Photoproduction and the Electromagnetic Moments of Charged ρ Mesons*†

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A gauge-invariant one-vector-meson-exchange (OVE) model is considered for the process $\gamma+n\rightarrow\rho^-+p$. The form of the model is determined by selecting the simplest extension of the OVE diagram consistent with gauge invariance. The model is used to calculate unpolarized differential cross sections between threshold and $E_{\gamma\text{lab}}=3.0$ GeV for various values of the electromagnetic moments of the ρ^- meson. It is concluded from the results of the calculation that, if the experimental data were found to exhibit features of the OVE contribution near threshold, then information might be obtained from the process $\gamma+n\rightarrow\rho^-+p$ regarding the electromagnetic moments, and hence the internal structure of the charged ρ mesons.

I. INTRODUCTION

It is possible that in the region near the threshold of the reaction $\gamma+n\rightarrow\rho^-+p$, the one-vector-meson-exchange (OVE) mechanism might provide the dominant contribution. The reaction might thereby provide information regarding the electromagnetic moments and hence the internal structure of the charged ρ mesons. In this paper we consider a model which we believe to represent the simplest gauge-invariant extension of the OVE model. We use this model to calculate unpolarized differential cross sections for incident photon energies between threshold and $E_{\gamma\text{lab}}=3.0$ GeV for various values of the electromagnetic moments of the ρ^- meson.

Previous work has been done on the OVE contribution by Berman and Drell,[1] who consider the special case in which the interaction takes place only through the magnetic moment of the ρ meson. Work has also been done by Joos and Kramer,[2] who set up general expressions for the vertices. Joos and Kramer's work is limited, however, to calculations for the case of neutral ρ-meson photoproduction, where the OVE contribution vanishes as a consequence of charge conjugation invariance at the $\rho^0\gamma\rho^0$ vertex.

Experimentally, the most accessible example of this reaction is the $\gamma+n\rightarrow\rho^-+p$ process which may be observed in liquid-deuterium–bubble-chamber experiments such as those presently in progress at DESY[3] and SLAC.[4]

For simplicity we have neglected the effects of the presence of the neutron in a deuteron and the contribution of s- and u-channel nucleon resonances. We have also omitted a discussion of the one-pion-exchange (OPE) diagram since its contribution is well known[1,2] and may be added directly to the cross section arising from the OVE model. The direct addition is made possible by the vanishing of the interference between the OPE and OVE amplitudes.[2,5] For values of $g_{\rho\pi\gamma}$ consistent with $\rho\rightarrow\pi+\gamma$ data and $SU(3)$ predictions, the OPE contribution is of the same order of magnitude as the OVE contribution.[5]

This paper is divided into six major sections. Section II is devoted to a summary of the kinematics and our choice of conventions. In Sec. III we define the model which will be considered and present expressions for the mass-shell forms of the vertices. In Sec. IV the expressions for the vertices are combined with appropriate pole terms to obtain expressions for the Born-term diagrams. Additional nonpole terms corresponding to the contact diagram are found and the simplest combination of terms consistent with gauge invariance is chosen and labeled the minimal gauge-invariant OVE model. In Sec. V we describe the method of calculation, the choice of the electromagnetic moments of the ρ meson considered in the calculation, and present our results. Our conclusions are then discussed in Sec. VI.

II. KINEMATICS

The reaction which we will consider is represented in Fig. 1. The notation which we will use to represent masses, momenta, helicities, polarization vectors and spinors is summarized in Table I.

The Dirac spinors and the polarization vectors satisfy the following normalizations and relations:

$$\bar{u}(p\lambda)u(p\lambda')=\delta_{2\lambda,2\lambda'}\,,\tag{1}$$

$$\zeta_\nu^*(q\lambda)\zeta^\nu(q\lambda')=-\delta_{\lambda\lambda'}\,,\tag{2}$$

$$\epsilon_\mu^*(k\lambda)\epsilon^\mu(k\lambda')=-\delta_{\lambda\lambda'}\,,\tag{3}$$

$$(\mathbf{p}-M)u(p\lambda)=0\,,\tag{4}$$

$$q_\nu\zeta^\nu(q\lambda)=0\,,\tag{5}$$

$$k_\mu\epsilon^\mu(k\lambda)=0\,,\tag{6}$$

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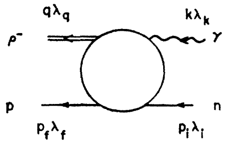

FIG. 1. Schematic diagram for the process $\gamma+n\rightarrow\rho^-+p$.

and

$$\sum_{\{\lambda\}} u(p\lambda)\bar{u}(p\lambda)=(\mathbf{p}+M)/2M\,, \tag{7}$$

$$\sum_{\{\lambda\}} \zeta^{\nu}(q\lambda)\zeta^{\mu *}(q\lambda)=-g^{\nu\mu}+q^{\nu}q^{\mu}/m^2\,, \tag{8}$$

$$\sum_{\{\lambda\}} \epsilon^{\nu}(k\lambda)\epsilon^{\mu *}(k\lambda)=-g^{\nu\mu}\,. \tag{9}$$

