Reggeized $U(6)\times U(6)\times O(3)$ and Absorptive Correction Cuts for $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ Charge-Exchange Reactions*

B. J. Hartley, R. W. Moore, and K. J. M. Moriarty†
Physics Department, Imperial College, London SW7, England
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Reggeized $U(6)\times U(6)\times O(3)$ with absorptive corrections is applied to some $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ charge-exchange processes. The differential cross sections for $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta^0n$ are used to fix the parameters of the ρ and A_2 trajectories, respectively. With these parameters, an absolute prediction is made for the process $K^-p\to\bar{K}^0n$. The agreement with experiment is good. The polarization parameter $P(t)$ and the Wolfenstein parameters $A(t)$ and $R(t)$ are also presented.

1. INTRODUCTION

Previous authors have shown that the differential cross-section data on high-energy two-body meson-baryon scattering in the peripheral region can be fitted by the exchange of a few Regge poles in the t channel. However, it is believed that the structure in the angular momentum plane is not limited simply to poles. In particular, there must also be cuts present. A simple way of introducing Regge cuts is by applying absorptive corrections to the pole amplitudes. We use Reggeized $U(6)\times U(6)\times O(3)$ to provide significant constraints among the Regge residues.

In Sec. 2 we present the general formalism of $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ reactions in terms of the t-channel M-functions and the s-channel helicity amplitudes and give an outline of the derivation of the Reggeized supermultiplet amplitudes. The model and its connection with ordinary Regge pole theory and the $U(6,6)$ absorptive peripheral model are presented in Sec. 3. In Sec. 4 we indicate our parametrization of the elastic scattering data and present a table of the parameters used. The nonlinear parametrization of the trajectory function is given in Sec. 5. We conclude in Sec. 6 with a brief discussion of the results of the model as applied to the reactions:

$$\pi^-p\to\pi^0n\,,\quad \pi^-p\to\eta^0n\,,\quad K^-p\to\bar{K}^0n\,.$$

2. FORMALISM

For reactions of the type $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ we can write the M-function[1] as

$$M=A+QB\,,$$

where A and B are the invariant amplitudes and Q is half the sum of the initial and final meson four-momenta. It is often more convenient to work in terms of the invariant amplitudes A' and B, where

$$A'=A+\frac{E_{\text{lab}}+t/4m_1}{1-t/4m_1^2}B\,,$$

where E_{lab} is the laboratory energy of the incoming meson, t is the four-momentum transfer, and m_1 is the mass of the target nucleon.

The independent s-channel helicity amplitudes are then given by

$$\phi_{++}=(1/4\pi w)\cos\tfrac{1}{2}\theta\,[m_1A+(E_1w-m_1^2)B]\,,$$
$$\phi_{+-}=(1/4\pi w)\sin\tfrac{1}{2}\theta\,[E_1A+m_1(w-E_1)B]\,,$$

where θ is the s-channel center-of-mass scattering angle and w is the center-of-mass energy.

The helicity-nonflip and helicity-flip amplitudes can be decomposed in the helicity representation of Jacob and Wick[2] to give

$$\phi_{++}(s,t)=\sum_J (2J+1)T_{++}{}^J(s)d_{\frac{1}{2}\frac{1}{2}}{}^J(\theta)\,,$$
$$\phi_{+-}(s,t)=\sum_J (2J+1)T_{+-}{}^J(s)d_{-\frac{1}{2}\frac{1}{2}}{}^J(\theta)\,,$$

where

$$d_{\frac{1}{2}\frac{1}{2}}{}^J(\theta)=[P_{l+1}(\cos\theta)+P_l(\cos\theta)]/[2(1+\cos\theta)]^{1/2}\,,$$
$$d_{-\frac{1}{2}\frac{1}{2}}{}^J(\theta)=[P_{l+1}(\cos\theta)-P_l(\cos\theta)]/[2(1-\cos\theta)]^{1/2}\,,$$

with $J=l+\frac{1}{2}$.

The application of absorptive corrections to the Regge pole amplitudes ϕ_{++} and ϕ_{+-} produces the modified helicity amplitudes ϕ'_{++} and ϕ'_{+-}. In terms of these modified helicity amplitudes, the differential cross section, the polarization parameter $P(t)$, and the Wolfenstein parameters[3] $A(t)$ and $R(t)$ can be written

$$d\sigma/dt=(\pi/q^2)(|\phi'_{++}|^2+|\phi'_{+-}|^2)\,,$$
$$P(t)=2\,\mathrm{Im}(\phi'_{++}\phi'^*_{+-})/(|\phi'_{++}|^2+|\phi'_{+-}|^2)\,,$$
$$A(t)=-2\,\mathrm{Re}[\phi'_{++}\phi'^*_{+-}]/(|\phi'_{++}|^2+|\phi'_{+-}|^2)\,,$$
$$R(t)=(|\phi'_{++}|^2-|\phi'_{+-}|^2)/(|\phi'_{++}|^2+|\phi'_{+-}|^2)\,,$$

where q is the center-of-mass three-momentum.

Following Ref. 4, we adopt the $U(6)\times U(6)\times O(3)$ classification scheme which is well satisfied by the known meson and baryon resonances.[5] For meson-

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† Science Research Council Research Fellow.
[1] See any standard textbook, e.g., R. J. Eden, *High Energy Collisions of Elementary Particles* (Cambridge University Press, New York, 1967).
[2] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[3] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).
[4] R. Delbourgo, A. Salam, and J. Strathdee, Phys. Rev. **179**, 1487 (1968); **172**, 1727 (1968); **187**, 1999 (1969).
[5] Q. Shafi, Nuovo Cimento **62**, 290 (1969); J. Sultoon, J. Phys. (to be published).

baryon scattering, which goes by the exchange of excitation number N, the relevant effective Lagrangians for the two $U(6)\times O(2)$ invariant 3-point couplings are

(a) meson-meson-meson supermultiplet vertex:

$$(6,\bar{6};0)_{\frac{1}{2}p+q'}-(6,\bar{6};0)_{\frac{1}{2}p-q'}-(6,\bar{6};N)_{-p},$$

$$\mathfrak{L}_{(a)}=\mu^{1-N}\Phi_A{}^B(\tfrac{1}{2}p+q')\Phi_C{}^D(\tfrac{1}{2}p-q')$$
$$\times\Big[h_0{}^{(-)}\delta_B{}^C\delta_D{}^A+h_0'{}^{(-)}\mu^{-2}q'_B{}^Aq'_D{}^C$$
$$+\mu h_1{}^{(+)}\Big(\delta_B{}^C\frac{\partial}{\partial q'_A{}^D}+\delta_D{}^A\frac{\partial}{\partial q'_C{}^B}\Big)$$
$$+\mu h_1{}^{(-)}\Big(\delta_B{}^C\frac{\partial}{\partial q'_A{}^D}-\delta_D{}^A\frac{\partial}{\partial q'_C{}^B}\Big)\Big]\Phi_{(N)}(-p,q');$$

(b) baryon-baryon-meson supermultiplet vertex:

$$(56,1;0)_{-\frac{1}{2}p+q}-(\bar{5}\bar{6},1;0)_{\frac{1}{2}p+q}-(6,\bar{6};N)_p,$$

$$\mathfrak{L}_{(b)}=m^{-N}\bar{u}^{(ACD)}(\tfrac{1}{2}p+q)u_{(BCD)}(-\tfrac{1}{2}p+q)$$
$$\times(g_0\delta_A{}^B+mg_1\partial/\partial q_B{}^A)\Phi_{(N)}(p,q),$$

where m and μ are the masses associated with the $(56,1;0)$ and $(6,\bar{6};0)$ multiplets, respectively. For $0^-\frac{1}{2}^+\rightarrow 0^-\frac{1}{2}^+$ charge-exchange scattering, the g_0 and h_0 couplings do not contribute.

