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High-Energy Behavior of Bethe-Salpeter Amplitudes: Spin and Nonplanar Kernels*

RICHARD A. BRANDT AND MARTIN FEINROTH
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

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We generalize our previous work on the asymptotic behavior of solutions to the Bethe-Salpeter equation to include spin and nonplanar kernels. Our aim is to develop simple heuristic methods for studying the asymptotic behavior of infinite sums of complicated Feynman diagrams. In the simplest case of πN scattering, we still obtain Regge asymptotic behavior with explicit formulas for the trajectory functions for a certain class of planar kernels. However, in higher-spin cases like $N\bar{N}$ scattering, the heuristically determined trajectory functions are given by divergent integrals. A modified procedure is developed for this situation, which leads to an asymptotic behavior that is not of the Regge form. We also treat the nonplanar X kernel and show how our method leads to the Gribov-Pomeranchuk essential singularity.

I. INTRODUCTION

In a previous paper,[1] to be referred to as I, we introduced new techniques to study the asymptotic behavior of solutions to Bethe-Salpeter (B-S) equations. Our methods were more heuristic and intuitive than rigorous, but correspondingly more simple and direct than the standard approaches.[2] We were able to establish Regge behavior for the amplitude defined by an arbitrary planar kernel in ϕ^3 theory in an approximation equivalent to summing the leading high-energy term in each order. Our results agreed with the previous calculations in the few places where the direct summation had been carried out.

The purpose of the present paper is to indicate how our techniques can be extended to more complicated cases involving spin and nonplanar kernels. In such cases it is prohibitively difficult to carry out directly the summation of even the leading terms in each order with the usual methods.[3] If further progress along these lines is to be made, therefore, it seems necessary that new methods be introduced. We shall accordingly sacrifice the rigor of the usual approach in favor of our heuristic procedures in the hope that our results will provide at least some indication of the correct asymptotic behavior.

As in I, the reliability of our approach can at present be determined only by comparing our conclusions with the known results of the standard procedures. In this paper we shall, therefore, consider cases that have already been treated by the standard methods. We shall thus obtain few new results but will however, be able essentially to reproduce older results with far greater ease.

We hope that our methods, and extensions and corrections of them, can be used to develop an intuition concerning the asymptotic behavior of infinite sums of complicated Feynman diagrams. We feel that our success indicates the usefulness of our approach, and therefore we suggest that our methods be used to treat other scattering processes of physical interest.

In Sec. II we briefly discuss the B-S equation corresponding to particles with arbitrary spin and indicate the application of our procedure. We stress the changes brought about by the presence of spin. In Secs. III–V we treat processes giving rise to progressively more complicated asymptotic behaviors. Section III deals with πN scattering with scalar exchange in the ladder approximation. The spin effects in this case do not alter the fact that the asymptotic behavior is of the Regge form. Extensions to more general kernels are briefly discussed. In Sec. IV we show how higher-spin effects do alter the form of our asymptotic integral equations. The solutions of the new equations do not, in general, exhibit Regge asymptotic behavior. This is illustrated with reference to $N\bar{N}$ scattering with π exchange in the ladder approximation. In Sec. V we return to the scattering of spinless particles, but now consider the effect of a nonplanar kernel—the X. By suitably taking into account the presence of the left-hand cut, we derive an asymptotic integral equation and illustrate how its solution corresponds to an essential singularity in the J plane. Concluding remarks are given in Sec. VI.

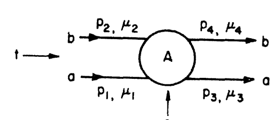

FIG. 1. Kinematics of the general B-S amplitude.

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[1] R. Brandt and M. Feinroth, Phys. Rev. **176**, 1985 (1968). We adopt here the notations and conventions of this paper.

[2] For a summary, see R. J. Eden *et al.*, *The Analytic S-Matrix* (Cambridge University Press, Cambridge, 1966). See also I.

[3] For partial (inconclusive) attempts to study the vector-spinor theory [first studied in connection with Reggeization by M. Gell-Mann *et al.*, Phys. Rev. **133B**, 145 (1964)] in this way, see J. Polkinghorne, J. Math. Phys. **5**, 1491 (1964); H. Cheng and T. T. Wu, Phys. Rev. **140**, B465 (1965); and J. V. Greenman, J. Math. Phys. **7**, 1782 (1966), and **8**, 26 (1967). More recently, H. Cheng and T. T. Wu [Phys. Rev. Letters **22**, 666 (1969); Phys. Rev. **182**, 1852 (1969)] have similarly studied high-energy elastic scattering in quantum electrodynamics and found "after 16 months and more than 2000 pages of calculations" complete results only through eighth order.

II. SPIN

In order to illustrate some general features of the B-S equation for particles with spin, we consider the scattering of particles with arbitrary spin. We will treat the spin complications rather formally in this section. The details in the cases of πN and $N\bar{N}$ scattering will be given in Secs. III–VI.

We consider the elastic scattering of particles a and b in the t channel, allowing the particles to have arbitrary spin.[4] The off-shell amplitude may be written as

$$A_{\mu_3\mu_4,\mu_1\mu_2}(p_3,p_4;\, p_1,p_2) \equiv A_{\mu_3\mu_4,\mu_1\mu_2}(p_i)\,, \tag{2.1}$$

where the initial state is composed of particle a with momentum p_1 and a set of Lorentz (and/or spinor) indices $\{\mu_1\}$ together with particle b of momentum p_2 and indices $\{\mu_2\}$. Similarly, the final state has particle a with p_3 and $\{\mu_3\}$, and particle b with p_4 and $\{\mu_4\}$. The kinematics are shown in Fig. 1. The high-energy limit we are interested in is $s \to \infty$ for fixed t.