We have adopted the metric and Dirac γ-matrix conventions of Bjorken and Drell[6] so that the diagonal elements of $g^{\nu\mu}$ are given by $(1, -1, -1, -1)$ and the γ matrices are defined as

$$\gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{10}$$

$$\boldsymbol{\gamma}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix}, \tag{11}$$

$$\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\gamma^5=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}. \tag{12}$$

We have also defined the Mandelstam invariants s, t, and u in the standard fashion:

$$s=(p_i+k)^2=(p_f+q)^2\,, \tag{13}$$

$$t=(q-k)^2=(p_f-p_i)^2\,, \tag{14}$$

$$u=(q-p_i)^2=(k-p_f)^2\,. \tag{15}$$

With our choice of conventions the unpolarized differential photoproduction cross section in the c.m. frame is given by the expression

$$\frac{d\sigma}{d\Omega}=\frac{M^2|\mathbf{q}_{\mathrm{c.m.}}|}{4s|\mathbf{k}_{\mathrm{c.m.}}|}\sum_{\{\lambda\}}\frac{1}{(4\pi)^2}|\mathfrak{M}_{fi}|^2\,, \tag{16}$$

where $\mathfrak{M}_{fi}$ is the covariant transition matrix given by

$$\begin{aligned}\mathfrak{M}_{fi}&=\mathfrak{M}_{fi}(p_f\lambda_f q\lambda_q;\,p_i\lambda_i k\lambda_k)\\ &=\zeta_\nu^*(q\lambda_q)\bar{u}(p_f\lambda_f)I^{\nu\mu}u(p_i\lambda_i)\epsilon_\mu(k\lambda_k)\,.\end{aligned} \tag{17}$$

TABLE I. Kinematical notation.

Particle	Four-momentum	Helicity	Polarization vectors and spinors	Mass
proton	p_f	λ_f	$\bar{u}(p_f\lambda_f)$	M
neutron	p_i	λ_i	$u(p_i\lambda_i)$	M
ρ meson	q	λ_q	$\zeta_\nu^*(q\lambda_q)$	m
photon	k	λ_k	$\epsilon_\mu(k\lambda_k)$	0

[6] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).

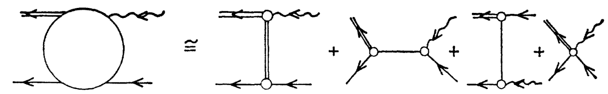

FIG. 2. Diagrammatic representation of the OVE model.

The general form of $I^{\nu\mu}$ is restricted not only by covariance, but also by gauge invariance (charge conservation) which requires that

$$\zeta_\nu^*(q\lambda_q)\bar{u}(p_f\lambda_f)I^{\nu\mu}u(p_i\lambda_i)k_\mu=0\,. \tag{18}$$

The determination of the form of $I^{\nu\mu}$, arising from the OVE model which we are considering, will be the primary object of the following sections.

III. OVE MODEL AND VERTEX EXPRESSIONS

The model which we wish to consider is the simplest extension of the amplitude representing the OVE diagram, consistent with gauge invariance. As we would expect from other electromagnetic processes, we find that it is necessary to couple the OVE diagram to the crossed-channel Born terms and a contact term in order to satisfy gauge invariance. We must therefore consider the contribution from the diagrams shown in Fig. 2.

The vertices which appear in the diagrams considered in Fig. 2 are shown in Fig. 3. The form of the contact term will be considered later. The expressions corresponding to the vertices in Fig. 3 are derived from general space-time invariance principles and can be written in the form

$$\begin{aligned}\mathcal{V}_{N\gamma N}=-ie\epsilon_\mu(k\lambda_k)\bar{u}(p'\lambda')[\mathfrak{F}_1(k^2,p^2,p'^2)(p+p')^\mu\\ +\mathfrak{F}_2(k^2,p^2,p'^2)\gamma^\mu]u(p\lambda)\,,\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{V}_{N\rho N}=-ig_{N\rho N}\zeta_\nu(q\lambda_q)\bar{u}(p'\lambda')[\mathcal{G}_1(q^2,p^2,p'^2)(p+p')^\nu\\ +\mathcal{G}_2(q^2,p^2,p'^2)\gamma^\nu]u(p\lambda)\,,\end{aligned} \tag{20}$$

$$\begin{aligned}\mathcal{V}_{\rho\gamma\rho}=ie\epsilon_\mu(k\lambda_k)\zeta_\alpha^*(q'\lambda_{q'})\{\mathcal{K}_1(k^2,q^2,q'^2)[(q+q')^\mu g^{\alpha\beta}]\\ +\mathcal{K}_2(k^2,q^2,q'^2)[k^\alpha g^{\beta\mu}-k^\beta g^{\alpha\mu}]+\mathcal{K}_3(k^2,q^2,q'^2)\\ \times[(q+q')^\mu k^\alpha k^\beta/m^2]\}\zeta_\beta(q\lambda_q)\,.\end{aligned} \tag{21}$$

The $\mathfrak{F}$'s, $\mathcal{G}$'s, and $\mathcal{K}$'s are form factors which reduce to coupling constants for mass-shell values of the momenta. In our calculations we will neglect the possible variation of these form factors and represent them by their corresponding coupling constants.