The following decompositions for the $U(6,6)$ fields are used:

$$u_{(ABC)}(p)_{(\mu_1\cdots\mu_{2N})}$$
$$=(1/6\sqrt{2}m)\{[(\not{p}+m)\gamma_5C]_{\alpha\beta}\epsilon_{abd}N_{c\gamma}{}^d{}_{(\mu_1\cdots\mu_{2N})}$$
$$+\text{cyclic perm}\},$$

$$\Phi_A{}^B{}_{(\mu_1\cdots\mu_N)}(p)=(1/2\sqrt{2}\mu)\{(\not{p}+\mu)\gamma_5p_{(\mu_1\cdots\mu_N)}\}_A{}^B.$$

The covariant T matrix is

$$T_{MB}=\mathfrak{L}_{(a)}\mathfrak{L}_{(b)},$$

and the fully contracted propagator for the meson supermultiplet is

$$\Delta_{(N)}=(\Phi_{(N)}(p,q)\Phi_{(N)}(-p,q'))$$
$$=[1/(t-M^2)](\mathbf{q}\cdot\mathbf{q}')^{N+1},$$

where M is the mass associated with the exchange.

To Reggeize the amplitude, we extract phase factors $(1\pm e^{-i\pi N})$ from $h^{(\pm)}$ and make the replacement

$$N\rightarrow\alpha-1,$$
$$t-M^2\rightarrow\sin\pi(\alpha-1)=-\pi/[\Gamma(\alpha)\Gamma(1-\alpha)].$$

The Gell-Mann ghost-eliminating mechanism is introduced by dividing by $\Gamma(\alpha)$. In the high-energy limit, we obtain

$$T=(1+M/2\mu)(1+2m/M)$$
$$\times[-(\tfrac{2}{3}-M/2m)(s/2m\mu)(\bar{N}N)_F$$
$$-(s/2m\mu)(\bar{N}N)_D+\mu^{-1}(1-t/4m^2)(\bar{N}\mathbf{q}'N)_{D+2F/3}]$$
$$\times[\beta_-\Gamma(1-\alpha_-)(1-e^{-i\pi\alpha_-})(s/2m\mu)^{\alpha_--1}(PP)_F$$
$$+\beta_+\Gamma(1-\alpha_+)(1+e^{-i\pi\alpha_+})(s/2m\mu)^{\alpha_+-1}(PP)_D],$$

where the $+$ and $-$ refer to the even- and odd-signature trajectories respectively. "Gribov doubling" is used to remove the $M=\sqrt{t}$ kinematic singularity. Hence, we have

$$B=\mu^{-1}(1+m/\mu)(1-t/4m^2)g_{D+2F/3}$$
$$\times[\beta_-h_F\Gamma(1-\alpha_-)(1-e^{-i\pi\alpha_-})(s/2m\mu)^{\alpha_--1}$$
$$+\beta_+h_D\Gamma(1-\alpha_+)(1+e^{-i\pi\alpha_+})(s/2m\mu)^{\alpha_+-1}]$$

and

$$A'=(1+t/4m\mu)g_F[\beta_-h_F\Gamma(1-\alpha_-)(1-e^{-i\pi\alpha_-})(s/2m\mu)^{\alpha_-}$$
$$+\beta_+h_D\Gamma(1-\alpha_+)(1+e^{-i\pi\alpha_+})(s/2m\mu)^{\alpha_+}],$$

where the g's and h's are baryon-baryon-meson and meson-meson-meson couplings, respectively.

Pair-wise equal-mass kinematics are assumed in this derivation.

3. MODEL

In recent years the simple Regge pole approach to peripheral high-energy inelastic reactions has enjoyed some success.[6] However, there are some features which cannot be explained without the addition of *ad hoc* assumptions,[7] notably,

(1) In reactions where only one Regge pole can be exchanged, the polarization is predicted to be identically zero.[8] For $\pi^-p\rightarrow\pi^0n$, only one known pole can be exchanged, namely, the ρ. However, recent experimental data[9] at high energy show that there is a positive polarization for small momentum transfers.

(2) In reactions where the π can be exchanged, the data indicate a strong forward peak of width $\simeq m_\pi{}^2$. Previously, this has been explained by complicated conspiracy relations. However, recently it has been shown by Le Bellac[10] that conspiracies predict dips in the forward direction for reactions such as

$$\pi N\rightarrow\rho\Delta,\quad KN\rightarrow K^*\Delta,\quad \pi N\rightarrow f^0\Delta.$$

These dips are not observed experimentally.

The $U(6,6)$ peripheral absorption model has been successful in explaining two-body and pseudo-two-body production processes which go through 0^- exchange.[11]

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[6] See, e.g., review article by L. Bertocchi, in *Proceedings of the International Conference on Elementary Particles, Heidelberg, 1967*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 197.

[7] Some of the problems associated with the simple Regge pole approach are discussed by R. C. Arnold, Argonne National Laboratory, Report No. ANL/HEP 6804 (1968).

[8] A formulation of the Regge amplitudes in the Khuri-Kretzschmar framework leads to nonzero polarization for the exchange of a single Regge pole. See Meng Ta-Chung, H. G. Schlaile, and R. Strauss, Nucl. Phys. **B7**, 133 (1968).

[9] P. Bonamy, P. Boregeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri, and L. Vincelli, Phys. Letters **23**, 501 (1966).

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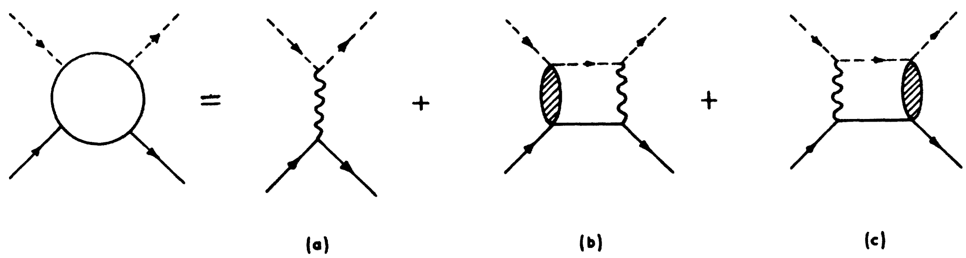

FIG. 1. Pole + cut diagram.

However, for nonzero spin exchange,[12] both the energy and momentum transfer dependence of the model are in disagreement with experiment. Now, the exchange of high spin is just the area in which simple Regge-pole approaches have been successful.[6] It is well known that for small momentum transfers an elementary and a Reggeized pion exchange give essentially the same results. Thus, a Reggized higher-symmetry scheme would give all the good results of the $U(6,6)$ peripheral absorption model of evasive 0^- exchange, but with improved results for higher-spin exchange. Such a scheme has been developed.[4]

By using $U(6,6)$ symmetry[13] in the ordinary absorption model, we were able to fix uniquely the Born diagrams involving the exchange of the **35** mesons for $\bar{35}\times 56$ or $\bar{56}\times 56$ scattering and **56** baryons for $\bar{35}\times 56$ scattering. Similarly, by using Reggeized supermultiplet theory we are able to fix most of the parameters for the Born diagrams for $\bar{35}\times 56$ and $\bar{56}\times 56$ scattering. A few parameters remain which cannot be determined by the theory, namely, the trajectory function $\alpha(t)$ and the residue $\beta(t)$ associated with each trajectory. The power of the Reggeized supermultiplet theory lies in the fact that once these parameters have been determined for, say, the ρ trajectory by a fit to the data for, say, $\pi^- p \to \pi^0 n$, the ρ trajectory is then fixed for all the above mentioned processes. However, the problems of the simple Regge-pole approach[7] remain.

TABLE I. Absorption coefficients.

