Calculation of KN and $\bar{K}N$ Scattering Lengths and Phase Shifts by a Pole-Dominance Method*

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Recently, a procedure for representing a two-particle scattering amplitude as a sum of resonances in all three channels was derived by Schwarz. In addition to satisfying crossing symmetry, such a formula has a large domain of validity in the (s,t,u) plane. This method is applied here to KN and $\bar{K}N$ scattering. The experimental parameters for the scattering are calculated in the low-energy region. They are (1) the s-wave scattering lengths for KN isospin-zero and -one channels; (2) the S_{01}, P_{01}, S_{11}, P_{11}, P_{13}, and D_{13} phase shifts (S_{ij} denotes the s-wave isospin i and angular momentum $\frac{1}{2}j$) for scattering in the energy region up to center-of-mass momentum 0.3 BeV/c; (3) the real part of the s-wave scattering lengths for $\bar{K}N$ isospin-zero and -one channels. In addition, a sum rule relating amplitudes of the KN channel and the $\bar{K}N$ channel is obtained. It is well satisfied for the real part of the amplitude and is not satisfied for the imaginary part.

I. INTRODUCTION

THEORETICAL understanding of $\bar{K}N$ scattering is hindered by the presence of inelastic channels, even at the threshold, where approximations such as effective-range expansions can be successfully applied to πN scattering. Also, the existence of the $Y_0^*(1405)$ and $Y_1^*(1385)$ resonances near the $\bar{K}N$ threshold makes it even more important to have an understanding of the unphysical cuts.

Experiments show that the $\bar{K}N$, $\Sigma\pi$, $\Lambda\pi$, and $\Lambda\pi\pi$ channels are all strongly coupled, and we cannot treat them separately. An effective-range analysis of the multichannel-scattering problem has been completed by Kim.[1] In his analysis, it was found that the $Y_1^*(1385)$ does not couple appreciably to the $\bar{K}N$ channel and the $Y_0^*(1405)$ may be regarded as a bound state of the $\bar{K}N$ system. The knowledge of the unphysical cut thus obtained enabled him to calculate the coupling constants $g^2_{\Lambda pK^-}/4\pi$ and $g^2_{p\Sigma^0K^-}/4\pi$ by saturating once-subtracted dispersion relations. It was also found that the high-energy contribution to the forward dispersion relation was large. This implies that we cannot saturate dispersion integrals with low-lying resonances as is commonly done for the highly convergent amplitudes $(A-, B+)$ in the πN problem.

In this paper, we point out that some further progress can be made toward understanding the low-energy behavior of KN and $\bar{K}N$ scattering using the pole-dominance method (PDM) recently discussed by Schwarz.[2] The PDM gives low-energy-scattering amplitudes as a sum of low-energy resonance contributions from all three channels without any double counting. Furthermore, within its region of validity, the formula is insensitive to the high-energy contributions. Thus we have scattering amplitudes that involve only low-energy resonance parameters and avoid the problem of poor convergence. By evaluating the amplitude at the threshold of the KN and $\bar{K}N$ channels, we obtain scattering lengths for both the KN and $\bar{K}N$ channels. Furthermore, having obtained explicit analytic functions, one can deduce the energy and angular dependences of the scattering amplitudes. By doing a partial-wave analysis, the energy dependence of the KN phase shifts S_{11}, P_{11}, S_{01}, P_{03}, and D_{03} is obtained.

In Sec. II, the PDM is formulated for the equal-mass spinless case, and in Sec. III, it is generalized to the KN problem. In Sec. IV, we calculate the scattering lengths and in Sec. V the phase shifts.

II. FORMULATION OF PDM

In order to avoid unnecessary complications, only the equal-mass and spin-zero case is considered in this section.[3] Consider elastic scattering of two particles of mass μ, and let the scattering amplitude be denoted by $F^I(t,s)$.[4] The fixed-t dispersion relation, neglecting subtraction and Born terms, is

$$F^I(t,s)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}F^I(t,s')}{s'-s}ds'+\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}F^I(t,u')}{u'-u}du'. \quad (2.1)$$

The partial-wave decomposition is

$$F^I(t,s)=\sum_{J=0}^{\infty}(2J+1)a_J{}^I(t)P_J(z_t), \quad (2.2)$$

where $z_t=1+s/2q_t^2$, q_t is the c.m. three-momentum in the t channel, $s=-2q_t^2(1-z_t)$, and $t=4(q_t^2+\mu^2)$. We calculate $a_J{}^I(t)$ by the Froissart-Gribov formula, if the

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[1] J. K. Kim, Phys. Rev. Letters 19, 1074 (1968); 19, 1079 (1968); Von Hippel and J. K. Kim, *ibid.* 20, 1303 (1968).
[2] J. H. Schwarz, Phys. Rev. 175, 1852 (1968).

[3] This section does not contain anything new; the basic idea was presented in Ref. 2.
[4] $F^I(t,s)$ denotes isospin I in the channel indicated by the variable appearing on the left.

integrals converge:

$$a_J{}^I(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\mathrm{Im}F^I(t,s')Q_J\left(1+\frac{s'}{2q_t{}^2}\right)\frac{ds'}{2q_t{}^2}$$
$$-\frac{1}{\pi}\int_{4\mu^2}^{\infty}\mathrm{Im}F^I(t,u')Q_J\left(-1-\frac{u'}{2q_t{}^2}\right)\frac{du'}{2q_t{}^2}. \quad (2.3)$$

(q^2 and p^2 denote $\mathbf{q}\cdot\mathbf{q}$ and $\mathbf{p}\cdot\mathbf{p}$ throughout this paper.) Although this formula usually arises in the discussion of complex J, we will use it only for the physical value of J.

We note that (2.3) is derived from the fixed-t dispersion relation of the form (2.1). Therefore, if (2.3) does not converge for some J, (2.1) will not converge. This implies that whenever (2.1) is divergent, we can blame it on low partial waves, provided that the scattering amplitude is bounded by a polynomial so that the Froissart-Gribov formula exists for $J>J_0$ for some J_0.