The $\mathfrak{F}$'s are the familiar nucleon electromagnetic form factors which are related to the electric and magnetic mass-shell form factors for the case $M_f=M_i=M$ by the

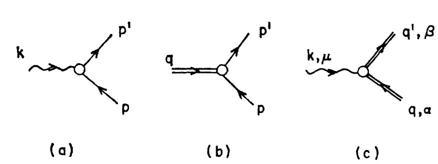

FIG. 3. Vertex diagrams for the (a) $N\gamma N$ vertex, (b) $N\rho N$ vertex, and (c) $\rho\gamma\rho$ vertex.

expressions

$$\mathcal{F}_2(k^2,M^2,M^2)=F_1(k^2,M^2,M^2)+2MF_2(k^2,M^2,M^2), \quad (22)$$

$$\mathcal{F}_1(k^2,M^2,M^2)=-F_2(k^2,M^2,M^2), \quad (23)$$

where

$$\begin{aligned} eF_1{}^p(0,M^2,M^2)&=e, \\ eF_1{}^n(0,M^2,M^2)&=0, \\ eF_2{}^p(0,M^2,M^2)&=e\mu^p/2M, \\ eF_2{}^n(0,M^2,M^2)&=e\mu^n/2M. \end{aligned} \quad (24)$$

The superscripts p and n refer to the proton and neutron, respectively. The values of the anomalous nucleon magnetic moments are taken to be[7] $\mu^p=1.79$ and $\mu^n=-1.91$ in units of the proton magneton.

The $\mathcal{G}$'s are the analogous terms for the $N\rho N$ coupling, which are related to a pair of terms G_1 and G_2 in the manner of Eqs. (22) and (23),

$$\mathcal{G}_2(q^2,M^2,M^2)=G_1(q^2,M^2,M^2)+2MG_2(q^2,M^2,M^2), \quad (25)$$

$$\mathcal{G}_1(q^2,M^2,M^2)=-G_2(q^2,M^2,M^2), \quad (26)$$

where we take[8]

$$\begin{aligned} G_1(q^2,M^2,M^2)g_{n\rho^-p}&=g_{n\rho^-p} \\ &=-\sqrt{2}g_{p\rho^0p} \\ &=-\tfrac{1}{2}\sqrt{2}f_\rho \end{aligned} \quad (27)$$

and

$$2.1\le f_\rho{}^2/4\pi\le 2.7. \quad (28)$$

For the purposes of this calculation we will use the value $f_\rho{}^2/4\pi=2.4$. We have used the vector-meson-dominance model of Sakurai[9] to obtain values for G_2 from the electromagnetic coupling. Vector dominance is expressed by the equation

$$eF_2{}^{\text{isovector}}(k^2)=(em^2/f_\rho)(m^2-k^2)^{-1}f_\rho G_2, \quad (29)$$

which yields (in the limit $k^2\to 0$) the result

$$\begin{aligned} G_2&\approx F_2{}^{\text{isovector}} \quad (k^2=0) \\ &=\tfrac{1}{2}(\mu^p-\mu^n)/2M \\ &=1.85/2M. \end{aligned} \quad (30)$$

We make the general assumption that

$$G_2(q^2,p^2,p'^2)=G_2, \quad (31)$$

which is consistent with the assumption of the absence of structure in strong interactions, and which will be shown later to be required for the satisfaction of gauge invariance in our model.

We thus obtain from Eqs. (25)–(27), (30), and (31) the suggested values

$$\mathcal{G}_1=-0.985 \quad (32)$$

and

$$\mathcal{G}_2=2.85 \quad (33)$$

The $\mathcal{H}$'s are less familiar terms. The connection between these functions and the static electromagnetic moments of a vector particle has been studied by several authors.[10] The $\mathcal{H}$ factors may be expressed in terms of e_ρ, the charge of the ρ meson, as follows:

(i) Electric-charge form factor

$$e\mathcal{H}_1(0,m^2,m^2)=e_\rho. \quad (34)$$

(ii) Magnetic-dipole form factor

$$e\mathcal{H}_2(0,m^2,m^2)=e_\rho(1+\mu_\rho{}^{\text{anomalous}}). \quad (35)$$

(iii) Electric quadrupole form factor

$$e\mathcal{H}_3(0,m^2,m^2)=\tfrac{1}{2}e_\rho(Q_\rho+\mu_\rho{}^{\text{anomalous}}). \quad (36)$$