The B-S equation for this amplitude is (see Fig. 2)

$$A_{\mu_3\mu_4,\mu_1\mu_2}(p_i) = B_{\mu_3\mu_4,\mu_1\mu_2}(p_i) - \frac{i}{(2\pi)^4}\int d^4k \times \sum_{\mu_i'} A_{\mu_3\mu_4;\mu_3'\mu_4'}(p_3,p_4;\, k_2,k_4) \frac{P_{\mu_4'\mu_2'}{}^b(k_4)}{k_4^2 - m_b^2 + i\epsilon} \frac{P_{\mu_3'\mu_1'}{}^a(k_2)}{k_2^2 - m_a^2 + i\epsilon} \times B_{\mu_1'\mu_2',\mu_1\mu_2}(k_2,k_4;\, p_1,p_2)\,, \tag{2.2}$$

where $(k^2 - m_a^2)^{-1} P_{\mu\nu}{}^a(k)$ and $(k^2 - m_b^2)^{-1} P_{\mu\nu}{}^b(k)$ are suitable propagators for a and b, respectively, and $B_{\mu_3\mu_4,\mu_1\mu_2}(p_i)$ is a kernel whose properties will be discussed later. As in I, we use the variables

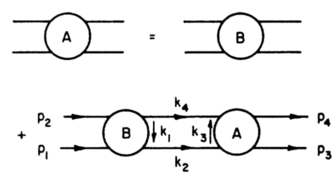

FIG. 2. B-S equation.

$$\begin{aligned} &k_1 = k - p_1, \quad k_2 = k, \quad k_3 = k - p_3, \quad k_4 = p_1 + p_2 - k, \\ &\kappa_i = k_i^2, \quad \rho_i = p_i^2, \quad s = (p_1 - p_3)^2, \quad t = (p_1 + p_2)^2. \end{aligned} \tag{2.3}$$

Before we take the high-energy limit, we must reduce (2.2) to a system of equations for invariant amplitudes. Therefore, we introduce the following decompositions:

$$A_{\mu_3\mu_4,\mu_2\mu_1}(p_i) = \sum_j A_j(s,t;\, \rho_i) O_{\mu_3\mu_4,\mu_1\mu_2}{}^j(p_1,p_3,p_4)\,, \tag{2.4a}$$

$$B_{\mu_3\mu_4,\mu_2\mu_1}(p_i) = \sum_j B_j(s,t;\, \rho_i) O_{\mu_3\mu_4,\mu_1\mu_2}{}^j(p_1,p_3,p_4)\,. \tag{2.4b}$$

The O^{i}'s are a complete set of independent tensor and/or spinor covariants formed from the vectors p_1, p_3, and p_4 together with the appropriate γ matrices if one of the particles is a fermion. After substituting (2.4a) and (2.4b) in (2.2), we obtain the following set of equations for the invariant amplitudes:

$$A_j(s,t;\, \rho_i) = B_j(s,t;\, \rho_i) - \frac{i}{(2\pi)^4}\int d^4k \sum_{kl} \frac{C_{jkl}(s,t;\, \rho_i,\kappa_i) B_k(\kappa_1,t;\, \kappa_2,\kappa_4,\rho_1,\rho_2) A_l(\kappa_3,t;\, \rho_3,\rho_4,\kappa_2,\kappa_4)}{(\kappa_2 - m_a^2 + i\epsilon)(\kappa_4 - m_b^2 + i\epsilon)}\,. \tag{2.5}$$

The coupling coefficients C_{jkl} are determined by

$$\sum_{\{\mu'\}} O_{\mu_3\mu_4,\mu_3'\mu_4'}{}^l(k_2,p_3,p_4) P_{\mu_4'\mu_2'}{}^b(k_4) P_{\mu_3'\mu_1'}{}^a(k_2) O_{\mu_1'\mu_2',\mu_1\mu_2}{}^k(p_1,k_2,k_4) = \sum_j C_{jkl}(s,t;\, \rho_i,\kappa_i) O_{\mu_3\mu_4,\mu_1\mu_2}{}^j(p_1,p_3,p_4) + \cdots\,, \tag{2.6}$$

where the sum over $\{\mu'\}$ goes over all primed Lorentz indices. The omitted terms on the right-hand side of (2.6) are covariants involving k_μ, which either vanish by symmetric integration in (2.5) or are already effectively included. It is a straightforward, though tedious, procedure to calculate the C_{jkl}'s from (2.6). We shall illustrate this in Sec. III for the case of π-N scattering with scalar exchange.

We are now ready to take the high-energy limit of the B-S equation(s) (2.5). As in I, we change measures from d^4k to

$$\prod_{i=1}^{4} d\kappa_i\,.$$

The Jacobian is given by

$$J = \theta(D)/4D^{1/2}\,, \tag{2.7}$$

where

$$D = -s^2 \Delta(\kappa_2,\kappa_4,t) + E(s,t;\, \kappa_i,\rho_i) \tag{2.8}$$

and

$$\Delta(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\,. \tag{2.9}$$

[4] We introduce cutoffs into the propagators for these particles wherever they are needed.

$E(s,t;\,\kappa_i,p_i)$ is linear in s and is given explicitly in I. Equation (2.5) then becomes

$$A_j(s,t;\,p_i)=B_j(s,t;\,p_i)-\frac{i}{4(2\pi)^4}\int\prod_{i=1}^{4}d\kappa_i\frac{\theta(D)}{D^{1/2}}\sum_{kl}\frac{C_{jkl}(s,t;\,p_i,\kappa_i)B_k(\kappa_1,t;\,\kappa_2,\kappa_4,p_1,p_2)A_l(\kappa_3,t;\,p_3,p_4,\kappa_2,\kappa_4)}{(\kappa_2-m_a{}^2+i\epsilon)(\kappa_4-m_a{}^2+i\epsilon)}. \tag{2.10}$$

To proceed further, we depart from the general case. Since we are trying to extend the heuristic method of I, we are only interested in an equation that reproduces the correct leading asymptotic behavior in each order of iteration. In the examples we have looked at so far, it is always possible to choose a linear combination of the amplitudes (called $\tilde{A}$) so that there is a single asymptotic integral equation for that amplitude. Furthermore, in these examples, the new coupling coefficients $\tilde{C}_j$ have the property that, at high s, they are independent of s, κ_1, κ_3, and the external masses, i.e.,

$$\lim_{s\to\infty}\tilde{C}_j(s,t;\,p_i,\kappa_i)=\tilde{C}_j(\kappa_2,\kappa_4,t). \tag{2.11}$$

The B-S equation then becomes

$$\tilde{A}(s,t;\,p_i)=\tilde{B}(s,t;\,p_i)-\frac{i}{4(2\pi)^4}\int\prod_{i=1}^{4}d\kappa_i\frac{\theta(D)}{D^{1/2}}\sum_{k}\frac{\tilde{C}_k(\kappa_2,\kappa_4,t)\tilde{B}_k(\kappa_1,t;\,\kappa_2,\kappa_4,p_1,p_2)\tilde{A}(\kappa_3,t;\,p_3,p_4,\kappa_2,\kappa_4)}{(\kappa_2-m_a{}^2+i\epsilon)(\kappa_4-m_b{}^2+i\epsilon)}. \tag{2.12}$$