Channel	p_{lab}	ν^{-1} (GeV^{-1})	c_1	c_2
$\pi^- p$	5.9	0.26	0.79	0.111
	9.8	0.26	0.74	0.087
	11.2	0.26	0.73	0.084
	13.3	0.26	0.72	0.079
	18.2	0.26	0.71	0.067
$K^- p$	5.0	0.26	0.74	
	7.1	0.26	0.70	
	9.5	0.26	0.67	
	12.3	0.26	0.65	

Now, it has been shown by a number of authors[7,14] that these difficulties can be overcome by absorptive corrections to the Reggeized Born diagram. In this paper we use a Reggeized supermultiplet theory[4] to reduce the number of free parameters, and absorptive corrections to avoid *ad hoc* additions to simple Regge pole theory.

We now describe in general the procedure for calculating Regge cuts which have been parametrized as absorptive corrections to the two-body scattering amplitude. We make a partial-wave analysis of the s-channel helicity amplitudes $\phi_{\lambda\mu}(s,t)$. If α and β denote the difference of helicities in the final and initial states, respectively, the partial-wave expansion is

$$\phi_{\lambda\mu}(s,t) = \sum_J (2J+1) T_{\lambda\mu}{}^J(s) d_{\alpha\beta}{}^J(\theta)$$

and the partial-wave amplitudes are given by

$$T_{\lambda\mu}{}^J(s) = \frac{1}{2}\int_{-1}^{+1} \phi_{\lambda\mu}(s,t) d_{\alpha\beta}{}^J(\theta) d(\cos\theta) .$$

The modified production amplitude is approximated by the traditional Watson formula[15]

$$T'_{\lambda\mu}{}^J = \tfrac{1}{2} \sum_{\lambda'} (S_{\lambda\lambda'}{}^{\text{el}J} T_{\lambda'\mu}{}^J + T_{\lambda\lambda'}{}^J S_{\lambda'\mu}{}^{\text{el}J}) ,$$

where $T'^J(s)$ is the pole partial-wave amplitude modified by absorptive corrections, $S^{\text{el}J}$ is the S-matrix

TABLE II. Parameters of the ρ and A_2 trajectories.

Trajectory	ρ	A_2
α_0	-0.871	-0.936
α_1	1.415	1.420
α_2 (GeV/c)$^{-2}$	0.632	0.607
β (GeV/c)$^{-1}$	1.953	3.300
No. of data points	86	50
χ^2	554.7	127.4

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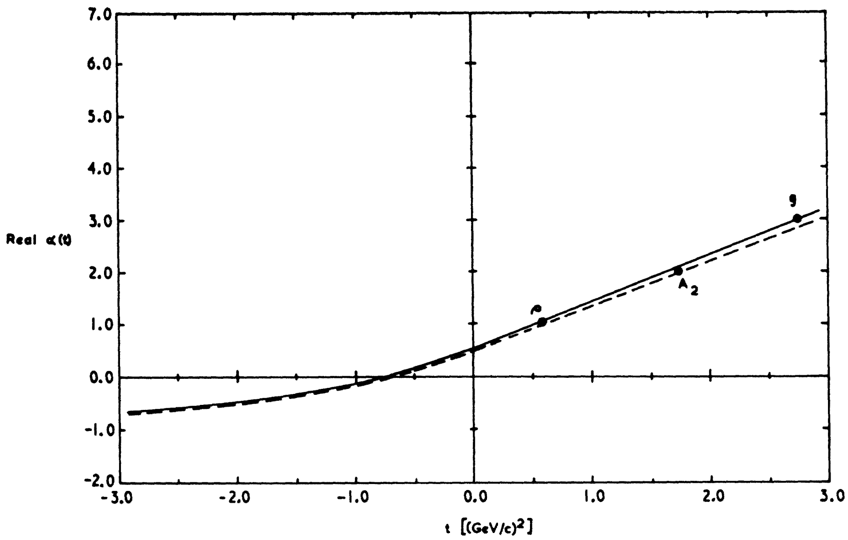

FIG. 2. Plot of $\alpha(t)$ against t for the ρ trajectory (———) and the A_2 trajectory (– – – –). Parameters from Table II.

element for elastic scattering, and T^J is the pole partial-wave amplitude. Writing

$$S^{\mathrm{el}J}=1+2i\rho T^{\mathrm{el}J},$$

where ρ is the phase-space factor, we obtain

$$T'_{\lambda\mu}{}^J(s)=T_{\lambda\mu}{}^J(s)+i\rho\sum_{\lambda'}\left[T_{\lambda\lambda'}{}^{\mathrm{el}J}(s)T_{\lambda'\mu}{}^J(s)+T_{\lambda\lambda'}(s)T_{\lambda'\mu}{}^{\mathrm{el}J}(s)\right].$$

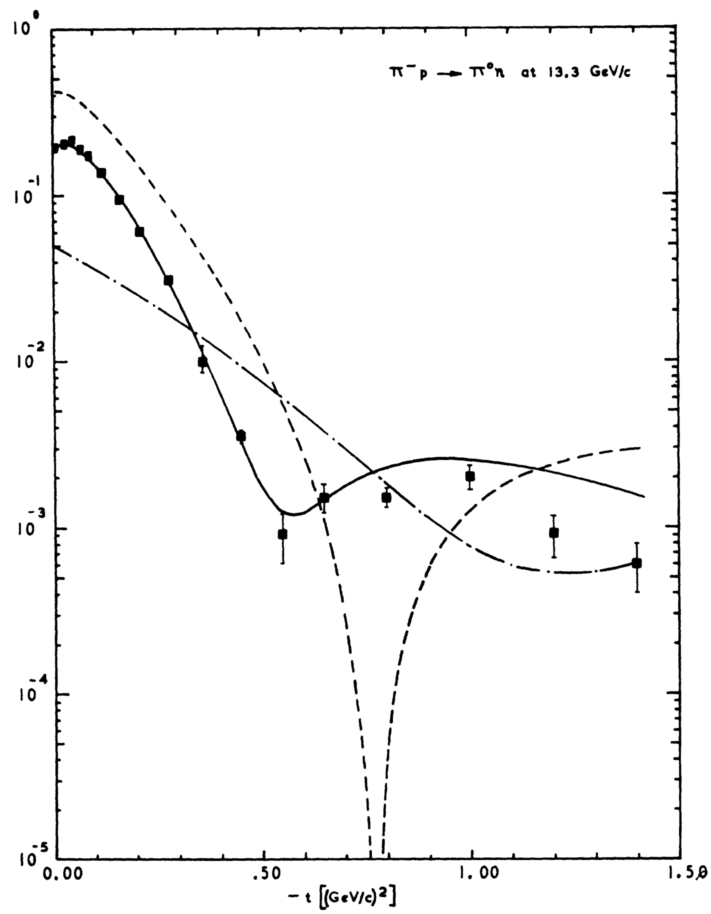

FIG. 3. Contributions from pole (– – – –), cut (—·—), and pole+cut (———) to the differential cross section for $\pi^- p \to \pi^0 n$. Data from A. V. Stirling, P. Sondregger, J. Kirz, P. Falk-Vairant, O. Guisain, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters **14**, 763 (1965); Phys. Letters **20**, 75 (1966); M. A. Wahlig and I. Mannelli, Phys. Rev. **168**, 155 (1968).