The quantities $\mu_\rho{}^{\text{anomalous}}$ and Q_ρ are related to the static magnetic dipole moment operator $\mathbf{M}$ and the electric quadrupole moments Q_ρ' by the equations

$$\mathbf{M}\equiv(e_\rho/2m)(1+\mu_\rho{}^{\text{anomalous}})\mathbf{S}, \quad (37)$$

where $\mathbf{S}$ is the spin operator and

$$Q_\rho'\equiv Q_\rho/m^2. \quad (38)$$

IV. BORN TERMS AND GAUGE INVARIANCE

A. Procedure

Expressions may be constructed for the amplitudes corresponding to the Born-term diagrams shown in Fig. 2. by use of the vertex expressions given in Eqs. (19)–(21). Inspection of the resulting combination of expressions reveals that gauge invariance may be obtained only if the G functions are independent of any variation in the parameters q^2, p^2, and p'^2 and, therefore, correspond to constant $\mathcal{G}_1$ and $\mathcal{G}_2$. This provides a justification for the assumption of constancy made in Eqs. (27) and (31). The inspection of the resulting expressions also reveals that gauge invariance requires the addition of two types of pole-free terms which can be attributed to the contact diagram represented in Fig. 2. The first type of term is linear in either a magnetic dipole or an electric quadrupole moment and can be added independently to the expression representing a particular diagram. The second type of term is linear in the electric charge and must be added to the coupled combination of charge terms from all the Born-term diagrams.

We will proceed in Secs. IV B–IV D by considering the expressions corresponding to each diagram and discussing the appropriate necessary additions of the first type. In Sec. IV E we will consider the combination of charge terms and determine the form of the term of the second type that must be included.

The total combination of terms will correspond to a gauge-invariant representation of the model. The simplest combination of terms which includes the

[7] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[8] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1967).
[9] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).
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features of both the OVE model and gauge invariance will be discussed in Sec. IV F.

B. OVE Diagram

The OVE diagram gives an expression of the form

$$\mathfrak{M}_{\mathrm{OVE}}=\zeta_\nu^*(q\lambda_q)\bar{u}(p_f\lambda_f)I_{\mathrm{OVE}}{}^{\nu\mu}u(p_i\lambda_i)\epsilon_\mu(k\lambda_k)\,, \quad (39)$$

where

$$\begin{aligned}I_{\mathrm{OVE}}{}^{\nu\mu}=&-ieg_{n\rho^-p}[\mathcal{H}_1(2q^\mu-k^\mu)g^{\nu\alpha}\\&+\mathcal{H}_2(k^\alpha g^{\nu\mu}-k^\nu g^{\mu\alpha})+(\mathcal{H}_3/m^2)(2q^\mu-k^\mu)k^\nu k^\alpha]\\&\times[(g_{\alpha\kappa}-(q-k)_\alpha(q-k)_\kappa/m^2)/(t-m^2)]\\&\times[\mathcal{G}_1(p_1+p_f)^\kappa+\mathcal{G}_2\gamma^\kappa]. \quad (40)\end{aligned}$$

If we set $Q=p_i+p_f$, we obtain

$$\begin{aligned}I_{\mathrm{OVE}}{}^{\nu\mu}=&[-ieg_{n\rho^-p}/(t-m^2)][\mathcal{G}_1\mathcal{H}_1Q^\nu(2q^\mu-k^\mu)\\&+\mathcal{G}_2\mathcal{H}_1\gamma^\nu(2q^\mu-k^\mu)+\mathcal{G}_1\mathcal{H}_2(g^{\nu\mu}k\cdot Q-k^\nu Q^\mu)\\&+\mathcal{G}_2\mathcal{H}_2(g^{\nu\mu}\boldsymbol{k}-k^\nu\gamma^\mu)+\mathcal{G}_1(\mathcal{H}_3/m^2)(2k^\nu q^\mu k\cdot Q)\\&+\mathcal{G}_2(\mathcal{H}_3/m^2)(2k^\nu q^\mu\boldsymbol{k})]. \quad (41)\end{aligned}$$

Since $2k\cdot q=-(t-m^2)$, pole-free terms of the first type may be labeled $I_{\mathrm{OVEC}}{}^{\nu\mu}$ and added in Eq. (41) to the quadrupole terms linear in $\mathcal{G}_1\mathcal{H}_3$ and $\mathcal{G}_2\mathcal{H}_3$, giving the result

$$\begin{aligned}I_{\mathrm{OVE}}{}^{\nu\mu}+I_{\mathrm{OVEC}}{}^{\nu\mu}=&[-ieg_{n\rho^-p}/(t-m^2)]\\&\times\{\mathcal{G}_1\mathcal{H}_1Q^\nu(2q^\mu-k^\mu)+\mathcal{G}_2\mathcal{H}_1\gamma^\nu(2q^\mu-k^\mu)\\&+\mathcal{G}_1\mathcal{H}_2(g^{\nu\mu}k\cdot Q-k^\nu Q^\mu)+\mathcal{G}_2\mathcal{H}_2(g^{\nu\mu}\boldsymbol{k}-k^\nu\gamma^\mu)\\&+\mathcal{G}_1(\mathcal{H}_3/m^2)[2k^\nu(q^\mu k\cdot Q-Q^\mu k\cdot q)]\\&+\mathcal{G}_2(\mathcal{H}_3/m^2)[2k^\nu(q^\mu\boldsymbol{k}-\gamma^\mu k\cdot q)]\}. \quad (42)\end{aligned}$$

