$$+\langle 0|A_\nu^{\beta}(0)|m\pi^\alpha\rangle\langle mp_2|V_\mu^{\text{em}}(0)|p_1\rangle[\delta^3(\mathbf{p}_n)/(yE'-\mu-p_n^0)]+\text{c.t.}\}$$

$$-\sum_l\{\langle 0|A_\nu^{\beta}(0)|lp_1\rangle\langle lp_2|V_\mu^{\text{em}}(0)|\pi^\alpha\rangle[\delta^3(\mathbf{p}_1+\mathbf{p}_n)/(yE'-p_1^0-p_n^0)]+\text{c.t.}$$

$$+\langle 0|A_\nu^{\beta}(0)|lp_1\pi^\alpha\rangle\langle lp_2|V_\mu^{\text{em}}(0)|0\rangle[\delta^3(\mathbf{p}_1+\mathbf{p}_n)/(yE'-p_1^0-\mu-p_n^0)]+\text{c.t.}\}). \quad \text{(B3)}$$

The intermediate states n, m, and l have again baryon numbers one, zero, and minus one, respectively. The corresponding equation for $U_\mu^{\text{em},\beta\alpha}$ can be obtained from Eq. (B3) by replacing $A_\nu^\beta(0)$ by $-iD^\beta(0)$.

We remark that if $|n\rangle$ is a one-nucleon state, then the first term has a singularity at $y=0$, while its crossed term has a double pole at $y=\mu/E'=y_0$. Similarly, if $|n\rangle$ is a one-nucleon state, the second term has a double pole at $y=\mu/E'=y_0$, while its crossed term has a singularity at $y=0$. Finally, if $|m\rangle$ is a one-pion state, the fifth term has a single pole at $y=\mu/E'=y_0$.

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Note on Parallel Boson Trajectories*

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Direct empirical testing wlth boson masses is proposed for the popular hypothesis—obtained indirectly from virtual boson exchange—that all boson trajectories are parallel and that most are exchange-degenerate, also. It turns out that presently well-determined resonances are not quite sufficient to decide for or against the hypothesis, but that the next stage of measurement should be critical. In particular, four charged-meson states with $J^P=2^-$ should be sought in the mass region ~1.4–1.6 BeV, where experiments to date show a dearth of resonances. If the hypothesis of a single slope is correct, present data indicate the universal trajectory type to be $\frac{1}{2}(J+L)\approx u+\alpha(0)$, where u is in BeV^2, and J and L are the total and orbital angular momenta, respectively, on a baryon-antibaryon model. All trajectories can be made exchange degenerate except for the ^1L_L case. The intercepts $\alpha(0)$ do not generally assume half-integer values, but $\alpha_\pi+3\alpha_\eta\approx 4\alpha_\kappa$ for all orbital types, which suggests little over-all SU_3 octet-singlet mixing between trajectories.

EMPIRICAL boson analyses have so far[1,2] assumed exchange degeneracy throughout and have yielded some distinctly nonparallel Regge trajectories. Recent arguments suggest that all rising linear trajectories should be parallel.[3,4] We try to incorporate this requirement in a fit to present data, still maintaining the maximum degree of exchange degeneracy. The result is to encompass most current data in the simple formula given in the Abstract. Some points of doubt remain and will be discussed below; the main challenge is experimental.

The leading boson trajectory for $I=1$ is always taken[5] as the exchange-degenerate sequence with a slope of about 0.90–0.95 BeV^{-2}. There are other boson trajectories[1,2] with slopes in the range 1.0±0.1 BeV^{-2}. These slopes also characterize baryon trajectories. We therefore assume a universal slope

$$d\alpha/du=1\pm 0.1\ \text{BeV}^{-2}. \quad (1)$$

On a baryon-antibaryon model, the spectroscopic notation for the leading trajectory is $^3L_{L+1}$; the other normal trajectory is $^3L_{L-1}$ and presumably starts with $J^P=0^+$ for the $\delta(975)$. On this basis, Eq. (1) would yield higher members of this exchange-degenerate trajectory in the A_2, R, S, T, $\cdots$ regions. In fact, the A_2 shows a split with two peaks; the R_1 and R_2 or $\pi'(1640)$[6] form a double peak and the leading trajectory has $J^P=4^+$ at the S, so a second resonance must account for the 2π decay mode found in this mass region. It is an immediate step to combine both cases into a single degenerate normal trajectory

$$\tfrac{1}{2}(J+L)=u-0.2, \quad (2)$$

where u is in BeV^2, as is assumed throughout. Individual states fluctuate around Eq. (2) with 0.1 as the