The high-energy limit now proceeds just as in Sec. 5 of I. We assume that the kernel is given by a finite sum of two-particle irreducible planar Feynman diagrams. For sufficiently small external masses, the $\tilde{B}_k(s,t;\,p_i)$ will satisfy dispersion relations in s with only right-hand cuts:

$$\tilde{B}_k(s,t;\,p_i)=\int_0^\infty da\,\frac{\sigma_k(a,t;\,p_i)}{s-a-i\epsilon}. \tag{2.13}$$

We neglect possible complex singularities when the masses are varied. Furthermore, we assume that the asymptotic behavior of the $\tilde{B}_k$'s is given by

$$\tilde{B}_k(s,t;\,p_i)\sim\tilde{b}_k(t)s^{-q_k}(\ln s)^{p_k} \tag{2.14}$$

for integer $p_k\geq 0$ and integer $q_k\geq 1$. Using (2.13) and (2.14) in (2.12), we obtain in the high-energy limit (see I, Sec. 5)

$$\tilde{A}(s,t)=\tilde{b}(t)s^{-q}(\ln s)^p+\sum_k K_k(t)\tilde{b}_k(t)s^{-q_k}\times\int_1^s d\kappa\,(\ln s/\kappa)^{p_k}\kappa^{q_k-1}\tilde{A}(\kappa,t), \tag{2.15}$$

where, formally,

$$K_k(t)=\text{“}\lim_{\eta\to 0}\text{”}\frac{1}{(q_k-1)!}\left(-\frac{\partial}{\partial m_a{}^2}-\frac{\partial}{\partial m_b{}^2}\right)^{q_k-1}\times\int\frac{d\kappa_2d\kappa_4\tilde{C}_k(\kappa_2,\kappa_4,t)\theta[-\Delta(\kappa_2,\kappa_4,t)-\eta]}{(\kappa_2-m_a{}^2+i\epsilon)(\kappa_4-m_b{}^2+i\epsilon)[-\Delta(\kappa_2,\kappa_4,t)]^{1/2}}. \tag{2.16}$$

In (2.15) we have set $q=\min(q_k)$ and $p=\max(p_k)$. Since we are only interested in the leading asymptotic behavior in each order, we can restrict the sum over k in (2.15) to those values of k for which $p_k=p$ and $q_k=q$. Equation (2.15) is then a simple Volterra equation with a unique solution of the Regge form. The leading trajectory is given by

$$\alpha(t)=-q+R(t), \tag{2.17}$$

where

$$R(t)=[p!\sum K_k(t)\tilde{b}_k(t)]^{1/(p+1)}. \tag{2.18}$$

This is in almost complete analogy with the scalar case treated in I. The important difference is that the functions $K_k(t)$ defined by (2.16) may not exist. In the scalar case when $\tilde{C}_k(\kappa_2,\kappa_4,t)=1$, there is never any problem. However, if $\tilde{C}_k(\kappa_2,\kappa_4,t)$ grows too fast for large κ_2 and κ_4, then (2.16) will diverge, and the heuristic method of I breaks down. In the case of πN scattering with scalar exchange, we shall see that the integral corresponding to (2.16) does converge and the heuristic method gives Regge asymptotic behavior. In the case of $N\bar{N}$ scattering with pion exchange, however, the integral (2.16) diverges, and a modified analysis is required. The result in that case is that the asymptotic behavior is not of the Regge form. The general modified analysis will be discussed in Sec. V.

III. πN SCATTERING

We now consider πN scattering with the exchange of a scalar particle (σ). We neglect isospin and restrict ourselves to the ladder approximation for the B-S equation. We shall discuss generalizations of the kernel at the end of the section. This process has been investigated in the multiperipheral model by Amati, Stanghellini, and Wilson,[5] and our treatment of the kinematics will be similar to theirs.

The scattering amplitude may be written

$$T_{\lambda_2,\lambda_1}(p_2,q_2;\,p_1,q_1)=\bar{u}_{\lambda_2}(p_2)A(p_2,q_2;\,p_1,q_1)u_{\lambda_1}(p_1), \tag{3.1}$$

where q_1 (q_2) is the initial (final) pion momentum, and p_1 (p_2) and λ_1 (λ_2) are the initial (final) nucleon mo-

[5] D. Amati, A. Stanghellini, and K. Wilson, Nuovo Cimento **28**, 635 (1963).

mentum and helicity. The B-S equation in the ladder approximation is

$$A(p_2,q_2;\,p_1,q_1)=-\frac{gG}{s-1}+\frac{igG}{(2\pi)^4}\times\int d^4k\,\frac{A(p_2,q_2;\,k_2,k_4)(\gamma\cdot k_2+m)}{(k_2^2-1+i\epsilon)(k_4^2-1+i\epsilon)(k_1^2-1+i\epsilon)},\tag{3.2}$$

where g and G are the $\pi\pi\sigma$ and $NN\sigma$ coupling constants, respectively; for convenience we have set the masses equal to 1. The equation is shown graphically in Fig. 3. The amplitude is then expanded as follows:

$$A(p_2,q_2;\,p_1,q_1)=\sum_{j=1}^{4}A_j(s,t;\,p_1,p_2)O_j(p_1,p_2,q_2),\tag{3.3}$$

where

$$\begin{aligned}O_1(p_1,p_2,q_2)&=1,\\ O_2(p_1,p_2,q_2)&=\gamma\cdot q_2,\\ O_3(p_1,p_2,q_2)&=(\gamma\cdot p_1-m),\\ O_4(p_1,p_2,q_2)&=\gamma\cdot q_2(\gamma\cdot q_2-m).\end{aligned}\tag{3.4}$$

The ρ_i's are the external masses, and we have taken the final particles on shell. Putting the expansion (3.3) into (3.2), we obtain the set of scalar equations

$$A_j(s,t;\,\rho_1\rho_2)=-\delta_{j1}\frac{gG}{s-1}+\frac{igG}{(2\pi)^4}\int d^4k\times\sum_{k=1}^{4}\frac{C_{jk}(s,t;\,\rho_i,\kappa_i)A_k(\kappa_3,t;\,\kappa_2,\kappa_4)}{(k_2^2-1+i\epsilon)(k_4^2-1+i\epsilon)(k_1^2-1+i\epsilon)},\tag{3.5}$$

where the coupling coefficients C_{jk} are defined by

$$O_k(l_2,p_2,q_2)(\gamma\cdot k_2+m)=\sum_{j=1}^{4}C_{jk}(s,t;\,\rho_i,\kappa_i)O_j(p_1,p_2,q_2).\tag{3.6}$$