C. *s*-Channel Nucleon-Pole Diagram

The *s*-channel nucleon-pole diagram gives an expression of the form

$$\mathfrak{M}_s=\zeta_\nu^*(q\lambda_q)\bar{u}(p_f\lambda_f)I_s{}^{\nu\mu}u(p_i\lambda_i)\epsilon_\mu(k\lambda_k)\,, \quad (43)$$

where

$$\begin{aligned}I_s{}^{\nu\mu}=&-ieg_{n\rho^-p}[\mathcal{G}_1(2p_f+q)^\nu+\mathcal{G}_2\gamma^\nu]\\&\times[(\boldsymbol{p}_i+\boldsymbol{k}+M)/(s-M^2)][\mathcal{F}_1{}^n(2p_i+k)^\mu+\mathcal{F}_2{}^n\gamma^\mu] \quad (44)\\=&[-ieg_{n\rho^-p}/(s-M^2)]\\&\times\{\mathcal{G}_1\mathcal{F}_1{}^n2[(2M)(2p_i+k)^\mu+(2p_i+k)^\mu\boldsymbol{k}]p_f{}^\nu\\&+\mathcal{G}_2\mathcal{F}_1{}^n[(2M)(2p_i+k)^\mu\gamma^\nu+(2p_i+k)^\mu\gamma^\nu\boldsymbol{k}]\\&+\mathcal{G}_1\mathcal{F}_2{}^n2[(2p_i+k)^\mu+(\boldsymbol{k}\gamma^\mu-k^\mu)]p_f{}^\nu\\&+\mathcal{G}_2\mathcal{F}_2{}^n[(2p_i+k)^\mu\gamma^\nu+\gamma^\nu(\boldsymbol{k}\gamma^\mu-k^\mu)]\}. \quad (45)\end{aligned}$$

Since $2k\cdot p_i=s-M^2$, pole-free terms of the first type may be labeled $I_{sC}{}^{\nu\mu}$ and added to the magnetic dipole terms linear in $\mathcal{F}_1{}^n=-F_2{}^n$, giving the form

$$\begin{aligned}I_s{}^{\nu\mu}+I_{sC}{}^{\nu\mu}=&[-ieg_{n\rho^-p}/(s-M^2)]\\&\times\{\mathcal{G}_1\mathcal{F}_1{}^n2[(2M)(2p_i+k)^\mu+(2p_i+k)^\mu\boldsymbol{k}\\&-(s-M^2)\gamma^\mu]p_f{}^\nu+\mathcal{G}_2\mathcal{F}_1{}^n[(2M)(2p_i+k)^\mu\gamma^\nu\\&+(2p_i+k)^\mu\gamma^\nu\boldsymbol{k}-(s-M^2)\gamma^\nu\gamma^\mu]\\&+\mathcal{G}_1\mathcal{F}_2{}^n2[(2p_i+k)^\mu+(\boldsymbol{k}\gamma^\mu-k^\mu)]p_f{}^\nu\\&+\mathcal{G}_2\mathcal{F}_2{}^n[(2p_i+k)^\mu\gamma^\nu+\gamma^\nu(\boldsymbol{k}\gamma^\mu-k^\mu)]\}. \quad (46)\end{aligned}$$

D. *u*-Channel Nucleon-Pole Diagram

The *u*-channel nucleon-pole diagram gives an expression of the form

$$\mathfrak{M}_u=\zeta_\nu^*(q\lambda_q)\bar{u}(p_f\lambda_f)I_u{}^{\nu\mu}i(p_i\lambda_i)\epsilon_\mu(k\lambda_k)\,, \quad (47)$$

where

$$\begin{aligned}I_u{}^{\nu\mu}=&-ieg_{n\rho^-p}[\mathcal{F}_1{}^p(2p_f+k)^\mu+\mathcal{F}_2{}^p\gamma^\mu]\\&\times[(\boldsymbol{p}_f-\boldsymbol{k}+M)/(u-M^2)][\mathcal{G}_1(2p_i-q)^\nu+\mathcal{G}_2\gamma^\nu]. \quad (48)\end{aligned}$$

