Self-Coupling of a Two-Level System by a Mirror

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We discuss, from a classical as well as a quantum-mechanical point of view, the effect of the coupling of an excited two-level system with itself due to emission and absorption of electric-dipole radiation, the emitted radiation being reflected by a nearby mirror. Expressions familiar from resonance-scattering theory are found for the shift of the transition frequency and linewidth from their values in the absence of the mirror. This renormalization effect leads to spatial modulation in the transition frequency and width of the excited state due to its resonant interaction with itself via the radiation field reflected from the mirror. This formalism is applied to some recent experiments with monomolecular dye layers. The experimental values of the fluorescence lifetime are in good agreement with the theory for distances from 500 to 6000 Å.

I. INTRODUCTION

A REFLECTOR, brought within a distance of order λ (the wavelength of electromagnetic radiation) changes the radiation pattern of a current distribution oscillating at an angular frequency $\omega=2\pi c/\lambda$. This fact is amply exploited in the design of radio wave and microwave antennas.[1] It is plausible (though certainly not as widely known) that the same effect persists at a level at which the radiating system is described by quantum mechanics, i.e., by an atom or ion in its excited state, so that the oscillating classical current distribution is replaced by the transition current.

In this paper, we intend to give a discussion of this effect in the optical region in connection with its recent experimental observation.[2,3] The observation was made possible by a special technique for the deposition of monomolecular layers of potential radiators (Eu^{3+} ions) at well-controlled distances (down to 20 Å) from a metal mirror.

Several viewpoints, all physically equivalent, are possible for the interpretation of this phenomenon. We list these in passing. In the jargon of field theory, one can speak of renormalization of the energy levels and width of the oscillator (two-level system) by its interaction with the radiation field, modulated by the nearby reflector. The self-energy modifies both the real and the imaginary parts of the energy eigenvalue, leading to shifts from its value in the absence of the mirror. The directional dependence of the radiation on the orientation of the transition dipole with respect to the mirror normal (parallel or perpendicular to it) introduces an additional distinguishing feature in the renormalization.

Alternatively, one can consider the problem in terms of multiple-scattering theory. Part of the originally emitted light wave is reflected back to the emitter and scattered from it with a finite probability for reabsorption. This probability can be incorporated into the theory by adding an imaginary part in the resonant-scattering amplitude. If we neglect the recoil of the atom or equivalently, assume the radiating system to be held fixed in its environment—an hypothesis well justified in the case of the reported experiments with Eu^{3+} ions bound in an organic cage[2,3], then the multiple-scattering integral equations reduce to a set of coupled algebraic equations. These are readily solved by standard methods as first discussed by Chew and Wick[4] and by Brueckner[5] in connection with s-wave scattering from two fixed scattering centers. The solutions for the scattering amplitude lead directly to resonant amplitudes with correspondingly shifted eigenfrequencies and widths.

One can also consider the effect to be a manifestation of the coherence between the emitted radiation and the two-level system. If we view the emitted radiation as a wave packet (photon) of size c/γ_0, it overlaps itself and the radiating system, because of reflection from the mirror and produces constructive and destructive interference in the emission pattern of the excited ion. Some care has to be exercised in this viewpoint, since from the quantum-statistical point of view we are discussing an ensemble of two-level systems, and the photon either is emitted into the solid angle subtended by the mirror, and hence suffers reflection, or is emitted directly into unobstructed space. The resulting interference pattern is due to the coherent addition of the probability amplitudes for the two types of emission direction in a manner similar to the analysis of the double-slit experiment for a single photon.

Treating the problem now in terms of the appropriate nonrelativistic quantum mechanics for the emitter, we find it advantageous to avail ourselves of an old trick, going back to Sommerfeld's[6] discussion of the propagation of radio waves along the earth's surface. We replace the mirror by an image system located a distance D behind the mirror. This approximation assumes the mirror to have reflectivity $\rho=1$ (corresponding to infinite conductivity) and a constant phase

[1] Microwave Antenna Theory and Design, edited by S. Silver, M. I. T. Radiation Laboratory Series, 1963 (unpublished).
[2] K. H. Drexhage, Habilitationschrift, University of Marburg, 1965 (unpublished).
[3] K. H. Drexhage, H. Kuhn, and F. D. Schafer, Ber. Bunsen Ges. Phys. **72**, 329 (1968).
[4] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952).
[5] K. A. Brueckner, Phys. Rev. **89**, 834 (1953); also M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 762.
[6] A. Sommerfeld, Ann. Physik **28**, 665 (1909).