To find the C_{jk}'s, we follow Ref. 5 and write the momentum integration variable as

$$k_2=\alpha p_1+\beta p_2+\gamma q_2+\delta r,\tag{3.7}$$

where the vector r is orthogonal to p_1, p_2, and q_2. Those terms on the left of (3.6) that are linear in r will vanish in the integral (3.5) by symmetric integration and may therefore be dropped. The C_{jk}'s can then be found in terms of α, β, and γ and are given by

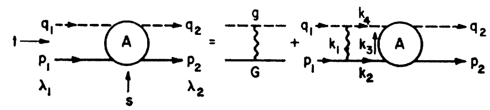

FIG. 3. B-S equation for πN scattering in the ladder approximation.

$$C_{jk}=\begin{bmatrix}1+\alpha+\beta & \gamma+\beta(t-2) & \kappa_2-1 & 0\\ \gamma & 1+\alpha-\beta & 0 & \kappa_2-1\\ \alpha & 0 & 0 & 0\\ 0 & \alpha & 0 & 0\end{bmatrix}.\tag{3.8}$$

Finally, α, β, and γ can be expressed in terms of s, t, ρ_i, and κ_i. For large s the result is

$$\begin{aligned}\alpha&=(1/2s)(2\kappa_3-\kappa_2-\kappa_4-2+t)+O(s^{-2}),\\ \beta&=(1/2t)(\kappa_2-\kappa_4+t)+O(s^{-1}),\\ \gamma&=(1/2t)(\kappa_2-\kappa_4+t)+O(s^{-1}).\end{aligned}\tag{3.9}$$

Since we are looking for an asymptotic solution to the set of equations (2.5), we substitute the asymptotic form of the C_{jk} given by (2.8) and (2.9) into (2.5). Note that we are dropping terms like κ_3/s compared to 1 even though κ_3 becomes comparable to s in the integration region. The justification for this is that we are restricting ourselves to a weak-coupling solution and can check in our final equations that terms like κ_3/s will not contribute to the leading asymptotic behavior in any order of iteration.

The set of equations (2.5) can now be reduced to two sets of coupled equations by taking the following linear combinations:

$$\begin{aligned}\tilde{A}_1&=A_1+(t^{1/2}-1)A_2,\\ \tilde{A}_2&=A_1-(t^{1/2}+1)A_2,\\ \tilde{A}_3&=A_3+(t^{1/2}-1)A_4,\\ \tilde{A}_4&=A_3-(t^{1/2}+1)A_4.\end{aligned}\tag{3.10}$$

Changing variables to the κ_i via the Jacobian (2.8), our equations become

$$\begin{aligned}\tilde{A}_1(s,t;\,\rho_1,\rho_2)&=-\frac{gG}{s-1}+\frac{igG}{4(2\pi)^4}\int\frac{(\prod_i d\kappa_i)\theta(D)}{(\kappa_1-1)(\kappa_2-1)(\kappa_4-1)D^{1/2}}\\ &\qquad\times\{[1+(4t)^{-1/2}(\kappa_2-\kappa_4+t)]\tilde{A}_1(\kappa_3,t;\,\kappa_2,\kappa_4)+[\kappa_2-1]\tilde{A}_3(\kappa_3,t;\,\kappa_2,\kappa_4)\},\\ \tilde{A}_3(s,t;\,\rho_1,\rho_2)&=\frac{igG}{8(2\pi)^4s}\int\frac{(\prod_i d\kappa_i)\theta(D)[2\kappa_3-\kappa_2-\kappa_4-2+t]\tilde{A}_1(\kappa_3,t;\,\kappa_2,\kappa_4)}{(\kappa_1-1)(\kappa_2-1)(\kappa_4-1)D^{1/2}};\\ \tilde{A}_2(s,t;\,\rho_1,\rho_2)&=-\frac{gG}{s-1}+\frac{igG}{4(2\pi)^4}\int\frac{(\prod_i d\kappa_i)\theta(D)}{(\kappa_1-1)(\kappa_2-1)(\kappa_4-1)D^{1/2}}\\ &\qquad\times\{[1-(4t)^{-1/2}(\kappa_2-\kappa_4+t)]\tilde{A}_2(\kappa_3,t;\,\kappa_2,\kappa_4)+[\kappa_2-1]\tilde{A}_4(\kappa_3,t;\,\kappa_2,\kappa_4)\},\\ \tilde{A}_4(s,t;\,\rho_1,\rho_2)&=\frac{igG}{8(2\pi)^4s}\int\frac{(\prod_i d\kappa_i)\theta(D)[2\kappa_3-\kappa_2-\kappa_4-2+t]\tilde{A}_2(\kappa_3,t;\,\kappa_2,\kappa_4)}{(\kappa_1-1)(\kappa_2-1)(\kappa_4-1)D^{1/2}}.\end{aligned}\tag{3.11}$$

Because of the factor s^{-1} in the equations for $\tilde{A}_3$ and $\tilde{A}_4$, these amplitudes will be asymptotically nonleading with respect to $\tilde{A}_1$ and $\tilde{A}_3$ in each order of iteration. Therefore, in the approximation of keeping only the leading term to each order of iteration, we set $\tilde{A}_3=\tilde{A}_4=0$ in (3.11). We then have two uncoupled equations for $\tilde{A}_1$ and $\tilde{A}_2$, which differ from the scalar B-S equation only by the factors $[1\pm(4t)^{-1/2}(\kappa_2-\kappa_4+t)]$. As promised in Sec. II, this factor is independent of κ_1 and κ_3, and we can therefore apply the method of I to obtain the asymptotic equations

$$\tilde{A}^{\pm}(s,t)=-\frac{gG}{s}-\frac{gGK^{\pm}(t)}{s}\int_1^s d\kappa \tilde{A}^{\pm}(\kappa,t)\,, \quad (3.12)$$

where

$$K^{\pm}(t)=\lim_{\eta\to 0}\frac{1}{32\pi^3}\times\int\frac{d\kappa_2 d\kappa_4\theta[-\Delta(\kappa_2,\kappa_4,t)-\eta][1\pm(4t)^{-1/2}(\kappa_2-\kappa_4+t)]}{(\kappa_2-1+i\epsilon)(\kappa_4-1+i\epsilon)[-\Delta(\kappa_2,\kappa_4,t)]^{1/2}}, \quad (3.13)$$