Denoting the trajectory of the leading Regge pole or cut in the t channel by $\alpha(t)$, we have

$$\mathrm{Im}F^I(t,s)\xrightarrow[|s|\to\infty]{} s^{\alpha(t)}\times(\text{function of } t)$$

(neglecting possible logarithms). Choosing J_0 such that $\alpha(t)-J_0-1<0$, we obtain

$$F^I(t,s)=\sum_{J=0}^{J_0}(2J+1)a_J{}^I(t)P_J(z_t)$$
$$+\frac{1}{\pi}\int_{4\mu^2}^{N}\frac{\mathrm{Im}F^I(t,s')}{s'-s}ds'+\frac{1}{\pi}\int_{4\mu^2}^{N}\frac{\mathrm{Im}F^I(t,u')}{u'-u}du'$$
$$-\sum_{J=0}^{J_0}(2J+1)P_J(z_t)a_J{}^{I,N}(t), \quad (2.4)$$

where

$$a_J{}^{I,N}(t)=\frac{1}{\pi}\int_{4\mu^2}^{N}\mathrm{Im}F^I(t,s')Q_J\left(1+\frac{s'}{2q_t{}^2}\right)\frac{ds'}{2q_t{}^2}$$
$$-\frac{1}{\pi}\int_{4\mu^2}^{N}\mathrm{Im}F^I(t,u')Q_J\left(-1-\frac{u'}{2q_t{}^2}\right)\frac{du'}{2q_t{}^2}, \quad (2.5)$$

and we have neglected the term

$$\sum_{J=J_0+1}^{\infty}(2J+1)P_J\left(1+\frac{s}{2q_t{}^2}\right)$$
$$\times\left[\frac{1}{\pi}\int_N^{\infty}\frac{ds'}{2q_t{}^2}\mathrm{Im}F^I(t,s')Q_J\left(1+\frac{s'}{2q_t{}^2}\right)\right.$$
$$\left.-\frac{1}{\pi}\int_N^{\infty}\frac{du'}{2q_t{}^2}\mathrm{Im}F^I(t,u')Q_J\left(-1-\frac{u}{2q_t{}^2}\right)\right], \quad (2.6)$$

which can be made as small as we wish by an appropriate choice of N and J_0. Equation (2.4) is a representation of $F^I(t,s)$, provided we have some independent method to calculate $a_J{}^I$ for $J\leqslant J_0$. Note that any approximation used to evaluate (2.4) will not require the knowledge of the high-energy behavior of the amplitude.

We choose to evaluate $a_J{}^I(t)$ for $J\leqslant J_0$ using partial-wave dispersion relations. Defining $b_J{}^I(t)=a_J{}^I(t)/q_t{}^{2J}$, again neglecting subtractions, we can write

$$b_J(t)=\frac{1}{\pi}\int_{-\infty}^{t_0}\frac{\mathrm{Im}b_J{}^I(t')}{t'-t}dt'+\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}b_J{}^I(t')}{t'-t}dt', \quad (2.7)$$

where t_0 denotes the position at which the left-hand cut begins. We will show that the left-hand-cut contribution to (2.7), when put into the first term of (2.4), cancels the last term of (2.4). The left-hand discontinuity function is given by the usual formula

$$\mathrm{Im}b_J{}^I(t)=\frac{1}{2}\int_{4\mu^2}^{4\mu^2-t}\frac{ds'}{2q_t{}^{2J+2}}\mathrm{Im}F^I(t,s')P_J\left(1+\frac{s'}{2q_t{}^2}\right)$$
$$-\frac{1}{2}\int_{4\mu^2}^{4\mu^2-t}\frac{du'}{2q_t{}^{2J+2}}\mathrm{Im}F^I(t,u')P_J\left(-1-\frac{u'}{2q_t{}^2}\right). \quad (2.8)$$

If resonances of the s and u channels are put into (2.8), each resonance of spin J_R gives a term which behaves like t^{J_R-J-1} as $t\to-\infty$. High-spin resonances will, therefore, give divergences. However, we have assumed polynomial boundedness of $b_J(t)$, and since it has cuts and poles only on the real axis, the unitarity bound $b_J(t)$ (which can be applied for $t\to\infty$) must hold for all directions on the complex t plane.[5] In other words, many resonance contributions, which individually give divergent asymptotic behavior (for $t\to-\infty$), will cancel each other in such a way that the unitarity bound is satisfied in all directions. We are, therefore, justified in taking only low-mass-exchange contributions to (2.8) when unitarity implies good convergence of the partial-wave dispersion relation. When narrow resonances are put into (2.8), we have

$$\mathrm{Im}b_J{}^I(t)=\sum_{i=1}^{\infty}\frac{\pi g_i{}^2}{4q_t{}^{2J+2}}P_{J_i}\left(1+\frac{t}{2q_s{}^2}\right)P_J\left(1+\frac{m_i{}^2}{2q_t{}^2}\right)$$
$$\times\theta(4\mu^2-m_i{}^2-t)-\sum_{i=1}^{\infty}\frac{\pi g_i{}^2}{4q_t{}^{2J+2}}P_{J_i}\left(1+\frac{t}{2q_u{}^2}\right)$$
$$\times P_J\left(-1-\frac{m_i{}^2}{2q_t{}^2}\right)\theta(4\mu^2-m_i{}^2-t). \quad (2.9)$$

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[5] Using the Sugawara-Kanazawa theorem [M. Sugawara and A. Kanazawa, Phys. Rev. **123**, 1895 (1961)], provided that the amplitude approaches some limit along the left-hand cut.

When narrow resonances are put into (2.5), we have

$$\frac{a_J^{I,N}}{q_t^{2J}}=\sum_{i=1}^{n}\frac{g_i^2}{2q_t^{2J+2}}P_J\left(1+\frac{t}{2q_s^2}\right)Q_J\left(1+\frac{m_i^2}{2q_t^2}\right)-\sum_{i=1}^{n}\frac{g_i^2}{2q_t^{2J+2}}P_J\left(1+\frac{t}{2q_u^2}\right)Q_J\left(-1-\frac{m_i^2}{2q_t^2}\right),\quad(2.10)$$

where $m_n^2<N$. Thus, to this approximation (high-mass exchange can be neglected),

$$\mathrm{Im}b_J^I(t)=\mathrm{Im}a_J^{I,N}/q_t^{2J},\quad(2.11)$$

and if both sides of (2.11) vanish as $t\to-\infty$, we have

$$\frac{q_t^{2J}}{\pi}\int_{-\infty}^{0}\frac{\mathrm{Im}b_J^I(t')}{t'-t}dt'=a_J^{I,N}(t).\quad(2.12)$$

$\mathrm{Im}b_J(t)\to0$ only when $J_i-J<1$ for all $i=1,\ldots,n$. When this inequality is not satisfied, then (2.12) is true only up to an additive polynomial in t, and (2.5) becomes

$$F^I(t,s)=\frac{1}{\pi}\int_{4\mu^2}^{N}\frac{\mathrm{Im}F^I(t,s')}{s'-s}ds'+\frac{1}{\pi}\int_{4\mu^2}^{N}\frac{\mathrm{Im}F^I(t,u')}{u'-u}du'+\sum_{J=0}^{J_0}(2J+1)P_J\left(1+\frac{s}{2q_t^2}\right)\frac{q_t^{2J}}{\pi}\times\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}b_J(t')}{t'-t}dt'+\sum_{J=0}^{J_0}(2J+1)P_J\left(1+\frac{s}{2q_t^2}\right)\times(\text{polynomial in }t).\quad(2.13)$$

If the right-hand-cut integrals are also approximated by narrow-resonance contributions, we can write $F^I(t,s)$ as a sum of poles in all three channels and terms due to possible subtractions.