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mean deviation. The abnormal triplet trajectory ${}^3L_L$ has its first member A_1 on Eq. (2) and will be assumed also degenerate; higher members could then be R_3 for ${}^3F_3$ and the ρ–ρ resonance at[7] 2.04 BeV as ${}^3G_4$. The first two members of the ${}^1L_L$ trajectory are the π and B mesons; to maintain Eq. (1), we must abandon exchange degeneracy for this specific case. This seems mainly to mean that 90° phase shifts in nucleon–nucleon scattering must be regarded as (weak) resonant states of a system with baryon number 2. The corresponding trajectories are

$$\tfrac{1}{2}(J+L)=u-0.25\pm0.25\,, \qquad (3)$$

with $\pm$ for even and odd L, respectively. The ${}^1F_3$ member of this sequence would be the coherently produced 5π peak[8] at 1.95 BeV. Within uncertainties, one may regard Eqs. (2) and (3) as identical except for the fluctuating term ±0.25.

Turning to the $I=\frac{1}{2}$ trajectories, we find the ${}^3S_1(890)$ and[9] ${}^3P_0(1150)$ K^* mesons not very degenerate, but we nonetheless take an average (mass)^2 value, $u\approx1.05$ BeV^2. The ${}^3P_2$ and ${}^3D_1$ masses are much closer, both appearing as unresolved components[10] of $K^*(1420)$. The corresponding trajectory is

$$\tfrac{1}{2}(J+L)=u-0.5\,. \qquad (4)$$

The ${}^3P_1$ member of this series is presumably the C meson at about 1.24 BeV, indicating that ${}^3L_L$ states also lie on Eq. (4). By analogy with Eq. (3) we write for ${}^1L_L$ states of $I=\frac{1}{2}$:

$$\tfrac{1}{2}(J+L)=u-0.5\pm0.25\,, \qquad (5)$$

which is about right for the K meson and yields a ${}^1P_1$ as K^* (1.33), where there is a prominent peak in the Q region.

Normal $I=0$ trajectories like the ω meson must follow Eq. (2) like their $I=1$ partners by the most basic arguments of exchange degeneracy. We assume the same for the corresponding ${}^3L_L$ $I=0$ trajectory. The ${}^1L_L$ trajectory in this system starts with the X^0 and must certainly be different, but nothing more is known.

The η-type $I=0$ meson shows good degeneracy between the ${}^3S_1$ (ϕ) and ${}^3P_0$ (S^*) states, and there is no evidence against 1^- admixture to the predominant 2^+ of the f'; thus,

$$\tfrac{1}{2}(J+L)=u-0.6\,. \qquad (6)$$

For ${}^3P_1$ the $D(1285)$ is in good agreement with Eq. (6). The corresponding ${}^1L_L$ trajectory is

$$\tfrac{1}{2}(J+L)=u-0.6\pm0.25\,, \qquad (7)$$

which is about right for the η, no higher members of the sequence being yet established.

If we write the residual terms in Eqs. (2), (4), and (6) as α_π, α_κ, and α_η, then the SU_3 relation is satisfied as given in the abstract: $-0.2+3(-0.6)\approx4(-0.5)$. This implies little SU_3 singlet-octet mixing among trajectories on the whole. Deviations of individual L, J masses from the average trajectories presumably require special explanations, the most immediate of which is slight curvature of the trajectories as $u\to0$. For example, among ${}^3S_1$ mesons the biggest deviation is exhibited by the $K^*(890)$, which according to Eq. (4) should have a mass closer to 1 BeV.

Several difficulties may be discerned. The principal trajectory Eq. (2) does not in any of its alternatives satisfy the condition[11] $J=\frac{1}{2}$ for $u\to0$. This condition is important to some arguments for parallelism. Again, slight trajectory curvature for small u might be invoked. A few extra bosons are known that do not fit into the scheme outlined: the $A_{1.5}$, E, and perhaps one in the R region.[6] Of course, daughter trajectories are to be expected, but it is not yet easy to see how to arrange them. The most crucial feature of the present assignment, however, is the location of $J^P=2^-$ states for $I=1$ and $I=\frac{1}{2}$. For $I=1$ the $G=-$ state is expected at about 1.41 BeV, the $G=+$ state at about 1.49 BeV; the two corresponding states for K^* at 1.50 and 1.58 BeV. Experimentally, no $I=1$ or $\frac{1}{2}$ resonances have been reported at all in this region, much less any with $J^P=2^-$. If the resonances are not evoked by concerted effort, the above scheme will need revision, probably in the direction of the quark or baryon-antibaryon models.[12]

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