and we have set $\tilde{A}^{+}=\tilde{A}_1$ and $\tilde{A}^{-}=\tilde{A}_2$. The function defined in (3.13) can easily be seen to be finite, and so the method of I is indeed applicable. The unique solutions to (3.12) are

$$\tilde{A}^{\pm}(s,t)=-gGs^{\alpha^{\pm}(t)}\,, \quad (3.14)$$

where

$$\alpha^{\pm}(t)=-1+gGK^{\pm}(t)\,. \quad (3.15)$$

These trajectories agree with the result of Ref. 5 for the weak-coupling limit of the multiperipheral model. Actually, for the equal-mass case we have considered, the term involving $(\kappa_2-\kappa_4)$ in the numerator of (3.13) vanishes in the limit $\eta=0$. We reinstate unequal masses μ and m for the pion and nucleon and obtain

$$K^{\pm}(t)=(gG/16\pi^3)\{mF_1(t)\pm\tfrac{1}{2}(\surd t)\times[F_1(t)+F_2(t)]\}\,, \quad (3.16)$$

where

$$F_1(t)=2\ln\left\{\frac{[(m-\mu)^2-t]^{1/2}+[(m+\mu)^2-t]^{1/2}}{2(m\mu)^{1/2}}\right\}\Big/\{[(m-\mu)^2-t][(m+\mu)^2-t]\}^{1/2}, \quad (3.17)$$

and

$$F_2(t)=(1/t)[\ln(\mu/m)+(m^2-\mu^2)F_1(t)]\,. \quad (3.18)$$

Note that the trajectories $\alpha_{\pm}(t)$ coincide at $t=0$, become complex conjugates of each other for $t<0$, and satisfy the Gribov[6] condition $\alpha^{+}(\surd t)=\alpha^{-}(-\surd t)$.

We may easily generalize these results to the case where the kernel contains only the covariant O_1 but is an otherwise arbitrary "planar kernal," i.e.,

$$B(p_2,q_2;\,p_1,q_1)=B_1(s,t;\,\rho_1,\rho_2)\,,\qquad B_i(s,t;\,\rho_1,\rho_2)=0,\quad i=2-4 \quad (3.19)$$

$$B_1(s,t;\,\rho_1,\rho_2)=\int_0^\infty da\,\frac{\sigma(a,t;\,\rho_1,\rho_2)}{s-a-i\epsilon}, \quad (3.20)$$

and

$$B_1(s,t;\,\rho_1,\rho_2)\to b_1(t)[(\ln s)^p/s^q],\quad \text{as}\quad s\to\infty\,. \quad (3.21)$$

The calculation of the coupling coefficients proceeds exactly as above, and one again obtains decoupled equations similar to (3.11). The asymptotic behavior is now given by

$$\tilde{A}_1\sim s^{\alpha+(t)},\quad \tilde{A}_3\sim s^{\alpha-(t)}\,, \quad (3.22)$$

where

$$\alpha_{\pm}(t)=-q+[p!b_1(t)G^{\pm}(t)]^{1/(p+1)}\,, \quad (3.23)$$

and

$$G^{\pm}(t)=\frac{1}{(q-1)!}\left(-\frac{\partial}{\partial m^2}-\frac{\partial}{\partial\mu^2}\right)^{q-1}K^{\pm}(t)\,, \quad (3.24)$$

with $K^{\pm}(t)$ given by (3.16).

This analysis can also be carried out when the kernel contains only the covariant $B_2(s,t;\,\rho_1,\rho_2)$. Unfortunately, an arbitrary planar Feynman diagram contains all four covariants (3.4), and then one must use the $4\times4\times4$ object $C_{jkl}(s,t;\,\rho_i,\kappa_i)$. We have not attempted to solve this general case, but it would seem reasonable that, given a specific set of B_i's, one could find some linear combination of the A_i's that decouple the integral equations. Our analysis could then be applied to give the appropriate Regge trajectories.

IV. HIGHER SPIN

In this section we shall consider planar kernels for which (2.16) is divergent. Then (2.15) must be replaced by the result of a more careful analysis of Eq. (2.12). The resultant equation will not imply Regge asymptotic behavior in general, but will give rise to fixed cuts, etc., in the complex J plane. After briefly considering the general case and presenting some simple examples, we apply our heuristic techniques to $N\bar{N}$ scattering.

We consider Eq. (2.15), where, for simplicity, we include only one term in the sum. We include a regularization factor $[\Lambda/(\kappa+\Lambda)]^{1/2}$ inside the κ integration in (2.16) to render the integral finite. Then for large Λ we will have a behavior of the form

$$K(t)\sim L(t)\Lambda^{q'}(\ln\Lambda)^{p'}\,, \quad (4.1)$$

where $q'\geq 0$. In this case we expect (2.15) to be replaced by

$$\tilde{A}(s,t)=\tilde{b}(t)s^{-q}(\ln s)^p+\Lambda(t)s^{q'-q}(\ln s)^{p'}\times\int_1^s d\kappa\left(\ln\frac{s}{\kappa}\right)^p\kappa^{q-1}\tilde{A}(\kappa,t)\,. \quad (4.2)$$

[6] V. N. Gribov, Proceedings of the 1962 International Conference on High-Energy Physics, CERN, p. 547 (unpublished).

The solution to this equation will *not*, in general, be of the Regge form.

We expect the integral equation (4.2) to result when the "trajectory function" $K(t)$ has the behavior (4.1) for the reason that was the basis of the heuristic analysis of I. There we saw that

$$I(s)\equiv\int_0^\infty da\,\frac{\sigma(a)}{s+a}\sim\frac{1}{s}\int_0^s da\sigma(a)$$

for suitable functions $\sigma(a)$. Thus, the effect of a divergent integral $\int_0^\infty da\,\sigma(a)$ on the formal high-s behavior

$$\frac{1}{s}\int_0^\infty da\,\sigma(a)$$

of $I(s)$ leads to the replacement

$$\int_0^\infty da\,\sigma(a)\leftrightarrow\int_0^s da\,\sigma(a).$$

In I this procedure was applied to the (divergent) κ_3 integration, but not to the (κ_2,κ_4) integration, since that was already convergent. In the present situation, however, the (κ_2,κ_4) integration can be divergent, and then the above procedure applied to this integration leads to the result (4.2).