Another way of looking at (2.13) is as the result of subtracting the fixed-t dispersion relation until only low-energy contributions are significant. In doing this, however, we have extracted the t dependence of the subtraction terms. This was done through added information such as analyticity of partial-wave amplitudes and convergence arguments.

III. FORMULATION OF PDM FOR KN SCATTERING

The unequal-mass problem is complicated by the analytic properties of the partial-wave amplitudes. This complication can be avoided, however, if we choose the t channel to be $K\bar{K}\to N\bar{N}$. Then we will be considering the partial-wave amplitude of $K\bar{K}\to N\bar{N}$. Since the initial and final states both have equal masses, pseudothreshold and circular cuts do not appear. The special role played by the t channel is only of a kinematical nature. The final result, with contributions from all three channels, is quite symmetric and does not depend on the choice of the channel in which the partial-wave expansion is made. To take account of spin and isospin, we define the amplitudes[6]

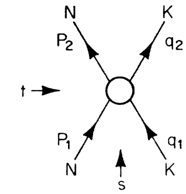

FIG. 1. Definition of variables in scattering.

$$T_i(p_2,q_2|p_1,q_1)=-A_i(s,t)+\tfrac{1}{2}i(q_1+q_2)\cdot\gamma B_i(s,t)$$

and

$$\bar{T}_i(p_2,-q_1|p_1,-q_2)=-\bar{A}_i(s,t)+\tfrac{1}{2}i(q_1+q_2)\cdot\gamma\bar{B}_i(s,t),$$

where four-momenta are defined in Fig. 1, and i is $\frac{1}{2}$ or $-\frac{1}{2}$ so that T_i and $\bar{T}_i$ describe the following reactions:

$$T_{1/2}:\ K^+p\to K^+p,\qquad \bar{T}_{1/2}:\ K^-p\to K^-p,$$
$$T_{-1/2}:\ K^0p\to K^0p,\qquad \bar{T}_{-1/2}:\ \bar{K}^0p\to\bar{K}^0p.$$

Here we have assumed isospin conservation, and thus all other KN and $\bar{K}N$ elastic reactions may be written in terms of those listed above. If we define

$$\mathcal{A}_i^\pm(s,t)=A_i(s,t)\pm\bar{A}_i(s,t),$$
$$\mathcal{B}_i^\pm(s,t)=B_i(s,t)\pm\bar{B}_i(s,t),$$

these amplitudes behave under interchange of u and s just as the invariant amplitudes of πN scattering:

$$\mathcal{A}_i^\pm(s,u)\rightleftarrows\pm\mathcal{A}_i^\pm(u,s),$$
$$\mathcal{B}_i^\pm(s,u)\rightleftarrows\pm\mathcal{B}_i^\pm(u,s),$$

under $s\rightleftarrows u$. The PDM will be formulated in terms of the amplitudes $\mathcal{A}^\pm$, $\mathcal{B}^\pm$. The fixed-t dispersion relations, which are required in the generalized form of (2.3), can be written for $\mathcal{A}_i(s,t)$ and $\mathcal{B}_i(s,t)$ just as in the πN case. The important thing to show here is the cancellation of (1) the low-energy contribution to the low-partial-wave amplitudes, expressed in terms of the Froissart-Gribov formula with cutoff N, and (2) the left-hand-cut contribution to the partial-wave dispersion relation evaluated by inserting only low-mass exchanges.

For example, consider the amplitude $\mathcal{B}_i^-(s,t)$.[6]

(1) [Low-energy contribution to the low-partial-wave expansion for $\mathcal{B}_i^-(s,t)$]

$$=8\pi\sum_{\text{odd }J}^{J_0}\frac{(J+\frac{1}{2})(pq)^{J-1}}{[J(J+1)]^{1/2}}\times f_{i-}^{-J,N}(t)P_J'(z_t),\quad(3.1)$$

where $z_t=(p^2+q^2+s)/2pq$ and $f_{i\pm}^{\pm J}(t)$ is the Jth partial-wave amplitude. The lower $\pm$ signs stands for

[6] These amplitudes, their isospin properties, and the dispersion relations are discussed in detail by D. Amati and B. Vitale, Nuovo Cimento 17, 190 (1957). For t-channel partial-wave expansion see W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1603 (1960).

the helicity

$$\begin{Bmatrix}0\\1\end{Bmatrix}$$

of the $N\bar{N}$ state. The upper $\pm$ sign corresponds to the $\pm$ sign of $\mathcal{B}_i^{\pm}$ for which the expansion is made. The N indicates that we calculate the expression using the Froissart-Gribov formula and we cut off the integral at N.

(2) (Left-hand-cut contribution to the partial-wave

$$\text{dispersion relation})=8\sum_{\text{odd }J}^{J_0}\frac{(J+\frac{1}{2})}{[J(J+1)]^{1/2}}\times(pq)^{J-1}P_J'(z_t)\int_{-\infty}^{t_0}\frac{\operatorname{Im}f_{i-}^{-J}(t)}{t'-t}dt'. \quad (3.2)$$

The cancellation can be seen to occur exactly in the same way as shown in Sec. II. Here we concern ourselves with the possible polynomial that must be added. Take J_0 to be 1. [Note that then our result for $\mathcal{B}_i^-(s,t)$ will be valid for $t<t'$, where $\alpha(t')=3$.] Then,

$$\operatorname{Im}f_-^{-1}(t)=\frac{\sqrt{2}}{24\pi}\bigg[\int_{(M+m)^2}^{\mathcal{L}(t)}\frac{ds'}{2pq}\operatorname{Im}\mathcal{B}^-(t,s')+u\text{ integral}+\int_{(M+m)^2}^{\mathcal{L}(t)}\frac{ds'}{2pq}\operatorname{Im}\mathcal{B}^-(t,s')\times P_2\left(\frac{s'+p^2+q^2}{2pq}\right)+u\text{ integral}\bigg], \quad (3.3)$$

where $\mathcal{L}(t)=-(\frac{1}{2}t-M^2-m^2+2pq)$. The s- and u-channel contributions to the left-hand cut may be obtained by expanding $\mathcal{B}^-(u,t)$ in terms of s- and u-channel partial waves:

$$\mathcal{B}^{\pm}(u,t)=8\pi W\bigg(\frac{1}{(W+M)^2-m^2}\sum_{l=0}^{\infty}[g_{l+}^{\pm}(u)P_{l+1}'(\cos\theta_u)-g_{l-}^{\pm}(u)P_{l-1}'(\cos\theta_u)]+\frac{1}{(W-M)^2-m^2}\times\sum_{l=0}^{\infty}[g_{l-}^{\pm}(u)-g_{l+}^{\pm}(u)]P_l'(\cos\theta_u)\bigg), \quad (3.4)$$

where $W=\sqrt{u}$ and $g_{l\pm}^{\pm}(u)$ is a partial-wave amplitude. The upper sign corresponds to that of $\mathcal{B}^{\pm}(u,t)$. The lower $\pm$ sign corresponds to the helicity

$$\begin{Bmatrix}+\frac{1}{2}\\-\frac{1}{2}\end{Bmatrix}$$

of the $\bar{K}N$ state.