In a manner strictly analogous to that used in the *s*-channel case, we use the fact that $2k\cdot p_f=-(u-M^2)$, to introduce pole-free terms of the first type which are labeled $I_{uC}{}^{\nu\mu}$ and added to the magnetic dipole terms linear in $\mathcal{F}_1{}^p=-F_2{}^p$. The results of this addition are given by

$$\begin{aligned}I_u{}^{\nu\mu}+I_{uC}{}^{\nu\mu}=&[-ieg_{n\rho^-p}/(u-M^2)]\\&\times\mathcal{G}_1\mathcal{F}_1{}^p2[(2M)(2p_f-k)^\mu-(2p_f-k)^\mu\boldsymbol{k}\\&-(u-M^2)\gamma^\mu]p_i{}^\nu+\mathcal{G}_2\mathcal{F}_1{}^p[(2M)(2p_f-k)^\mu\gamma^\nu\\&-(2p_f-k)^\mu\boldsymbol{k}\gamma^\nu-(u-M^2)\gamma^\mu\gamma^\nu]\\&+\mathcal{G}_1\mathcal{F}_2{}^p2[(2p_f-k)^\mu-(\gamma^\mu\boldsymbol{k}-k^\mu)]p_i{}^\nu\\&+\mathcal{G}_2\mathcal{F}_2{}^p[(2p_f-k)^\mu\gamma^\nu-(\gamma^\mu\boldsymbol{k}\gamma^\nu-k^\mu\gamma^\nu)]\}. \quad (49)\end{aligned}$$

E. Channel Coupling and Charge Conservation

In Secs. IV B–IV D we obtained a set of expressions which are fully gauge-invariant except for their respective charge (electric monopole) terms. The charge terms may be collected into two terms linear in the coefficients $\mathcal{G}_1$ and $\mathcal{G}_2$:

$$\begin{aligned}D_1{}^{\nu\mu}=&-ieg_{n\rho^-p}\mathcal{G}_1\\&\times\left(\mathcal{H}_1\frac{Q^\nu(2q-k)^\mu}{t-m^2}+(2M\mathcal{F}_1{}^p+\mathcal{F}_2{}^p)\frac{2p_i{}^\nu(2p_f-k)^\mu}{u-M^2}\right.\\&\left.+(2M\mathcal{F}_1{}^n+\mathcal{F}_2{}^n)\frac{2p_f{}^\nu(2p_i+k)^\mu}{s-M^2}\right), \quad (50)\end{aligned}$$

$$\begin{aligned}D_2{}^{\nu\mu}=&-ieg_{n\rho^-p}\mathcal{G}_2\\&\times\left(\mathcal{H}_1\frac{\gamma^\nu(2q-k)^\mu}{t-m^2}+(2M\mathcal{F}_1{}^p+\mathcal{F}_2{}^p)\frac{\gamma^\nu(2p_f-k)^\mu}{u-M^2}\right.\\&\left.+(2M\mathcal{F}_1{}^n+\mathcal{F}_2{}^n)\frac{\gamma^\nu(2p_i+k)^\mu}{s-M^2}\right). \quad (51)\end{aligned}$$

The assumed constancy of $\mathcal{G}_1$ and $\mathcal{G}_2$, discussed earlier, is necessary for the extraction of the $\mathcal{G}$'s as common factors. We use expressions in Eqs. (22) and (23) to replace the $\mathcal{F}$-factor combinations occurring in the expressions (50) and (51) with the electric charge terms,

$$F_1{}^{p,n}=2M\mathcal{F}_1{}^{p,n}+\mathcal{F}_2{}^{p,n}. \quad (52)$$

If we apply the gauge-invariance condition $k_\mu D_2{}^{\nu\mu}=0$ to Eq. (51), we obtain the relation

$$F_1{}^n-F_1{}^p-\mathcal{H}_1=0\,, \quad (53)$$

which is the statement that charge must be conserved in the reaction.

If we represent $\mathcal{H}_1$ in $D_1^{\nu\mu}$ by Eq. (53) and impose the gauge-invariance condition $D_1^{\nu\mu}k_\mu=0$, we find that it is necessary to include a pole-free term of second type, having the form $\mathcal{G}_1\mathcal{C}g^{\nu\mu}$ where $\mathcal{C}$ is to be determined. This addition yields the gauge-invariant replacement for $D_1^{\nu\mu}$,

$$D_1'^{\nu\mu}=D_1^{\nu\mu}-ieg_{n\rho^-p}\mathcal{G}_1\mathcal{C}g^{\nu\mu}. \tag{54}$$

The value for $\mathcal{C}$ found by performing the contraction $D_1'^{\nu\mu}k_\mu=0$ is

$$\mathcal{C}=-(F_1^p+F_1^n). \tag{55}$$

F. Minimal Gauge-Invariant OVE Model

It is easily seen from the results of Sec. IV E that the simplest gauge-invariant extension of the OVE diagram includes Eq. (42) and those parts of Eqs. (51) and (54) absent from Eq. (42). This combination is then defined as the minimal gauge-invariant OVE model.