change $\delta=\pi$ on reflection for the electric field amplitude. The problem then reduces to the resonant interaction of two two-level systems separated by a distance $R=2D$ treated by Lyuboshitz.[7,8] The substitution of the image system in its ground state for the effect of the mirror vividly expresses the essential self-coupling of the two states of the originally excited ion through its coupling to the electromagnetic field reflected from the mirror. One then introduces properly symmetrized or antisymmetrized eigenstates of the zero-order Hamiltonian. The evaluation of the matrix elements of the dipole-interaction Hamiltonian between the entire electric field (including the static and induction fields, since we are focusing on the near-field region as far as coupling is concerned) and the matter system leads to the real and imaginary parts of the self-energy, which correspond to shifts of the transition frequency and width. The two-level system can be treated as having spin 1 in its upper state and spin 0 in its lower state, to account for the electric-dipole character of the transition.

The formulas found for the change of frequency and width as a function of $\kappa=kR=4\pi D/\lambda$ by either classical or quantum-mechanical reasoning are identical, and agree well with measurements of the Eu^{3+} fluorescence lifetime after taking account of their random orientation in the monomolecular dye later by averaging with the appropriate weight factors of $\frac{1}{3}$ for the parallel alignment with the mirror normal and $\frac{2}{3}$ for the perpendicular case. Because the density of radiators is kept sufficiently low, the neglect of interaction between neighboring ions seems amply justified. For distances smaller than 500 Å, deviations from the derived expressions are observed, probably due to the onset of different forms of deexcitation. The shift in the resonance frequency was not looked for in the experiments mentioned, but should in principle be observable for sufficiently small distances, since it goes like $\Delta\omega\sim\gamma_0/\kappa^3$, where γ_0 is the free-space linewidth.

It is easy to see from the various interpretations that the effect disappears for $D\gg\lambda$, because of the limited range of the dipole-dipole interaction.

II. CLASSICAL TREATMENT OF THE INTERACTION OF AN OSCILLATING, DAMPED DIPOLE WITH A MIRROR

From a classical point of view, we may replace the excited ion by a damped oscillator emitting electric-dipole radiation in front of a mirror. Simulating the effect of the mirror by substituting an image system at a distance D behind the mirror, we can now use some of the results obtained by Lyuboshitz[7,8] on the resonant interaction of two identical dipole emitters separated by a distance $R=2D$, which is of the same order of magnitude as the wavelength of the emitted radiation

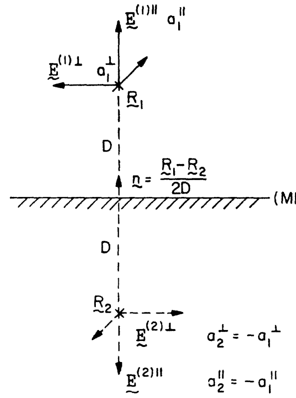

FIG. 1. Excited two-level system at $\mathbf{R}_1$, and the image system at $\mathbf{R}_2$, simulating the effect of the mirror. The two possible orientations of the dipole axis, $E^{(1)\perp}$ and $E^{(1)\|}$, and the phase conventions for the corresponding scattering amplitudes $a_1^{\|}=a_1^{\perp}=a=(3/4k)\gamma_0/(\omega_0-\omega-\frac{1}{2}i\gamma_0)$, $a_2^{\perp}=-a_1^{\perp}$, $a_2^{\|}=a^{1\|}$, are indicated.