TABLE I. The asymptotic contributions of Λ and $Y_0^*(1405)$ to the left-hand cut of the partial-wave dispersion relations (as $t\to-\infty$).

$\operatorname{Im}f_-^{-1}(t)\to\text{const}\times t^{-1}$
$\operatorname{Im}f_+^{+0}/p^2\to\text{const}\times t^{-1}$
$\operatorname{Im}[f_+^{-1}(t)-(m/\sqrt{2})f_-^{-1}(t)]/p^2\to\text{const}\times t^{-2}$

We are concerned with the behavior of $\operatorname{Im}f_-^{-1}(t)$ when u-channel resonances are put into (3.3). For example, the $Y_0^*(1405)$, S_{01}, and the $Y_0^*(1520)$, D_{03} resonances give the contribution in the limit $t\to-\infty$

$$\begin{aligned}&[\operatorname{Im}f_-^{-1}(t)]_{Y_0^*(1405)}\to\eta t^{-1}\theta(\mathcal{L}(t)-(1405\text{ MeV})^2),\\&[\operatorname{Im}f_-^{-1}(t)]_{Y_0^*(1520)}\to\eta'\theta(\mathcal{L}(t)-(1520\text{ MeV})^2),\end{aligned} \quad (3.5)$$

where η and η' are constants. By unitarity, we have

$$|f_-^{-1}(t)|\leqslant ct^{-1}\quad\text{as } t\to\infty,$$

where c is some constant. Thus by the assumption of analyticity, the contribution of $Y_0^*(1520)$ and other high-spin resonances will cancel in such a way that the unitarity bound is preserved. Thus we neglect contributions from resonances above 1500 MeV in (3.3). The approximation is very good near the $\bar{K}N$ threshold, since the $Y_0^*(1405)$ contribution is an order of magnitude larger than those from high-spin resonances. At KN threshold, the Λ contribution dominates $Y_0^*(1405)$ and all other higher resonances.[7]

When (3.1) and (3.2) were calculated with $N<(1500\text{ MeV})^2$, (3.1)=(3.2), and we have

$$\mathcal{B}_i^-(s,t)=\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\operatorname{Im}\mathcal{B}_i^-(s',t)}{s'-s}ds'+\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\operatorname{Im}\mathcal{B}_i^-(u',t)}{u'-u}du'+\frac{12}{\sqrt{2}}\int_{4\mu^2}^{\infty}\frac{\operatorname{Im}f_{i-}^{-}(t')}{t'-t}dt'. \quad (3.6)$$

In a manner similar to that described for $\mathcal{B}_i^-(s,t)$, we can obtain expressions for $\mathcal{A}_i^{\pm}(s,t)$ and $\mathcal{B}_i^+(s,t)$. From Table I, we see that the contributions of Λ and $Y_0^*(1405)$ give convergent left-hand-cut integrals to the partial-wave dispersion relations in all cases. The unitarity bounds (see Table II) imply that the right-

TABLE II. Unitarity bounds on the partial-wave amplitudes (as $t\to\infty$).

$\lvert f_-^{-1}(t)\rvert<\text{const}\times t^{-1}$
$\lvert f_+^{+0}(t)/p^2\rvert<\text{const}\times t^{-1/2}$
$\lvert[f_+^{-1}(t)-(m/\sqrt{2})f_-^{-1}(t)]/p^2\rvert<\text{const}\times t^{-3/2}$

[7] Kim's results indicate that both $Y_1^*(1385)$ and Σ couples very weakly to the $\bar{K}N$ isospin-one channel. So, to this approximation we do not consider $I=1$ $\bar{K}N$ resonances.

hand-cut integral converges without a subtraction. Then

$$\mathcal{B}_i^+(s,t)=\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{B}_i^+(s',t)}{s'-s}ds'+\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{B}_i^+(u',t)}{u'-u}du', \quad (3.7)$$

$$\mathcal{A}_i^+(s,t)=\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{A}_i^+(s',t)}{s'-s}ds'+\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{A}_i^+(u',t)}{u'+u}du'+4\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}f_{+i}^{+0}(t')}{p'^2(t'-t)}dt', \quad (3.8)$$

$$\mathcal{A}_i^-(s,t)=\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{A}_i'(s',t)}{s'-s}ds'+\frac{1}{\pi}\int_{(M+m)^2}^{N}\frac{\mathrm{Im}\mathcal{A}_i^-(u',t)}{u'-u}du'-12pq\cos\theta_t \times\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}f_+^{-1}(t')-(m/\sqrt{2})\,\mathrm{Im}f_-^{-1}(t')}{p'^2(t'-t)}dt'. \quad (3.9)$$

In (3.9), we have written a dispersion relation for the combination

$$[f_+^-(t)-(m/\sqrt{2})f_1^{-1}(t)]/p^2$$

rather than for the individual amplitudes. No pole is introduced at $p^2=0$.

The integral

$$\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}f_+^{+0}(t')}{p'^2(t'-t)}dt',$$

which appears in (3.8), may not be approximated by low-energy contributions alone since the unitarity bound, given in Table II, does not imply rapid convergence. In such a case, we choose to make one subtraction in the partial-wave dispersion relation.

We now discuss the various integrals that appear in (3.6) through (3.9). The u channel is highly inelastic, as we have previously seen. The unphysical region from the $\pi\Lambda$ threshold to the $\bar{K}N$ threshold has been studied by Kim.[1] The results indicate that it is reasonable to approximate the contribution to the cut by the Y_0^* resonance. Furthermore, the Y_1^* resonance does not contribute appreciably to $\bar{K}N$ scattering. Also, the Λ and Σ poles contribute with

$$g^2{}_{p\Lambda K^-}/4\pi=16\pm2.5\,,\quad g^2{}_{p\Sigma^0K^-}/4\pi\approx0\,.$$

No s-channel resonances are included, as no positive-strangeness ones have been firmly established to date. If one has been missed, it must couple very weakly to the KN channel. The total cross sections for KN scattering are constant in energy at about 20 mb (up to 20-BeV laboratory kinetic energy of the kaon). The background contribution in the u channel, which is about 50 mb within the energy range of our consideration, was neglected in the above narrow-resonance approximation. Therefore, it is consistent to drop the s-channel integral completely.