This is shown diagrammatically in Fig. 1., where the first term corresponds to (a), the second term to (b), and the third term to (c). If we assume that the elastic scattering is the same in the final state as in the initial state, and that it is pure nonflip, we have

$$T'_{\lambda\mu}{}^J(s)=T_{\lambda\mu}{}^J(s)+2ipT_{\lambda\lambda}{}^{\mathrm{el}J}(s)T_{\lambda\mu}{}^J(s)\,.$$

Having calculated the partial-wave amplitudes $T'_{\lambda\mu}{}^J(s)$, we resum the partial-wave expansion to obtain the modified helicity amplitudes

$$\phi'_{\lambda\mu}(s,t)=\sum_J\,(2J+1)T'_{\lambda\mu}{}^J(s)d_{\alpha\beta}{}^J(\theta)\,.$$

The amplitudes $\phi'_{\lambda\mu}(s,t)$ are to be compared with experiment.

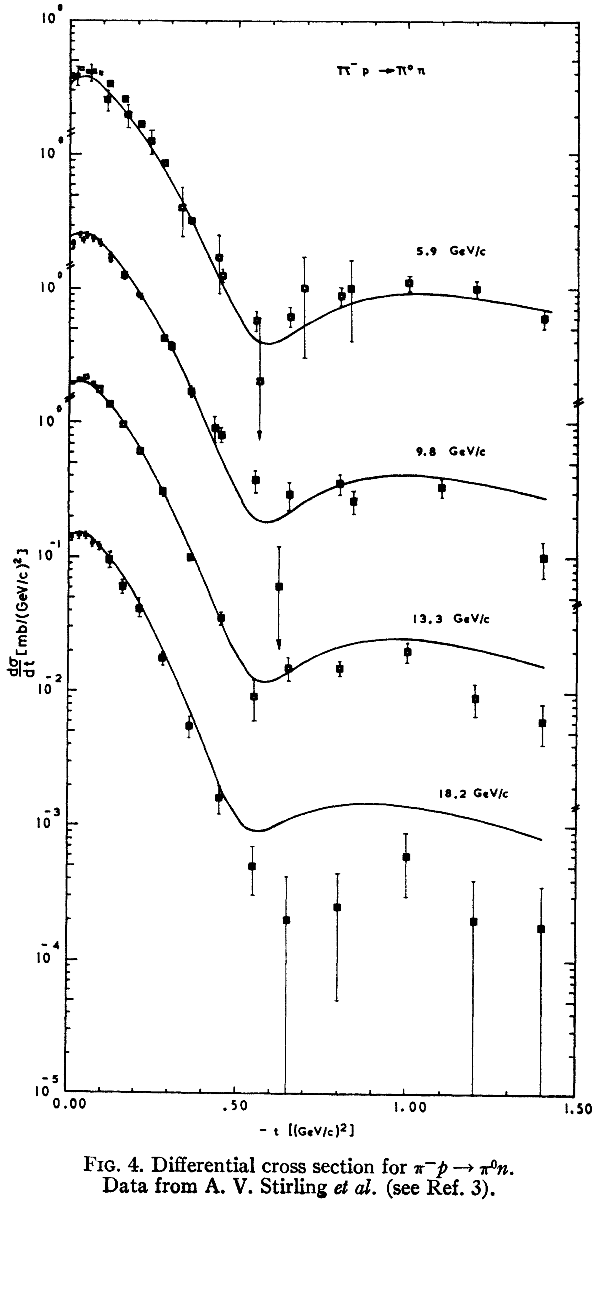

FIG. 4. Differential cross section for $\pi^-p\to\pi^0n$. Data from A. V. Stirling *et al.* (see Ref. 3).

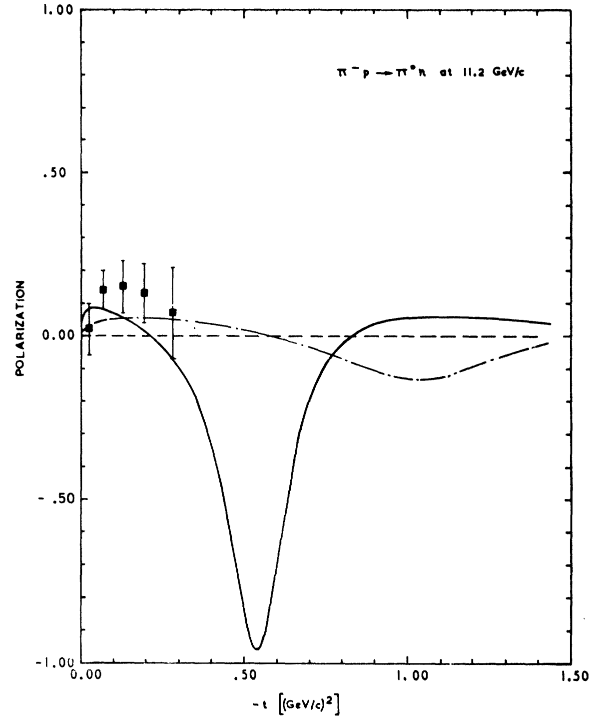

FIG. 5. Contributions from pole (– – – –), cut (—·—·), and pole+cut (———) to the polarization for $\pi^-p\to\pi^0n$. Data from Ref. 9.

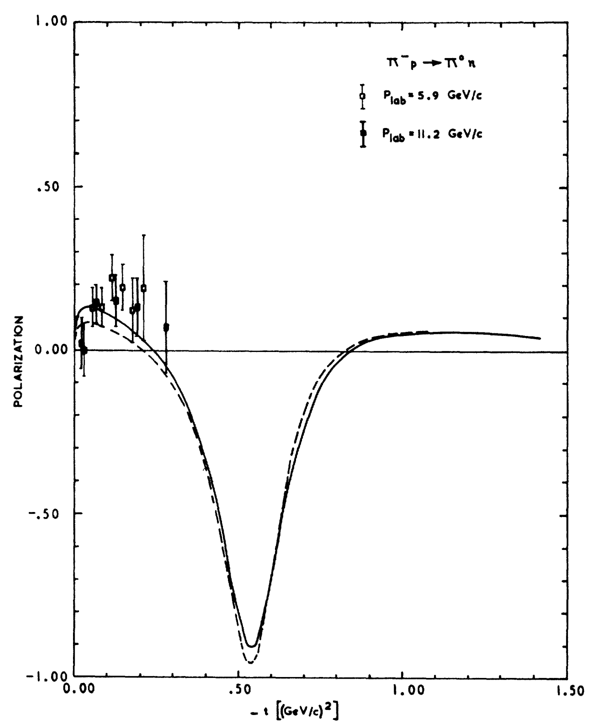

FIG. 6. Polarization for $\pi^-p\to\pi^0n$ at 5.9 (———) and 11.2 GeV/c (– – – –). Data from Ref. 9.

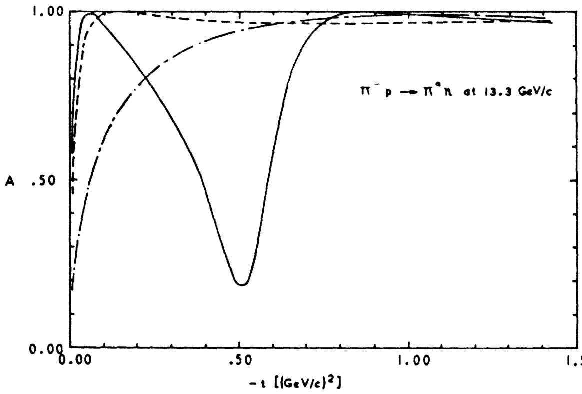

FIG. 7. $A(t)$ for $\pi^-p \rightarrow \pi^0 n$: pole (- - - -), cut (—·—·), and pole+cut (———).