(Fig. 1). We note from Fig. 1 that, because of the reflection symmetry of the arrangement, we have to use an out-of-phase oscillator for $\mathbf{R}\perp\mathbf{e}^{(1)}$, and an in-phase oscillator for $\mathbf{R}\|\mathbf{e}^{(1)}$, where $\mathbf{e}^{(1)}$ is in the direction of the dipole axis, or, equivalently, is parallel to the polarization vector of the original excitation field. For a single center, the scattering amplitude near resonance has the form

$$T(\omega)=(3/4k)[\gamma_0/(\omega_0-\omega-\tfrac{1}{2}i\gamma_0)](\mathbf{e}^{(1)}\cdot\mathbf{e}^{(2)*}),\qquad(1)$$

where $k=\omega/c$ and $\gamma_0=\frac{2}{3}e^2\omega_0^2/mc^3$ is taken from the classical theory of radiation damping, and $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)*}$ are polarization vectors before and after the scattering. The crucial physical quantity of interest is the polarizability tensor $\epsilon_{\alpha\beta}$, since in the dipole approximation the scattering amplitude is proportional to the contraction of this tensor, with $e_\alpha^{(0)}$ and $e_\beta^{(2)*}$ corresponding to the scalar product of the induced dipole moment and the scattered field. In the isotropic case (for a single symmetric center), $\epsilon_{\alpha\beta}\sim\delta_{\alpha\beta}$ and the induced dipole moment is in the direction of the incident field. The presence of the second center (or the mirror) reduces the spherical symmetry to cylindrical symmetry because the total field of an oscillating dipole at the location of the second oscillator contains both longitudinal and transverse components. The field of an oscillating dipole at $R_{1,2}$ seen at $R_{2,1}$ is given by the classical formula

$$\mathbf{E}_{1,2}=e^{\mp i\mathbf{k}\cdot\mathbf{R}}[\mathbf{d}_{1,2}k^2+\nabla(\mathbf{d}_{1,2}\nabla)]e^{ikR}/R,\qquad(2)$$

where $\mathbf{R}=\mathbf{R}_2-\mathbf{R}_1$ ($R=|\mathbf{R}|$) is the radius vector between the two centers, and $\mathbf{d}_{1,2}$ are their dipole moments. The original dipole moment $\mathbf{d}_1\sim\mathbf{e}^{(1)}$, but due to the presence of the second center, the most general polarizability tensor that can be formed from the available vectors is

$$\epsilon_{\alpha\beta}^{(1,2)}=\alpha_{1,2}\delta_{\alpha\beta}+\beta_{1,2}\eta_\alpha\eta_\beta,\qquad(3)$$

[7] V. L. Lyuboshitz, Zh. Eksperim. i Teor. Fiz. **52**, 926 (1967) [English transl.: Soviet Phys.—JETP **25**, 612 (1967)].
[8] V. L. Lyuboshitz, Zh. Eksperim. i Teor. Fiz. **53**, 1630 (1967) [English transl.: Soviet Phys.—JETP **26**, 937 (1968)].

with $\boldsymbol{\eta}=\mathbf{R}/|\mathbf{R}|$ and $\alpha_{1,2}$, $\beta_{1,2}$ are as yet undetermined coefficients. Writing the induced dipole moments $\mathbf{d}_{1,2}$ in terms of the total-field, individual-center scattering amplitude $a_{1,2}$, and using Eqs. (2) and (3), we find

$$\mathbf{d}_{1,2}=a_{1,2}(\mathbf{e}^{(1)}+\mathbf{E}_{2,1}), \tag{4}$$

with

$$a_{1,2}=\pm a=\pm(3/4k)\gamma_0/(\omega_0-\omega-\tfrac{1}{2}i\gamma_0).$$

$a_{1,2}$ expresses the resonance nature of the process, and the + or − sign depends on whether we consider oscillation in or out of phase, depending on the original orientation, as mentioned earlier. Substituting from (2) and (3) we find, for the coefficients $A_i=k^2\alpha_i$ and $B_i=k^2\beta_i$ ($i=1,2$), the coupled equations

$$A_{1,2}=a_{1,2}\left[1+\frac{A_{2,1}e^{ikR}}{R}\left(1+\frac{i}{\kappa}-\frac{1}{\kappa^2}\right)e^{\pm i\mathbf{k}\cdot\mathbf{R}}\right],$$

$$B_{1,2}=\frac{-a_{1,2}e^{ikR\pm i\mathbf{k}\cdot\mathbf{R}}}{R}\left[A_{2,1}\left(1+\frac{3i}{\kappa}+\frac{3}{\kappa^2}\right)-\frac{2B_{2,1}}{\kappa}\left(-i+\frac{1}{\kappa}\right)\right]. \tag{5}$$