Consider the t-channel contribution. From (3.6) through (3.9), the $J=0$ and $J=1$ amplitudes must be considered. We first assume that the s-wave $K\bar{K}$ contribution is negligible compared to the $J=1$ contribution. Then, realizing that the agreement of the model with experiment still has room for improvement, we consider the contribution of the σ, an s-wave $\pi\pi$ resonance whose existence is not yet firmly established. We will show that in some phase shifts a σ contribution can improve the agreement. Also, the σ does not require any new parameter for the calculation of the scattering length. The ρ contribution is[8,9]

$$[\mathcal{A}_1^-(t,s)]_\rho=-6\pi\gamma_\rho(\mu_\rho(s-u)/M(m_\rho^2-t))\,,$$
$$[\mathcal{B}_1^-(t,s)]_\rho=12\pi(1+2\mu_\rho)\gamma_\rho/(m_\rho^2-t)\,.$$

The ρ does not contribute to $\mathcal{A}_1^+(t,s)$ and $\mathcal{B}_1^+(t,s)$, because of symmetry considerations, and ω and ϕ contribute to only the $\mathcal{B}_0^-(t,s)$ amplitude

$$\frac{12\pi\gamma_\omega'}{m_\omega^2-t}+\frac{12\pi\gamma_\phi}{m_\phi^2-t}=[\mathcal{B}_0^-(t,s)]_{\omega,\phi}\,.$$

In determining γ_ω' and γ_ϕ, we must know how ω and ϕ couple to the $N\bar{N}$ state. Various theoretical predictions from dispersion relations do not agree well with each other.[10–13] There is no experimental measurement to date. In our problem, therefore, we talk about the effective contribution of the ω and the ϕ. Since the ω and ϕ contributions always appear in the same linear combination, they can be approximated by

$$\frac{12\pi\gamma_\omega'}{m_\omega^2-t}+\frac{12\pi\gamma_\phi}{m_\phi^2-t}\approx\frac{12\pi\gamma_\omega}{\frac{1}{4}(m_\omega+m_\phi)^2-t},$$

for example, in the $\mathcal{B}^-(t,s)$ amplitude. Since γ_ω' and γ_ϕ are not known and must be treated as parameters, this approximation will reduce the number of parameters from two to one. This approximation is reasonable, far away from the t-channel poles. We evaluate scattering lengths at $t=0$ and phase shifts at $t\leqslant0$. The σ

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[8] The coupling constants γ_ρ and μ_ρ are those defined in Ref. 2. $\gamma_\rho=1.53$ and $\mu_\rho=1.85$, taking $\Gamma_\rho=120$ MeV and assuming Sakurai's universality [Ann. Phys. (N. Y.) **11**, 1 (1960)].

[9] Subscripts on $A_I(t,s)$ denote isospin in the t channel. It is related to $\mathcal{A}_i(s,t)$ by Clebsch-Gordan coefficients and a crossing matrix.

[10] D. Y. Wong, Rev. Mod. Phys. **39**, 622 (1967).

[11] D. Y. Wong, Phys. Rev. **138B**, 156 (1965).

[12] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

[13] G. Kopp and G. Kramer, Phys. Letters **19**, 593 (1965).

resonance contributes only to the $\mathcal{A}^+(t,s)$ amplitude:

$$[\mathcal{A}_0^+(t,s)]_\sigma = 12\pi\gamma_\sigma/(m_\sigma^2-t).$$

Since we evaluate the expression far away from $t=m_\sigma^2$, we may approximate the contribution as that of a narrow resonance.

IV. CALCULATION OF S-WAVE SCATTERING LENGTH

In this section, the scattering length for KN and real part of the $\bar{K}N$ scattering length are calculated using the $\mathcal{A}_i^\pm(s,t)$ and $\mathcal{B}_i^\pm(s,t)$ amplitudes discussed in the previous section. The relation between the s-channel s-wave scattering length and the T matrix is given by the well-known formula

$$T_I'(s,t=0)=\left[A_I(s,t=0)+\frac{s-u}{4M}B_I(s,t=0)\right]_{s=(M+m)^2} =4\pi\left(1+\frac{m}{M}\right)a_I^{KN}.$$

Also, it is trivial to write this expression in terms of $\mathcal{A}_i^\pm(u,t=0)$ and $\mathcal{B}_i^\pm(u,t=0)$. Using the $s\to u$ and $t\to u$ crossing matrices

$$C^{s\to u}=\frac{1}{2}\begin{pmatrix}-1 & 3\\ 1 & 1\end{pmatrix},\quad C^{t\to u}=\frac{1}{2}\begin{pmatrix}-1 & 3\\ -1 & -1\end{pmatrix},\quad C^{t\to s}=\frac{1}{2}\begin{pmatrix}-1 & -3\\ -1 & 1\end{pmatrix}, \tag{4.1}$$

and the results (3.6)–(3.9) of the previous section, we obtain the following for physical values of u:

$$T_0'(u,t=0)=\frac{1}{\pi}\int_{(M_\Lambda+\mu)^2}^{N}\frac{\mathrm{Im}T_0'(u,t=0)}{u'-u} -\frac{3\pi\gamma_\omega^2(u-s)}{2Mm_\omega'^2}-\frac{9\pi\gamma_\rho\mu_\rho(u-s)}{Mm_\rho^2} +\frac{9\pi\gamma_\rho(1+2\mu_\rho)(u-s)}{2Mm_\rho^2}-\frac{6\pi\gamma_\sigma}{m_\sigma^2}. \tag{4.2}$$

The integral may be written in terms of the Λ pole and the Y_0^* contribution. Evaluating the expression at $u=(M+m)^2$, $t=0$, we obtain

$$T_0'(u=(M+m)^2,t=0)=2\frac{M_\Lambda-M-m}{M_\Lambda^2-(M+m)^2}g_{\Lambda N\bar{K}}^2 +\frac{1}{\pi}\int_{(M_\Lambda+\mu)^2}^{(M+m)^2}\frac{\mathrm{Im}T_0(u')}{u'-(M+m)^2}du'-\frac{6\pi m\gamma_\omega}{m_\omega'^2} +\frac{18m\gamma_\rho}{m_\rho^2}-\frac{6\pi\gamma_\sigma}{m_\sigma^2}. \tag{4.3}$$