4. ELASTIC SCATTERING

We employ a complex Gaussian form for the nonflip elastic scattering amplitude

$$S_{++}{}^J = 1-(c_1+ic_2)e^{-J(J+1)/\nu^2 q^2},$$

where ν is the radius of the interaction, and c_1 and c_2 are related to the opacity of the target particle. We set $S_{+-}{}^J=0$, since the elastic scattering is dominated for small momentum transfer by the non-helicity-flip amplitude. Using the impact-parameter representation,[16] we find that

$$\phi_{++}(s,t) = q(e^{t/4\nu^2}/2\nu^2)(ic_1 - c_2).$$

Using the optical theorem, we find c_1 from

$$c_1 = \nu^2 \sigma_{\text{tot}}/2\pi$$

and ν is determined from the observed exponential slope of $(d\sigma/dt)_{\text{elastic}}$. The quantity c_2 is given by the relation

$$\text{Re}\phi_{++}(t=0)/\text{Im}\phi_{++}(t=0) = -c_2/c_1.$$

The ratio of the real part to the imaginary part of the amplitude at $t=0$ for π^-p elastic scattering is taken from Ref. 17.

Since there are no data on the ratio of real to imaginary parts of the K^-p elastic scattering amplitude, we take $c_2=0$, i.e., a purely real Gaussian. The absorption coefficients are shown in Table I.

Since there are no data for the final-state elastic scattering, we assume that they are the same as in the

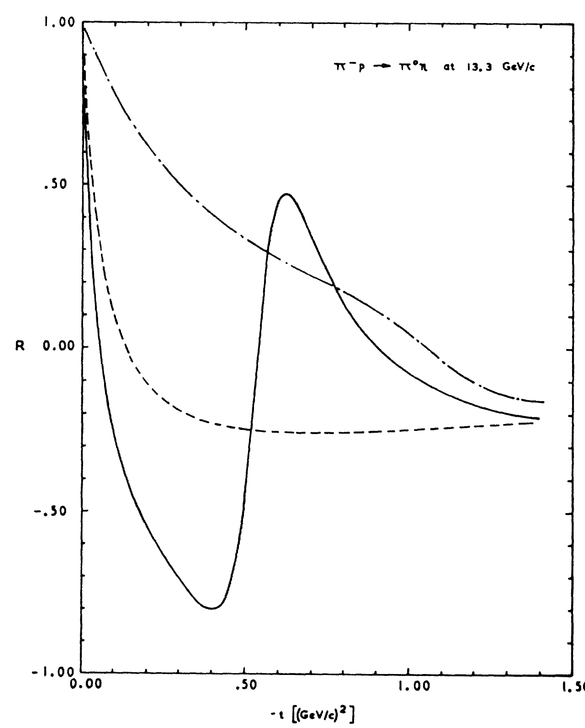

FIG. 8. $R(t)$ for $\pi^-p \rightarrow \pi^0 n$: pole (- - - -), cut (—·—), and pole+cut (———).

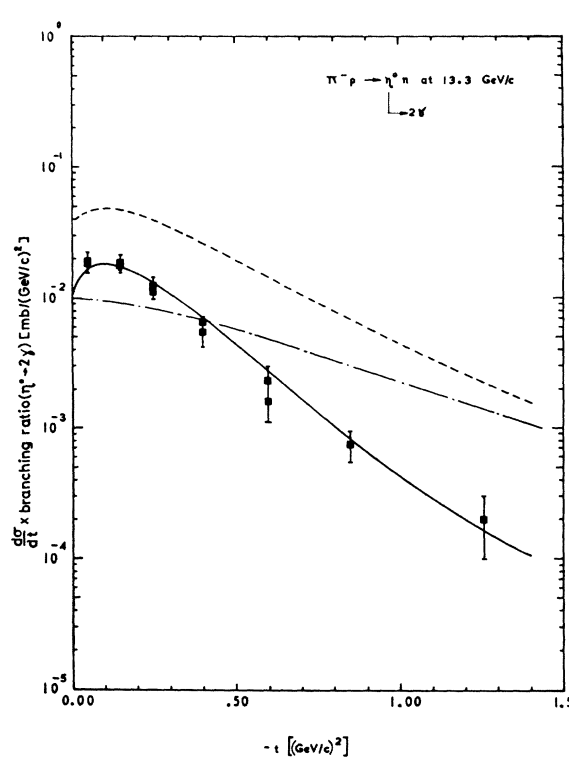

FIG. 9. Contributions from pole (- - - -), cut (—·—·), and pole+cut (———) to the differential cross section for $\pi^-p \rightarrow \eta^0 n$. The theoretical curve has been multiplied by the branching ratio ($\eta^0 \rightarrow 2\gamma$). Data from O. Guisan, J. Kirz, P. Sonderegger, A. V. Stirling, P. Borgeaud, C. Bruneton, and P. Falk-Vairant, Phys. Letters **18**, 200 (1965).

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initial state. In any case, it would be surprising if the parameters were radically different.

5. TRAJECTORY

There are indications both from potential scattering and from perturbation theory that the Regge trajectory $\alpha(t)$ is nonlinear, at least in the scattering region.

It is known[18] that for scattering from "reasonable" potentials, e.g., a Yukawa potential, the resultant Regge trajectory $\alpha(t)$ is such that

$$\alpha(t) \to -\tfrac{1}{2}N - \tfrac{1}{2}, \quad \text{as } t \to -\infty,$$

where N is a positive integer. The leading trajectory behaves as

$$\alpha(t) \to -1 \quad \text{as} \quad t \to -\infty.$$

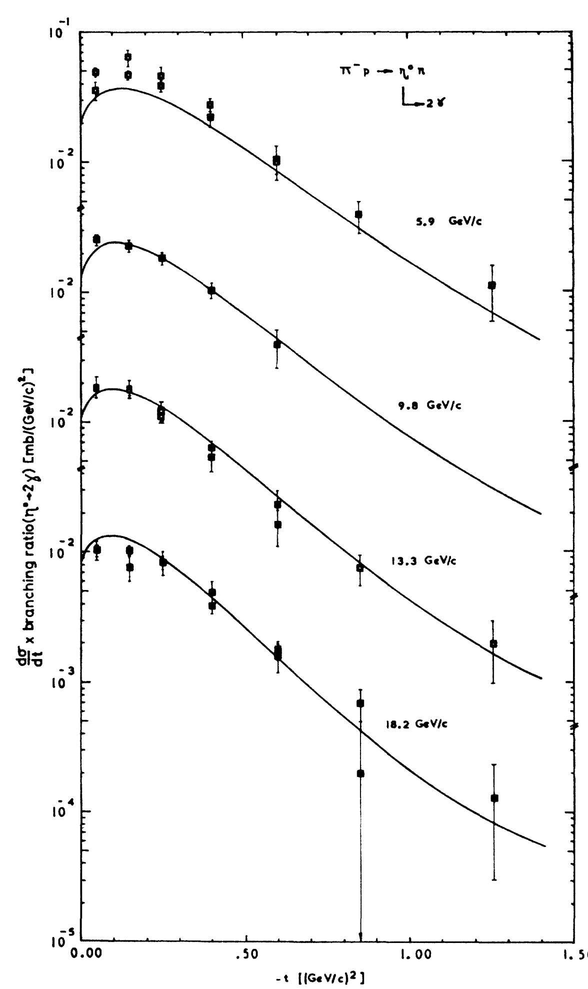

FIG. 10. Differential cross section for $\pi^- p \to \eta^0 n$. The theoretical curve has been multiplied by the branching ratio ($\eta^0 \to 2\gamma$). Data from Guisan *et al.* (see Ref. 9).