The solutions are readily found to be

$$A_{1,2}=\frac{a[1+(ae^{ikR}/R)(1+i/\kappa-1/\kappa^2)e^{\pm i\mathbf{k}\cdot\mathbf{R}}]}{[1-(a/R)(1+i/\kappa-1/\kappa^2)e^{ikR}][1+(a/R)(1+i/\kappa-1/\kappa^2)e^{ikR}]},$$
$$B_{1,2}=-\frac{ae^{ikR}}{R}\,\frac{(1+3i/\kappa-3/\kappa^2)[A_{2,1}e^{\pm i\mathbf{k}\cdot\mathbf{R}}+(2a/\kappa R)A_{1,2}(i+1/\kappa)e^{2ikR}]}{[1+(2a/\kappa R)(1/\kappa-i)e^{ikR}][1-(2a/\kappa R)(1/\kappa-i)e^{ikR}]}. \tag{6}$$

The total scattering amplitude is given in terms of $A_{1,2}$ and $B_{1,2}$ by

$$T(\omega)=k^2\epsilon_{\alpha\beta}e_\alpha{}^{(2)*}e_\beta{}^{(1)}=[A_1(\mathbf{e}^{(1)}\cdot\mathbf{e}^{(2)*})+B_1(\mathbf{e}^{(1)}\cdot\boldsymbol{\eta})(\mathbf{e}^{(2)*}\cdot\boldsymbol{\eta})]e^{-i\mathbf{q}\cdot\mathbf{R}_1}+[A_2(\mathbf{e}^{(1)}\cdot\mathbf{e}^{(2)*})+B_2(\mathbf{e}^{(1)}\cdot{}^*\boldsymbol{\eta})(\mathbf{e}^{(2)*}\cdot\boldsymbol{\eta})]e^{-i\mathbf{q}\cdot\mathbf{R}_2}, \tag{7}$$

where $\mathbf{q}=\mathbf{k}'-\mathbf{k}$ is the momentum transfer, and $|\mathbf{k}'|=|\mathbf{k}|$. We distinguish between the two possible dipole orientations $\mathbf{e}^{(1)}\|\boldsymbol{\eta}$ and $\mathbf{e}^{(1)}\perp\boldsymbol{\eta}$. For the former case, we find that for $\mathbf{q}=0$,

$$T(\omega)=A_1+A_2+B_1+B_2=\frac{2a\sin^2(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{R})}{1+(2a/\kappa R)(1/\kappa-i)e^{ikR}}+\frac{2a\cos^2(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{R})}{1-(2a/\kappa R)(1/\kappa-i)e^{ikR}}. \tag{8}$$

Keeping only the in-phase amplitude with appropriately shifted real and imaginary parts of the resonance denominators, we obtain

$$\omega_1=\omega_0-\tfrac{3}{2}\gamma_0(\kappa^{-3}\cos\kappa+\kappa^{-2}\sin\kappa),$$
$$\gamma_1=\gamma_0+3\gamma_0(\kappa^{-3}\sin\kappa-\kappa^{-2}\cos\kappa). \tag{9}$$

For the latter case,

$$T(\omega)=A_1+A_2=\frac{2a\sin(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{R})}{1+(a/R)(1+i/\kappa-1/\kappa^2)e^{ikR}}+\frac{2a\cos^2(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{R})}{1-(a/R)(1+i/\kappa-1/\kappa^2)e^{ikR}}, \tag{10}$$

and picking out only the out-of-phase resonance amplitude, we find for the renormalized eigenfrequency and width in this case

$$\omega_2=\omega_0-\tfrac{3}{4}\gamma_0\left[\left(\frac{1}{\kappa^3}-\frac{1}{\kappa}\right)\cos\kappa+\frac{1}{\kappa^2}\sin\kappa\right],$$
$$\gamma_2=\gamma_0+\tfrac{3}{2}\gamma_0\left[\left(\frac{1}{\kappa^3}-\frac{1}{\kappa}\right)\sin\kappa-\frac{1}{\kappa^2}\cos\kappa\right]. \tag{11}$$