Similarly, we obtain

$$T_1(u=(M+m)^2,t=0)=-\frac{6\pi m\gamma_\omega}{m_\omega'^2} -\frac{6\pi m\gamma_\rho}{m_\rho^2}-\frac{6\pi\gamma_\sigma}{m_\sigma^2}, \tag{4.4}$$

$$T_0'(s=(M+m)^2,t=0)=-\frac{M_\Lambda-M-m}{M_\Lambda^2-(M+m)^2}g_{\Lambda\rho\bar{K}}^2 -\frac{1}{2\pi}\int_{(M_\Lambda+\mu)^2}^{(M+m)^2}\frac{\mathrm{Im}T_0(u')}{u'-(M-m)^2}du'-\frac{6\pi m\gamma_\omega}{m_\omega'^2} -\frac{18\pi m\gamma_\rho}{m_\rho^2}-\frac{6\pi\gamma_\sigma}{m_\sigma^2}, \tag{4.5}$$

$$T_1'(s=(M+m)^2,t=0)=\frac{M_\Lambda-M-m}{M_\Lambda^2-(M-m)^2}g_{\Lambda\rho\bar{K}}^2 +\frac{1}{2\pi}\int_{(M_\Lambda+\mu)^2}^{(M+m)^2}\frac{\mathrm{Im}T_0(u')}{u'-(M-m)^2}du'-\frac{6\pi m\gamma_\omega}{m_\omega'^2} +\frac{6\pi m\gamma_\rho}{m_\rho^2}-\frac{6\pi\gamma_\sigma}{m_\sigma^2}. \tag{4.6}$$

In these equations, the $Y_0^*(1405)$ contribution is left in an integral form. We will use the narrow-resonance form for all except (4.3), which we evaluate at the $\bar{K}N$ threshold, 1440 MeV. At the KN threshold, $Y_0^*(1405)$ is reasonably far away, so it can be treated as a narrow resonance. Using Kim's result[1] for $\mathrm{Im}T_{Y_0^*}$,

$$\mathrm{Re}T_{Y_0^*}(u=(m+M)^2,t=0) =\frac{1}{\pi}\int_{(M_\Lambda+\mu)^2}^{(M+m)^2}\frac{\mathrm{Im}T_{Y_0^*}(u')}{u'-(M+m)^2}du'=-(60\pm6)F. \tag{4.7}$$

The error comes from the background contribution which we neglected. We evaluate the $Y_0^*(1405)$ coupling constant using (4.7). Since the $Y_0^*(1405)$ is an S_{01} state,

$$g_{Y_0^*}^2/4\pi=2g_{Y_0^*}'^2/[(M_{Y_0^*}+M)^2-m^2]=0.6.$$

Thus we can write the Y_0^* contribution to the amplitudes $A_0(u,t)$ and $B_0(u,t)$ as

$$[A_0(u,t)]_{Y_0^*}=g_{Y_0^*}^2(M_{Y_0^*}+M)/(M_{Y_0^*}^2-u),$$
$$[B_0(u,t)]_{Y_0^*}=g_{Y_0^*}^2/(M_{Y_0^*}^2-u).$$

Then in view of (4.1), (4.3)–(4.6) become numerically, in units of fermis,

$$\begin{aligned}
&(a)\quad 4\pi(1+m/M)a_0^{KN}=12.2-1.7-11.4+C_0,\\
&(b)\quad 4\pi(1+m/M)a_1^{KN}=-12.2+1.7+3.8+C_0,\\
&(c)\quad 4\pi(1+m/M)\,\mathrm{Re}a_0^{\bar{K}N}=31.6-60+11.4+C_0,\\
&(d)\quad 4\pi(1+m/M)\,\mathrm{Re}a_1^{\bar{K}N}=-3.8+C_0,
\end{aligned} \tag{4.8}$$

where

$$C_0=-\left(\frac{3\pi m\gamma_\omega}{m_\omega'^2}+\frac{3\pi\gamma_\sigma}{m_\sigma^2}\right).$$

The theoretical error in each term is $\approx 10\%$. Using an experimental value[7]

$$4\pi(1+m/M)\,\mathrm{Re}a_1^{\bar{K}N}=-2.5 \quad \text{for (4.8(d))},$$
$$C_0=1.3\pm0.3.$$

We now use this value to compare the other three equations with experiment.[1,14] The results are summarized in Table III. The large error in the calculation of a_0^{KN} is due to cancellation of various quantities in the first equation of (4.8).

The reason for good agreement in the KN s-wave scattering lengths may be understood from (4.3)–(4.6) or more easily from (4.8). We have

$$T_1^{\bar{K}N}(u=(M+m)^2,t=0)=\tfrac{1}{2}[T_0^{KN}(s=(M+m)^2,t=0)+T_1^{KN}(s=(M+m)^2,t=0)]. \quad (4.9)$$

In terms of scattering lengths,

$$a_1^{\bar{K}N}=\tfrac{1}{2}(a_0^{KN}+a_1^{KN}).$$

Experimentally,

left-hand side of (4.9)
$=[(-0.13\pm0.05)+i(0.51\pm0.03)]$ F,

right-hand side of (4.9) $=(-0.13\pm0.05)$ F.

We mentioned above that the Σ and the $Y_1^*(1398)$ do not couple appreciably to $\bar{K}N$ scattering. Thus the left-hand side of (4.9) is given by the t-channel contribution only. The right-hand side is also a linear combination which is given just by the t-channel contribution. In such a case, we have

$$T_1^{\bar{K}N}(u,s)=\tfrac{1}{2}[T_0^{KN}(s,u)+T_1^{KN}(s,u)].$$

The PDM is not applicable to the imaginary part of the amplitudes. In fact, all PDM amplitudes are real, since the narrow-resonance approximation is used to evaluate the integrals. Thus if we take (4.9) to be the relationship between real parts for the scattering amplitudes, the agreement is evidence that the Σ and $Y_1^*(1385)$ do not couple appreciably.

TABLE III. PDM predictions of s-wave scattering length and their experimental values. KN scattering lengths were obtained from Goldhaber *et al.* (Ref. 14), and $\bar{K}N$ scattering lengths were obtained from Kim (Ref. 1).

	PDM prediction	Experimental value
a_0^{KN}	0.02 ± 0.1	0.03 ± 0.03
a_1^{KN}	-0.28 ± 0.1	-0.29 ± 0.02
$\mathrm{Re}a_0^{\bar{K}N}$	-0.85 ± 0.5	-1.65 ± 0.04

[14] G. Goldhaber *et al.*, Phys. Rev. **134**, B1111 (1964); in Lawrence Radiation Laboratory Report No. UCRL-18322, 1968 (unpublished).