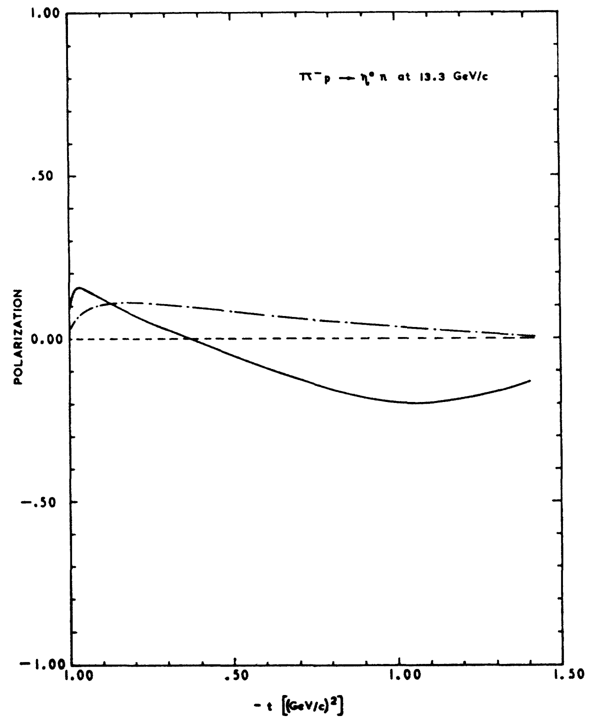

FIG. 11. Polarization for $\pi^- p \to \eta^0 n$: pole (– – – –), cut (—·—·), and pole+cut (———).

In Chap. 12 of Ref. 18 the typical highly nonlinear variation of $\alpha(t)$ is shown for various trajectories.

In the relativistic domain, perturbation theory[19] shows that the typical trajectory obtained by summing Feynman diagrams is of the form

$$\alpha(t) = -N + \sum_{n=1}^{\infty} g^{2n} K_n(t),$$

where $K_n(t) \to 0$ as $t \to -\infty$. Again for the leading trajectory $\alpha(t) \to -1$ as $t \to -\infty$.

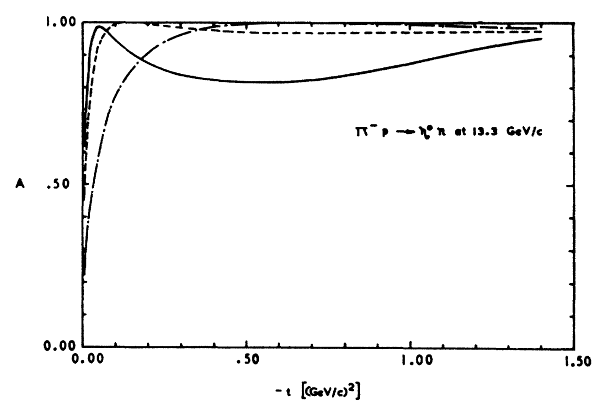

FIG. 12. $A(t)$ for $\pi^- p \to \eta^0 n$: pole (– – – –), cut (—·—·), and pole+cut (———).

[18] R. G. Newton, *The Complex j-Plane; Complex Angular Momentum in Non-Relativistic Quantum Theory* (W. A. Benjamin, Inc., New York, 1964).

[19] R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, New York, 1966).

Owen *et al.*[20] have recently carried out an interesting calculation in which they fitted the parametric form

$$d\sigma/dt \sim s^{2\alpha(t)-2}$$

to the $\pi^- p$ elastic scattering data from the forward to the backward region, and found that $\alpha(t)$ tends to approximately -1 as $t \rightarrow -\infty$.

In view of these facts, we have parametrized our trajectory for t negative by the relation

$$\alpha(t) = \alpha_0 + \alpha_1 e^{\alpha_2 t}.$$

We see that this gives

$$\alpha(t) \rightarrow \alpha_0 \quad \text{as} \quad t \rightarrow -\infty$$

as suggested above. In the peripheral region our trajectory becomes

$$\alpha(t) \simeq (\alpha_0 + \alpha_1) + (\alpha_1 \alpha_2) t,$$

which corresponds to the standard linear parametrization of the Regge trajectory.

From Table II we see that in the scattering region

$$|\alpha(t)| < 1.$$

Thus the only nonsense point which occurs is

$$\alpha(t) = 0.$$

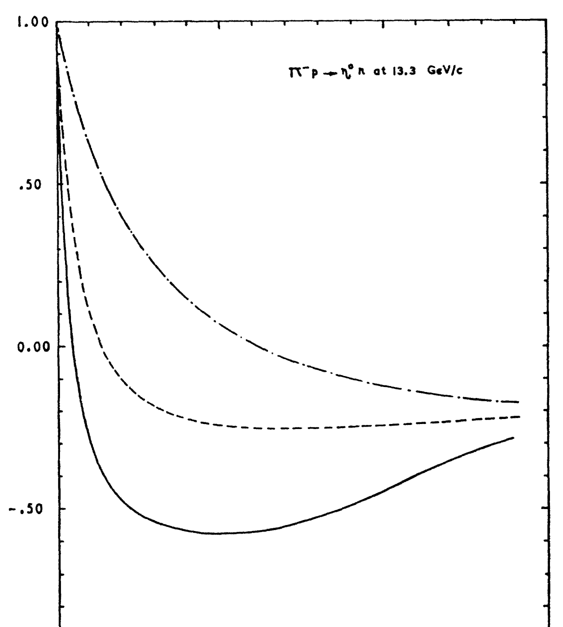

FIG. 13. $R(t)$ for $\pi^- p \rightarrow \eta^0 n$: pole (– – – –), cut (—·—·), and pole+cut (———).

[20] D. P. Owen, F. C. Peterson, J. Orear, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubenstein, Phys. Rev. **181**, 1794 (1969).

Since for the odd-signature trajectory this is a wrong-signature point, using the Gell-Mann mechanism we obtain a zero in the flip and nonflip amplitudes for the pole graph at this point. For the even-signature trajectory, this is a right-signature point, and no such zero occurs in the amplitudes.

6. DISCUSSION AND RESULTS

For mass-splitting in $U(6)\times U(6)\times O(3)$, we took $\mu = 0.417$ GeV/c^2, the average of the 0^- nonet, and $m = 1.150$ GeV/c^2, the average of the $\frac{1}{2}^+$ octet.

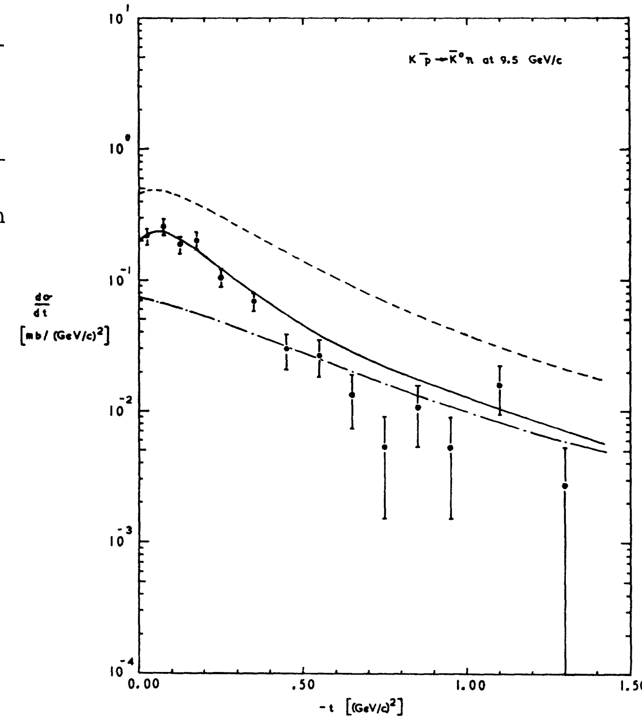

FIG. 14. Contributions from pole (– – – –), cut (—·—·), and pole+cut (———) to the differential cross section for $K^- p \rightarrow \bar{K}^0 n$. Data from P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. H. West, P. Zanella, W. Beusch, W. Fisher, B. Gobbi, M. Pepin, and E. Polgar, Phys. Letters **23**, 396 (1966).