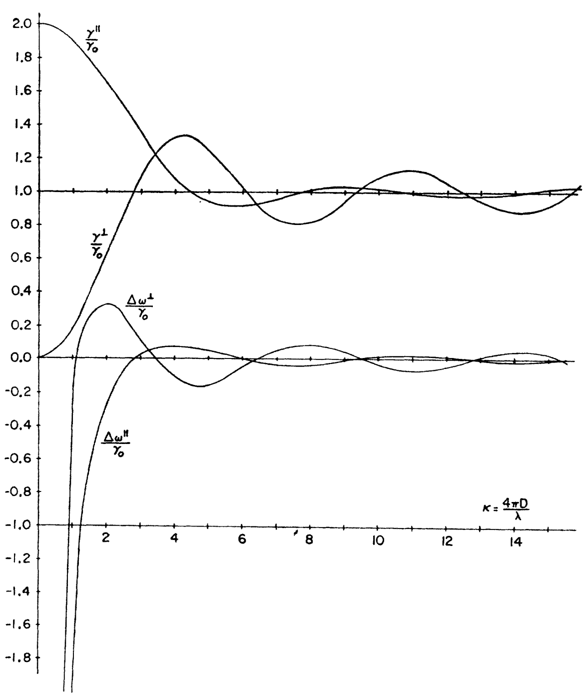

FIG. 2. Plot of renormalized frequency shifts and widths $\Delta\omega_R{}^{\|}/\gamma_0=[(\omega_1-\omega_0)/\gamma_0]$, $(\gamma_R{}^{\|}/\gamma_0)$; $(\Delta\omega_R{}^{\perp}/\gamma_0)=[(\omega_2-\omega_0)/\gamma_0]$ and $(\gamma_R{}^{\perp}/\gamma_0)$ versus $\kappa=4\pi D/\lambda$ [Eqs. (9) and (11)] for the orientations $\mathbf{E}^{(1)\|}$, $\mathbf{E}^{(1)\perp}$.

We note that interference between multiply scattered waves periodically modulates the resonance frequency and width of the dipole oscillators as a function of the parameter $\kappa = kR = 4\pi D/\lambda$.

In Fig. 2, we have plotted Eqs. (9) and (11) for the renormalized frequency and width for the two distinct cases. For a comparison with the actually measured fluorescence lifetime, which is proportional to $1/\gamma_{\text{tot}}$, we form

$$\gamma_{\text{tot}} = \tfrac{1}{3}\gamma_1 + \tfrac{2}{3}\gamma_2 \tag{12}$$

to account for the two possible orientations $\mathbf{e}^{(1)} \perp \boldsymbol{\eta}$ and only one $\mathbf{e}^{(1)} \| \boldsymbol{\eta}$ in Fig. 3. We note the good agreement between the two curves except for small distances where the experimental curves have additional structure. Because the actual observations were made with metallic mirrors (gold, silver, and copper), and the layered structure had a refractive index $n = 1.53$, the interface between the structure and air introduces a second partially reflecting mirror, whose effect we have not accounted for here. Detailed examination of this effect in a classical calculation improves the agreement.[9] As mentioned earlier, no attempt was made to detect the shift of the fluorescence frequency in the experimental work.

III. QUANTUM-MECHANICAL TREATMENT

The proper framework for the treatment of the emission and absorption of electromagnetic radiation from an excited system of atomic dimensions is nonrelativistic quantum mechanics. Again we adopt the viewpoint that the mirror simulates an image system located at a distance D behind the symmetry plane (mirror), and consider the change in the energy levels and widths of two two-level systems coupled by their interaction with the electromagnetic field,[10] which will be treated to second order in the fine-structure constant α, in the dipole approximation.[7] We account for the electric-dipole nature of the transition by assigning spin 1, $m_z = +1, 0, -1$ to the upper level and spin 0 and opposite parity to the lower level, and taking the mirror normal $\boldsymbol{\eta} = \mathbf{R}/|R|$ as the quantization axis, although the actual transition in the experimental work is a ${}^5D_0 \to {}^7F_2$ transition in the Eu^{3+} ion[11] at $\lambda = 6120$ Å. While the upper level is triply degenerate for the free two-level system, the presence of the second (image) system removes this degeneracy, in view of the coupling of the two systems by the radiation field. We characterize the unperturbed wave functions for the isolated system by $\phi_m^{(i)}$ ($m = +1, 0, -1$) and $\phi^{(0)}(i)$ ($i = 1, 2$) for the excited states and the ground state, respectively, and proceed to form symmetric and antisymmetric linear combinations for the coupled system to describe its collective excitations, which we