V. CALCULATION OF KN PHASE SHIFTS

In this section, phase shifts for KN scattering are calculated using the PDM. The $\bar{K}N$ channel is inelastic, and we cannot calculate the imaginary part of the scattering lengths. As mentioned in Sec. IV, a simple interpretation of the imaginary part of the $I=1$ $\bar{K}N$ channel is elusive. Thus, we discuss only KN-channel phase shifts. Experimentally, the KN-channel inelasticity is zero up to 0.4-BeV/c c.m. kaon momentum.[15] We have calculated the phase shifts up to this value. For some phase shifts, however, we restricted the calculation to the region below 0.3-BeV/c c.m. momentum because, as will be discussed below, if the value of the phase shift becomes too large, the approximation

$$e^{i\delta}\sin\delta\approx\delta$$

will not be satisfied.

The starting point of the calculation is the relation between $f_l^{\pm}(s)$ (the s-channel partial-wave amplitudes for $J=1\pm\frac{1}{2}$) and the invariant amplitudes A and B:

$$f_l^{\pm}(s)=\frac{(W+M)^2-m^2}{16\pi W^2}[A_l+(W-M)B_l]+\frac{(W-M)^2-m^2}{16\pi W^2}[-A_{l\pm1}+(W+M)B_{l\pm1}], \quad (5.1)$$

where

$$A_l=\frac{1}{2}\int_{-1}^{1}A(z)P_l(z)dz,\quad B_l=\frac{1}{2}\int_{-1}^{1}B(z)P_l(z)dz. \quad (5.2)$$

The MacDowell symmetry,[16] used later, is $f_l^+(W)=-f_{l+1}^-(-W)$. Using this symmetry, we can calculate P_{11} from S_{11} by interchanging W and $-W$. Also, we will calculate P_{01} and D_{03} from S_{01} and P_{03}, respectively. Since

$$qf_l=e^{i\delta_l}\sin\delta_l$$

in the region where δ_l is small, we have

$$qf_l\approx\delta.$$

This approximation is good for $\delta_l\lesssim30$. Since we are calculating the KN scattering amplitude using the narrow-resonance approximation, we expect our calculation to be accurate in the region where the imaginary parts of the amplitudes are small. Thus we restrict ourselves to the region where $\delta_l\lesssim30$. Since there are no resonances in the low-energy part of the KN channel, there are regions in which our approximation is meaningful.

Consider the amplitudes for the $I=1$ KN channel. (The amplitude for the $I=0$ KN channel will be ob-

[15] A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Rev. **165**, 1770 (1968).

[16] S. W. McDowell, Phys. Rev. **116**, 774 (1959).

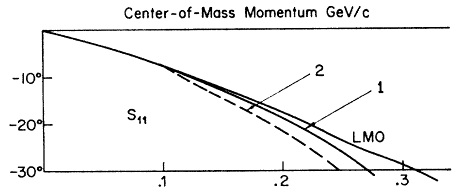

FIG. 2. S_{11} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Lea *et al.*

tained from the final result by changing the coefficients according to the isospin-crossing matrix.)[17]

$$A_1^{KN}(u,t)=\frac{(M-M_\Lambda)}{M_\Lambda^2-u}g_{\Lambda p\bar K}^2+\frac{(M_{Y_0^*}+M)}{M_{Y_0^*}^2-u}g_{Y_0^*}^2 -\frac{3\pi\gamma_\rho M_\rho(s-u)}{M(m_\rho^2-t)}-\frac{6\pi\gamma_\sigma}{m_\rho^2-t}, \tag{5.3}$$

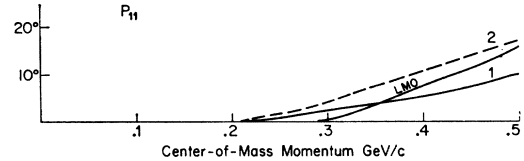

FIG. 3. P_{11} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Lea *et al.*

$$B_1^{KN}(u,t)=\frac{1}{M_\Lambda^2-u}g_{\Lambda p\bar K}^2+\frac{g_{Y_0^*}^2}{M_{Y_0^*}^2-u} -\frac{6\pi\gamma_\rho(1+2\mu_\rho)}{m_\rho^2-t}-\frac{6\pi\gamma_\omega}{m_\omega^2-t}. \tag{5.4}$$

We let

$$z_\rho=1+\frac{m_\rho^2}{2p^2},\quad z_\Lambda=-1-\frac{\Sigma-u-M_\Lambda^2}{2p^2}.$$

z_ω, z_σ, and $z_{Y_0^*}$ are defined like z_ρ and z_Λ, respectively. Then we obtain

$$\begin{aligned}
f_0^+(I=1)={}&\frac{(M+W)^2-m^2}{16\pi W^2}\Bigg[\alpha\frac{g_{\Lambda p\bar K}^2}{2p^2}(M_\Lambda-W)Q_0(z_\Lambda)+\alpha\frac{g_{Y_0^*}^2(M_{Y_0^*}+W)}{2p^2}Q_0(z_{Y_0^*})-\beta\frac{3\pi\gamma_\rho\mu_\rho}{M}\left(1+\frac{\Sigma-m_\rho^2-2s}{2p^2}Q_0(z_\rho)\right)\\
&+\beta(W-M)\frac{3\pi\gamma_\rho(1+2\mu_\rho)}{p^2}Q_0(z_\rho)-(W-M)\frac{3\pi\gamma_\omega}{p^2}Q_0(z_\omega)-\frac{3\pi\gamma_\sigma}{p^2}Q_0(z_\sigma)\Bigg]+\frac{(W-M)^2-m^2}{16\pi W^2}\Bigg[\alpha\frac{g_{\Lambda p\bar K}^2}{2p^2}(-M_\Lambda-W)Q_1(z_\Lambda)\\
&+\alpha\frac{g_{Y_0^*}^2}{2p^2}(W-M_{Y_0^*})Q_1(z_{Y_0^*})-\frac{3\pi\gamma_\rho\mu_\rho}{M}\beta\frac{\Sigma-m_\rho^2-2s}{2p^2}Q_1(z_\rho)-(W+M)\frac{3\pi\gamma_\omega}{p^2}Q_1(z_\omega)\\
&+\frac{(W+M)3\pi\beta\gamma_\rho(1+2\mu_\rho)}{p^2}Q_1(z_\rho)+\frac{3\pi\gamma_\sigma}{p^2}Q_1(z_\sigma)\Bigg],
\end{aligned}\tag{5.5}$$

where $\alpha=\beta=1$. We obtain $f_0^+(I=0)$ by setting $\alpha=-1$ and $\beta=-3$.

The phase shifts may be obtained by evaluating (5.5). The couplings γ_ω and γ_σ are related through the $\bar KN$ scattering length.

$$-\left(\frac{6\pi m\gamma_\omega}{m_\omega'^2}+\frac{6\pi\gamma_\sigma}{m_\sigma^2}\right)=C_0=1.3\pm0.3\text{ F}.$$

We first assumed that the σ does not contribute to $\bar KK\to N\bar N$. The partial waves S_{11}, P_{11}, S_{01}, and P_{01} are plotted in Figs. 2–5, respectively.