There is an abundance of experimental data on the charge-exchange reactions:

$$\pi^- p \rightarrow \pi^0 n,$$
$$\pi^- p \rightarrow \eta^0 n \quad (\eta^0 \rightarrow 2\gamma \text{ mode}),$$

at a sufficiently high energy for us to hope that in this region the asymptotic Regge form is a good approximation to the two-body scattering amplitude. These two reactions are important in that for each, only one trajectory has the appropriate quantum numbers to be exchanged, namely, the ρ and A_2 trajectories, respectively. We fix the residue $\beta(t)$ and the trajectory function $\alpha(t)$ for each of these trajectories from a χ^2 fit of the pole+cut prediction to the experimental differential cross-section data, and then predict the experimental results for the reaction $K^-p \rightarrow \bar{K}^0 n$, which proceeds via both ρ- and A_2- trajectory exchange.

Parametrizing the trajectories as discussed in Sec. 5 and taking the residues to be constants, we have four free parameters for each trajectory. We fitted these

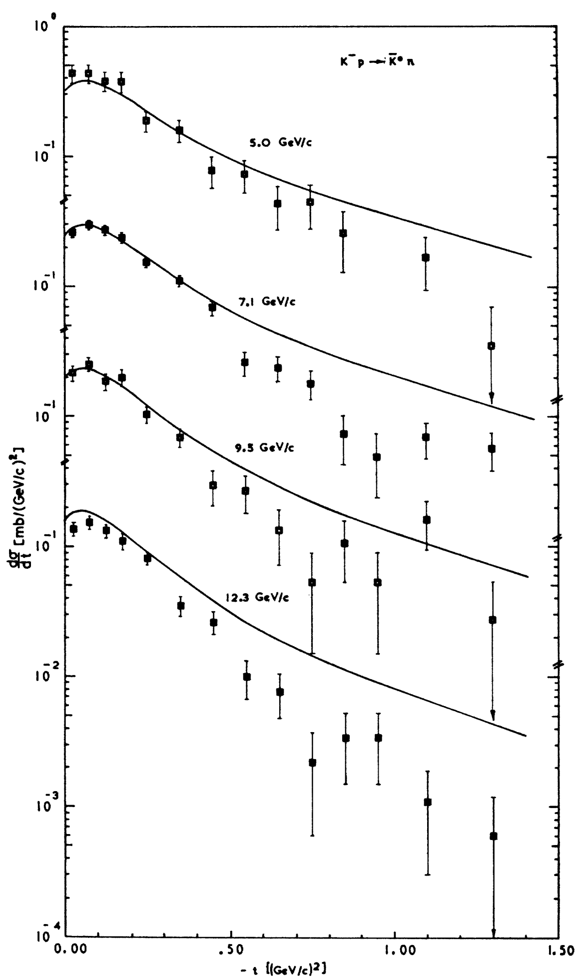

FIG. 15. Differential cross section for $K^-p \rightarrow \bar{K}^0 n$. Data from P. Astbury *et al.* (see Ref. 14).

using MINUIT (CERN Program Library No: D506). We took the branching ratio $\eta^0 \rightarrow 2\gamma$ to be 0.381. The results of this minimization procedure are shown in Table II.

It is generally accepted that for those trajectories where several resonances are known, the trajectory for positive t is linear. Since our trajectory parameters were determined from the scattering data, that is, t negative, we feel that it is a reasonable procedure to

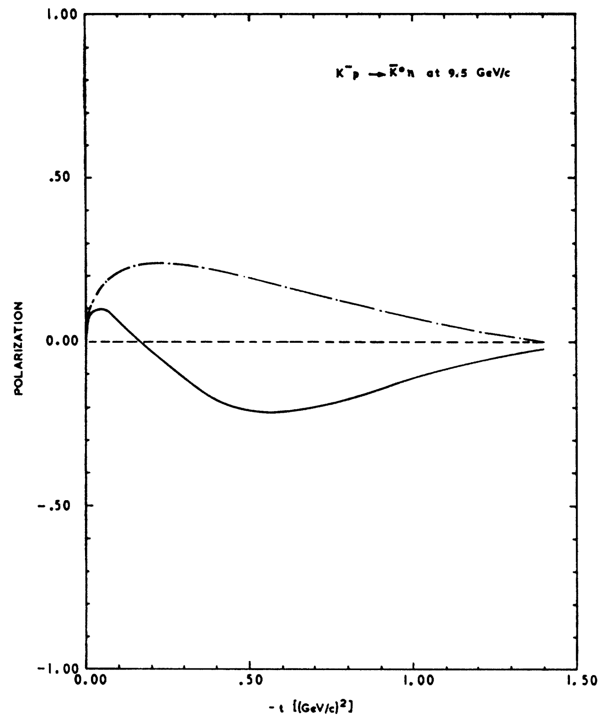

FIG. 16. Polarization for $K^-p \rightarrow \bar{K}^0 n$: pole (- - - -), cut (—·—·), and pole+cut (———).

represent $\alpha(t)$ for t positive by extrapolating the tangent at $t=0$. A Chew-Frautschi plot is presented in Fig. 2. The expected position of the ρ-Regge recurrence, the g meson (3^-), is also indicated. The ρ and A_2 trajectories thus determined pass surprisingly close to the physical ρ and A_2 mesons. We note that, although no such constraint was imposed in the data fitting, the ρ and A_2 trajectories are almost exchange degenerate.

Consider first the reaction $\pi^-p \rightarrow \pi^0 n$. In Fig. 3 we show the contributions to the momentum transfer distributions from the pole, the cut, and the pole+cut. We see that the cut has the effect of producing the "turnover" near the forward direction, and filling in the nonsense zero caused by the wrong-signature point $\alpha(t)=0$, and moving the minimum in to form the dip near $t \simeq -0.6$ (GeV/c)^2.

Figure 4 illustrates the energy dependence of the momentum-transfer distribution. The decrease in normalization of the experimental data with increasing energy and the t dependence are very well represented.

Having fixed the parameters from the data on the differential cross section, we predicted the polarization for this reaction. In Fig. 5 we show the contributions from the pole, the cut, and the pole+cut. The polarization from the pole alone is, of course, identically zero. In Fig. 6 we show the energy variation of the polarization. We note that the predicted polarization has the correct features, namely, it is positive for small t, of about the correct magnitude, and decreases with in-

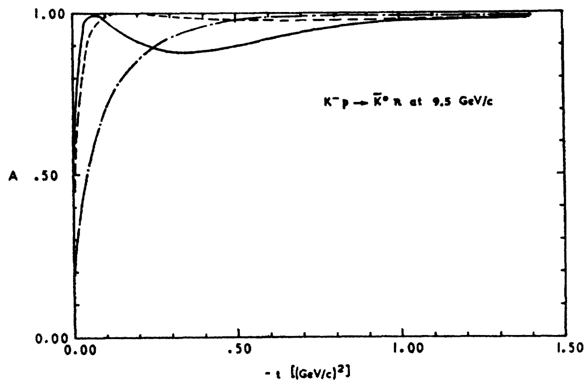

FIG. 17. $A(t)$ for $K^-p \to \bar{K}^0n$: pole (- - - -), cut (—·—·), and pole+cut (———).

creasing energy. We obtain about -90% polarization near $t=-0.55$ (GeV/c)^2. The data do appear to indicate a downward trend, but clearly data at larger momentum transfers are required to see whether this effect is present experimentally.

In Figs. 7 and 8 we present the predictions for the Wolfenstein parameters A and R, respectively.