V. CALCULATIONS AND CHOICE OF PARAMETERS

A. Method of Calculation

The minimal gauge-invariant OVE model has been used to calculate unpolarized differential cross sections in the energy region between $E_{\gamma\text{lab}}=1.08$ (threshold) and 3.0 GeV for which the dominance of OVE is most likely expected to occur. In performing the calculations we fix the $N\rho N$ vertex by setting $f_\rho^2/4\pi=2.4$ and consider the consequences of varying the magnitudes of the electromagnetic moments of the ρ^- meson, μ_ρ and Q_ρ. The trace calculations have been carried out by several independent methods. The first technique used was a brute-force Fortran-IV method developed by the author, and described in the dissertation.[5] More recently the trace calculations have been performed with a special short program for Dirac-matrix algebra devised by Campbell.[11] Certain special cases have been checked by hand.

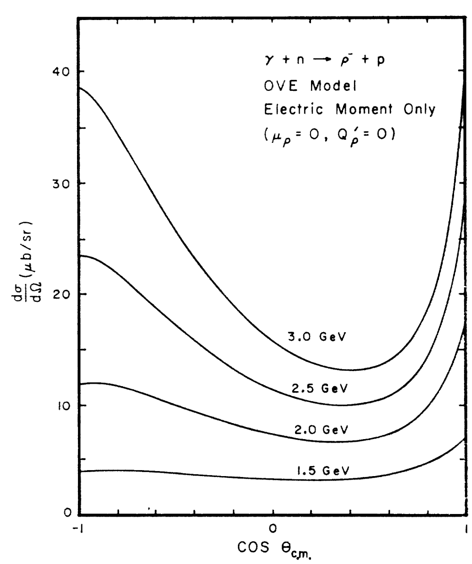

FIG. 4. Differential cross sections $d\sigma/d\Omega$ in the c.m. system for $\gamma+n\rightarrow\rho^-+p$ for $E_{\gamma\text{lab}}=1.5$–3.0 GeV. Only the electric moment coupling of the ρ^- is considered ($\mu_\rho=0$, $Q_\rho'=0$).

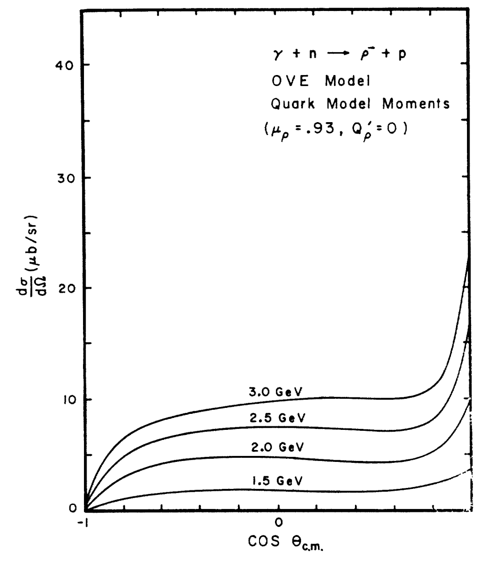

FIG. 5. Differential cross sections $d\sigma/d\Omega$ in the c.m. system for $\gamma+n\rightarrow\rho^-+p$ for $E_{\gamma\text{lab}}=1.5$–3.0 GeV. The electromagnetic moments of the ρ^- are derived from a simple S-wave quark model ($\mu_\rho=0.93$, $Q_\rho'=0$).

B. Choices of Parameters

We have considered three combinations of the moments μ_ρ and Q_ρ'.

1. *Electric Charge Coupling*

First, we consider the case in which the coupling is entirely that of the electric charge ($\mu_\rho=0$, $Q_\rho'=0$). This combination has no particular theoretical justification, but serves as a reference for comparison with the finite μ_ρ and Q_ρ' calculations. The results are shown in Fig. 4.

2. *Magnetic Moment Coupling*

Second, we consider the case in which we have a finite magnetic dipole moment, but no electric quadrupole moment. It is interesting that three independent approaches undertaken to obtain a reasonable value for μ_ρ yield almost identical predictions. If we assume that the magnetic moment of the ρ^- meson has the value of its natural magnetic moment (i.e., $\mu_\rho^{\text{anomalous}}=0$), we obtain $\mu_\rho=1$.

If we use a simple S-wave quark model in the spirit of Dalitz,[12] we obtain $\mu_\rho=|\frac{1}{3}\mu^p|=0.93$ and $Q_\rho'=0$.

[11] J. A. Campbell, J. Comput. Phys. **2**, 412 (1968).

[12] R. H. Dalitz, in *High Energy Physics*, edited by C. Dewitt and M. Jacob (Gordon and Breach, Science Publishers, Inc., New York, 1965), p. 253.

If we consider the ρ^- meson, in the spirit of Fermi and Yang,[13] as an S-wave bound state (or resonance) of a $\bar{p}n$ pair, we obtain

$$\mu_\rho = |\mu^p - \mu^n| = 0.88 \quad \text{and} \quad Q_\rho' = 0.$$

The results of the calculation for the parameters predicted by the quark model are shown in Fig. 5.