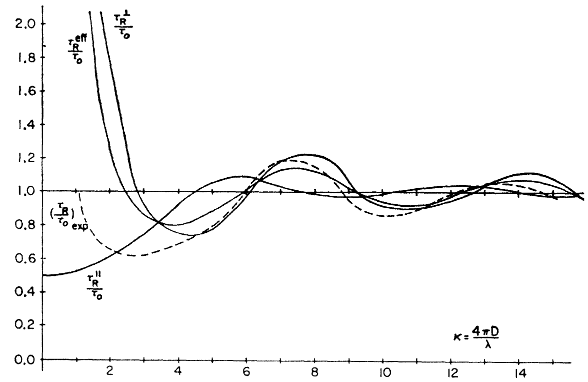

FIG. 3. Plot of the renormalized lifetimes $\tau_R^{\|}/\tau_0 = \gamma^0/\gamma_R^{\|}$ and $\tau_R^{\perp}/\tau_0 = \gamma_0/\gamma_R^{\perp}$, and the effective lifetime $\tau_R^{\text{eff}}/\tau_0 = \frac{1}{3}(\tau_R^{\|} + 2\tau_R^{\perp})/\tau_0 = \gamma_0/\gamma_R^{\text{eff}}$ according to theory (solid line) and experiment (Refs. 2 and 3) with a silver mirror for which $|\rho| \approx 1$ (dashed line).

denote by

$$\Psi_m^{(\pm)}(1,2) = \tfrac{1}{2}\sqrt{2}[\phi_m(1)\phi^{(0)}(2) \pm \phi_m(2)\phi^{(0)}(1)]. \tag{13}$$

The interaction Hamiltonian in dipole approximation is taken from semiclassical radiation theory in the form

$$H_I = -\mathbf{d}^{(2)} \cdot \mathbf{E}^{(1)} = -k^2[2(-i/\kappa^2 + 1/\kappa^3)\mathbf{d}^{(1)\|} \cdot \mathbf{d}^{(2)\|} + (1 + i/\kappa - 1/\kappa^2)\mathbf{d}^{(1)\perp} \cdot \mathbf{d}^{(2)\perp}]e^{ikR}/R, \tag{14}$$

where $d^{(i)\|}$ and $d^{(i)\perp}$ are the parallel and perpendicular components of the dipole operators for $i = 1, 2$; $\mathbf{E}^{(1)}$ is the electric field produced by the transition dipole of system 1. Evaluating the interaction Hamiltonian between the various states $\Psi_m(1,2)$, we find the self-energy $\Delta E_m^{(\pm)}$ of the system due to its resonant coupling with the electromagnetic field:

$$\begin{aligned}\Delta E_m^{(\pm)} &= \langle \Psi_m^{(\pm)} H_I \Psi_m^{(\pm)} \rangle \\ &= \pm\tfrac{3}{4}\gamma_0(-1/\kappa - i/\kappa^2 + 1/\kappa^3)e^{ikR}, \quad m = \pm 1 \\ &= \pm\tfrac{3}{2}\gamma_0(i/\kappa^2 - 1/\kappa^3)e^{ikR}, \quad m = 0\end{aligned} \tag{15}$$

corresponding to the transition dipole's being oriented perpendicular or parallel to $\boldsymbol{\eta}$, for $m = \pm 1$ and $m = 0$, respectively. The free-space width γ_0 is given by Fermi's "Golden rule" as $\gamma_0 = \frac{4}{3}k^3|\mathbf{d}^{(i)}|^2$, with

$$\mathbf{d}^{(i)} = \langle \phi_m(i)\mathbf{d}^{(i)}\phi^{(0)}(i) \rangle,$$

independent of the value of m. If we separate real and imaginary parts in Eq. (15) and recall our mirror phase convention, i.e., antisymmetric and symmetric combinations for $\mathbf{d}^{\perp}(m = \pm 1)$ and $\mathbf{d}^{\|}(m = 0)$, we find the same frequency and width renormalization as in Eqs. (9) and (11): $\Delta\omega_{1,2} = \omega_{1,2} - \omega_0$, $\Delta\gamma_{1,2} = \gamma_{1,2} - \gamma_0$. We are, of course, simply face to face with the correspondence principle, from which the semiclassical radiation theory is derived. The introduction of wave functions and operators only serves to define the dipole moment in quantum-mechanical language, while the radiation field is treated classically. The symmetry considerations take care of the parity change and the angular momentum carried away by the radiation field.