As a simple illustration, we consider the equation

$$A(s)=g^2+g^2s^{-1}(\ln s)^r\int_1^s d\kappa\,A(\kappa).$$

Keeping only the leading term in each iteration, the Neumann series becomes

$$g^2\sum_{n=0}^{\infty}[g^2(\ln s)^r]^n=g^2[1-g^2(\ln s)^r]^{-1}.$$

Thus

$$A(s)\xrightarrow[s\to\infty]{}(\ln s)^{-r},$$

and we obtain an asymptotic behavior that does not correspond to a pole in the complex J plane.

Another interesting example is

$$A(s)=1+\int_0^s d\kappa\,A(\kappa).$$

The Neumann series is

$$\sum_{n=0}^{\infty}\frac{s^n}{n!}=e^s.$$

This behavior is again not of the Regge form.

As a final example, we consider the equation

$$A(s)=gs^{-1}+2N^2s^{-1}\ln s\int_1^s d\kappa\,A(\kappa),\qquad(4.3)$$

which will be of use to us in our discussion of $N\bar{N}$ scattering. The corresponding (exact) Neumann series is

$$A(s)=\frac{g}{s}\sum_{n=0}^{\infty}\frac{(2N^2\ln^2 s)^n}{(2n-1)!!}.\qquad(4.4)$$

The solution (4.4) can be expressed in terms of the error function, which has the series expansion[7]

$$\operatorname{erf}(z)=\frac{2}{\sqrt{\pi}}e^{-z^2}\sum_{n=0}^{\infty}\frac{2^nz^{2n+1}}{(2n+1)!!}.\qquad(4.5)$$

We find

$$A(s)=gs^{-1}+gN\pi^{1/2}s^{-1}\ln s\exp(N^2\ln^2 s)\operatorname{erf}(N\ln s).\qquad(4.6)$$

Use of the property[8]

$$\operatorname{erf}(z)\to 1$$

(for $z\to\infty$ in $|\arg z|<\frac{1}{4}\pi$) gives the asymptotic behavior

$$A(s)\xrightarrow[s\to\infty]{}gN(\sqrt{\pi})s^{-1}\ln s\exp(N^2\ln^2 s).\qquad(4.7)$$

Again, this behavior does not correspond to a Regge pole in the complex J plane.

Let us now consider $N\bar{N}$ scattering in the ladder approximation with pion exchange. Swift and Lee[9] have shown that the leading high-energy behavior in this theory is given by a fixed cut. They worked with the "fusion" amplitudes[10] $\{U_i\}$ and found that the B-S equations for the partial-wave amplitudes $\{U_i{}^J\}$, as a consequence of parity and "spin exchange" invariance, completely decoupled in the high-energy $(J\to 0)$ limit in the approximation of keeping only the most singular terms $\{\tilde{U}_{i,n}{}^J\}$ in each order n. They then summed these terms and found that $\tilde{U}_i{}^J\equiv\sum_n\tilde{U}_{i,n}{}^J$ had a fixed cut in J leading to large-s behavior of the form

$$(\ln s)^{3/2}s^G.\qquad(4.8)$$

As an illustration of how similar results can be obtained in our formalism, we consider an s-space analog of typical Swift-Lee partial-wave equation. In terms of the κ variables, we find

$$A(s,t;\rho_1,\rho_2)=\frac{-g^2}{s-1}+\frac{ig^2}{4(2\pi)^4}\times\int\prod_i d\kappa_i\frac{A(\kappa_3,t;\kappa_2,\kappa_4)}{(\kappa_1-1)(\kappa_2-1)(\kappa_4-1)}\frac{\theta(D)}{D^{1/2}}\frac{(\kappa_2-\kappa_4)^2}{4t}.\qquad(4.9)$$

The factor $(\kappa_2-\kappa_4)^2/4t$ corresponds to the $\tilde{C}_j$ in Eq. (2.12) and represents the effect of spin. The appropriate

[7] See, e.g., M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, Inc., New York, 1965), p. 297.

[8] Reference 7, p. 298.

[9] A. R. Swift and B. W. Lee, Phys. Rev. **131**, 1857 (1963).

[10] These amplitudes were introduced by M. Gourdin, Nuovo Cimento **7**, 338 (1958). Their relation to the usual helicity amplitudes [M. Goldberger *et al.*, Phys. Rev. **120**, 2250 (1960)] is given in Ref. 9.

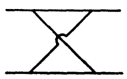

FIG. 4. X-diagram.

trajectory function (2.16) is thus formally

$$K(t)=\frac{1}{32\pi^3}\int\frac{d\kappa_2 d\kappa_4\theta(-\Delta)}{(\kappa_2-1)(\kappa_4-1)(-\Delta)^{1/2}}\frac{(\kappa_2-\kappa_4)^2}{4t}. \quad (4.10)$$

To investigate the properties of (4.10), we change integration variables to

$$x=\kappa_2-\kappa_4,\quad y=\kappa_2+\kappa_4.$$

Then

$$\Delta=x^2-2ty+t^2,$$

so that (4.10) becomes

$$K(t)=\frac{1}{(4\pi)^3 t}\int_{-\infty}^{\infty}dx\, x^2\int_{-\infty}^{y_0}dy(y+x-2)^{-1}$$
$$\times(y-x-2)^{-1}(-x^2+2ty-t^2)^{-1/2}, \quad (4.11)$$

where

$$y_0=(t^2+x^2)/2t.$$

For large x, the y integral is easily seen to behave as $\sim x^{-3}$, so that (4.11) is logarithmically divergent. Thus, in accordance with our general remarks above, we expect (4.9) to have precisely the form (4.3) in the high-energy limit. A more detailed argument that (4.3) follows from (4.9) can be given along the lines of the heuristic analysis of I. The calculation is, however, tedious, and we omit it here.

Since the solution (4.6) of (4.3) does not correspond to a Regge pole, the above anslysis constitutes a heuristic but simple argument for the existence of more complicated J-plane singularities in the ladder approximation to $N\bar{N}$ scattering. Our expression (4.7) is rather different from the Swift-Lee–type result (4.8), but this is expected, since our starting equations are not the same. Furthermore, our approximation schemes are quite distinct. Swift and Lee summed the most singular partial-wave amplitudes $U_n{}^J$ in each order and then transformed the sum U^J back to s space to obtain their high-energy approximation $U(s)$, whereas our approximation amounted to summing directly the leading high-energy behaviors in s space. In fact, one easily sees from the Swift-Lee expressions for $U_n{}^J$ that the sum of their leading high-energy behavior $U_n(s)$ (the Sommerfeld-Watson transform of their $U_n{}^J$'s) must be an analytic function of $\ln s$ and hence cannot be of the form (4.8).[11]

We need not emphasize the lack of rigor in the discussions of this section, which renders our results only suggestive. We feel, nevertheless, that the simplicity and intuitive appeal of our approach make it a possibly useful method for the study of more complicated singularities in the complex J plane.