3. *Magnetic Moment Plus Electric Quadrupole Coupling*

Last, we consider the effect of adding a finite electric quadrupole moment to the magnetic moment considered above. We have obtained a reasonable value for Q_ρ' by considering the case in which the geometry of the charge distribution of the ρ^- meson is assumed to be equivalent to that of the deuteron, but the ρ-meson radius is equal to that of the nucleon, giving

$$Q_\rho' \approx (r_p/r_d)^2 Q_d'. \tag{56}$$

The value for the ρ^--meson quadrupole moment suggested by this simple model using the experimental values[14] $r_p = 0.77$ F, $r_d = 2.11$ F, and $Q_d' = 2.74$ mb, is $Q_\rho' = 0.37$ mb.

The results of the calculation obtained assuming the combination of parameters ($\mu_\rho = 0.93$, $Q_\rho' = 0.37$ mb) are shown in Fig. 6. It is coincidental that the interference arising from this value of the quadrupole moment causes a near cancellation of the cross section in the forward direction. In order to exhibit a more typical example of the effect of a finite quadrupole moment on the cross section, we have also presented in Fig. 7 the results of the calculation for the combination of parameters ($\mu_\rho = 0.93$, $Q_\rho' = 0.18$ mb).

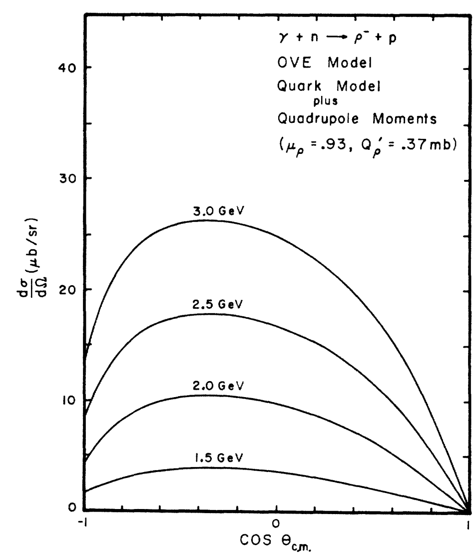

FIG. 6. Differential cross sections $d\sigma/d\Omega$ in the c.m. system for $\gamma+n \rightarrow \rho^-+p$ for $E_{\gamma\mathrm{lab}} = 1.5$–$3.0$ GeV. The electromagnetic moments of the ρ^- are derived from a simple quark model plus the quadrupole moment which might be expected for a ρ^- meson with the charge distribution of a deuteron and the radius of a proton ($\mu_\rho = 0.93$, $Q_\rho' = 0.37$ mb).

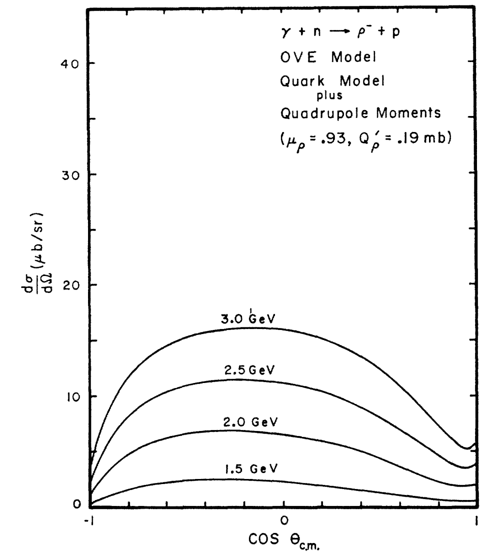

FIG. 7. Differential cross sections $d\sigma/d\Omega$ in the c.m. systems for $\gamma+n \rightarrow \rho^-+p$ for $E_{\gamma\mathrm{lab}} = 1.5$–$3.0$ GeV. The electromagnetic moments of the ρ^- are derived from a simple quark model plus one-half the quadrupole moment which might be expected for a ρ^- meson with the charge distribution of a deuteron and the radius of a proton ($\mu_\rho = 0.93$, $Q_\rho' = 0.19$ mb).

VI. DISCUSSION AND CONCLUSIONS

An investigation of the results of the calculations reveals some interesting consequences of the theory.

(1) The OVE model predicts for the $\gamma+p \rightarrow \rho^-+p$ process either a nonexistent or less dramatic forward peak, but a larger total cross section $[\sigma_T = \int (d\sigma/d\Omega) d\Omega]$ than those which have been found experimentally[15] for the process $\gamma+p \rightarrow \rho^0+p$.

(2) The shape of the unpolarized differential cross section predicted by the OVE model is quite sensitive to the values of the electromagnetic moments of the charged ρ mesons.

We may conclude that if the experimental data were found to reveal details characteristic of the OVE model, the $\gamma+n \rightarrow \rho^-+p$ process might then be useful in providing information regarding the magnitudes of the electromagnetic moments and hence, the structure of the charged ρ mesons.

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[13] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

[14] R. Hofstadter, Ann. Rev. Nucl. Sci. **3**, 231 (1957); H. G. Kolsky *et al.*, Phys. Rev. **81**, 1061 (1951).

[15] For recent results and bibliography see Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Rev. **175**, 1669 (1968).