[9] K. H. Drexhage (unpublished).

[10] J. Hamilton, Proc. Phys. Soc. (London) **A62**, 12 (1949).

[11] L. D. Derkacheva *et al.*, Usp. Fiz. Nauk **91**, 247 (1967) [English transl.: Soviet Phys.—Usp. **10**, 91 (1967)].

IV. CONCLUSIONS

We have shown, both by classical and by quantum-mechanical calculations, that an idealized mirror renormalizes both the frequency and the width of an excited atomic system via its interaction with the electromagnetic field. We have assumed that the wavelengths are comparable to the distance of the two-level system from the mirror. The mirror can be replaced effectively by a suitably placed image system, and it is demonstrated that the experimentally observed modulation of the lifetime of the excited system agrees with the theory over a wide range of values of $\kappa = 4\pi D/\lambda$, except for small values of D. This discrepancy is, in part, due to the approximations made ($|\rho| = 1, \delta = \pi$) and to the occurrence of a second partially reflecting mirror in the experiments: The interface between the monolayer structure and air acts as an additional partial reflector. At distances smaller than 500 Å it seems likely that nonradiative energy transfer also occurs. The effect described is, of course, not confined to the optical region,[8] and we intend to explore a variant of the general two-center interference phonomenon as a possible test of the parity-nonconserving part of the nucleon-nucleon potential.

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Gravitational Radiation Interaction between Two Separated Systems*

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Einstein's power-transfer theory is developed to exhibit the rate at which energy is transferred by gravitational radiation from an emitter to an absorber in terms of their lowest-order contributing multipole moments (quadrupole), the separation distance, and the emitter frequency. The description of the interaction is complete, and the need to calculate fields is obviated. Consistency is demonstrated by comparing the total power transferred between the two systems with the power flux over an infinite sphere which arises from their interaction. The gravitational linear momentum flux which arises from the interaction is also shown to be purely quadrupole. Bonnor and Rotenberg, Papapetrou, and Peres demonstrated the quadrupole-octupole dependence for linear momentum flux from a single system, and the present work illustrates that a lower-order flux can be achieved by a double system.

I. INTRODUCTION

GRAVITATIONAL radiation has existed as a theoretical construct for over 50 years,[1,2] but it is only within recent years that physicists have launched a serious effort to detect it.[3] Typically, one visualizes some time-dependent stress-energy distribution as a source of gravitational radiation and a second system which is affected by the source and thereby serves as a detector for the radiation.

Einstein[2] laid the foundation for the analysis of such a process and it is the primary aim of this paper to expand and generalize his work. An expression for the gravitational radiation power transferred between an arbitrary emitter-absorber combination in terms of the structure of each is developed within the framework of Einstein's linearized theory, for arbitrary relative orientations and an arbitrary separation distance. Assuming a simply periodic emitter, the time-averaged power transfer is deduced and is shown to be completely consistent with the flux of gravitational energy over an infinite sphere which arises from the interaction. The answers, moreover, are expressed in terms of the frequency of the radiation, the separation distance, and the lowest-order (quadrupole) multipole moments of the emitter and absorber which are effective in the interaction. Since these elements are familiar to all physicists (there are no gravitational field components or esoteric tensors in the final expression), it is hoped that they can be as readily appreciated by a broad spectrum of physicists as is the well-known quadrupole formula for

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[1] A. Einstein, Sitzber. Preuss. Akad. Wiss. Physik-Math. Kl., 688 (1916).
[2] A. Einstein, Sitzber. Preuss. Akad. Wiss. Physik.-Math. Kl., 154 (1918). See also W. Pauli, *Theory of Relativity* (Pergamon Press Ltd., London, 1958), pp. 175–178.
[3] See, for example, B. B. Braginskiĭ, Usp. Fiz. Nauk **86**, 433 (1966) [English transl.: Soviet Phys.—Usp. **8**, 513 (1966)], for appropriate references, and J. Weber, Phys. Rev. Letters **22**, 1320 (1969), who presents evidence for the discovery of gravitational radiation.