We carried out a similar analysis of the reaction $\pi^-p \to \eta^0n$. The results are shown in Figs. 9–13. We note that, since $\alpha(t)=0$ is a right-signature point for the A_2-trajectory, there is no nonsense zero in the differential cross section, as shown in Fig. 9. The

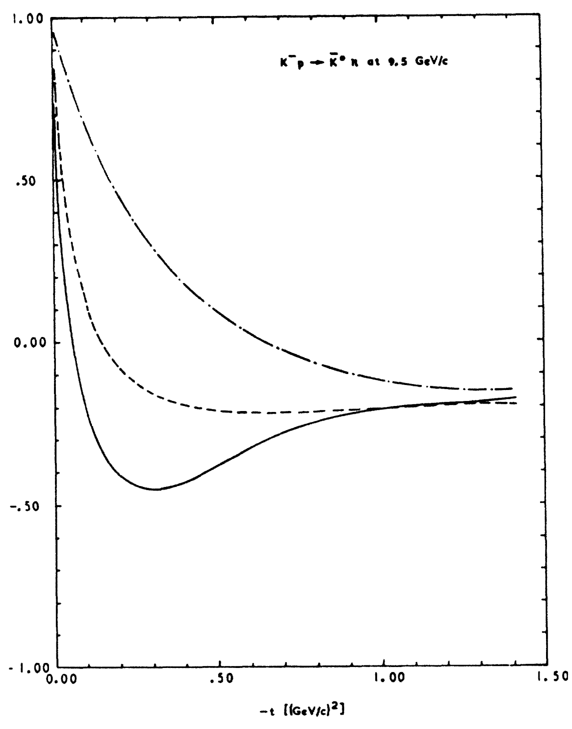

FIG. 18. $R(t)$ for $K^-p \to \bar{K}^0n$: pole (- - - -), cut (—·—·), and pole+cut (———).

effect of the cut is to produce a sharper decrease in the momentum-transfer distribution, and to decrease the normalization, giving a distribution with a turnover near the forward direction. The predictions for the Wolfenstein parameters P, A, and R for $\pi^-p \to \eta^0n$ are presented in Figs. 11, 12, and 13, respectively.

Having determined the parameters for the ρ and A_2 trajectories, we proceeded to predict the experimental results for the reaction $K^-p \to \bar{K}^0n$. Figure 14 shows the $\rho+A_2$ contributions to the momentum transfer distribution from the pole, the cut, and the pole+cut. The correct t dependence is reasonably well reproduced. Figure 15 shows the energy dependence of the differential cross section. Considering that there are no additional adjustable parameters for this process, the momentum transfer distributions are in good agreement with the experimental data. Figs. 16, 17, and 18 show, respectively, the predictions for the parameters P, A, and R.

Rather than use the impact-parameter representation, we employed an exact partial-wave summation using 30 partial waves calculated with a 48-point Gaussian quadrature. As a check on the accuracy, some calculations were repeated with a 96-point Gaussian and 50 partial waves. The results were found not to change significantly.

This work differs from that of other authors in several respects.

A significant difference is in the numerical procedures employed:

(i) By using an exact partial-wave series rather than attempting an analytic integration of the Regge amplitudes in the absorptive eikonal formulation, we can retain the exact form of the Regge amplitudes; e.g., we do not have to drop trignometric functions or Γ functions, which are an essential part of the physics.

(ii) This work is part of a program to explain all two-body hadronic interactions using Regge pole theory and higher-symmetry schemes. Certain parameters cannot be determined by the higher-symmetry scheme alone. Since these will be fixed in extending calculations to other processes, a minimization procedure was used to obtain the best values.

The $U(6)\times U(6)\times O(3)$ symmetry fixes the ratio of the helicity-flip amplitude to the helicity-nonflip amplitude. By doing this we obtained an absolute prediction for the polarization.

Unlike other authors, we have not used linear trajectories but the parametrization discussed in Sec. 5, nor have we assumed exchange degeneracy although, as discussed above, the results clearly indicate its applicability.

Calculations have now been successfully extended to the reactions

$$p\bar{p}\to n\bar{n},\quad pn\to np,$$

which are dominated by π exchange at small momentum transfers.[21,22] The results will be published shortly.

[21] J. H. R. Migneron and K. Moriarty, Phys. Rev. Letters **18**, 978 (1967).
[22] B. J. Hartley, J. D. Jenkins, R. W. Moore, and K. J. M. Moriarty (to be published).

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Lagrangian Forms of the Dynamical Theory of Currents*

S. Deser
Brandeis University, Waltham, Massachusetts 02154
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Simple Lagrangian formulations of Sugawara models are given, which directly exhibit the "gauge-component" nature of the currents. It is also shown that there is no corresponding theory for tensor currents.

I. INTRODUCTION

The dynamical theory of currents[1] bypasses the usual Lagrangian mechanism of local field theory in favor of a framework involving the physical currents directly. The dynamics governing the currents j^μ is determined by the stress tensor $T^{\mu\nu}$ as a function of j^μ and by the equal-time commutation relations (ETC) among the j^μ. Thus, only the Heisenberg equations $\partial_\mu j_\alpha \equiv i[j_\alpha,\int T^0{}_\mu d^3r]$ are assigned, with no corresponding Euler-Lagrange equations, which is why the ETC must be separately given. Consistency is no longer checked between Heisenberg and Lagrange equations, but rather between the ETC and the Poincaré algebra requirements on $T^{\mu\nu}$. For example, the Schwinger terms in $[j_0,j_i]$ ETC give rise to the required $T^{0i}\partial_i\delta(\mathbf{r})$ terms in the $[T^{00},T^{00}]$ ETC.

This attractive scheme has been exhaustively analyzed for stress tensors quadratic[2] in currents carrying $SU_n\times SU_n$ symmetry; it has been shown to be a particular limit of the corresponding massive Yang-Mills field theories[3] and also to be equivalent to a Lagrangian theory of spin-zero fields,[4,5] which furnish representations of the algebra. In Sec. II, we shall give directly a transparent Lagrangian form in terms of the currents themselves, which illuminates the basis for the earlier derivations and the close relation between the currents and the gauge or "longitudinal" parts of massive Yang-Mills fields.

Section III deals with possible generalization to higher-spin currents: In view of the close relation of vector currents to gauge components, it might be thought that symmetric tensor currents would correspond to gauge parts of that massive spin-2 field, and, by appropriate extension, of the full gravitational field. However, we show that for a variety of reasons there is no (nontrivial) "dynamical theory of tensor currents."

* Work supported in part by the U. S. Air Force, OAR, under OSR Grant No. 368-67.
[1] H. Sugawara, Phys. Rev. **170**, 1659 (1968); C. M. Sommerfield, *ibid.* **176**, 2019 (1968).
[2] For discussion of more general $T^{\mu\nu}$, see S. Deser and J. Rawls, following paper, Phys. Rev. **187**, 1935 (1969).
[3] K. Bardakci, Y. Frishman, and M. B. Halpern, Phys. Rev. **170**, 1353 (1968).
[4] K. Bardakci and M. B. Halpern, Phys. Rev. **182**, 1542 (1968).
[5] S. Coleman, D. Gross, and R. Jackiw, Phys. Rev. **180**, 1355 (1969).

II. LAGRANGIAN FORM OF SUGAWARA THEORY

We exhibit a particularly simple Lagrangian form of the Sugawara model, which preserves the form of $T^{\mu\nu}$. The resulting Euler-Lagrange equations will be the same as the Heisenberg equations in the Sugawara theory, while the current ETC will follow from the canonical commutation relations dictated by the action principle.

Consider first, for simplicity, the Abelian case of a single current V^μ, with

$$T^{\mu\nu}=V^\mu V^\nu-\tfrac{1}{2}\eta^{\mu\nu}V_\alpha V^\alpha,\tag{2.1}$$

where, as throughout, all required symmetrization is understood, and the usual over-all constant C is set to unity. This stress tensor is the variation of an "action"

$$I_1=-\tfrac{1}{2}\int d^4x(-g)^{1/2}g^{\mu\nu}V_\mu V_\nu\tag{2.2}$$