how the different

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