V. NONPLANAR KERNAL

As a final final application of our methods, we consider the B-S equation with the nonplanar X kernel (see Fig. 4). The solution to this equation has already been shown[12] to have an essential singularity in the complex J plane at $J=-1$—the so-called Gribov-Pomeranchuk[13] singularity. These rigorous calculations involved a rather complicated J-plane analysis. We shall show how the same result emerges in our formalism in a much simpler, but heuristic, way. We shall also obtain an integral equation for the residue functions of the Regge trajectories tending to $J=-1$.

The integral equation in question is

$$A(s,t;\rho_1,\rho_2)=X(s,t;\rho_1,\rho_2,1,1)$$
$$-\frac{i}{4(2\pi)^4}\int\frac{d\kappa_2 d\kappa_4}{(\kappa_2-1)(\kappa_4-1)}\int d\kappa_3\, A(\kappa_3,t;\kappa_2,\kappa_4)$$
$$\times\int d\kappa_1\, X(\kappa_1,t;\rho_1,\rho_2,\kappa_2,\kappa_4)\frac{\theta(D)}{D^{1/2}}, \quad (5.1)$$

and we write the dispersion relation

$$X(s,t;\rho,\kappa)=\int_4^{\infty}ds'\frac{\sigma(s',t;\rho,\kappa)}{s'-s-i\epsilon}+\int_{-\infty}^{s_L}ds'\frac{\sigma(s',t;\rho,\kappa)}{s'-s+i\epsilon}, \quad (5.2)$$

where $\rho\leftrightarrow(\rho_1,\rho_2)$, and $\kappa\leftrightarrow(\kappa_2,\kappa_2)$. As shown in I, the κ_1 integral can be done exactly and gives, in the large-s limit,

$$\int d\kappa_1\, X(\kappa_1,t;\rho,\kappa)\frac{\theta(D)}{D^{1/2}}\sim i\pi F(t;\rho,\kappa)\frac{\theta(C)}{\sqrt{C}}, \quad (5.3)$$

where

$$F(t;\rho,\kappa)\equiv\int_4^{\infty}ds'\,\sigma(s',t;\rho,\kappa)-\int_{-\infty}^{s_L}ds'\,\sigma(s',t;\rho,\kappa), \quad (5.4)$$

and

$$C=\alpha\kappa_3{}^2+\beta\kappa_3+\gamma, \quad (5.5)$$

with

$$\alpha=0(1),\quad \beta=0(s),\quad \gamma=-s^2\Delta(\kappa_2,\kappa_4,t)+O(s). \quad (5.6)$$

As discussed in I, it is the difference in the signs of the $i\epsilon$'s in the two terms in (5.2) that is responsible for the occurrence of the *difference* in (5.4). This means that

[11] A typical Swift-Lee expression [cf. Eq. (22) of Ref. 9] is $U_n{}^J\sim J^{1-2n}(2n-3)!/(n-2)!n!$, and this corresponds in s space to asymptotic behaviors $\sim s^{-1}(\ln s)^{2n-1}/(n-2)!n!(2n-1)$, which sum to an analytic function of $\ln s$.

[12] F. Kaschluhn and W. Zoellner, Nuovo Cimento **34**, 1618 (1964); A. D. Contogouris, *ibid.* **36**, 250 (1965).

[13] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters **2**, 239 (1962).

the large-s behavior

$$X\sim J(t)(\ln^2 s/s^2)\,, \tag{5.7}$$

which is governed by the sum

$$\int_4^\infty ds'\,\sigma+\int_{-\infty}^{sL} ds'\,\sigma=0 \tag{5.8}$$

is not relevant in (5.1) and invalidates the heuristic approach of I.

Nevertheless, we can proceed by applying the heuristic analysis to the equation obtained by using the correct result (5.3) in (5.1). In view of (5.6), we obtain

$$\tilde{A}(s,t;\,\rho)=J(t)\frac{\ln^2 s}{s^2}+\frac{1}{64\pi^3 s}\times\int\frac{d\kappa_2 d\kappa_4 F(t;\,\rho,\kappa)\theta(-\Delta)}{(\kappa_2-1)(\kappa_4-1)(-\Delta)^{1/2}}\int_1^s d\kappa_3\,\tilde{A}(\kappa_3,t;\,\kappa)\,. \tag{5.9}$$

The new features here are (i) the mass dependence of the asymptotic amplitude $\tilde{A}$—a consequence of the relevancy of the mass-dependent function (5.4) rather than the mass-independent function (5.7) in (5.9); and (ii) the difference of a factor s between the Born term and the remainder in (5.9)—a consequence of the fact that "pinch" contributions give the leading high-energy terms.

We substitute the function

$$\tilde{A}(s,t;\,\rho)=s^{\alpha(t)}f(t;\,\rho) \tag{5.10}$$

in (5.9) and, assuming that $\alpha(t)>-1$, obtain the asymptotic homogeneous integral equation

$$f(t;\,\rho)=\frac{1}{64\pi^3(\alpha+1)}\times\int\frac{d\kappa_2 d\kappa_4 F(t;\,\rho,\kappa)}{(\kappa_2-1)(\kappa_4-1)}\frac{\theta(-\Delta)}{(-\Delta)^{1/2}}f(t;\,\kappa)\,. \tag{5.11}$$

In the usual way, this equation now leads to the accumulation of an infinite number of trajectories at $\alpha(t)=-1$ and, therefore, to the existence of an essential singularity in the J plane at $J=-1$. This is because (5.11) is essentially a homogeneous Fredholm integral equation with a nonseparable symmetric kernel and hence[14] posseses a countable infinity of eigenvlaues $[64\pi^3(\alpha_n+1)]^{-1}$, $n=1,\,2,\,3,\,\cdots$, with an accumulation point at infinity, i.e., for $\alpha_n\to-1$. For each eigenvalue $\alpha_n(t)$, Eq. (5.11) determines the corresponding $f_n(t;\,\rho)$, which is simply related to the residue function of the Regge pole in the J plane.