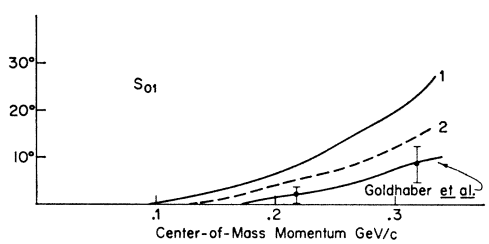

FIG. 4. S_{01} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Goldhaber *et al.*

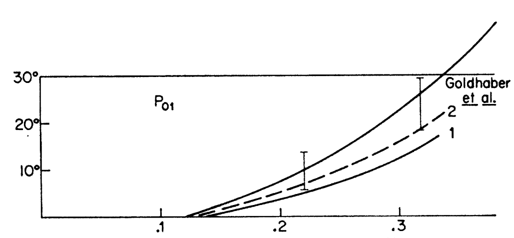

FIG. 5. P_{01} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Goldhaber *et al.*

[17] Results are compared with experimental results of G. Goldhaber *et al.*, Phys. Rev. **134**, B1111 (1964), for $I=0$ and A. Lea *et al.*, (LMO) (Ref. 15) for $I=1$.

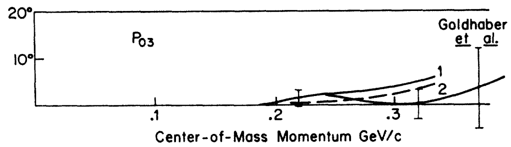

FIG. 6. P_{03} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Goldhaber *et al.* (see Ref. 14).

The effect of the σ on the high-energy behavior is investigated by choosing various values of γ_ω and γ_σ such that (5.6) is satisfied. In Fig. 4, we have plotted the result, in which we let $\gamma_\omega=4$ F, so that the high-energy behavior of the phase shift S_{01} fits the experimental results. This has the following effects on the other waves:

(a) The agreement between theory and experiment is improved for P_{01} (Fig. 5).

(b) The agreement between theory and experiment for the S_{11} and P_{11} phase shifts is better when the σ does not contribute (Figs. 2 and 3, respectively).

The other phase shifts do not show any drastic dependence on the ω and σ couplings. Thus we were not able to determine from these considerations whether the σ should or should not exist. The result does not change when we vary the mass of the σ in the region 400–800 MeV.

Experimentally, the phase shifts are determined by fitting experimental differential cross sections. This procedure gives a unique solution for the s waves. Higher waves, however, have the Fermi-Yang ambiguity. Only the solution which is consistent with the polarization measurements[14] is shown in Figs. 2–7.

It is also of interest to calculate the P_{03} phase shift to make sure that the result is consistent with the particular solution chosen above. The formula for the P_{03} phase shift may be obtained by replacing Q_0 and Q_1 in (5.5) by Q_1 and Q_2, respectively. The D_{03} phase shift may be obtained from that of P_{03} by the MacDowell symmetry.

The calculated P_{03} phase shift is shown in Fig. 6. The D_{03} phase shift is shown in Fig. 7.

We see from Figs. 2–7 that the PDM explains the qualitative features of KN scattering up to about 0.3-GeV/c c.m. momentum. Since the phase shifts are not sensitive to the values of γ_ω and γ_σ (when these are allowed to vary over a reasonable range), we can say that the qualitative features are explained without any arbitrary parameters.

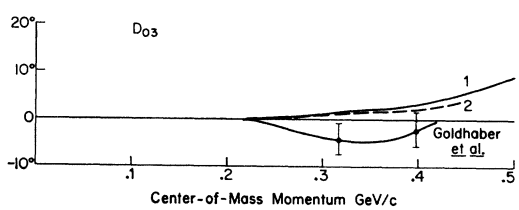

FIG. 7. D_{03} phase shift. (1) No σ contribution ($\gamma_\omega=-1.3$ F); (2) σ contribution included ($\gamma_\omega=+4$ F). The calculated phase shifts are compared with the results of the phase-shift analysis of Goldhaber *et al.* (see Ref. 14).

VI. CONCLUSION

The PDM, by the nature of its approximations, gives real scattering amplitudes. For the channels with small inelasticity, it successfully predicts scattering lengths and low-energy phase shifts. For very inelastic channels, it predicts real parts for the scattering lengths.

In our particular case, the KN phase shifts S_{01}, P_{01}, P_{03}, D_{03}, S_{11}, and P_{11}[32], the KN s-wave scattering lengths, and the real part of the $\bar{K}N$ s-wave scattering lengths were calculated. Because of the lack of experimental data, the couplings of ω, ϕ, and σ in $K\bar{K}\to N\bar{N}$ had to be treated as parameters. In the evaluation of scattering lengths, we have one arbitrary parameter. Choosing the parameter so that $\mathrm{Re}a_1^{\bar{K}N}$ has the value given by experiment, we calculated $\mathrm{Re}a_0^{\bar{K}N}$, a_1^{KN}, and a_0^{KN} (Table III).

The phase shifts up to 0.3-GeV/c c.m. momentum were calculated under various assumptions on the contribution of a σ meson. The results, shown in Figs. 2–7, are not very sensitive to the strength of the σ coupling. Also the value for the σ coupling which fits one phase shift does not improve the fit to all the other phase shifts. Thus we conclude that the PDM, without further modification or additional experimental information, is not able to give a definite prediction on the coupling of the σ in $K\bar{K}\to N\bar{N}$. We point out, however, that the PDM did enable us to calculate the qualitative features of all the phase shifts considered. Furthermore, if we look at qualitative features only, which are not sensitive to σ, we have only one adjustable parameter.

In the process of deriving the PDM amplitudes, we saw that the sum rule

$$T_1^{\bar{K}N}(u,t)=\tfrac{1}{2}[T_0^{KN}(s,t)+T_1^{KN}(s,t)]$$

must be satisfied [in terms of the PDM, since all amplitudes are real, we really have $\mathrm{Re}T_1^{\bar{K}N}(u,t)$ on the left-hand side of the equation]. Besides the assumptions of the PDM, we used the fact that the isospin-one poles (Σ and Y_1^*) do not couple strongly to the $\bar{K}N$ channel.[18] The fact that the sum rule is well satisfied is evidence supporting Kim's conclusion that the Σ and Y_1^* couplings to the $\bar{K}N$ channel are small.

ACKNOWLEDGMENT

It is a pleasure to thank Dr. John H. Schwarz for suggesting the problem and his guidance throughout this work.

[18] This relation does not depend on the narrow-resonance approximation.