Our heuristic momentum-space analysis is thus seen to suggest the same results as the rigorous J-plane calculations. We have, in addition, obtained the simple homogeneous integral equation (5.11) for the residue functions. This equation may be of use in understanding some properties of the Gribov-Pomeranchuk singularity.

VI. CONCLUDING REMARKS

We have tried to extend our work on asymptotic solutions to B-S equations to include spin and nonplanar kernels. We have seen in the simplest generalization to scalar-spinor scattering that spin is not an essential complication. We simply obtain a set of equations that must first be decoupled before the Regge trajectories can be obtained. However, in the case of fermion-fermion scattering (and presumably all higher-spin problems), the heuristically determined trajectory functions are given by divergent integrals, and one is led to integral equations which imply asymptotic behaviors corresponding to more complicated singularities in the J plane. Similarly, when the kernel is given by the nonplanar X graph, the heuristic method again breaks down, and one obtains a homogeneous integral equation for which the Regge trajectories $\alpha_n(t)$ (with an accumulation point at $\alpha=-1$) are the eigenvalues and the Regge residue functions are essentially the eigenfunctions.

These different effects have analogs in the standard asymptotic analysis of individual Feynman graphs. In the case of π-N scattering with scalar exchange, the asymptotic behavior of the $(n+1)$-rung ladder diagram is still given by the end point contribution corresponding to contraction of each of the rungs. The asymptotic ladder diagrams therefore sum up to give Regge behavior just as in the scalar case. In situations with higher spin, the asymptotic behavior is governed by the so-called "displacement contributions" and "singularity contributions."[15] These effects are still generalized end-point–type contributions in the Feynman parameter integration, and the contributions of the different ladders still add up coherently to give an asymptotic behavior corresponding to fixed cuts, etc. In the case of the nonplanar X kernel, the asymptotic behavior of the iterations is governed by "pinch" contributions. The individual iterations still behave like $s^{-1}(\ln)^n$, but the coefficients are so uncorrelated that an infinite number of Regge terms [with $\alpha=-1+O(g^2)$] is needed to produce this series. This gives rise to the essential singularity at $J=-1$.

Our methods in this paper have certainly not been rigorous, but we hope they may be used to gain an intuition about the complicated effects that result from summing infinite sets of diagrams. One of the most interesting questions to which these methods may have some application is that of the Reggeization of the

[14] See, e.g., R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Wiley-Interscience, Inc., New York, 1953), Vol. 1, Chap. III.

[15] P. G. Federbush and M. T. Grisaru, Ann. Phys. (N. Y.) **22**, 299 (1963); J. C. Polkinghorne, J. Math. Phys. **5**, 1491 (1964).

nucleon in vector-spinor theory.[3] Here the presence of spin and nonplanar graphs, together with radiative corrections and renormalization, all play important roles in showing that the nucleon lies on a Regge trajectory through sixth order of perturbation theory. Attempts to extend this result to higher orders have met with great difficulty,[3] and perhaps our methods will prove more useful here than the usual techniques.

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Pion-Nucleon Charge-Exchange Scattering and the Crossover Phenomenon with $M=0$ ρ and $M=1$ ρ' Trajectories*

SHU-YUAN CHU† AND BIPIN R. DESAI
Department of Physics, University of California, Riverside, California 92502

AND

D. P. ROY
CERN, Geneva, Switzerland

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A Regge-pole model with ($M=0$) ρ and ($M=1$) ρ' trajectories is used to fit the πN charge-exchange differential cross-section and polarization data. The model automatically predicts the crossover phenomenon around $t=-0.2$ GeV^2 in πN elastic scattering without assuming any zeros in the nonflip ρ residue. Thus it avoids difficulties of such zeros due to factorization. The parameters of the ρ and ρ' are consistent with the nucleon electromagnetic form factors and predict the mass of ρ' to be around 1.1 GeV.

I. INTRODUCTION

In high-energy πN, KN, and NN elastic scattering, it is found that the cross-section difference $d\sigma/dt(AB \to AB) - d\sigma/dt(\bar{A}B - \bar{A}B)$ changes sign[1] around $t=t_0 \sim -0.2$ GeV^2 (here $\bar{A}$ means the antiparticle of A). This "crossover" phenomenon in the previous Regge-pole models had been attributed to the presence of a zero at $t=t_0$ in the helicity-nonflip residue functions of the ρ- and the ω-exchange amplitudes.[2–4] There are two objections to such a zero: (1) Factorization would imply such a zero to be present in the ω residue functions for all channels,[4,5] which contradicts the $\gamma p \to \pi^0 p$ data[6,7]; (2) factorization also implies (assuming simple zeros) that the $\rho\pi\pi$ residue vanishes at t_0. Since the same $\rho\pi\pi$ residue appears in both πN residues, this in turn would imply that in, addition to helicity nonflip, the helicity-flip πN residue is also zero at t_0. This is in contradiction with the experimental πN charge-exchange data. Alternative explanations to avoid the difficulty of a zero have been suggested.[7]

We propose that the relevant πN data can be explained in terms of the (usual) $M=0$ Regge trajectory together with a conspiring $M=1$ ρ' trajectory with otherwise the same quantum numbers as the ρ tra-

TABLE I. Summary of results.

Parameters			
A_ρ (GeV mb$^{1/2}$)	-5.34	$A_{\rho'}$ (GeV^{-1} mb$^{1/2}$)	-25.7
B_ρ (GeV^{-2})	3.92	$B_{\rho'}$ (GeV^{-2})	0.92
C_ρ (GeV^{-1} mb$^{1/2}$)	-5.50	$C_{\rho'}$ (GeV^{-1} mb$^{1/2}$)	-9.67
D_ρ (GeV^{-2})	1.81	$D_{\rho'}$ (GeV^{-2})	0.43
α_ρ	$0.60+0.83t$	$\alpha_{\rho'}$	$0.32+0.64t$

χ^2 comparison		This work	Ref. 8
$d\sigma/dt$	χ^2	84.4	97.4
Data points		70	57
$p(d\sigma/dt)$	χ^2	6.8	3.6
Data points		12	12
Total	χ^2	91.2	101
No. of points		82	69
No. of parameters		12	11

Crossover point		Residue at the ρ pole		
E_L (GeV)	t (GeV^2)		Calculated	Ref. 2
3	-0.19	nonflip	0.64	0.87
7	-0.21	flip	4.0	3.98
11	-0.22			
15	-0.23			

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† Present address: Department of Physics, Indiana University, Bloomington, Ind. 47401.

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